# MATRICES -3

## EXERCISE – 3(h)

**A. Solve the following system of equations.**

**i) By using Cramer's rule and matrix inversion method when the coefficient matrix is non-singular.**

**ii) Using Gauss-Jordan method. Also determine whether the system has a unique solution or infinite number of solutions or no solution and find solutions if exist.**

**1. 5x – 6y + 4z = 15**
**7x + 4y – 3z = 19**
**2x + y + 6z = 46**

**Hint :** $x = \frac{\Delta_1}{\Delta}, y = \frac{\Delta_2}{\Delta}, z = \frac{\Delta_3}{\Delta}$

**Sol. i) Cramer's rule :**

$$\Delta = \begin{vmatrix} 5 & -6 & 4 \\ 7 & 4 & -3 \\ 2 & 1 & 6 \end{vmatrix}$$

$$= 5(24+3) + 6(42+6) + 4(7-8)$$

$$= 135 + 288 - 4 = 419$$

$$\Delta_1 = \begin{vmatrix} 15 & -6 & 4 \\ 19 & 4 & -3 \\ 46 & 1 & 6 \end{vmatrix}$$

$$= 15(24+3) + 6(114+138) + 4(19-184)$$

$$= 405 + 1512 - 660 = 1917 - 660 = 1257$$

$$\Delta_2 = \begin{vmatrix} 5 & 15 & 4 \\ 7 & 19 & -3 \\ 2 & 46 & 6 \end{vmatrix}$$

$$= 5(114+138) - 15(42+6) + 4(322-38)$$

$$= 1260 - 720 + 1136 = 1676$$

$$\Delta_3 = \begin{vmatrix} 5 & -6 & 15 \\ 7 & 4 & 19 \\ 2 & 1 & 46 \end{vmatrix}$$

$$= 5(184-19) + 6(322-38) + 15(7-8)$$

$$= 825 + 1074 - 15 = 2529 - 15 = 2514$$

$$x = \frac{\Delta_1}{\Delta} = \frac{1527}{419} = 3$$

$$y = \frac{\Delta_2}{\Delta} = \frac{1676}{419} = 4$$

$$z = \frac{\Delta_3}{\Delta} = \frac{2514}{419} = 6$$

Solution is x = 3, y = 4, z = 6.

**ii) Matrix inversion method :**

Hint : $A^{-1} = \frac{\text{Adj}A}{\det A}$

$$A = \begin{bmatrix} 5 & -6 & 4 \\ 7 & 4 & -3 \\ 2 & 1 & 6 \end{bmatrix}$$

$$A_1 = \begin{vmatrix} 4 & -3 \\ 1 & 6 \end{vmatrix} = 24 + 3 = 27$$

$$B_1 = -\begin{vmatrix} 7 & -3 \\ 1 & 6 \end{vmatrix} = -(42 + 6) = -48$$

$$C_1 = \begin{vmatrix} 7 & 4 \\ 2 & 1 \end{vmatrix} = 7 - 8 = -1$$

$$A_2 = -\begin{vmatrix} -6 & 4 \\ 1 & 6 \end{vmatrix} = -(-36 - 4) = 40$$

$$B_2 = \begin{vmatrix} 5 & 4 \\ 2 & 6 \end{vmatrix} = 30 - 8 = 22$$

$$C_2 = -\begin{vmatrix} 5 & -6 \\ 2 & 1 \end{vmatrix} = -(5 + 12) = -17$$

$$A_3 = \begin{vmatrix} -6 & 4 \\ 4 & -3 \end{vmatrix} = 18 - 16 = 2$$

$$B_3 = -\begin{vmatrix} 5 & 4 \\ 7 & -3 \end{vmatrix} = -(-15 - 28) = 43$$

$$C_3 = \begin{vmatrix} 5 & -6 \\ 7 & 4 \end{vmatrix} = 20 + 42 = 62$$

$$\text{Adj } A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} 27 & 40 & 2 \\ -48 & 22 & 43 \\ -1 & -17 & 62 \end{bmatrix}$$

Det A = $\Delta$ = 419

$$A^{-1} = \frac{AdjA}{DetA} = \frac{1}{419}\begin{bmatrix} 27 & 40 & 2 \\ 48 & 22 & 43 \\ -1 & -17 & 62 \end{bmatrix}$$

$$x = A^{-1}D = \frac{1}{419}\begin{bmatrix} 27 & 40 & 2 \\ -48 & 22 & 43 \\ -1 & -17 & 62 \end{bmatrix}\begin{bmatrix} 15 \\ 19 \\ 46 \end{bmatrix}$$

$$= \frac{1}{419}\begin{bmatrix} +405+760+92 \\ -720+418+1978 \\ -15-323+2852 \end{bmatrix}$$

$$= \frac{1}{419}\begin{bmatrix} 1257 \\ 1676 \\ 2514 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$$

$\therefore$ Solution is x = 3, y = 4, z = 6.

**iii) Gauss-Jordan method :**

Augmented matrix is $A = \begin{bmatrix} 5 & -6 & 4 & 15 \\ 7 & 4 & -3 & 19 \\ 2 & 1 & 6 & 46 \end{bmatrix}$

$R_2 \to 5R_2 - 7R_1$, $R_3 \to 5R_3 - 2R_1$

$$A \sim \begin{bmatrix} 5 & -6 & 4 & 15 \\ 0 & 62 & -43 & -10 \\ 0 & 17 & 22 & 200 \end{bmatrix}$$

$R_1 \to 31R_1 + 3R_2$, $R_3 \to 62R_3 - 17R_2$

$$A \sim \begin{bmatrix} 155 & 0 & -5 & 435 \\ 0 & 62 & -43 & -10 \\ 0 & 0 & 2095 & 12570 \end{bmatrix}$$

$$R_3 \to R_3\left(\frac{1}{2095}\right)$$

$$A \sim \begin{bmatrix} 155 & 0 & -5 & 435 \\ 0 & 62 & -43 & -10 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

$R_1 \to R_1 + 5R_3$, $R_2 \to R_2 + 43R_3$

$$A \sim \begin{bmatrix} 155 & 0 & 0 & 465 \\ 0 & 62 & 0 & 248 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

$$R_1 \to R_1\left(\frac{1}{155}\right) R_2 \to R_2\left(\frac{1}{62}\right)$$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

$\therefore$ Unique solution exists.

Solution is x = 3, y = 4, z = 6.

**2.** **x + y + z = 1**
**2x + 2y + 3z = 6**
**x + 4y + 9z = 3**

**I.** **i) Cramer's rule**

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 4 & 9 \end{vmatrix}$$

$$= 1(18-12) - 1(18-3) + 1(8-2)$$

$$= 6 - 15 + 6 = -3$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 6 & 2 & 3 \\ 3 & 4 & 9 \end{vmatrix}$$

$$= 1(18-12) - 1(54-9) + 1(24-6)$$

$$= 6 - 45 + 18 = -21$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 6 & 3 \\ 1 & 3 & 9 \end{vmatrix}$$

$$= 1(54-9) - 1(18-3) + 1(6-6)$$

$$= 45 - 15 = 30$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 6 \\ 1 & 4 & 3 \end{vmatrix}$$

$$= 1(6-24) - 1(6-6) + 1(8-2)$$

$$= -18 - 0 + 6 = -12$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-21}{-3} = 7$$

$$y = \frac{\Delta_2}{\Delta} = \frac{30}{-3} = -10$$

$$z = \frac{\Delta_3}{\Delta} = \frac{-12}{-3} = 4$$

Solution is x = 7, y = –10, z = 4.

**ii) Matrix inversion method :**

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $D = \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}$

$$A_1 = \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = 18 - 12 = 6$$

$$B_1 = -\begin{vmatrix} 2 & 3 \\ 1 & 9 \end{vmatrix} = -(18 - 3) = -15$$

$$C_1 = \begin{vmatrix} 2 & 2 \\ 1 & 4 \end{vmatrix} = 8 - 2 = 6$$

$$A_2 = -\begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} = -(9 - 4) = -5$$

$$B_2 = \begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} = 9 - 1 = 8$$

$$C_2 = -\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = -(4 - 1) = -3$$

$$A_3 = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 3 - 2 = 1$$

$$B_3 = -\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = -(3 - 2) = -1$$

$$C_3 = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 2 - 2 = 0$$

$$\text{AdjA} = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}$$

$$\text{DetA} = \Delta = -3$$

$$A^{-1} = \frac{AdjA}{DetA} = -\frac{1}{3}\begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}$$

$$x = A^{-1}D = \frac{1}{3}\begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}$$

$$= -\frac{1}{3}\begin{bmatrix} 6-30+3 \\ -15+48-3 \\ 6-18+0 \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} -21 \\ 30 \\ -12 \end{bmatrix} = \begin{bmatrix} 7 \\ -10 \\ 4 \end{bmatrix}$$

$\therefore$ Solution is x = 7, y = –10, z = 4.

**iii) Gauss-Jordan method:**

Augmented matrix is $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 3 & 6 \\ 1 & 4 & 9 & 3 \end{bmatrix}$

$R_2 \to R_2 - 2R_1, R_3 \to R_3 - R_1$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 4 \\ 0 & 3 & 8 & 2 \end{bmatrix}$$

$R_3 \to R_3 - 8R_2, R_1 \to R_1 - R_2$

$$A \sim \begin{bmatrix} 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 3 & 0 & -30 \end{bmatrix}$$

$R_3 \to R_3\left(\frac{1}{3}\right)$

$$A \sim \begin{bmatrix} 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & -10 \end{bmatrix}$$ Unique solution exists

$R_1 \to R_1 - R_3, R_2 \leftrightarrow R_3$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$\therefore$ Solution is x = 7, y = –10, z = 4.

**3.** **x – y + 3z = 5**
**4x + 2y – z = 0**
**–x + 3y + z = 5**

**Sol. (i) Cramer's rule :**

$$\Delta = \begin{vmatrix} 1 & -1 & 3 \\ 4 & 2 & -1 \\ -1 & 3 & 1 \end{vmatrix}$$

$$= 1(2+3)+1(4-1)+3(12+2)$$

$$= 5+3+42 = 50$$

$$\Delta_1 = \begin{vmatrix} 5 & -1 & 3 \\ 0 & 2 & -1 \\ 5 & 3 & 1 \end{vmatrix}$$

$$= 5(2+3)+1(0+5)+3(0-10)$$

$$= 25+5-30 = 0$$

$$\Delta_2 = \begin{vmatrix} 1 & 5 & 3 \\ 4 & 0 & -1 \\ -1 & 5 & 1 \end{vmatrix}$$

$$= 1(0+5)-5(4-1)+3(20-0)$$

$$= 5-15+60 = 50$$

$$\Delta_3 = \begin{vmatrix} 1 & -1 & 5 \\ 4 & 2 & 0 \\ -1 & 3 & 5 \end{vmatrix}$$

$$= 1(10-0)+1(20-0)+5(12+2)$$

$$= 10+20+70 = 100$$

$$x = \frac{\Delta_1}{\Delta} = \frac{0}{50} = 0$$

$$y = \frac{\Delta_2}{\Delta} = \frac{50}{50} = 1$$

$$z = \frac{\Delta_3}{\Delta} = \frac{100}{50} = 2$$

$\therefore$ Solution is x = 0, y = 1, z = 2/

**ii) Matrix inversion method :**

Let $A = \begin{bmatrix} 1 & -1 & 3 \\ 4 & 2 & -1 \\ -1 & 3 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $D = \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$

$$A_1 = \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = 2+3 = 5$$

$$B_1 = -\begin{vmatrix} 4 & -1 \\ -1 & 1 \end{vmatrix} = -(4-1) = -3$$

$$C_1 = \begin{vmatrix} 4 & 2 \\ -1 & 3 \end{vmatrix} = 12+2 = 14$$

$$A_2 = -\begin{vmatrix} -1 & 3 \\ 3 & 1 \end{vmatrix} = -(-1-9) = 10$$

$$B_2 = \begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix} = 1+3 = 4$$

$$C_2 = -\begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix} = -(3-1) = -2$$

$$A_3 = \begin{vmatrix} -1 & 3 \\ 2 & -1 \end{vmatrix} = 1-6 = -5$$

$$B_3 = -\begin{vmatrix} 1 & 3 \\ 4 & -1 \end{vmatrix} = -(-1-12) = 13$$

$$C_3 = \begin{vmatrix} 1 & -1 \\ 4 & 2 \end{vmatrix} = 2+4 = 6$$

$$\therefore \text{Adj}A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} 5 & 10 & -5 \\ -3 & 4 & 13 \\ 14 & -2 & 6 \end{bmatrix}$$

$$\det A = a_1A_1 + b_1B_1 + c_1C_1$$

$$= 1 \cdot 5 - 1 \cdot (-30 + 3 \cdot 14$$

$$= 5 + 3 + 42 = 50$$

$$A^{-1} = \frac{\text{Adj}A}{\det A} = \frac{1}{50}\begin{bmatrix} 5 & 10 & -5 \\ -3 & 4 & 13 \\ 14 & -2 & 6 \end{bmatrix}$$

$$X = A^{-1}D = \frac{1}{50}\begin{bmatrix} 5 & 10 & -5 \\ -3 & 4 & 13 \\ 14 & -2 & 6 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$$

$$= \frac{1}{50}\begin{bmatrix} 25+0-25 \\ -15+0+65 \\ 70+0+30 \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 \\ 50 \\ 100 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore$ Solution is x = 0, y = 1, z = 2.

**iii) Gauss Jordan method :**

Augmented matrix is $\begin{bmatrix} 1 & -1 & 3 & 5 \\ 4 & 2 & -1 & 0 \\ -1 & 3 & 1 & 5 \end{bmatrix}$

By $R_2 \to R_2 - 4R_1, R_3 \to R_3 + R_1$

$$A \sim \begin{bmatrix} 1 & -1 & 3 & 5 \\ 0 & 6 & -13 & -20 \\ 0 & 2 & 4 & 10 \end{bmatrix}$$

$$R_3 \to R_3\left(\frac{1}{2}\right)$$

$$A \sim \begin{bmatrix} 1 & -1 & 3 & 5 \\ 0 & 6 & -13 & -20 \\ 0 & 1 & 2 & 5 \end{bmatrix}$$

$R_1 \to R_1 + R_3, R_2 \to R_2 - 6R_3$

$$A \sim \begin{bmatrix} 1 & 0 & 5 & 10 \\ 0 & 0 & -25 & -50 \\ 0 & 1 & 2 & 5 \end{bmatrix}$$

$$R_2 \to R_2\left(\frac{-1}{25}\right)$$

$$A \sim \begin{bmatrix} 1 & 0 & 5 & 10 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 2 & 5 \end{bmatrix}$$

$R_1 \to R_1 - 5R_2, R_3 \to R_3 - 2R_2$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Unique solution

$\therefore$ Solution is x = 0, y = 1, z = 2.

**4.** **2x + 6y + 11 = 0**
**6x + 20y – 6z + 3 = 0**
**6y – 18z + 1 = 0**

**Sol**. $\Delta = \begin{vmatrix} 2 & 6 & 0 \\ 6 & 20 & -6 \\ 0 & 6 & -18 \end{vmatrix}$

$= 2(-360 + 36) - 6(-108 - 0)$

$= -648 + 648 = 0$

$\therefore$ Cramer's rule and matrix inversion method cannot be used. $\because \Delta = 0$

**ii) Gauss Jordan method :**

Augmented matrix is $\begin{bmatrix} 2 & 6 & 0 & -11 \\ 6 & 20 & -6 & -3 \\ 0 & 6 & -18 & -1 \end{bmatrix}$

$R_2 \rightarrow R_2 - 3R_1$

$$A \sim \begin{bmatrix} 2 & 6 & 0 & -11 \\ 0 & 2 & -6 & 30 \\ 0 & 6 & -18 & -1 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 3R_2$

$$A \sim \begin{bmatrix} 2 & 6 & 0 & -11 \\ 0 & 2 & -6 & 30 \\ 0 & 0 & 0 & -93 \end{bmatrix}$$

$\rho(A) = 2, \rho(AB) = 3$

$\rho(A) \neq \rho(AB)$

$\therefore$ The given system of equations do not have a solution.

**5.** **2x – y + 3z = 9**
**x + y + z = 6**
**x – y + z = 2**

**Sol. i) Cramer's rule :**

$$\Delta = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= 2(1+1)+1(1-1)+3(-1-1)$$

$$= 4+0-6=-2$$

$$\Delta_1 = \begin{vmatrix} 9 & -1 & 3 \\ 6 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= 9(1+1)+1(6-2)+3(-6-2)$$

$$= 18+4-24=-2$$

$$\Delta_2 = \begin{vmatrix} 2 & 9 & 3 \\ 1 & 6 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= 2(6-2)-9(1-1)+3(2-6)$$

$$= 8-0-12=-4$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 9 \\ 1 & 1 & 6 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= 2(2+6)+1(2-6)+9(-1-1)$$

$$= 16-4-18=-6$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-2}{-2} = 1, \; y = \frac{\Delta_2}{\Delta} = \frac{-4}{-2} = 2$$

$$z = \frac{\Delta_3}{\Delta} = \frac{-6}{-2} = 3$$

Solution is x = 1, y = 2, z = 3.

**ii) Matrix inversion method :**

Let $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $D = \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$

$$A_1 = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 1+1 = 2$$

$$B_1 = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$$C_1 = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1-1 = -2$$

$$A_2 = -\begin{vmatrix} -1 & 3 \\ -1 & 1 \end{vmatrix} = -(-1+3) = -2$$

$$B_2 = \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = 2-3 = -1$$

$$C_2 = -\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} = -(-2+1) = 1$$

$$A_3 = \begin{vmatrix} -1 & 3 \\ 1 & 1 \end{vmatrix} = -1-3 = -4$$

$$B_3 = -\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = -(2-3) = 1$$

$$C_3 = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2+1 = 3$$

$$\text{AdjA} = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ 0 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$$

$$\begin{aligned} \text{DetA} &= a_1A_1 + b_1B_1 + c_1C_1 \\ &= 2(2) - 1\cdot 0 + 3(-2) \\ &= 4 - 6 = -2 \end{aligned}$$

$$A^{-1} = \frac{\text{AdjA}}{\text{DetA}} = -\frac{1}{2}\begin{bmatrix} 2 & -2 & -4 \\ 0 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$$

$$X = A^{-1}D = -\frac{1}{2}\begin{bmatrix} 2 & -2 & -4 \\ 0 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} 18-12-8 \\ -6+2 \\ -18+6+6 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -2 \\ -4 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Solution is x = 1, y = 2, z = 3.

**iii) Gauss-Jordan method :**

Augmented matrix $A = \begin{bmatrix} 2 & -1 & 3 & 9 \\ 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \end{bmatrix}$

$R_1 \to R_1 - 2R_2, R_3 \to R_3 - R_1$

$$A \sim \begin{bmatrix} 0 & -3 & 1 & -3 \\ 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & -4 \end{bmatrix}$$

$R_3 \to R_3\left(-\frac{1}{2}\right)$ gives $A \sim \begin{bmatrix} 0 & -3 & 1 & -3 \\ 1 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 \end{bmatrix}$

$R_1 \to R_1 + 3R_3, R_2 \to R_2 - R_3$

$$A \sim \begin{bmatrix} 0 & 0 & 1 & 3 \\ 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$R_2 \to R_2 - R_1$ gives $A \sim \begin{bmatrix} 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \end{bmatrix}$

By $R_1 \leftrightarrow R_2, R_2 \leftrightarrow R_2 - R_3$ we get

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$\therefore$ The given equations have a unique solution.
Solution is x = 1, y = 2, z = 3.

**6.** **2x – y + 8z = 13**
**3x + 4y + 5z = 18**
**5x – 2y + 7z = 20**

**Sol. i) Cramer's rule :**

$$\Delta = \begin{vmatrix} 2 & -1 & 8 \\ 3 & 4 & 5 \\ 5 & -2 & 7 \end{vmatrix}$$

$$= 2(28+10)+1(21-25)+8(-6-20)$$

$$= 76-4-208 = -136$$

$$\Delta_1 = \begin{vmatrix} 13 & -1 & 8 \\ 18 & 4 & 5 \\ 20 & -2 & 7 \end{vmatrix}$$

$$= 13(28+10)+1(126-100)+8(-36-80)$$

$$= 494+26-928 = -408$$

$$\Delta_2 = \begin{vmatrix} 2 & 13 & 8 \\ 3 & 18 & 5 \\ 2 & 20 & 7 \end{vmatrix}$$

$$= 2(126-100)-13(21-25)+8(60-90)$$

$$= 52+52-240 = -136$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 13 \\ 3 & 4 & 18 \\ 5 & -2 & 20 \end{vmatrix}$$

$$= 2(80+36)+1(60-90)+13(-6-20)$$

$$= 232-30-338 = -136$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-408}{-136} = 3$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-136}{-136} = 1$$

$$z = \frac{\Delta_3}{\Delta} = \frac{-136}{-136} = 1$$

$\therefore$ Solution is x = 3, y = 1, z = 1.

**ii) Matrix inversion method :**

Let $A = \begin{bmatrix} 2 & -1 & 8 \\ 3 & 4 & 5 \\ 5 & -2 & 7 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $D = \begin{bmatrix} 13 \\ 18 \\ 20 \end{bmatrix}$

$$A_1 = \begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix} = 28 + 10 = 38$$

$$B_1 = -\begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} = -(21 - 25) = 4$$

$$C_1 = \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = -6 - 20 = -26$$

$$A_2 = -\begin{vmatrix} -1 & 8 \\ -2 & 7 \end{vmatrix} = -(-7 + 16) = -9$$

$$B_2 = \begin{vmatrix} 2 & 8 \\ 5 & 7 \end{vmatrix} = (14 - 40) = -26$$

$$C_2 = -\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = -(-4 + 5) = -1$$

$$A_3 = \begin{vmatrix} -1 & 8 \\ 4 & 5 \end{vmatrix} = -5 - 32 = -37$$

$$B_3 = -\begin{vmatrix} 2 & 8 \\ 3 & 5 \end{vmatrix} = -(10 - 24) = 14$$

$$C_3 = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 8 + 3 = 11$$

$$\text{Adj}\, A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} 38 & -9 & -37 \\ 4 & -26 & 14 \\ -26 & 1 & 11 \end{bmatrix}$$

$$\begin{aligned} \text{Det}\, A &= a_1A_1 + b_1B_1 + c_1C_1 \\ &= 2 \cdot 38 + (-1)4 + 8(-26) \\ &= 76 - 4 - 208 = -136 \end{aligned}$$

$$A^{-1} = \frac{\text{Adj}\,A}{\text{Det}\,A} = -\frac{1}{136}\begin{bmatrix} 38 & -9 & -37 \\ 4 & -26 & 14 \\ -26 & 1 & 11 \end{bmatrix}$$

$$X = A^{-1}D = -\frac{1}{280}\begin{bmatrix} 38 & -9 & -37 \\ 4 & -26 & 14 \\ -26 & 1 & 11 \end{bmatrix}\begin{bmatrix} 13 \\ 18 \\ 20 \end{bmatrix}$$

$$= -\frac{1}{136}\begin{bmatrix} 494 & -162 & -740 \\ 52 & -468 & +280 \\ -338 & -18 & +220 \end{bmatrix}$$

$$= -\frac{1}{136}\begin{bmatrix} -408 \\ -136 \\ -136 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

Solution is x = 3, y = 1, z = 1.

**iii) Gauss Jordan method :**

Augmented matrix is $A = \begin{bmatrix} 2 & -1 & 8 & 13 \\ 3 & 4 & 5 & 18 \\ 5 & -2 & 7 & 20 \end{bmatrix}$

$R_2 \to 2R_2 - 3R_1, R_3 \to 2R_3 - 5R_2$ we get

$$A \sim \begin{bmatrix} 2 & -1 & 8 & 13 \\ 0 & 11 & -14 & -3 \\ 0 & 1 & -26 & -25 \end{bmatrix}$$

$R_1 \to R_1 + R_3, R_2 \to R_2 - 11R_3$, we get

$$A \sim \begin{bmatrix} 2 & 0 & -18 & -12 \\ 0 & 0 & 272 & 272 \\ 0 & 1 & -26 & -25 \end{bmatrix}$$

$R_2 \to R_2\left(\frac{1}{272}\right)$ we get

$$A \sim \begin{bmatrix} 2 & 0 & -18 & -12 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & -26 & -25 \end{bmatrix}$$

$R_1 \to R_1 + 18R_2, R_3 \to R_3 + 26R_2$, we get

$$A \sim \begin{bmatrix} 2 & 0 & 0 & 6 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$R_2 \leftrightarrow R_3, R_1\left(\frac{1}{2}\right)$ we get $A \sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$\therefore$ The given equations have a unique solution and solution is x = 3, y = 1, z = 1.

**7. 2x – y + 3z = 8**
**–x + 2y + z = 4**
**3x + y – 4z = 0**

**Sol. i) Cramer's rule :**

$$\Delta = \begin{vmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{vmatrix}$$

$$= 2(-8-1)+1(4-3)+3(-1-6)$$

$$= -18+1-21 = -38$$

$$\Delta_1 = \begin{vmatrix} 8 & -1 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -4 \end{vmatrix}$$

$$= 8(-8-1)+1(-16-0)+3(4-0)$$

$$= -72-16+12 = -76$$

$$\Delta_2 = \begin{vmatrix} 2 & 8 & 3 \\ -1 & 4 & 1 \\ 3 & 0 & -4 \end{vmatrix}$$

$$= 2(-16-0)-8(4-3)+3(-0-12)$$

$$= -32-8-36 = -76$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 8 \\ -1 & 2 & 4 \\ 3 & 1 & 0 \end{vmatrix}$$

$$= 2(0-4)+1(0-12)+8(-1-6)$$

$$= -8-12-56 = -76$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-76}{-38} = 2$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-76}{-38} = 2$$

$$z = \frac{\Delta_3}{\Delta} = \frac{-76}{-38} = 2$$

$\therefore$ Solution is x = 2, y = 2, z = 2.

**ii) Matrix inversion method :**

Let $A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $D = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$

$$A_1 = \begin{vmatrix} 2 & 1 \\ 1 & -4 \end{vmatrix} = -8 - 1 = -9$$

$$B_1 = -\begin{vmatrix} -1 & 1 \\ 3 & -4 \end{vmatrix} = -(4 - 3) = -1$$

$$C_1 = \begin{vmatrix} -1 & 2 \\ 3 & 1 \end{vmatrix} = -1 - 6 = -7$$

$$A_2 = -\begin{vmatrix} -1 & 3 \\ 1 & -4 \end{vmatrix} = -(4 - 3) = -1$$

$$B_2 = \begin{vmatrix} 2 & 3 \\ 3 & -4 \end{vmatrix} = -8 - 9 = -17$$

$$C_2 = -\begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} = -(2 + 3) = -5$$

$$A_3 = \begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix} = -1 - 6 = -7$$

$$B_3 = -\begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} = -(4 + 3) = -7$$

$$C_3 = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 4 - 1 = 3$$

$$\text{Adj}A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} -9 & -1 & -7 \\ -1 & -17 & -7 \\ -7 & -5 & 3 \end{bmatrix}$$

$$\text{Det } A = a_1A_1 + b_1B_1 + c_1C_1$$

$$= 2(-9) - 1(-1) + 3(-7)$$

$$= -18 + 1 - 21 = -38$$

$$A^{-1} = \frac{AdjA}{DetA} = -\frac{1}{38}\begin{bmatrix} -9 & -1 & -7 \\ -1 & -17 & -7 \\ -7 & -5 & 3 \end{bmatrix}$$

$$X = A^{-1}D = -\frac{1}{38}\begin{bmatrix} -9 & -1 & -7 \\ -1 & -17 & -7 \\ -7 & -5 & 3 \end{bmatrix}\begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

$$= -\frac{1}{38}\begin{bmatrix} -72-4 \\ -8-68 \\ -56-20 \end{bmatrix} = -\frac{1}{38}\begin{bmatrix} -76 \\ -76 \\ -76 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

Solution is x = 2, y = 2, z = 2.

**iii) Gauss Jordan method :**

Augmented matrix is $A = \begin{bmatrix} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{bmatrix}$

$R_1 \to R_1 + 2R_2$, $R_3 \to R_3 + 3R_2$, we get

$$A \sim \begin{bmatrix} 0 & 3 & 5 & 16 \\ -1 & 2 & 1 & 4 \\ 0 & 7 & -1 & 12 \end{bmatrix}$$

$R_2 \to 3R_2 - 2R_1, R_3 \to 3R_3 - 7R_1$, we have

$$A \sim \begin{bmatrix} 0 & 3 & 5 & 16 \\ -3 & 0 & -7 & -20 \\ 0 & 0 & -38 & -76 \end{bmatrix}$$

$R_3 \to R_3\left(-\frac{1}{38}\right)$, we get

$$A \sim \begin{bmatrix} 0 & 3 & 5 & 16 \\ -3 & 0 & -7 & -20 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_1 \to R_1 - 5R_2, R_2 \to R_2 + 7R_3$, we get

$$A \sim \begin{bmatrix} 0 & 3 & 0 & 6 \\ -3 & 0 & 0 & -6 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_1 \to R_1\left(\frac{1}{3}\right), R_2 \to R_2\left(-\frac{1}{3}\right), R_1 \to R_2$ we get

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$\therefore$ The given equations have a unique solution and solution is x = 2, y = 2, z = 2.

**8. Solve x + y + z = 9**
**2x + 5y + 7z = 52**
**2x + y – z = 0**

**Sol. i) Cramer's rule :**

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & -7 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= 1(-5-7) - 1(-2-14) + 1(2-10)$$

$$= -12 + 16 - 8 = -4$$

$$\Delta_1 = \begin{vmatrix} 9 & 1 & 1 \\ 52 & 5 & 7 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= 9(-5-7) - 1(-52-0) + 1(52-0)$$

$$= -108 + 52 + 52 = -4$$

$$\Delta_2 = \begin{vmatrix} 1 & 9 & 1 \\ 2 & 52 & 7 \\ 2 & 0 & -1 \end{vmatrix}$$

$$= 1(-52-0) - 9(-2-14) + 1(0-104)$$

$$= -52 + 144 - 104 = -20$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 9 \\ 2 & 5 & 52 \\ 2 & 1 & 0 \end{vmatrix}$$

$$= 1(0-52) - 1(0-104) + 9(2-10)$$

$$= -52 + 104 - 72 = -20$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-4}{-4} = 1$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-12}{-4} = 3$$

$$z = \frac{\Delta_3}{\Delta} = \frac{-20}{-4} = 5$$

Solution is x = 1, y = 3, z = 5

**ii) Matrix inversion method :**

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $D = \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$

$$A_1 = \begin{vmatrix} 5 & 7 \\ 1 & -1 \end{vmatrix} = -5 - 7 = -12$$

$$B_1 = -\begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix} = -(-2 - 14) = 16$$

$$C_1 = \begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix} = 2 - 10 = -8$$

$$A_2 = -\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -(-1 - 1) = 2$$

$$B_2 = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -1 - 2 = -3$$

$$C_2 = -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -(1 - 2) = 1$$

$$A_3 = \begin{vmatrix} 1 & 1 \\ 5 & 7 \end{vmatrix} = 7 - 5 = 2$$

$$B_3 = -\begin{vmatrix} 1 & 1 \\ 2 & 7 \end{vmatrix} = -(7 - 2) = -5$$

$$C_3 = \begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix} = 5 - 2 = 3$$

$$\text{Adj}\,A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$$

$$\text{Det}\,A = a_1A_1 + b_1B_1 + c_1C_1$$
$$= 1(-12) + 1(16) + 1(-8)$$
$$= -12 + 16 - 8 = -4$$

$$A^{-1} = \frac{\text{Adj}A}{\text{Det}A} = -\frac{1}{4}\begin{bmatrix} -12 & +2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$$

$$X = A^{-1}D = -\frac{1}{4}\begin{bmatrix} -12 & +2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}\begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$$

$$= -\frac{1}{4}\begin{bmatrix} -108+104 \\ 144-156 \\ -72+52 \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} -4 \\ -12 \\ -20 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$\therefore$ Solution is x = 1, y = 3, z = 5.

**iii) Gauss Jordan method :**

Augmented matrix $A = \begin{bmatrix} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \end{bmatrix}$

$R_2 \to R_2 - 2R_1, R_3 \to R_3 - R_2$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & -4 & -8 & -52 \end{bmatrix}$$

$R_1 \to 3R_1 - R_2, R_3 \to 3R_3 + 4R_2$

$$A \sim \begin{bmatrix} 3 & 0 & -2 & -7 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & -4 & -20 \end{bmatrix}$$

$R_3 \to R_3\left(-\frac{1}{4}\right)$, we obtain

$$A \sim \begin{bmatrix} 3 & 0 & -2 & -7 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$R_1 \to R_1 + 2R_3, R_2 \to R_2 - 5R_3$, we get

$$A \sim \begin{bmatrix} 3 & 0 & 0 & 3 \\ 0 & 3 & 0 & 9 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$R_1 \to R_1\left(\frac{1}{3}\right), R_2 \to R_2\left(\frac{1}{3}\right)$ we have

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$\therefore$ The given equations have a unique solution and solution is x = 1, y = 3, z = 5.