GRAVITATION
4. ESCAPE AND ORBITAL VELOCITIES

POINTS TO REMEMBER

1. The minimum vertical velocity required for an object to escape from the gravitational influence of a planet is known as escape velocity. $V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$

2. If mass of the planet is M, radius of the planet is R, distance between the orbiting object and the planet is h, universal gravitational constant is G, then

   Orbital velocity $(v_0) = \sqrt{\frac{GM}{(R+h)}}$

   For smaller heights, Orbital velocity $(v_0) = \sqrt{gr}$

3. The orbital and escape velocities are independent of mass of the body (satellite) angle of projection.

4. Escape velocity is 1.414 times that of orbital velocity.

5. The escape velocity of an object from the earth' s surface is 11.2km/s. The orbital velocity of a satellite very close to the earth is 7.92km/s..

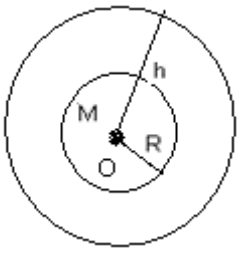
6. If the period of revolution of an artificial satellite is equal to the period of rotation of earth then such a satellite is called geostationary satellite. It appears stationary for an observer on the earth. Geo-stationary satellites have a variety of uses right from studying the atmosphere to the transmission of TV programmes.

SHORT ANSWERS

1. What is Orbital velocity? Obtain an expression for it.
(March 2011, June 2010, March 2010, March 2000, March 2902, March 2004)

Ans ; The velocity required for a body in order that it revolve round the earth in a circular orbit is known as orbital velocity V_0.

Expression: Consider a satellite of mass m on the surface of the earth .Let M be the mass of the earth and R be its radius. The satellite is revolving around the earth at a height h from the surface of the earth. The Gravitational force of attraction between the satellite and the earth supplies the necessary centripetal force for the satellite.



Gravitational force of attraction $F = \frac{GMm}{(R+h)^2}$

Centripetal force $F = \frac{mv_0^{\,2}}{(R+h)}$ where v_0 is orbital velocity.

$$\therefore \frac{mv_0^{\,2}}{(R+h)} = \frac{GMm}{(R+h)^2} \text{ or } v_0 = \sqrt{\frac{GM}{(R+h)}}$$

If the acceleration due to gravity is g at the height h , then $g = \frac{GM}{(R+h)^2}$

$$\therefore v_0 = \sqrt{g(R+h)}$$

If the satellite is very close to the surface of the earth, $\therefore v_0 = \sqrt{gR}$ = 7.92 km/s

2. What is escape velocity? Obtain expression for it. (June2009, March 2009, March2008, March 2003, March 2005, March 2007,)

Ans ; The minimum velocity with which a body must be projected upwards, so that it escapes from the force of attraction of earth is called escape velocity(V_e) of the body. Its value is 11.2 km/s

Expression:

Consider earth as a perfect sphere of mass M and radius R. Consider a body of mass m on the surface of the earth. The force of attraction of the earth on the body is given by $F = \frac{GMm}{R^2}$

The gravitational potential on the surface of the earth $= -\frac{GM}{R}$

The work done in bringing the from infinity to the surface of the earth is stored as gravitational potential energy.

Gravitational potential energy $= -\frac{GMm}{R}$ ---- (1)

The negative sign indicates that the body is bound to the earth. If V_e is the minimum velocity with which the body can escape from the force of attraction of earth ., then is called escape velocity of the body.

$KE\ of\ the\ body = \frac{1}{2}mV_e^2$ ---- (2)

From (1) and (2)

$$\frac{1}{2}mV_e^2 = \frac{GMm}{R} \qquad \Rightarrow V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} \quad \left(\because g = \frac{GM}{R^2}\right)$$

3. What is a geo stationary satellite? State its uses. (March 2001, March 2006, March 2007)

Ans ; A Satellite whose time period of revolution is equal to the time period of rotation of earth (i.e. 24 hours) is called geostationary satellite. This satellite appears to be at rest with respect to the earth. Radius of the geo– stationary orbit is approximately 42,400 km. Speed of geo– stationary satellite in it is 3.1 kms^{1}.

Uses:- 1) To study the upper layers of the atmosphere.
2) To forecast the changes in the atmosphere.
3) To know the shape and size of the earth.
4) To identify the minerals and natural resources present inside and on the surface of the earth.
5) To transmit the T.V. programmes to the distant places.
6) To study the properties of transmission of radio waves in the upper layers of the atmosphere and
7) To undertake extensive research work on the planets, satellites and comets etc., which are present in space.

4. There is no atmosphere on the moon's surface. Explain.

Ans. The escape velocity of a body on earth's surface is 11.2km/s. So atmosphere is present on surface of earth. But the escape velocity on the moon is 2.36 km/s which is less than the rms velocity of gas molecules. So gas molecules escape from moon and can not form the atmosphere.

VERY SHORT ANSWER ANSWERS

1. On what factors does orbital velocity depend?

Ans. Orbital velocity depends on mass of planet, radius of the planet and height of the object from the surface of the planet.

2. What is the expression for the orbital velocity of a satellite around a planet? What is its value around the earth?

Ans. $v_0 = \sqrt{\frac{GM}{(R+h)}}$ Where h is the height of the satellite from the surface of the planet. M is the mass and R is the radius of the planet. The value of orbital velocity on the earth is 7.92 km/s.

3. Express orbital velocity in terms of (i) G and (ii) g.

Ans. $v_0 = \sqrt{\frac{GM}{(R+h)}}$ Where h is the height of the satellite from the surface of the earth.

If the acceleration due to gravity is g at the height h , then $g = \frac{GM}{(R+h)^2}$

$$\therefore v_0 = \sqrt{g(R+h)}$$

If the satellite is very close to the surface of the earth, $\therefore v_0 = \sqrt{gR}$ = 7.92 km/s

4. To launch a satellite into its orbit how many minimum stages of a rocket are required?

Ans. To launch a satellite into its orbit two minimum stages of a rocket are required.

5. On what factors does escape velocity depend?

Ans. It depends on mass and radius of planet.

6. Express escape velocity in terms of (i) G and (ii) g.

Ans. $V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} \quad \left(\because g = \frac{GM}{R^2}\right)$

7. State the relation between the escape and orbital velocities. Does this relation change with height? (May2007, Apr 92, 95, Oct92, 93 Mar02, 04)

Ans. $v_0 = \sqrt{\frac{GM}{R}}$ and $V_e = \sqrt{\frac{2GM}{R}}$

$\therefore v_e = 2v_0$ This relation does not change for smaller heights.

8. If a nut becomes loose and gets detached from a satellite revolving around the earth, will it fall down to the earth or will it revolve in the same orbit as the satellite? Give reason for your answer?

Ans. The nut revolves in the same orbit as that of a satellite due to inertia of motion.

9. **An object projected with a velocity greater than or equal to 11.2Km/s will not come back to the earth. Explain.**

A. The escape velocity of the body on the earth is 11.2Km/s.If a body is projected with 11.2Km/s or greater than this velocity it will not come back to the earth and the body will escape from the influence of the earth.

10. "Hydrogen is in abundance around the sun and less around the earth ". Discuss.

Ans. The escape velocity of the body on the earth is 11.2Km/s. This value is less than rms velocity of Hydrogen where as the escape velocity of the body on the sun is 620 km/s. This value is very much greater than rms velocity of Hydrogen. Hence Hydrogen is in abundance around the sun and less around the earth.

11. What is the period of revolution of a geo - stationary satellite? Does it rotate from west to east or east to west?

Ans. The time period of geo stationary satellite is 24 hours. It revolves around the earth at a constant speed once per day over the equator from west to east in a equatorial plane.

12. What is the orbital radius of a geo stationary satellite? What is the height from the surface of the earth?

Ans. The orbital radius of geo stationary satellite is 42,250 km from the center of the earth and its height from surface of earth is 35,800km.

Exercise 1

1. Calculate the escape velocity of an object on planet earth.

Sol For the earth,

Radius (R) = 6.4×10^6 m, g = 9.8 m/s^2

Escape velocity (V_e) = ?

$$V_e = \sqrt{2gR} = \sqrt{2\times 9.8\times 6.4\times 10^6 m/s} = 11.2 \text{ km/s}$$

2. An artificial satellite is revolving in a circular orbit at height of 1200km above the surface of the earth. If the radius of the earth is 6400km, mass is 6×10^{24} kg ,find the orbital velocity. ($G=6.67\times10^{-11}$ Nm^2/kg^2).

Sol

Height of the satellite from the earth	h = 1200 km = 1200×10^3m
Radius of the earth	R = 6400Km = 6400×10^3m
Mass of the earth	M = 6×10^{24} kg
Universal Gravitational Constant	G = 6.67×10^{-11} Nm^2/kg^2
Orbital velocity of the satellite	V_o = ?

$$V_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{6.67\times10^{-11} \times 6\times10^{24}}{7600 \times 10^3}} = 7.26 \ km/s$$

3. Calculate the orbital velocity of a satellite, very close to the surface of the earth.

Sol Radius of the earth (R) = 64×10^5m

Acceleration due to gravity (g) = 9.8m/s^2

Orbital velocity of the satellite (V_0)=?

For a satellite close to earth,

$$V_0 = \sqrt{gR} = \sqrt{9.8\times64\times10^5} m/s = 7.92 \text{ km/s}$$

4. Express escape velocity in terms of radius and density of a planet.

Sol. Escape velocity = $V_e = \frac{2GM}{R} = \sqrt{2G \times \frac{4}{3}\frac{\pi R^3\rho}{R}} = \sqrt{\frac{8}{3}\pi GR^2\rho}$

5. Find the ratio of escape velocities of two planets if g values on the two planets are 9.8m/s^2 and 3.3m/s^2 and their radius are 6400 km and 3400 km respectively.

Sol. The escape velocity in terms of g and R is $V_e = \sqrt{2gr}$

The ratio of escape velocities $\frac{V_{e1}}{V_{e2}} = \sqrt{\frac{g_1 R_1}{g_2 R_2}}$

g_1 = g on the first planet= 9.8m/s^2

g_2 = g on the second planet = 3.3m/s^2

R_1= Radius of the first planet = 6400km

R_2= Radius of the second planet = 3400 km

$$\frac{V_{e1}}{V_{e2}} = \sqrt{\left(\frac{9.8}{3.3}\right)\left(\frac{6400}{3400}\right)} \quad \Rightarrow V_{e1} : V_{e2} = 2.364 : 1.$$

6. Find the ratio of escape velocities from the earth and the moon. (Mass of the earth = $6.0\times10^{24}\,kg$, Radius of the earth = 6400km, Mass of the moon = Radius of the moon = 1740Km.)

Sol. M_1 = Mass of the earth = $6.0\times10^{24}\,kg$; R_1 = Radius of the I planet = 6400 km.

M_1= Mass of the moon = $7.4\times10^{22}\,kg$; R_2 = Radius of the moon = 1740km

$$\frac{V_{e1}}{V_{e2}} = \sqrt{\left(\frac{6.0\ \times10^{24}}{7.4\ \times10^{22}}\right)\left(\frac{1740}{6400}\right)} = \frac{4.695}{1}$$

7. Show that the velocity of an object released from rest at infinity reaches the earth with a velocity equal to the escape velocity.

Sol. **Using** the law of conservation of energy,

(KE+PE) at infinity = (KE+PE) at the surface of the earth.

As the body is released from rest at infinity, it's KE and PE at infinity are zero. If v is its velocity on reaching the earth its KE becomes equal to $\frac{1}{2}mv^2$ and PE= $-\frac{GMm}{R}$

$$\therefore 0+0 = \frac{1}{2}mv^2 - \frac{GMm}{R}; \frac{1}{2}v^2 = \frac{GM}{R} \Rightarrow \textit{The velocity of the object}, v = \sqrt{\frac{2GM}{R}} = v_e$$

8. Calculate the orbital speed of a satellite revolving in a circular orbit at a height of 6400km from the surface of the earth. (Mass of the earth = 6×10^{24}kg. Radius of the earth = 6400km, G= 6.67×1 0^{11} Nm^2/kg^2)

Sol. $M = 6\times10^{24}\ \ kg;\ \ R = 6400km\ = 6400\times10^3 m = 6.4\times10^6 m; h = 6400km = 6.4x10^6 m;$

$G = 6.67\times10^{-11} Nm^2/kg^2$

The orbital speed, = $v_0 = \sqrt{\frac{Gm}{R+h}} = \sqrt{\frac{6.67\ \times10^{-11}\times6\times10^{24}}{\left(6.4\times10^6+6.4\times10^6\right)}} = \sqrt{\frac{40.02}{128}}\times10^4\, m/s$

9. Find the ratio of the orbital speeds of two satellites of the earth if the satellites are at heights 6400km and 19200 km. (Radius of the earth = 6400 km.)

Sol. Orbital speed, $V_0 = \sqrt{\frac{GM}{R+h}}$

The ratio of the orbital speeds of two satellites around the earth is

$\frac{V_{01}}{V_{02}} = \sqrt{\frac{R+h_2}{R+h_1}}$ where h_1 and h_2 are the heights of the satellites.

But, $h_1 = 6400$km, $h_2 = 19200$km, R=6400km.

$$\frac{V_{01}}{V_{02}} = \sqrt{\frac{6400+19200}{6400+6400}} = \sqrt{\frac{25600}{12800}} = \frac{\sqrt{2}}{1}$$

10. Find the ratio of the orbital speeds of two satellites one of which is rotating round the earth and the other around Mars close to their surface. (Mass of the earth = 6×10^{24} kg, Mass of the Mars = 6.4×10^{23} kg Radius of the earth 6400 km and Radius of the Mars = 3400km.)

Sol. The orbital speed of a satellite close to the surface of a planet, $v_0 = \sqrt{\frac{GM}{R}}$

The ratio of orbital speeds, $\frac{v_{01}}{v_{02}} = \sqrt{\left(\frac{M_1}{M_2}\right)\left(\frac{R_2}{R_1}\right)}$.

M_1 = Mass of the earth = $6\times10^{24}kg$; R_1 = radius of the earth = 6400km;

$$\frac{V_{01}}{V_{02}} = \sqrt{\left(\frac{6\times10^{24}}{6.4\times10^{23}}\right)\left(\frac{3400}{6400}\right)} = \sqrt{\frac{60\times34}{6.4\times64}} = \frac{2.232}{1}$$

11. A satellite is at a height of 25,600 km from the surface of the earth. If its orbital speed is 3.536 km/s, find its time period. (Radius of the earth=6400km).

Sol. $R = 6400km = 6.4\times10^6 m,\ \ h = 25,600km = 25.6\times10^6 m, v_0 = 3.536\times10^3 m/s.$

The time period, $T = \frac{2\pi(R+h)}{v_0} = \frac{2\pi(2.56+6.4)10^6}{3.536\times10^3} = 56,870s.$

12. What is the work done in taking an object of mass 1 kg from the surface of the earth to a height equal to the radius of the earth? (G= $6.67\times10^{-11}Nm^2/kg^2$, Radius of the earth=6400km, Mass of the earth =).

Sol. $G = 6.67\times10^{-11}Nm^2/kg^2,\ \ M = 6\times10^{24}kg;\ \ m = 1kg;\ \ h = R = 6400km = 6.4\times10^6 m.$

Using the law of conservation of energy,

$$\text{W} = \frac{-GMm}{R+h} - \left(\frac{-GMm}{R}\right) = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right) = GMm\left(\frac{R+h-R}{R(R+h)}\right) = \frac{GMmh}{R(R+h)}$$

$$\text{Work done} = \frac{GMmR}{R(R+R)} = \frac{GMm}{2R} = \frac{6.67\times10^{-11}\times6\times10^{24}\times1}{2\times6.4\times10^6} = 3.127\times10^7 J$$

EXERCISE 1

1. If the mass of the Venus is 0.814 times of the earth and radius is 0.967 times of the earth, find the escape velocity on the Venus.

Ans. Mass of Venus $M_v = 0.814M$

Radius of Venus $R_v = 0.967R$

Escape velocity of Earth = $\sqrt{\frac{2GM}{R}}$ = 11.2 km/s

Escape velocity of Venus

$$V_e = \sqrt{\frac{GM_v}{R_v}} = \sqrt{\frac{2G(0.814M)}{0.967R}} = \sqrt{\frac{0.814}{0.967}\left[\frac{2GM}{R}\right]} = \sqrt{\frac{0.814}{0.967}}(11.2) = 10.25 \text{ km/s}$$

2. A satellite orbits the earth at a height of 1000km, find its orbital speed.

Ans. Height of satellite h=1000km

Radius of earth $R = 6400km, G = 6.67\times\ 10^{-11} Nm^2/kg^2$;Mass of earth $M = 6\times10^{24} kg$

$\therefore$ Orbit velocity of satellite at height 'h ' is given by ,

$$V_o = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{6500\times10^3}} = 7.354 \quad km/s$$

3. A Satellite orbits the earth at a distance equal to the radius of the earth above its surface. Find (i) its speed (ii) its period.

Ans. Height of satellite h = R

Mass of earth $M = 6\times10^{24} kg$

Radius of earth R= 6400km

Speed of satellite (orbital velocity) $V_o = \sqrt{\frac{GM}{R+h}}$

$$V_o = \sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{2\times6400\times10^3}} = 5.592km/s$$

Also, $V_o = (R+h)\omega$

$$\therefore T = \frac{2\pi(R+h)}{V_o}$$

Or $$T = \frac{2\pi\times2\times6400\times10^3}{5.592\times10^9} = 4 \quad hours$$

4. Find the height of a satellite from the surface of the earth if its orbital velocity is 4 km/s.

Ans. $V_o = 4km/s; M = 6\times20^{24} kg$;R = 6400 km; $G = 6.67\times10^{-11} Nm^2/kg^2$

Let h be the height of satellite .

$$V_o = \sqrt{\frac{GM}{R+h}} \Rightarrow 4\times10^3 = \sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{R+h}}$$

$$6400\times10^3 + h = \frac{6.67\times10^{-11}\times6\times10^{24}}{16\times10^6} \Rightarrow h = 1.853\times10^7 m = 1.853\times10^4 km$$

5. Two satellites are revolving round the earth at different heights. The ratio of their orbital speeds is 2:1, if one of them is at a height of 100km. what the height the other satellite is.

Ans. $\frac{V_{01}}{V_{02}} = \frac{2}{1}$, $R = 6400km$

h_1 = 100 km, h_2 = ?

Orbital velocity $V_o = \sqrt{\frac{GM}{R+h}} \Rightarrow \frac{V_{01}}{V_{02}} = \sqrt{\frac{R+h_2}{R+h_1}} \Rightarrow \frac{2}{1} = \sqrt{\frac{6400+h_2}{6500}} \Rightarrow h_2 = 19,600km$

6. Find the ratio of the time periods of two satellites at heights 3600km, 13600km from the surface of the earth.

Ans. $h_1 = 3600km$; $h_2 = 13,600km$; $R = 6400km$

According to Kepler's third law , $T \quad \alpha \quad (R+h)^{3/2}$

$$\frac{T_1}{T_2} = \left(\frac{R+h_1}{R+h_2}\right)^{3/2} = \left(\frac{10,000}{20,000}\right)^{3/2} = \frac{1}{2\sqrt{2}} = \frac{1}{2.828}$$

7. Two satellites are at heights 1000 km, 2000km. Find the ratio of their orbital angular speed.

Ans. $h_1 = 1000km$; $h_2 = 2000km$; $R = 6400km$

$\therefore$ Orbital velocity $V_o = \sqrt{\frac{GM}{R+h}}$

But $V_o = (R+h)\omega = \sqrt{\frac{GM}{R+h}} \Rightarrow \omega \quad \alpha \frac{1}{(R+h)^{3/2}}$

$$\frac{\omega_1}{\omega_2} = \left(\frac{R+h_2}{R+h_1}\right)^{3/2} = \left(\frac{7600}{7500}\right)^{3/2} = \frac{1.209}{1}$$

8. Calculate speed of communication satellite that appears stationary from the place of equator.

Ans. For a geostationary satellite, h = 35,800 km

R+h = 6400 + 35,800 = 42, 250 km

Speed of the satellite $V_o = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{42,250\times10^3}} = 3.075 \quad km/s$

9. Find the number of revolutions made by a satellite close to the surface of the earth in one day.

Ans. When a satellite is revolving close to the surface of the earth,

Gravitational force between earth and satellite = Centripetal force.

$$\frac{GMm}{R^2} = mR\omega^2 \qquad \Rightarrow \quad \omega = \sqrt{\frac{GM}{R^3}}$$

$\therefore$ Frequency $f = \frac{1}{2\pi}\sqrt{\frac{GM}{R^3}}$

$$f = \frac{1}{2\pi}\sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{(64\times10^5)^3}} = 19\times10^{-4}\,Hz$$

Number of revolution made by satellite in one day = $(24\times60\times60)\times1.9\times10^{-4} = 16.982$

10. A satellite is revolving in a circular orbit around the earth at a height of 3200km. If the acceleration due to gravity at that height is 4.378 m/s^2. Find the orbital speed of the satellite.

Ans. H = 3200 km; $g_h = 4.378 \quad m/s^2$

$\therefore$ Orbital speed $V_o = \sqrt{gR} = \sqrt{4.38\times3200\times10^3} = 6.483km/s$

11. A satellite is revolving round a planet in a circular orbit with a velocity of 8km/s at a height where the value of acceleration due to gravity is $8m/s^2$. How high is the satellite from the planet's surface? (Radius of planet = 6000km).

Ans. Velocity of satellite $V_o = 8 \quad km/s, \quad R = 6000km$

$g_h = 8 \quad m/s^2$

$$V_o = \sqrt{g_h(R+h)} \Rightarrow 8\times10^3 = \sqrt{8\times(R+h)} \Rightarrow h = 2000 \quad km$$

12. Find the difference in PE of an object of mass 10kg when it is taken from a height of 6400 km to 12800 km from the surface of the earth.

Ans. $m = 10kg \quad ; \quad M = 6\times10^{24}kg ; R = 6400km ; h_1 = 6400km ; h_2 = 12800km$

Difference in potential energy $(\Delta PE) = PE_2 - PE_1 = -\dfrac{GMm}{R+h_2} - \left(-\dfrac{GMm}{R+h_1}\right)$

$$= GMm\left[\frac{1}{R+h_1} - \frac{1}{R+h_2}\right] = 6.67\times10^{-11}\times6\times10^{24}\times10\left[\frac{1}{12800\times10^3} - \frac{1}{19200\times10^3}\right] = 1.045\times10^8 \quad J$$

13. If an object is projected vertically upwards from the earth with a velocity of 2.5 km/s find the maximum height reached.

Ans. $V = 2.5 \quad km/s, \; R = 6400km, \; G = 6.67\times10^{-11}Nm^2/kg^2, \; M = 6\times10^{24} \quad kg$

Let 'h' be the maximum height reached by an object .

From the law of conservation of energy,

(PE + KE) at the surface = (PE + KE) at the height

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} \quad \Rightarrow \quad \text{h = 336.8 km}$$

14. Calculate the escape velocity of a body from the earth's surface. If the earth were made of wood, its mass would be only about 10% as much as it is now. What would be the escape velocity if the earth were made of wood.

Ans: $M = 6\times10^{24} \quad kg ; G = 6\times10^{-11}Nm^2/kg^2 ; R = 6400km$

$\therefore$ Escape velocity $V_e = \sqrt{\dfrac{2GM}{R}} = \sqrt{\dfrac{2\times6.67\times10^{-11}\times6\times10^{24}}{6.4\times10^5}}$ = 11.2 km/s

If earth were made of wood, then its

Mass is given as $M^1 = 10\% \quad M = 6\times10^{23}kg$ and R = 6400 km

$$\therefore V_e^1 = \sqrt{\frac{2GM^1}{R}} = \sqrt{\frac{2\times6.67\times10^{-11}\times6\times10^{23}}{64\times10^5}} = 3.536 \quad km/s$$

15. A communication satellite appears stationary from a place on the equator. Find the height of the satellite from the surface of the earth.

Ans. For a satellite to be stationary, its time period shoule be same as that of earth.

i.e., T = 24 hr,

$G = 6.67\times \ 10^{-11}Nm^2/kg^2 ; \; M = 6\times \ 10^{24}kg$

$$\therefore T = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$r = \left[GM \times \frac{T^2}{4\pi^2} \right]^{1/3} = \left[6.67 \times 10^{-11} \times 6 \times 10^{24} \times \frac{(86400)^2}{4\pi^2} \right]^{1/3}$$

$\therefore$ Distance to the satellite from centre of earth $r = R + h = 42,250 km$

$\therefore\ h = 42,250 - 6400 = 35,850 km$

16. Three particles each of mass m are placed at three corners of an equilateral triangle of side a. Find the work done in increasing the side length to 2a.

Ans. Initial potential energy of the system.

$$PE_i = U_{AB} + U_{BC} = U_{Ac} = -\frac{Gmm}{a} - \frac{Gmm}{a} - \frac{Gmm}{a} = \frac{3Gm^2}{a}$$

Final potential energy of the system $PE_f = U_{AB} + U_{BC} + U_{AC} = -\frac{Gmm}{2a} - \frac{Gmm}{2a} - \frac{Gmm}{2a} = \frac{3Gm^2}{2a}$

Work done in increasing the side length to 2a is $W = PE_f - PE_i = \frac{-3Gm^2}{2a} - \left[-\frac{Gm^2}{a} \right] = -\frac{Gm^2}{2a}$

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17. Find the PE of three objects of masses 1kg, 2kg and 3kg placed at the three vertices of an equilateral triangle of side 20cm.

Ans. $m_1 = 1\ \ kg\ , m_2 = 2\ \ kg\ , m_3\ =\ 3kg$

Potential energy of system = $U_{AB} + U_{BC} + U_{AC} = -\frac{Gm_1m_2}{a} - \frac{Gm_2m_3}{a} - \frac{Gm_3m_1}{a} = -\frac{G}{0.2}[2+6+3]$

ASSESS YOURSELF

1. What is the apparent weight of an object in a satellite that revolves around the earth?

Ans. Zero

2. What is the minimum number of satellites required to cover the entire area of the earth?

Ans. Three

3. When a body is projected from the surface of a planet with a velocity less than the orbital velocity , what path does it follow when it returns to the earth?

Ans. Parabolic path

4. When a body is projected with a velocity equal to the orbital velocity how does it move?

Ans. It moves in a circular orbit.

5. When a body is projected with a velocity greater than the orbital velocity but less than escape velocity how does it move?

Ans. It moves in an elliptical orbit

6. When a body is projected with a velocity equal to the escape velocity what path does it follow?

Ans. It escapes from the gravitational field of the planet following a parabolic path

7. When a body is projected with a velocity greater than the escape velocity how does it move?

Ans. It escapes from the gravitational field of the planet following a rectangular hyperbolic bath.

8. Why doesn' t the moon hit the earth because of earth's gravity?

Ans. Moon is also influenced by earth's gravity as stone does. Indeed the moon falls towards the earth' s surface and its surface is curving constantly away from it, so that they never get any closer.