GRAVITATION
1. BASIC FORCES IN NATURE

POINTS TO REMEMBER

1. Basing on the nature and relative strength the basic forces in nature are classified into four categories. They are 1) Gravitational force 2) Electromagnetic force 3) Strong Nuclear force 4) Weak nuclear force.

BASIC FORCE	RELATIVE STRENGTH	RANGE
Gravitational force	1	Long Range, Infinite
Weak nuclear force	10^{31}	Extremely Short Range <<1fm
Electromagnetic force	10^{36}	Long Range, Infinite
Strong Nuclear force	10^{38}	Short Range, 1fm

2. The gravitational force is responsible for the attraction between particles of various bodies having same mass.
3. The electromagnetic force is responsible for holding the atoms together that make up the molecules.
4. The strong nuclear force is responsible for holding the protons and neutrons in the nucleus.
5. The weak force is responsible for a type of radioactive decay known as beta decay.
6. If m_1 and m_2 are the masses of two bodies which are separated by a distance r, then the gravitational force of attraction F is given by

$$F = G \cdot \frac{m_1 m_2}{r^2}$$

In vector form, $$\vec{F} = G \cdot \frac{m_1 m_2}{r^3} \vec{r}$$

Where G is called universal gravitational constant and is equal to 6.67×10^{-11} Nm^2Kg^{-2}.

7. Universal gravitational constant does not depend upon the medium between the particles (or) any other factors. It is constant every where in the universe.
8. The uniform acceleration produced in a freely falling body due to the gravitational force of a planet is known as acceleration due to gravity (g)
9. Gravitational force exists even when the particles have no physical contact. This is called action at a distance or force at a distance.
10. To overcome the difficulties in the gravitational law, field concept is introduced. The space around a body in which its influence is there is called gravitational field.
11. According to Newton, gravitational field is the distortion (disturbance) of space due to the presence of matter. Gravitational field, just like photons, is considered as energy packets called graviton. The rest mass energy of gravitation is zero and it travels with velocity of light. Graviton is a mass less BOSON with spin 2 units.
12. Gravitational field strength on the surface of the earth is equal to the acceleration due to gravity. Though both quantities are equal in magnitude having the same directions, they should be treated as two separate physical quantities.

SHORT ANSWER ANSWERS

1. State the different types of basic forces in nature?

Ans. The basic forces in nature can be classified into four types depending upon their nature and relative strength. They are

1) Gravitational force 2) Electromagnetic force 3) Strong Nuclear force 4) Weak Nuclear force.

Gravitational Forces:

a) These are long range attractive forces.

b) These are weak forces and are appreciable only when the interacting objects are massive.

c) These are independent of presence of other bodies and the medium between the bodies.

d) These are conservative forces.

e) These form action reaction pairs.

f) Gravitational force exists even when there is no physical contact between them.

Electromagnetic Force:

a)These are long range forces

b) These have intermediate strength between electric and magnetic forces.

C) According to quantum field theory electromagnetic force between two charges is communicated by exchange of Photons.

Strong Nuclear Forces:

a) These are responsible for holding protons and neutrons in the nucleus.

b) These are between proton – proton, proton-neutron and neutron – neutron

c) These are short range attractive forces.

d) If the distance between the nucleons is less thano.4 fm, these become repulsive.

e) These are communicated by exchange of π-mesons.

Weak Nuclear Forces:

a) These are responsible for β-deay and similar process involving fundamental particles.

b) These act between leptons and hadrons.

C) These are short range forces communicated by weak bosons.

2. Among the following, pick out the basic forces involved.

a) Force of friction b) Forces between two neutrons in side the nucleus c) β-decay d) Muscular strength e) Moon revolving round the earth f) Surface tension g) Tension on a string h) Tides

Ans. a) Force of friction --- Electromagnetic Force

b) Forces between two neutrons in side the nucleus -- Nuclear Forces

c) β-decay --- Nuclear Forces

d) Muscular strength --- Electromagnetic Force

e) Moon revolving round the earth -- Gravitational force

f) Surface tension --- Electromagnetic Force

g) Tension on a string --- Electromagnetic Force

h) Tides -- Gravitational force

3. Discuss the nature of gravity.

Ans. Every body in the universe attracts every other body. This force of attraction between the bodies having a certain mass is called gravitational force or gravity.

1. Gravitational force between any two bodies depends upon the product of the masses and the distance between them.
2. Gravitational force does not depend on the nature of attracting bodies and is always attractive.
3. Gravitational force between a pair of bodies is a central force. (i.e., it acts along the line joining their centers of masses)
4. Gravitational force is a long range force.
5. Gravitational force is a weakest of all four basic forces.
6. Gravitational force is conservative force. (i.e. The work done by the gravitational force does not depends upon the path traversed)

VERY SHORT ANSWER ANSWERS

1. Which is the weakest force of all the basic forces in nature?

Ans. Gravitational Force is the weakest force.

2. Which is the strongest force of all the basic forces in nature?

Ans. Nuclear force is the strongest force of all the basic forces in nature.

3. Among the following, which of the forces are long range?

(i) Force between two nucleons, (ii) Force between two electric charges, (iii) Force between the earth and the moon.

Ans. The gravitational force between earth and moon is a long range force

4. State the units and dimensions of universal gravitational constant.

Ans. Dimensional formula of universal gravitational constant is $M^{-1}L^{3}T^{-2}$ and its unit is $Nm^{2}Kg^{-2}$.

5. State the limitation of Newton's third law briefly.

Ans. Newton's laws do not apply when

a) Velocities of objects are comparable with velocity of light and
b) Gravitational fields are very strong.

6. Define gravitational field.

Ans. The space around a mass in which its gravitational force of attraction is felt is known as gravitational field.

7. Define gravitational field strength. Is it a scalar or vector?

Ans. Gravitational field strength at a point in a gravitational field is defined as the gravitational force per unit mass placed at that point and it is a vector quantity.

EXERCISE 1

1. Calculate the ratio of electromagnetic and gravitational forces between two electrons. (Charge of the electron = e = 1.6×10^{-19}C, mass of the electron = m = 9.1×10^{-31}kg, permittivity of free space ε_0 = 9 × 10^9 $\frac{N-m^2}{C^2}$, universal gravitational constant G = 6.67×10^{-11} $\frac{N-m^2}{kg^2}$)

Sol: Electric force $F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$ and Gravitational force $F_g = \frac{Gm^2}{r^2}$

$$\frac{F_e}{F_g} = \frac{e^2}{4\pi\varepsilon_0 G(m)^2} = \frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{6.67\times10^{-11}\times\left(9.1\times10^{-31}\right)^2} = 4.17\times10^{42}$$

2. The gravitational force between two identical objects at a separation of 1m is 0.0667 mgwt. Find the masses of the objects (G = 6.67×10^{-11} Nm^2/kg^2 and g = 10 m/s^2)

Sol: $m_1 = m_2 = m$; r = 1 m; F = 0.0667 mg wt = 0.0667 × 10^{-6} × 10 N = 6.67×10^{-7} N

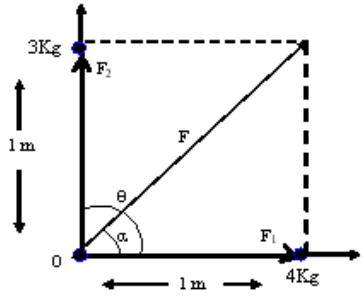
$$F = \frac{Gm_1m_2}{r^2}$$

$$6.67\times10^{-7} = \frac{6.67\times10^{-11}m^2}{(1)^2} \Rightarrow \text{m} = 10^2 = 100\text{kg}.$$

3. Two objects of masses 4 kg and 3 kg are placed along X and Y axes at respectively at a distance of 1m from the origin. An object of mass 1 kg is kept at origin of the co-ordinate system. Calculate the resultant gravitational force of attraction on the object at the origin.

Sol. Force on 1 kg mass at the origin due to 4 kg mass is given by

$$F_X = \frac{Gm_1m_2}{r^2} = \frac{G\times4\times1}{(1)^2} = 4\text{G}$$ (along positive X - axis.)



Force on 1 kg mass at the origin due to 3 kg mass is given by

$F_Y = \frac{G\times3\times1}{1^2}$ =3G directed along positive Y - axis.

From the parallelogram law of vectors,

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{(4G)^2 + (3G)^2 + (2)(4G)(3G)\cos90^0} = \sqrt{16G^2 + 9G^2} = 5G$$

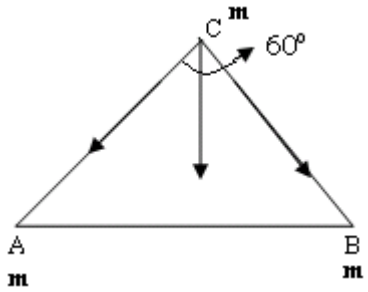
Let the resultant makes an angle α with X – axis

$$\tan\alpha = \frac{F_2}{F_1} = \frac{3}{4} = 0.75 \Rightarrow \alpha = 36^0 52'$$

4. Three particles of identical masses 'm' are kept at the vertices of an equilateral triangle of each side length 'a'. Find the gravitational force of attraction on any one of the particles.

Sol: The force of attraction on the particle C due to A is given by,

$$F_A = \frac{Gm \times m}{a^2} = \frac{Gm^2}{a^2} \quad \text{along} \quad CA$$



The force of attraction on the particle C due to B is given by,

$$F_B = \frac{Gm \times m}{a^2} = \frac{Gm^2}{a^2} \quad \text{along} \quad CB$$

Resultant of these two forces can be found by parallelogram law of vectors.

$$F = \sqrt{F_A^2 + F_B^2 \quad + 2F_A F_B \cos\theta} = \sqrt{\left(\frac{Gm^2}{a^2}\right)^2 + \left(\frac{Gm^2}{a^2}\right)^2 + 2^0\left(\frac{Gm^2}{a^2}\right) \quad \left(\frac{Gm^2}{a^2}\right)\cos 60}$$

$$= \sqrt{\left(\frac{Gm^2}{a^2}\right)^2 + \left(\frac{Gm^2}{a^2}\right)^2 + \left(\frac{Gm}{a^2}\right)^2} = \sqrt{3\left(\frac{Gm^2}{a^2}\right)^2} = \sqrt{3}\frac{Gm^2}{a^2}$$

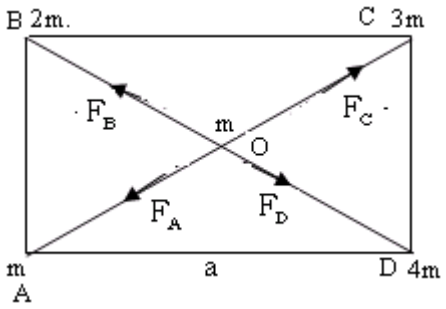
$$\text{Tan} \ \alpha = \frac{F_B \ \sin\theta}{F_A + F_B \cos\theta} = \frac{\frac{Gm^2}{a^2}\sin 60^0}{\frac{Gm^2}{a^2} + \frac{Gm^2}{a^2}\cos 60^0} = \frac{1}{\sqrt{3}} \Rightarrow \alpha = 30^0$$

i.e., along the perpendicular bisector to the opposite side.

5. Four particles of masses m, 2 m, 3m and 4m are placed at the corners of a square of side length a. Find the gravitational force on a particle of mass m placed at the centre of the square.

Sol Let the masses m, 2m, 3m, and 4m are placed at the vertices of a square ABCD respectively. At the centre 'O ' a mass m is placed. Force due to the mass at m on the mass at the centre is given by given by

$$F_A = \frac{G \times m \times m}{\left(\frac{a}{\sqrt{2}}\right)^2} = \frac{2Gm^2}{a^2} \text{along} \quad OA.$$



Force due to the mass at 3m on the mass at the centre is given by

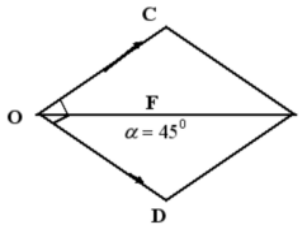
$$F_C = \frac{G \times m \times 3m}{\left(\frac{a}{\sqrt{2}}\right)^2} = \frac{6Gm^2}{a^2} \text{along} \quad OC$$

F_A and F_C are oppositely directed.

Their resultant force is given by

$$F_{AC} = F_C - F_A = \frac{4Gm^2}{a^2} \quad \text{along} \quad OC.$$

Similarly the force due to the masses 2m and 4m are



$$F_B = \frac{G \times m \times 2m}{\left(\frac{a}{\sqrt{2}}\right)^2} = \frac{4Gm^2}{a^2} \text{along} \quad OB \quad \text{and}$$

$$F_D = \frac{G \times m \times 4m}{\left(\frac{a}{\sqrt{2}}\right)^2} = \frac{8Gm^2}{a^2} \text{along} \quad OD$$

Their resultant force, $F_{DB} = F_D - F_B = \frac{4Gm^2}{a^2}$ along OD.

The resultant of F_{AC} and F_{DB} is of magnitude

$F = \sqrt{(F_{AC})^2 + (F_{DB})^2}$ As these two forces are perpendicular to each other.

$$F = 4\sqrt{2}\frac{Gm^2}{a^2}$$

$$\text{Tan}\ \alpha = \frac{F_{AC}}{F_{DB}} = 1; \quad \alpha = 45^0 \quad \text{i.e.,}$$

parallel to AD.

6. Find the point at which the gravitational force acting on any mass is zero due to the earth and the moon system. The mass of the earth is approximately 81 times the mass of the moon and the distance between the earth and the moon is 3, 85,000km (Such a point is called neutral point).

Sol: Let m_1 and m_2 be the masses of the earth and the moon separated by a distance d. Consider an object of mass m at a point P which is at a distance x from m_1.

The force due to the mass m_1 on the mass m is

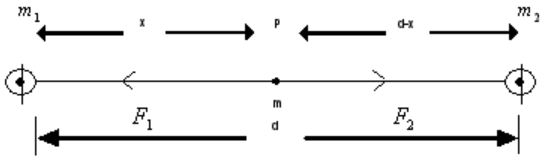
$$F_1 = \frac{Gmm_1}{X^2} \text{towards} \quad m_1$$

The force due to the mass m_2 on the mass m is

$$F_2 = \frac{Gmm_2}{(d-x)^2} towards \quad m_2$$

If the resultant force on the mass m is to be zero, F_1 must be equal to F_2 in magnitude as the two forces are oppositely directed.

$$\frac{Gmm_1}{X^2} = \frac{Gmm_2}{(d-x)^2}, \left(\frac{d-x}{x}\right)^2 = \frac{m_2}{m_1}$$



$$\frac{d-x}{x}=\sqrt{\frac{m_2}{m_1}};\ \frac{d}{x}-\frac{x}{x}=\sqrt{\frac{m_2}{m_1}};\ \frac{d}{x}-1=\sqrt{\frac{m_2}{m_1}};\ \frac{d}{x}=1+\sqrt{\frac{m_2}{m_1}}$$

$$x=\frac{d}{\sqrt{\frac{m_2}{m_1}}+1} \quad \text{where} \quad x \quad \text{is} \quad \text{from} \quad m_1.$$

Now m_1= mass of the moon =M; m_2=mass of the earth=81 M and d=3,85,000 Km

$$x=\frac{3,85,000}{\sqrt{\frac{81M}{M}}+1}=\frac{3,85,000}{9+1}=38,500 \quad km \quad from \quad the \quad moon.$$

7. Two particles of equal masses go round a circle of radius R under the action of their mutual gravitational attraction. Find the speed of each particle.

Sol. The two particles are to be diametrically opposite so that the gravitational force along the radius supplies the necessary centripetal force.

Gravitational force between the two particles = Centripetal force

$$\frac{GMm}{(2R)^2}=\frac{mV^2}{R}\Rightarrow V=\sqrt{\frac{GM}{4R}}$$

Exercise 2

1. Two spherical balls each of mass 1kg are placed 1cm apart. Find the gravitational force of attraction between them.

Ans. $m_1=m_2=1kg$ and $r=1cm=10^{-2}m$

From Newton law of gravitation,

$$F=\frac{Gm_1m_2}{r^2}=\frac{6.67\ \times10^{-11}\times1\times1}{10^{-4}}=6.67\times10^{-7}N$$

2. The mass of a ball is four times the mass of another ball. When these balls are separated by a distance of 10cm, the gravitational force between them is $6.67\times10^{-7}N$. Find the masses of the two balls.

Ans. Mass of first ball $m_1=4m$

Mass of second ball $m_2=m$

Distance between them $r=10cm=10^{-1}m$

Force between them F = $6.67\times10^{-7}N$

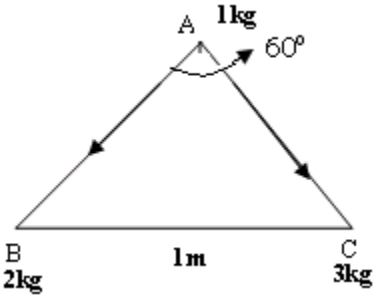
From Newton law of gravitation, $F=\frac{Gm_1m_2}{r^2}$

$$6.67\ \times\ 10^{-7}=\frac{6.67\times10^{-11}\times4m\times m}{10^{-2}}\ \Rightarrow\ \ m=5kg$$

$$\therefore m_1=20kg \quad and \quad m_2=5kg$$

3. Three spherical balls of masses 1kg, 2kg and 3kg are placed at the corners of an equilateral triangle of side 1m. Find the magnitude of the gravitational force exerted by 2kg and 3kg masses on 1kg mass.

Ans. From the figure.



The force of attraction of the 2 kg ball on 1kg ball is

$$F_1 = \frac{Gm_1m_2}{a^2} = \frac{G(1\times 2)}{1^2} = 2G, \ along \quad AB$$

Force of attraction of the 3kg ball on 1kg ball is

$$F_2 = \frac{Gm_1m_3}{a^2} = \frac{G(1\times 3)}{1^2} = 3G, \ along \quad AC$$

And Here $\theta = 60^0$

Now resultant force on 1kg is given by , $F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2Cos\theta}$

$$F = \sqrt{4G^2 + 9G^2 + 2(2G)(3G)Cos60^0} \Rightarrow F = \sqrt{19}G$$

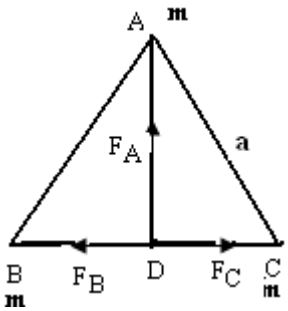
4. The force between two objects decreases by 36%. When the distance between them is increased by 4m, find the original distance between them.

Ans. From Newton law of universal gravitation, $F\alpha \ \frac{1}{r^2}$ or $\frac{F_1}{F_2} = \left(\frac{r_2}{r_1}\right)^2$

$$\frac{100}{64} = \left(\frac{r+4}{r}\right)^2 \Rightarrow \left(\frac{r+4}{r}\right) = \frac{10}{8} \Rightarrow \quad r = 16 \text{ m}$$

5. Three equal masses m are placed at the three vertices of an equilateral triangle of side'a' .Find the gravitational force exerted by this system on another particle of mass m is placed a) at the mid-point of a side and b) at the centroid of the triangle.

Ans. (a) The force of attraction of the particle at A on the particle at D, is



$$F_A = \frac{Gm^2}{\left(\frac{\sqrt{3}}{2}a\right)^2} = \frac{4}{3}\frac{Gm^2}{a^2} \text{ along DA}$$

The force of attraction of the particle at B on the particle at D, is

$$F_B = \frac{Gm^2}{(a/2)^2} = 4\frac{Gm^2}{a^2} \quad along \quad DB$$

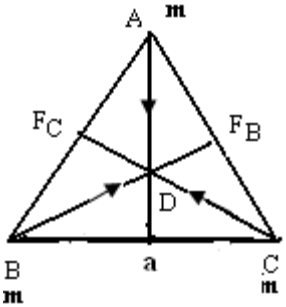
$$F_C = \frac{Gm^2}{(a/2)^2} = 4\frac{Gm^2}{a^2} \quad along \quad DC$$

Since F_B and F_C are equal and opposite they cancel each other.

Hence the resultant force acting on particle at mid point (D) of a side is

$$F_R = F_A = \frac{4}{3}\ \frac{GM^2}{a^2}$$

(b) The forces F_A, F_B and F_C acting on particle at the centroid 'D' of triangle are equal i.e., $F_A = F_B = F_C = F$



The angle between $F_B \& F_C$ is $\theta = 120^0$.

By using parallelogram law of vectors ,the resultant of $F_B \& F_C$ is

$$F^1 = \sqrt{F_B^2 + F_C^2 + 2F_B F_C \cos\ \theta} = \sqrt{F^2 + F^2 + 2F^2 \cos 120^0}$$

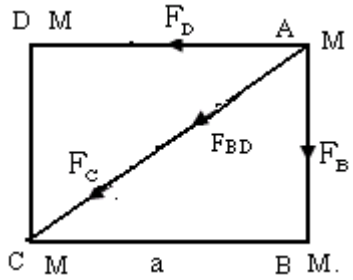
$$= \sqrt{2F^2 + 2F^2(-1/2)} = F \quad along \quad DA$$

Since this resultant is equal and opposite to F_A, the net force acting on particle at D is

$$F_R = F^1 - F_A = F - F = 0 \quad N$$

6. Four identical masses M are kept at the corners of a square of side ' a'. Find the gravitational force exerted on one of the mass by the other masses.

Ans. Let four equal masses M each are placed at the be vertices of a square ABCD of side 'a' as shown in figure. Let the forces acting on the mass at A by other three are $F_B, F_C \& F_D$ respectively.



Now, $F_B = \dfrac{GM^2}{a^2}$ along AB

$F_D = \dfrac{GM^2}{a^2}$, along AD

$F_C = \dfrac{GM^2}{\left(\sqrt{2}a\right)^2}$, along AC

The resultant $F_B \& F_D$ *is*

$$F_{BD} = \sqrt{F_B^2 + F_D^2 + 2F_B F_D \cos 90^0}$$

$$F_{BD} = \sqrt{\left(\frac{GM^2}{a^2}\right)^2 + \left(\frac{GM^2}{a^2}\right)^2} = \sqrt{2}\frac{GM^2}{a^2}$$

Also, $Tan\alpha = \dfrac{F_B \sin\ \theta}{F_D\ F_B \cos\theta} - \dfrac{F_B}{F_D} = 1 \qquad \Rightarrow \alpha = 45^0$ This is in the direction of F_C

Net resultant force $F_R = F_{BD} + F_C$

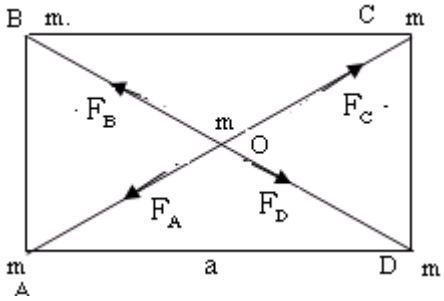
$$F_R = \sqrt{2}\frac{GM^2}{a^2} + \frac{GM^2}{2a^2} = \left(\sqrt{2} + \frac{1}{2}\right)\frac{GM^2}{a^2}$$

7. Four identical masses m are kept at the corners of a square of side a. Find the gravitational force exerted on a point mass m kept at the centre of the square.

Ans. Let four identical masses 'm' each are placed at the vertices of a square ABCD respectively. Let another mass 'm' is placed a t the centre 'O' .

Force on the mass at the centre 'O' due to mass at A is

$$F_A = \frac{Gmm}{\left(\frac{a}{\sqrt{2}}\right)^2} = \frac{2Gm^2}{a^2}$$ along OA.



Force on the mass at the centre due to the mass at C is

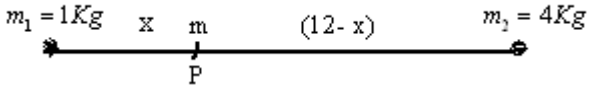
$$F_C = \frac{Gmm}{\left(a/\sqrt{2}\right)^2} = \frac{2GM^2}{a^2}$$ along OC.

The forces F_A and F_C are equal and opposite and hence their resultant force $F_A - F_C = 0$.

Similarly the forces due the masses at the opposite corners B and D are equal and opposite and their resultant force is also equal to zero. Hence the net resultant force acting on mass m at centre is zero.

8. Two spherical balls of mass 1kg and 4kg are separated by a distance of 12cm. Find the distance from 1kg at which the gravitational force on any mass become zero.

Ans. $m_1 = 1Kg$; $m_2 = 4Kg$; d = 12cm Consider an object of mass m at a point P which is at a distance 'x' from 1 kg



The force due to mass m_1 on the mass 'm' is $F_1 = \frac{Gmm_1}{x^2}$ towards m_1

The force due to mass m_2 on the mass ' m' is $F_2 = \frac{Gmm_2}{(d-x)^2}$ towards m_2

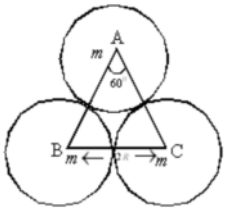
If resultant force on the mass m is to be zero, $F_1 = F_2$

$$\frac{Gmm_1}{x^2} = \frac{Gmm_2}{(d-x)^2} \Rightarrow \frac{1}{x^2} = \frac{4}{(12-x)^2} \Rightarrow x = 4cm$$

9. Three uniform sphere each of mass m and radius R are kept in such away that each touches the other two. Find the magnitude of the gravitational force on any of the spheres due to the other two.

Ans. The system is similar to three identical point mass placed at three corners of an equilateral triangle of side 2R

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Gravitational force acting on sphere A due to B is

$$F_B = \frac{Gm^2}{4R^2}$$ along AB

Similarly gravitational force acting on sphere A due to C is

$$F_C = \frac{Gm^2}{4R^2}$$ along AC

∴ Resultant force acting on sphere A is

$$F_R = \sqrt{F_B^2 + F_C^2 + 2F_BF_C\cos\theta} = \sqrt{\left(\frac{Gm^2}{4R^2}\right)^2 + \left(\frac{Gm^2}{4R^2}\right)^2 + 2\left(\frac{Gm^2}{4R^2}\right)^2 \frac{1}{2}} = \frac{\sqrt{3}}{4} \frac{Gm^2}{R^2}$$

ASSESS YOURSELF

1. Mention the type of the basic forces involved in the following situations?
(i) Earth revolving around the sun.
(ii) Electron revolving around the nucleus
(iii) Neutron changing into a proton emitting electron and antineutrino
(iv) The attractive forces between two protons inside the nucleus
(v) The repulsive force between two protons out side the nucleus

Ans. i) Gravitational force, ii) Electromagnetic force iii) Weak nuclear force, iv) Strong nuclear force and v) Electro magnetic force.

2. Arrange, the basic forces in the ascending order of their strengths and mention which of the forces ,weak nuclear and gravitational is weaker?

Ans. Gravitational force < weak nuclear force <electromagnetic force < strong nuclear force. Gravitational force is weaker than weak nuclear force.

3. Why can' t two persons of say 100kg standing 1m apart not experience any gravitational attraction between them?

Ans. According to Newton's law of gravitation the force of attraction between the two persons is 10^{-7} ,which is very small. Hence two persons of say 100kg standing 1m apart not experience any gravitational attraction.

4. What causes the tides in the oceans?

Ans. The gravitational force of attraction between the moon and the earth causes the tides.

5. If heavier bodies are attracted more strongly by the earth, why don't they fall faster than the lighter bodies?

Ans. Since the acceleration due to gravity (g) is same for both heavier and lighter bodies, both fall with the same rate.