GRAVITATION
2. VARIATION OF ' g'

POINTS TO REMEMBER

1. g and G are related on any planet as $g = \frac{GM}{R^2} = \frac{4}{3}\pi R\rho G$ Where M is the mass of the planet, ρ is the density and R is the radius of the planet.

2. The value of 'g' depends upon the altitude, depth, latitude and also on the shape of the earth planet.

3. The variation of g with height h from the surface of the earth is given by the equation $g_h = g\left(1-\frac{2h}{R}\right)$

4. The variation of ' g' with depth d from the surface of the earth is given by the equation $g_d = g\left[1-\frac{d}{R}\right]$

5. The acceleration due to gravity at a latitude angle ϕ is given by,
   $g_\phi = g - R\omega^2 \cos^2\phi$
   At equator $\phi = 0^0$, $g_\phi = g - R\omega^2$
   At poles, $\phi = 90^0$, $g_\phi = g$

6. g changes with the angular velocity of rotation of earth at equator as the rotational speed of the earth increases g at the equator decreases. If the earth stops rotating g becomes maximum.

7. The value of g at the poles is independent of rotational speed of the earth.

8. If the rotational speed of the earth increases by 17 times the present value, the bodies at the equator becomes weightless.

9. There are two types of reference frames (i) Inertial reference frame (ii) Non-inertial reference frame.
   The reference frame in which Newton's laws hold good is called inertial reference frame.
   Ex: The reference frame at rest.

10. The reference frame in which Newton's laws of motion do not hold good is called 'non -inertial reference frame'.
    Ex. The linearly accelerating frame or a rotating frame.

11. The acceleration of a body in inertial reference frame is caused by the real forces whereas the acceleration of a body in non-inertial reference frame is caused by the pseudo forces.

12. The property of a body that measures its resistance to acceleration is called its inertial mass.

13. The property of a body responsible for the gravitational force it exerts on another body is called its gravitational mass

14. The principle of equivalence states that 'no experiment performed inside an accelerated, closed system can distinguish between the effects of gravitational field and the effects of an accelerated motion.

LONG ANSWER QUESTIONS

1. What is acceleration due to gravity (g) and universal gravitational constant (G). Derive the relation between them? (May2008)

Ans. . **Acceleration due to gravity (g):**

The acceleration with which a body of certain mass falls towards the centre of a planet is called acceleration due to gravity 'g' on that planet.

Universal gravitational constant (G):

According to the law of universal gravitation, if m_1 and m_2 are the masses of two bodies and r is the distance between them, the gravitational force of attraction F which each exerts on the other is given by

F = $G \cdot \frac{m_1 m_2}{r^2}$ where G is called universal gravitational constant and it is equal to 6.67×10^{-11} Nm^2kg^{-2}.

Universal gravitational constant is numerically equal to the force of attraction between two bodies of masses 1kg each separated by a distance 1 meter in any medium.

Relation between g and G:

Consider the earth as a perfect homogeneous sphere of radius R. Let the total mass of the earth M is supposed to be concentrated at its centre. Consider a body of mass 'm' on the surface of the earth then the gravitational force (weight) acting on the body is given by

$F = mg$ ------- (1)

According to universal law of gravitation

$F = \frac{GMm}{R^2}$ ---- (2)

From (1) and (2) $mg = \frac{GMm}{R^2}$ Or $g = \frac{GM}{R^2}$

This is the relation between g and G which is applicable for any planet.

2. Derive expressions for the variation of acceleration due to gravity with depth and latitude. What would be the time period of rotation of the earth to make the bodies at equator weightless?

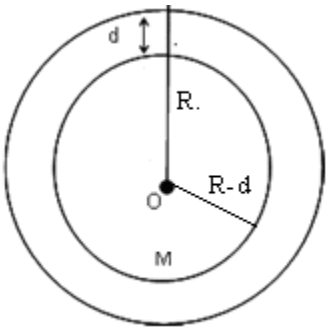
Variation of ' g' with depth:

Consider the earth as a perfect homogeneous sphere with centre O and radius R. Let the total mass of the earth is M and ρ be its uniform density.

The acceleration due to gravity on the surface of the earth given by, $g = \frac{GM}{R^2}$

But , the mass of the earth = Volume of the earth X density of the earth

$$M = \frac{4}{3}\pi R^3 \rho$$



$$\therefore g = \frac{4}{3}\pi R\rho G \qquad \text{------- (1)}$$

Now consider the body to be placed at a depth ' d' from the surface of the earth. Then the mass of the earth that attracts the body is of radius (R-d) only . Hence the acceleration due to gravity at depth 'd' is given by

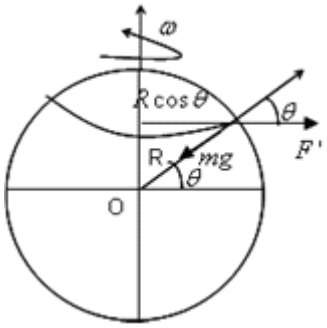
$$g_d = \frac{4}{3}\pi(R-d)\rho G \qquad \text{------- (2)}$$

From equations (1) and (2), $\frac{g_d}{g} = \frac{R-d}{R}$ Or $g_d = g\left[1-\frac{d}{R}\right]$

Hence the value of 'g' decreases with depth.

Variation of 'g' with latitude:

The earth notates from west to east about an axis passing through its north and south poles once in 24 hours. Let us consider a body of mass 'm' placed on the surface of the earth at latitude ϕ. Due to the rotation of the earth about its own axis the body experiences a centrifugal force F^1 as shown in the fig



Assuming the cross – sectional radius as R cosϕ

$F^1 = m(R\cos\phi)\omega^2 \qquad \Rightarrow F' = mR\omega^2 \cos\phi$

$\therefore$ Resultant force on the body is given by , $F_R = mg - F^1\cos\phi$

$mg_\phi = mg - (mR\omega^2\cos\phi)\cos\phi$

$\Rightarrow g_\phi = g - R\omega^2\cos^2\phi$

At equator $\phi = 0^0$, $\quad g_\phi = g - R\omega^2$

If the bodies are to be weightless at equator,

$$g - R\omega^2 = 0 \Rightarrow g - R\omega^2 \Rightarrow \omega = \sqrt{\frac{g}{R}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{g}{R}}$$

$$\therefore T = 2\pi\sqrt{\frac{R}{g}} \simeq 84\,\text{min}\,utes$$

SHORT ANSWER ANSWERS

1. State and explain Newton's law of gravitation on what factors 'G' depends?

Ans. **Statement:** Newton's law of gravitation states that "Every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Explanation: If m_1 and m_2 are the masses of two bodies which are separated by a distance r, then the gravitational force of attraction F is given by

F = $G\cdot\frac{m_1m_2}{r^2}$ where G is called universal gravitational constant and is equal to 6.67×10^{-11} Nm^2Kg^{-2}.

Universal gravitational constant does not depend upon the medium between the particles (or) any other factors. It is constant every where in the universe.

2. Deduce the relation between g and G.

Ans. Consider the earth as a perfect homogeneous sphere of radius R. Let the total mass of the earth M is supposed to be concentrated at its centre. Consider a body of mass 'm' on the surface of the earth then the gravitational force (weight) acting on the body is given by

$F = mg$ ------- (1)

According to universal law of gravitation

$F = \frac{GMm}{R^2}$ ---- (2)

From (1) and (2) $mg = \frac{GMm}{R^2}$ Or $g = \frac{GM}{R^2}$

This is the relation between g and G which is applicable for any planet.

3. Obtain an expression for the acceleration due to gravity at height 'h' from surface of planet.

Ans. Consider a planet of mass M and radius R. If g is acceleration due to gravity on the surface of planet, then

$$g = \frac{GM}{R^2} \text{------- (1)}$$

The acceleration due to gravity at that height. H from the surface of a planet is given by

$$g_h = \frac{GM}{(R+h)^2} \text{ ---- (2)}$$

From (1) and (2)

$$\frac{g_h}{g} = \frac{GM}{(R+h)^2} \cdot \frac{R^2}{GM} = \frac{R^2}{(R+h)^2} \Rightarrow g_h = g\left(\frac{R}{R+h}\right)^2 \Rightarrow g_h = g\left(1-\frac{2h}{R}\right)$$

The acceleration due to gravity decreases with the increase of height.

4. Derive an expression for the acceleration due to gravity at a depth'd'.

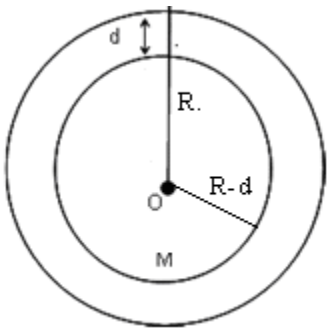
Ans. Consider the earth as a perfect homogeneous sphere with centre O and radius R. Let the total mass of the earth is M and ρ be its uniform density.

The acceleration due to gravity on the surface of the earth given by, $g = \frac{GM}{R^2}$

But, the mass of the earth = Volume of the earth X density of the earth

$$\text{M} = \frac{4}{3}\pi R^3 \rho$$

$$\therefore g = \frac{4}{3}\pi R \rho G \text{ ------- (1)}$$



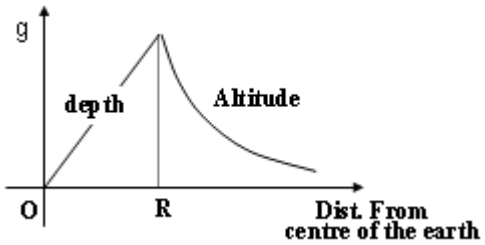
Now consider the body to be placed at a depth ' d' from the surface of the earth. Then the mass of the earth that attracts the body is of radius (R-d) only . Hence the acceleration due to gravity at depth 'd' is given by

$$g_d = \frac{4}{3}\pi (R-d)\rho G \text{ ------- (2)}$$

From equations (1) and (2), $\frac{g_d}{g} = \frac{R-d}{R}$ Or $g_d = g\left[1-\frac{d}{R}\right]$

Hence the value of ‘g’ decreases with depth.

5. Plot a graph between the acceleration due to gravity and the distance from the centre of the earth. State the nature of the obtained curve.



Ans. The variation of g with the distance r of a point from the centre of the earth is shown in the figure. The variation of ‘ g’ with depth is given by the equation

$$g_d = g\left[1-\frac{d}{R}\right]=\frac{g}{R}(R-d)=\frac{g}{R}r \;\; Or \; g_d \alpha r$$

At the centre of the earth (d = R), the value of g is zero. As depth from the surface of the earth increases, the value of g decreases. g_d Versus r graph is a straight line passing through the origin as shown in figure.

The variation of g with height h is given by the equation

$$g_h = g\left(\frac{R}{R+h}\right)^2=\frac{gR^2}{(R+h)^2}=\frac{gR^2}{r^2} \; Or \; g_h \alpha \frac{1}{r^2}$$ As the height from the surface of the earth increases, g decreases and g becomes maximum on the surface of the earth. Decrease in ‘g’ at small heights is more than the decrease in ‘g’ at small depths. But, at large heights decrease in ‘g’ is less than the decrease in g at large depths.

6. How does ’g’ change for the same values of height h and depth d?

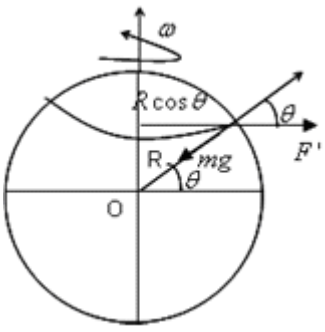
Ans. The variation of g with height is given by the equation $g_h = g\left(1-\frac{2h}{R}\right)$

The variation of ‘g’ with depth is given by the equation $g_d = g\left[1-\frac{d}{R}\right]$

For same value of d and h gravity decreases more at height compared to depth. g will be the same if ` d = 2h .

7. Derive an expression for the variation of ‘g’ with latitude.

Ans. The earth notates from west to east about an axis passing through its north and south poles once in 24 hours. Let us consider a body of mass ‘ m’ placed on the surface of the earth at latitude ϕ. Due to the rotation of the earth about its own axis the body experiences a centrifugal force F^1 as shown in the figure.



Assuming the cross – sectional radius as R cosϕ

$$F^1 = m(R\cos\phi)\omega^2 \qquad \Rightarrow F' = mR\omega^2\cos\phi$$

$\therefore$ Resultant force on the body is given by , $F_R = mg - F^1\cos\phi$

$$mg_\phi = mg - (mR\omega^2\cos\phi)\cos\phi$$

$$\Rightarrow g_\phi = g - R\omega^2\cos^2\phi$$

8. Deduce a relation between ' g' at poles and 'g' at equator.

Ans. The acceleration due to gravity at a latitude angle ϕ is given by,

$g_\phi = g - R\omega^2 \cos^2 \phi$

At equator $\phi = 0^0$, $g_\phi = g - R\omega^2$

At poles $\phi = 90^0$, $g_\phi = g$.

$\therefore g_E = g_P - R\omega^2$ or $R\omega^2 = g_P - g_E$

9. Discuss the effect of the shape of the earth on 'g'.

Ans. The relation between g and G is given by , $g = \frac{GM}{R^2}$

Earth is not a perfect sphere. Earth is flat at the poles and it is bulged at the equator. Hence the polar radius is lesser than the equatorial radius by 21 km. Hence g is greater at the Polar Regions than at the equatorial regions.

10. Define Gravitational field and gravitational field strength. State the units and dimensions of gravitational field strength.

Ans. Gravitational field: The space around a mass in which the gravitational force of attraction of it is felt is known as gravitational field.

Gravitational field strength: It is defined as the gravitational force per unit mass placed at a pointing the gravitational field. The unit of gravitational field is N/Kg and its Dimensional formula is $M^0L^1T^{-2}$

11. State the properties of gravitational field.

Ans. 1. Gravitational field of a body at a point is characterized either by a vector function of position called gravitational intensity or by a scalar function of position called gravitational potential

2. The basic property of gravitational field is the uniformity of accelerations of bodies in a field of given strength irrespective of their mass.

3. Gravitational field is vector quantity and hence adds according to the rules of vector addition.

4. The direction of a gravitational field at a point is given by the tangent to the line of force at that point.

12. Distinguish between inertial and non-inertial frames of reference.

Ans.

Inertial Frame of reference	**Non - inertial frame of reference**
1) A frame that is at rest or moving with uniform velocity Ex: Railway plat form.	1) A frame that moves with a uniform acceleration. Ex: Rotating platform.
2) In this Newton's laws of motion holds good.	2) In this Newton's laws of motion does not hold good.
3) Bodies are acted upon by real forces.	3) Bodies are acted upon by pseudo forces.

13. Distinguish between inertial mass and Gravitational mass. (March 2000)

Inertial mass	**Gravitational mass**
1. The mass determined using equation F=ma is called inertial mass of the body.	1. The mass determined using the equation W=mg is called gravitational mass of the body.
2. While determining inertial mass, the body is is at rest	2. While finding gravitational mass the body in motion
3. It is easy to determine the inertial mass because the body is at rest.	3. It is difficult to determine gravitational mass because the body is in motion
4. Inertial mass increases when the velocity of the body is comparable to that of light	4. Gravitational mass is constant at a particular place.

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14. State and explain the principle of equivalence.

Ans. **Principle:** A uniformly accelerated reference frame in the absence of any gravitational field is completely equivalent to a homogeneous gravitational field Einstein has proved that, physically these two reference frames are identical. This is the principle of equivalence.

Explanation: Consider a space ship on the surface of the earth, which is an inertial frame. When a body projected with in the space ship falls with acceleration due to gravity 'g'. If the space ship is out of the earth's gravitational force of attraction and if it is accelerating away from the earth with acceleration equal to 'g'. This is non inertial frame. Now, if any object is projected up within the ship, it falls with an acceleration 'g' .

VERY SHORT ANSWER ANSWERS

1. State vector form of Newton's law of gravitation.

Ans. If m_1 and m_2 are the masses of two bodies and r is the distance between them, the gravitational force of attraction F which each exerts on the other is given by

$F = = G \cdot \frac{m_1 m_2}{r^3} \bar{r}$ where G is called universal gravitational constant

2. The gravitational force of the earth on the moon is F. What is the gravitational force of the moon on the earth? Do these forces form an action - reaction pair?

Ans. If The gravitational force of the earth on the moon is F , then The gravitational force of moon on earth is also 'F'. These forces form action and reaction pair.

3. State the relation between g and G in terms of (i) mass and radius (ii) density and radius of a planet.

Ans. (i) The relation between g and G in terms of m and R is $g = \frac{GM}{R^2}$ Where R is radius and M is the mass of planet

(ii) The relation between g and G in terms of m and R $g = \frac{GM}{R^2} = \frac{4}{3}\pi R \rho G$

Where R is radius and ρ is the density of planet

4. Keeping the mass of the earth constant, if the radius of the earth decreases by 2%, what would be the change in 'g'?

Ans. $g = \frac{GM}{R^2} \Rightarrow \frac{\Delta g}{g} = -2\frac{\Delta R}{R} \Rightarrow \%\,change = -2 \times -2 = 4\%$ .Hence gravity increases by 4%

5. Keeping the density of the earth constant, if the radius of the earth decreases by 2% what would be the change in 'g'?

Ans. $g = \frac{4}{3}\pi R\rho G \Rightarrow \frac{\Delta g}{g} = \frac{\Delta R}{R} \Rightarrow \%\,change = 2\%$ .Hence if the radius decreases by 2% then gravity also decreases by 2%

6. As we go from one planet to another planet, how will (a) the mass and (b) weight of a body changes.

Ans. (a) The mass is constant (b) Weight (mg) of a body is proportional to the acceleration due to gravity.

7. Keeping the length of a simple pendulum the same, will the time period be the same on all planets? Give also the reason?

Ans. $T = 2\pi\sqrt{\frac{\ell}{g}}$ .Time period of a pendulum depends on the acceleration due to gravity. So it changes from planet to planet. .

8. Give the equation for the value of g at a height h above the surface of the earth. How does it change at small heights?

Ans. Variation of g at a height 'h' above the surface of earth is given by

$g_h = g\left(\frac{R}{R+h}\right)^2$

For smaller heights, $g_h = g\left(1 - \frac{2h}{R}\right)$

9. Give the equation for the value of g at a depth 'd ' from the surface of the earth. What is the value of 'g' at the centre of the earth?

Ans. The variation of 'g' with depth is given by the equation $g_d = g\left[1 - \frac{d}{R}\right]$

At the centre of the earth g becomes zero since depth d = R

10. For small and equal values of d an h, which is greater g_h or g_d ? Give reason for your answer.

Ans. Decrease in g at small heights is more than decrease in 'g' at smaller depths.

11. For large and equal values of d and h, which is greater g_h or g_d ? Give reason for your answer?

Ans. At larger heights the decrease in g is less compared to same depth. .

12. Why does value of g change with latitude?

Ans. The acceleration due to gravity at a latitude angle ϕ is given by,

$g_\phi = g - R\omega^2 \cos^2 \phi$

At equator $\phi = 0^0$, $g_\phi = g - R\omega^2$

At poles $\phi = 90^0$, $g_\phi = g$

13. Does g change with the angular velocity of rotation of the earth at (a) the equator and (b) the poles?

Ans. (a) g changes with the angular velocity of rotation of earth at equator. As the rotational speed of the earth increases g at the equator decreases. If the earth stops rotating g becomes maximum.

(b) The value of g at the poles is independent of rotational speed of the earth.

14. If bodies at a latitude at a latitude of 45^0 of the earth are to be weightless, what should be its time period of rotation?

Ans. The acceleration due to gravity at a latitude angle ϕ is given by,

$g_\phi = g - R\omega^2 \cos^2 \phi$

Now if $\phi = 45^0$, $g_\phi = g - R\omega^2 \cos^2 45 \Rightarrow g_\phi = g - \frac{R\omega^2}{2}$

The bodies to be weightless g_ϕ should be zero.

$$\therefore 0 = g - \frac{R\omega^2}{2} \Rightarrow g = \frac{R\omega^2}{2} \Rightarrow \omega^2 = \frac{2g}{R} \Rightarrow \omega = \sqrt{\frac{2g}{R}}$$

$$\text{But } T = \frac{2\pi}{\omega} \Rightarrow T = 2\pi\sqrt{\frac{R}{2g}}$$

15. If bodies at the equator of the earth are to be weightless. What should be its time period of rotation?

Ans. The acceleration due to gravity at a latitude angle ϕ is given by,

$g_\phi = g - R\omega^2 \cos^2 \phi$

At equator $\phi = 0^0$, $g_\phi = g - R\omega^2$

The bodies to be weightless g_ϕ should be zero.

$$\therefore 0 = g - R\omega^2 \Rightarrow g = R\omega^2 \Rightarrow \omega^2 = \frac{g}{R} \Rightarrow \omega = \sqrt{\frac{g}{R}}$$

$$\text{But } T = \frac{2\pi}{\omega} \Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$$

16. What are the factors that make 'g' the least at the equator?

Ans. The relation between g and G is given by, $g = \frac{GM}{R^2}$

Earth is not a perfect sphere. Earth is flat at the poles and it is bulged at the equator. Hence the equatorial radius is more than the polar radius by 21 km. Hence g is least at the equatorial regions.

17. What are the factors that make 'g' maximum at the poles?

Ans. The relation between g and G is given by, $g = \frac{GM}{R^2}$

Earth is not a perfect sphere. Earth is flat at the poles and it is bulged at the equator. Hence the polar radius is lesser than the equatorial radius by 21 km. Hence g is maximum at the Polar Regions.

18. If two places are at the same height from the mean sea level, one is on a mountain and the other in air. At which of these places 'g' will be greater? State the reason for your answer?

Ans. The gravity is more at mountain since the concentration of mass exists on mountain.

19. The weight of an object is more at the poles than at the equator. At which of these places we can get more sugar for the same weight? State the reason for your answer.

Ans. g is more at poles, less sugar we can obtain is less at equator, so we can obtain more sugar.

20. According to a stationary observer on the earth, which of the following are inertial frames of reference? (i) Merry go round, (ii) giant wheel (iii) A vehicle moving with uniform velocity and (iv) Your class room.

Ans. (i) Merry go round -- Non- inertial frame
(ii) giant wheel -- -- Non- inertial frame
(iii) A vehicle moving with uniform velocity -- Inertial frame
(iv) Your class room -- Inertial frame

21. In the expression for force F = ma, which mass - inertial mass or gravitational mass, does m represent? State the reason for your answer.

Ans. In the expression of F = ma, m represents inertial mass having the concept that mass of an object is a measure of inertia.

22. Give an example of the equivalence.

Ans. Consider a space ship on the surface of the earth, which is an inertial frame. When a body projected with in the space ship falls with acceleration due to gravity 'g'. If the space ship is out of the earth's gravitational force of attraction and if it is accelerating away from the earth with acceleration equal to ' g' . This is non inertial frame. Now, if any object is projected up within the ship, it falls with an acceleration 'g'.

EXERCISE 1

1. If the acceleration due to gravity on earth is 9.81 m/s^2 and the radius of the earth is 6370km. Find the mass of the earth? (G = 6.67×10^{-11} Nm^2 / kg^2)

Sol: g = 9.81 m/s^2, R = 6370 km = 6.370×10^6 m; G = 6.67×10^{-11} Nm^2 / kg^2

$$g = \frac{GM}{R^2} \Rightarrow M = \frac{gR^2}{G}$$

$$\text{Mass of the earth (M)} = \frac{9.81\left(6.37\times10^6\right)^2}{6.67\times10^{-11}} = 5.97\times10^{24}\,kg$$

2. If g on the surface of the earth is 9.8 m/s^2, find its value at a height of 6400km. (Radius of the earth = 6400 km)

Sol: h = 6400 km R = 6400 km and g = 9.8 m/s^2

$$\text{But, } g_h = g\left(\frac{R}{R+h}\right)^2 \Rightarrow g_h = 9.8\left(\frac{6400}{6400+6400}\right)^2 = 2.45\,m/s^2$$

3. Find the value of g at a height of 100 km from the surface of the earth. (Radius of the earth = 6400 km, g on the surface of the earth = 9.8 m/s^2)

Sol h =100km; R = 6400 km; and g = 9.8 m/s^2.

$$\text{For smaller heights, } g_h = g\left(1-\frac{2h}{R}\right) = 9.8\left(1-\frac{2\times100}{6400}\right) = 9.8\left(1-\frac{1}{32}\right) = 9.494\,m/s^2$$

4. If g on the surface of the earth is 9.8 m/s^2, find its value at a depth of 3200 km(Radius of the earth = 6400km)

Sol. g = 9.8 m/s^2 ; d = 3200 km ; R = 6400 km

$$g_d = g\left(1-\frac{d}{R}\right) = 9.8\left(1-\frac{3200}{6400}\right) = 4.9\ m/s^2$$

5. How much faster than its normal rate should the earth rotate about it axis so that the weight of the body at the equator becomes zero. (Radius of the earth = 6.4 ×10^6 m; g =9.8 m/s^2)

Sol The acceleration due to gravity at a latitude angle ϕ is given by

$$g_\phi = g - R\omega^2\cos^2\phi$$

At equator $\phi = 0$, $g_0 = g - R\omega^2\cos^2 0$

$$g_0 = g - R\omega^2 \quad \text{.....(i)}$$

Let ω changes to ω^1 so that apparent weight of the body at equator is zero i.e., $g_0 = 0$

$$\therefore 0 = g - R\omega'^2 \quad \text{Or } \omega' = \sqrt{\frac{g}{R}}$$

$$\therefore \frac{\omega'}{\omega} = \sqrt{\frac{g}{R}} \Big/ \omega$$

where ω is the present angular speed of rotation of the earth

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24X\,60X\,60}\,rad/s$$

$$\therefore \frac{\omega'}{\omega} = \left(\sqrt{\frac{9.8}{6.4X10^6}}\right)24X\,60X\,\frac{60}{2\pi} = 17 \Rightarrow \omega' = 17\omega$$

6. Calculate the height at which the value of acceleration due to gravity becomes 50% of that at the surface of the earth. (Radius of the earth = 6400 km).

Sol. g at height, $g_h = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2}$

$g_h = \dfrac{50}{100} g = 0.5g$

$0.5g = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2} \qquad \Rightarrow \left(1+\dfrac{h}{R}\right)^2 = \dfrac{g}{0.5g} = 2$

$h = 0.414 \quad R = 0.414 \times 6400 = 2650 \quad km.$

7. Find the depth at which the value of g becomes 25% of that at the surface of the earth. (Radius of the earth = 6400 km).

Sol. $g \quad at \quad depth, \quad g_d = g\left(1-\dfrac{d}{R}\right)$

But, $g_d = \dfrac{25}{100} g = 0.25g$

$0.25g = g\left(1-\dfrac{d}{R}\right) \quad \Rightarrow 1-\dfrac{d}{R} = \dfrac{0.25g}{g}$

$d = 0.75R = 0.75 \times 6400 = 4800 km.$

8. What is the time period of rotation of the earth around its axis so that the objects at the equator become weightless? (g=9.8 m/s^2, Radius of earth = 6400 km.)

Sol. g at the equator is, $g_0 = g - R\omega^2 = g$

If bodies are to become weightless at the equator,

$0 = g - R\omega^2 \Rightarrow \omega = \sqrt{\dfrac{g}{R}}.$

Time period of rotation, $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{R}{g}}$

$R = 6400 \times 10^3 m; \quad g = 9.8 \quad m/s^2$

$T = 2\pi \sqrt{\dfrac{6.4x10^6}{9.8}} = 5078 \quad s = 84 \text{minute} 38 s.$

EXERCISE 2

1. Calculate the value of acceleration due to gravity on the earth at a place having latitude of 30^0 $\left(g = 9.8 m/s^2\right)$.

Ans. Latitude angle $\phi = 30^0$, Radius of the earth R=6400km,

Angular velocity of earth $\omega = \frac{2\pi}{T} = \frac{2\pi}{24\times 60\times 60} rad/s$.

Acceleration due to gravity at latitude ϕ is given by

$$g_\phi = g - R\omega^2 Cos^2\phi = 9.8 - 6400 \times 10^3 \times \left(\frac{\pi}{12\times 3600}\right)^2 \times \frac{3}{4}$$

$$= 9.8 - 0.076 = 9.72 m/s^2$$

2. Find the angular velocity of the earth with which it has to rotate so that the acceleration due to gravity on 60° latitude becomes zero.

Ans. Given data

Latitude angle $\phi = 60^0$, Radius of the earth R=6400km,

$g_\phi = 0 \quad m/s^2$

$g_\phi = g - R\omega^2 Cos^2\phi$

$\therefore 0 = g - R\omega^2 Cos^2 60^0$

$$\therefore \omega^2 = \frac{g}{R\cos^2 60^0} = \frac{10}{6400\times 10^3 \times \frac{1}{4}} = \frac{10}{16\times 10^5}$$

$\therefore$ Angular velocity of the earth $\omega = 0.25\times 10^{-2} = 2.5\times 10^{-3} rad/s$

3. At a certain height above the earth's surface, the gravitational acceleration is 4% of its value at the surface of the earth. Determine the height.

Ans. gravitational acceleration at height 'h' is $g_h = 4\% \quad g$

$$g_h = g\left(1 - \frac{2h}{R}\right) \Rightarrow 4 = 100\left(1 - \frac{2h}{R}\right) \Rightarrow h = 4R = 4[6400] = 25600\text{km}$$

4. At a certain height, above the earth' s surface the gravitational attraction is 90% of its value at the surface, Determine the height of the point.

Ans. Gravitational attraction at height 'h' is 90% = 0.9 of Gravitational attraction on surface

i.e. $\frac{GMm}{(R+h)^2} = 0.9 \quad \frac{GMm}{R^2} \qquad \Rightarrow h = 346 km$

5. Find the altitude at which the acceleration due to gravity is 25% of that at the surface of the earth.

Ans. Given that $g_h = 25\% g$ and R = 6400 km

$$g = g\left[\frac{R}{R+h}\right]^2 \Rightarrow 25 = 100\left[\frac{R}{R+h}\right]^2 \Rightarrow h = R = 6400 km$$

6. An astronaut orbiting in a spaceship round the earth has centripetal acceleration $4.378 m/s^2$. Find the height of the spaceship.

Ans. Centripetal acceleration $a_c = 4.378 m/s^2$

but $a_c = \frac{GM}{(R+h)^2} = g\left[\frac{R}{R+h}\right]^2 \Rightarrow 4.378 = 9.8\left[\frac{R}{R+h}\right]^2 \Rightarrow h = 3200km$

7. Find the acceleration due to gravity in a mine of depth 640m, if the value at the surface of the earth is $9.8 m/s^2$.

Ans. $d = 640m$; $g = 9.8 m/s^2$; $R = 6400km$

Acceleration due to gravity at depth'd' is ,

$g_d = g\left(1-\frac{d}{R}\right) = 9.8 \quad \left(1-\frac{640}{6400\times10^3}\right) = 9.799 ms^{-2}$

8. Find the acceleration due to gravity at a depth of 1600km inside the earth.

Ans. $d = 1600m$, $g = 9.8 m/s^2$, $R = 6400km$

Acceleration due to gravity at depth'd' is ,

$g_d = g\left(1-\frac{d}{R}\right) = 9.8 \left[1-\frac{1600}{6400}\right] = 7.35 \ m/s^2$

9. What is the acceleration due to gravity at a point on the surface of the earth whose latitude is 45º. (g at poles = 9.823m/s^2).

Ans. $\phi = 45^0$;R = 6400 km; $\omega = \frac{2\pi}{T} = \frac{2\pi}{86400} \ rad/s$

$g_\phi = g - R\omega^2 \cos^2\phi = 9.823 - 64\times10^5 \times \frac{4\pi^2}{(86400)^2}\cos^2 45^0 = 9.806 \ m/s^2$

ASSESS YOURSELF

1. Cavendish is credited with weighing the earth. How did he do that?

Ans. Cavendish by determining the value of G and using the equation $g = \frac{GM}{R^2}$ determined the mass M of the earth.

2. What is the value of acceleration due to gravity and the weight of the body at the centre of the earth?

Ans. $g_d = g\left[1-\frac{d}{R}\right]$.Both acceleration due to gravity and the weight of the body at the centre of the earth are zero.

3. How does the gravity change inside the mines?

Ans. $g_d = g\left[1-\frac{d}{R}\right]$.Decreases

4. Consider two points. One above the surface of the earth and other inside the surface of the earth at the same distance of 1km. Where is 'g' greater?

Ans. Above the surface of the earth.

5. If the centre suddenly shrinks, keeping its mass same, does 'g' change at a point which is at a height from its surface before shrinking?

Ans. No.

6. According to Newton's third law of motion, if the earth pulls the stone, stone must also pull the earth. Then why doesn't the earth fall towards the stone?

Ans. Stone also pulls the earth with the same force as the earth pulls the stone, but due to the large mass of the earth its acceleration towards the stone is negligible and we just can't notice it.

7. Which type of frame of reference does a bus starting from rest represent?

Ans. Non inertial frame of reference

8. What is the apparent weight of a body in a freely falling lift?

Ans. Zero

9. Which mass gravitational or inertial does the mass of celestial bodies refer to?

Ans. Inertial mass

10. Does principle of equivalence applicable to electromagnetic phenomena?

Ans. Yes