ROTATORY MOTION

2. MOMENT OF INERTIA

POINTS TO REMBER

1. **Rigid body**: PA body which does not undergo any change in its shape and volume by the application of force, however strong, is called a rigid body. There are no perfect rigid bodies in nature.

2. **Moment of inertia:** The moment of inertia (I) of a rigid body about an axis is defined as the sum of product of masses of all particles of the body and the squares of their perpendicular distances from the axis of rotation.

   $I = \sum mr^2$ where m is the mass of each particle and r is the relative distance of the particle from the axis of rotation. **Unit:** kg –m^2 **Dimensions** : ML^2T^0

3. Moment of inertia of a rotating body depends on the relative distances of the particles from the axis of rotation.

4. **Radius of gyration**: Radius of gyration of a rigid body about an axis of rotation is the effective distance of a point where the total mass of the body is supposed to be concentrated from the axis of rotation, so that its moment of inertia would be same with the actual distribution of mass.

   $$K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + ... + r_n^2}{n}}$$

   Unit: m **Dimensions** : $M^0L^1T^0$

5. Radius of gyration depends on (i) Position of axis of rotation and (ii) Distribution of mass about the axis of rotation.

6. **Parallel axes theorem:** The moment of inertia of a rigid body about any axis is equal to the sum of moment of inertia about a parallel axis passing through its centre of mass and the product of the mass of the body and the square of the distance between the parallel axes. $I = I_G + mr^2$

7. **Perpendicular axes theorem:** The moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the plane lamina about two axes perpendicular to each other in its own plane and interesting each other at that point, where the axis perpendicular to the plane passes $I_z = I_x + I_y$

Body	Axis of rotation	Moment of inertia
1. Disc of radius R	1) through its centre and perpendicular	$\frac{MR^2}{2}$
	2) about its diameter	$\frac{MR^2}{4}$
	3) about a tangent to its own plane	$\frac{5MR^2}{4}$
	4) tangent perpendicular to the plane of	$\frac{3MR^2}{2}$
2. Annular ring or disc of outer and inner radii R and r	1) through its centre and perpendicular	$\frac{M(R^2+r^2)}{2}$
	2) about the diameter	$\frac{M(R^2+r^2)}{4}$
	3) about a tangent to its own plane	$\frac{M(5R^2+r^2)}{4}$
3. Solid cylinder of length L and	1) axis of cylinder	$\frac{MR^2}{2}$
	2) through its centre and perpendicular to	$M\left(\frac{L^2}{12}+\frac{R^2}{4}\right)$
	3) diameter of the face	$M\left(\frac{L^2}{3}+\frac{R^2}{4}\right)$
4. Thin rod of uniform length L	1) through its centre and perpendicular	$\frac{ML^2}{12}$
	2) through one end and perpendicular	$\frac{ML^2}{3}$
5. Solid sphere of radius R	1) about a diameter	$\frac{2}{5}MR^2$
	2) about a tangent	$\frac{7}{5}MR^2$
6. Hollow sphere of radius R	about a diameter	$\frac{2}{3}MR^2$
7. Thin circular ring of radius R	1) perpendicular to its plane and passing through its centre.	MR^2
	2) about its diameter	$\frac{MR^2}{2}$
8. Hollow cylinder of radius R	about axis of the cylinder	MR^2
9. Rectangular lamina of length l and breadth b	1) through its centre and perpendicular to its plane	$M\left(\frac{l^2}{12}+\frac{b^2}{12}\right)$
	2) through its centre and parallel to breadth along its own plane	$\frac{Ml^2}{12}$
	3) through its centre and parallel to length along its own plane	$\frac{Mb^2}{12}$
	4) edge of the length in the plane of the lamina	$\frac{Mb^2}{3}$
	5) edge of the breadth in the plane of the lamina	$\frac{Ml^2}{3}$
	6) perpendicular to the plane of the lamina and passing through the mid point of the edge of the length	$M\left(\frac{l^2}{12}+\frac{b^2}{3}\right)$
	7) perpendicular to the plane of the lamina and passing through the mid point of the edge of the breadth	$M\left(\frac{l^2}{3}+\frac{b^2}{12}\right)$
10. Plane elliptical lamina with the values of axes 2a and 2b	Perpendicular to the plane of the lamina and passing through its centre	$\frac{M}{4}(a^2+b^2)$

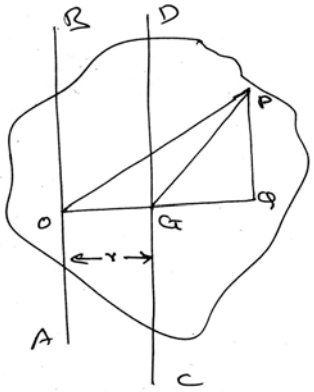
Short Answer Questions :

1. State and prove parallel axes theorem.(2010-June ,2010-March, 2007-May2007-March 2005-March,)

A. **Statement:** The moment of inertia of a rigid body about any axis is equal to the sum of moment of inertia about a parallel axis passing through its centre of mass and the product of the mass of the body and the square of the distance between the parallel axes.

$$I = I_G + mr^2$$

Proof: Consider a rigid body of mass M. Let I and I_G be the moment of inertia of the body about two parallel axes AB and CD. The axis CD passes through the centre of mass of the body. Let r be the perpendicular distance between the axes AB and CD. Consider a particle P of mass m. Extend OG and draw a perpendicular PQ on to OQ produced.



Moment of inertia of the body about AB

$$I = \sum m(OP)^2$$

Moment of inertia of the body about CD

$$I_G = \sum m(GP^2)$$

In the triangle OPQ,

$$OP^2 = OQ^2 + PQ^2 = (OG + GQ)^2 + PQ^2$$
$$= OG^2 + GQ^2 + 2.OG.GQ + PQ^2$$
$$= OG^2 + GP^2 + 2.OG.GQ \qquad (\because GQ^2 + PQ^2 = GP^2)$$

$$\therefore I = \sum m(OP)^2$$
$$= \sum m\left[OG^2 + GP^2 + 2.OG.GQ\right]$$
$$= \sum mr^2 + \sum mGP^2 + \sum m.2.OG.GQ$$

$$\therefore I = I_G + Mr^2 \qquad (\because \sum m = M)$$

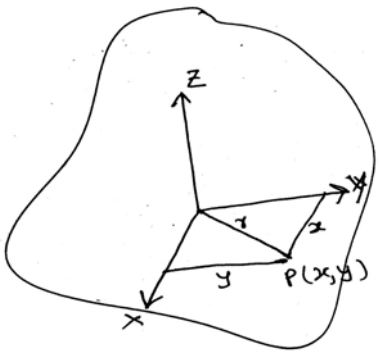
($\sum m.GQ = 0$ This is the sum of the moments of all masses about CM.)

2. State and prove perpendicular axes theorem. (2009-March,2008-May.2006-March,2005-June)

A. **Statement:** The moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the plane lamina about two axes perpendicular to each other in its own plane and interesting each other at that point, where the axis perpendicular to the plane passes

$$I_z = I_x + I_y$$

Proof: Let the moments of inertia of the plane lamina about X,Y and Z axes be I_X, I_Y and I_Z, the point of intersection being 'O'. Let X and Y axis lie in the plane of the lamina and Z axis is perpendicular to the lamina.



Consider a particle P of mass m in the X-Y plane whose co-ordinates are (x,y). The perpendicular distance of the particle from X and Y axis be y and x respectively.

$\therefore I_x = \sum my^2$

$I_y = \sum mx^2$

But $r^2 = x^2 + y^2$

$\therefore I_z = \sum mr^2 = \sum m\left(x^2 + y^2\right)$

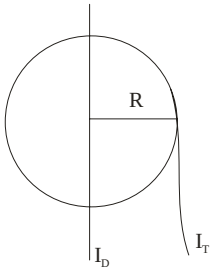
$\therefore I_z = I_x + I_y$

3. Find the moment of inertia of a circular ring about a tangent (i) in its plane and (ii) in a perpendicular plane using its formula for transverse axis.

A. The moment of inertia of a circular ring about its natural axis is given by $I = MR^2$ Where M is the mass and R is the radius of ring.

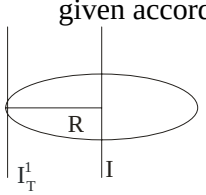
i) Moment of inertia of the ring about any diameter is given by

$$I_D = \frac{MR^2}{2}$$



According to parallel axis theorem, the moment of inertia about the tangent in its plane is given by $I_T = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$

ii) Moment of inertia about the tangential axis perpendicular to the plane of the ring is given according to parallel axis theorem.

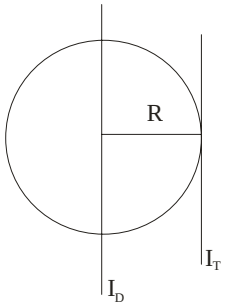


$I_T^1 = I + MR^2 = MR^2 + MR^2$

$I_T^1 = 2MR^2$

4. Find the moment of inertia of a circular disc about a tangent (i) in its plane and (ii) in a perpendicular plane using its formula about a transverse axis.

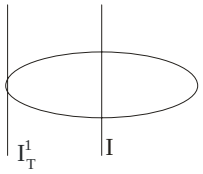
A. Moment of inertia of a circular disc about its natural axis is given by $I = \frac{MR^2}{2}$ where M is the mass and R is the radius of the disc



i) Moment of inertia of the disc about the diameter $I_D = \frac{MR^2}{4}$

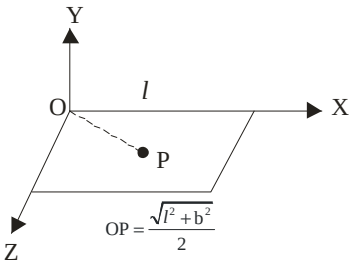
By using parallel axis theorem the moment of inertia of the disc about a tangential axis in the its plane $I_T = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2$

ii) Moment of inertia of the disc about a tangential axis perpendicular to the plane of the disc in given by $I_T^1 = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$



5. Obtain an expression for the moment of a thin rectangular lamina if the axis of rotation is perpendicular to the plane of the lamina and passing through are of its corner

A. The moment of inertia of the lamina about its natural axis is given by $I = \frac{m\left(l^2 + b^2\right)}{12}$



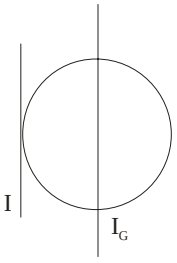
Moment of inertia of the lamina about an axis perpendicular to the plane and passing through one corner (Y-axis) is given from parallel axis theorem.

$$I' = \frac{m\left(l^2 + b^2\right)}{12} + m\left[\frac{\sqrt{l^2 + b^2}}{2}\right]^2$$

$$= \frac{m}{12}\left(l^2 + b^2\right) + \frac{m}{4}\left(l^2 + b^2\right) = \frac{m\left(l^2 + b^2\right)}{3}$$

6. Deduce an expression for the moment of inertia of a solid sphere about a tangent using parallel axis theorem.

A. Moment of inertia of a solid sphere about its natural axis $I_G = \frac{2}{5}mR^2$

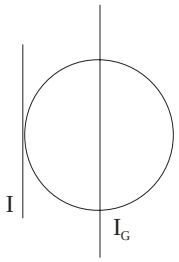


Now, according to parallel axis theorem, the moment of inertia of the sphere about the tangent $I = I_G + mr^2 = \frac{2}{5}mR^2 + mR^2 = \frac{7}{5}mR^2$

Where m is the mass and R is the radius of the sphere.

7. Deduce an expression for the moment of inertia of a hollow sphere about a tangent using parallel axis theorem.

A. Moment of inertia of a hollow sphere about it its natural axis $I_G = \frac{2}{3}mR^2$



Now according to parallel axis theorem, the moment of inertia about the tangent

$$I = I_G + mR^2 = \frac{2}{3}mR^2 + mR^2$$

$$\therefore I = \frac{5}{3}mR^2$$

Where m is the mass and R is the radius of the hollow sphere.

Very Short Answer Questions :

1. Compare moment of inertia and mass.

A. In translatory motion, the property of a body by virtue of which it opposes any state of rest or uniform motion along a straight line is called inertia..

In rotatory motion, the property of a rigid body by virtue of which it opposes any change of state of rest or of uniform rotatory motion about an axis is called rotational inertia or moment of inertia.

2. Define moment of inertia and explain its importance.

A. **Definition:** The moment of inertia (I) of a rigid body about an axis is defined as the sum of product of masses of all particles of the body and the squares of their perpendicular distances from the axis of rotation.

$I = \sum mr^2$ where m is the mass of each particle and r is the relative distance of the particle from the axis of rotation.

Importance: M**oment of inertia** of a body is the inability of a rigid body to change in its rotatory motion in the absence of an external torque.

3. "Moment of inertia changes when the axis of rotation is changed". Discuss.

A. M**oment of inertia** $(I) = \sum mr^2$ where m is the mass of each particle of a rigid body and r is the relative distance of the particle from the axis of rotation.

i.e. moment of inertia of a rotating body depends on the relative distances of the particles from the axis of rotation. Therefore it changes when the axis of rotation is changed.

4. Define radius of gyration. Mention the factors on which it depends. (2007-May)

A. **Radius of gyration (K)** of a rigid body about an axis of rotation is the effective distance of a point where the total mass of the body is supposed to be concentrated from the axis of rotation, so that its moment of inertia would be same with the actual distribution of mass.

$$K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + ... + r_n^2}{n}}$$

Radius of gyration depends on (i) Position of axis of rotation and

(ii) Distribution of mass about the axis of rotation.

5. Define moment of inertia? And write its unit. (2007-March 2005-March)

A. **Definition:** The **moment of inertia** (I) of a rigid body about an axis is defined as the sum of product of masses of all particles of the body and the squares of their perpendicular distances from the axis of rotation.

I = $\sum mr^2$ where m is the mass of each particle and r is the relative distance of the particle from the axis of rotation.

Unit: kg –m^2

6. Mention the factors on which moment of inertia depends.

A. M**oment of inertia** of a rigid body depends on

(i) Mass of body
(ii) Position of axis of rotation and
(iii) Distribution of mass about the axis of rotation

7. State the dimensional formula of i) moment of inertia, ii) radius of gyration.

A. (i) Moment of inertia ML^2T^0

(ii) Radius of gyration $M^0L^1T^0$

8. State the expressions for the moment of inertia of a circular ring and a circular disc about an axis passing through its centre and perpendicular to its plane.

A. $I_{ring} = MR^2$ *and* $I_{disc} = \frac{MR^2}{2}$ Where M and R are mass and radius of the respective bodies.

9. State the expressions for the moment of inertia of a hollow cylinder and a solid cylinder about the axis of symmetry.

A. $I_{hollow} = MR^2$ *and* $I_{solid} = \frac{MR^2}{2}$ Where M and R are mass and radius of the respective bodies.

Exercise 1

1. Two objects of masses 1 kg and 2 kg separated by a distance of 0.6 m are rotating about their center of mass. Find the moment of inertia of the system.

Sol : Let ' O' be the **center of mass of system of two particles.**

$$x_1 = \frac{m_2 d}{m_1 + m_2} = 0.6\left(\frac{2}{3}\right) = 0.4\,m \text{ and } x_2 = \frac{m_1 d}{m_1 + m_2} = 0.6\left(\frac{1}{3}\right) = 0.2\,m$$

Moment of inertia of the system about the center of mass

$$= m_1 x_1^{\,2} + m_2 x_2^{\,2} = \left(1 \times 0.4^2\right) + \left(2 \times 0.2^2\right) = 0.24\,Kgm^2$$

2. Three particles of masses 1 g, 2g and 3 g are at distances of 1 cm, 2 cm and 3 cm from the axis of rotation. Find i) the moment of inertia of the system and ii) the radius of gyration of the system.

Sol : $m_1 = 1g; m_2 = 2g; m_3 = 3g; r_1 = 1cm; r_2 = 2cm; r_3 = 3cm.$

i) The MI of the system, $I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$

$= 1 \times 1^2 + 2 \times 2^2 + 3 \times 3^2 = 1 + 8 + 27 = 36g\,cm^2.$

ii) The radius of gyration, $K = \sqrt{\frac{\mathbf{I}}{Total\ mass}} = \sqrt{\frac{36}{1+2+3}}$

3. Find the moment of inertia of a thin uniform rod about an axis perpendicular to its length and passing through a point which is at a distance of $\frac{l}{3}$ from one end. Find also radius of gyration about that axis.

Sol : i) Let a point be at a distance $\frac{l}{3}$ from one end. It is at a distance of $r = \frac{l}{6}$ from the center of the rod

Using parallel axes theorem, $I = I_0 + Mr^2 = \frac{Ml^2}{12} + M\left(\frac{l}{6}\right)^2 = \frac{Ml^2}{12}\left(1 + \frac{1}{3}\right) = \frac{Ml^2}{9}$.

ii) The radius of gyration, $K = \sqrt{\frac{I}{M}} = \sqrt{\frac{Ml^2}{9M}} = \frac{l}{3}$.

4. The radius of gyration of a body about an axis at a distance of 12 cm from its center of mass is 13 cm. Find its radius of gyration about a parallel axis through its center of mass.

Sol: Using parallel axes theorem,

$I = I_0 + Mr^2 \Rightarrow MK^2 = I_0 + Mr^2$

$\Rightarrow M(13)^2 = I_0 + M(12)^2 \Rightarrow I_0 = M\left(13^2 - 12^2\right) = \text{M (25)}.$

Its radius of gyration about a parallel axis through its center of mass,

$K = \sqrt{\frac{I_0}{M}} = \sqrt{\frac{M(25)}{M}} = \sqrt{25} = 5\,cm.$

5. Radius of gyration of a disc of mass 5 k g about a transverse axis passing through its center is 14.14 cm. Find its radius of gyration about its diameter and hence calculate its moment of inertia about its diameter.

Sol : Radius of gyration of a disc about a transverse axis passing through its center,

$K = \sqrt{\frac{I}{M}} = \sqrt{\frac{MR^2}{2M}} = \frac{R}{\sqrt{2}} = 14.14\,cm. \quad \left(\because I = MR^2/2\right)$

Radius of the disc, $R = 14.14 \times \sqrt{2} = 20\,cm.$

Radius of gyration of the disc about its diameter.

$K = \sqrt{\frac{I}{M}} = \sqrt{\frac{MR^2}{4M}} = \frac{R}{2} = \frac{20}{2} = 10\,cm$

Moment of inertia about its diameter = $\frac{MR^2}{4M} = \frac{5(0.2)^2}{4} = 5 \times 10^{-2}\,kg\,m^2$.

6. Find the ratio of the moments of inertia of two solid spheres of same mass but densities in the ratio 1: 8.

Sol: Let M_1 and M_2 be the masses of two solid spheres of densities d_1 and d_2 respectively.

But, $M_1 = M_2$

$\frac{4}{3}\pi R_1^3 \times d_1 = \frac{4}{3}\pi R_2^3 \times d_2 \Rightarrow \left(\frac{R_1}{R_2}\right)^3 = \frac{d_2}{d_1} = \frac{8}{1} \Rightarrow \frac{R_1}{R_2} = \frac{2}{1}.$

The ratio of moments of inertia of the spheres.,

$\frac{I_1}{I_2} = \frac{\frac{2}{5}M_1R_1^2}{\frac{2}{5}M_2R_2^2} = \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{2}{1}\right)^2 = \frac{4}{1}$.

7. A hoop of mass 500 g and radius 10 cm is placed on a nail. Find the moment of inertia of the hoop when it is rotated about the nail.

Sol : M = 0.5 kg ; R = 0.1 m.

When the hoop is rotated about the nail, the axis of rotation becomes the tangent perpendicular to its plane.

Moment of inertia, $I = 2MR^2 = 2\times 0.5(0.1)^2 = 0.01 kg\, m^2$.

Exercise 1

1. A particle of mass 1 g is revolving on a circle of radius 10 cm. Find its moment of inertia

A. m = 1g = 1 10^{-3} Kg; r = 10 cm = 0.1 m

I = m r^2 = 1× 10^{-3} × 0.1^2 = 10^{-5} Kg-m^2

2. Find the moment of inertia of a carbon monoxide molecule rotating about its center of mass. (Distance between carbon and oxygen atoms of the molecule = 1.12A, 1 amu=1.661 x10^{-27} kg.

A.. Let x_1 *and* x_2 be the distances of c. m. from c and o atoms respectively.

$x_1 = 1.12\left(\frac{16}{28}\right) = 0.64\overset{0}{A}$ and $x_2 = 1.12\left(\frac{12}{28}\right) = 0.48\overset{0}{A}$

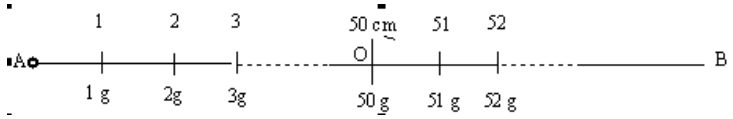
M.I of CO molecule about the c.m.

= 12 ×1.661× 10^{-27} $(0.64\times10^{-10})^2$ + 16 × 1.661×$10^{-27}$$(0.48 \times10^{-10})^2$

= 1.661×10^{-27} 10^{-20} $[(12\times0.64^2 + 16\times0.48^2)]$ = 1.429 ×10^{-46} Kg-m^2

3. Particles of masses 1 g, 2 g, 3 g,.....100 g, are kept at the marks 1 cm, 2 cm, 3 cm,...., 100 cm respectively on a meter scale. Find the moment of inertia of the system of particles about a perpendicular bisector of the meter scale.

A.



Let AB be the metre scale and c.m be the position of c.m.

$AC = \frac{1\times1+2\times2+3\times3+.....100\times100}{1+2+3+.....+100} = \frac{201}{3} = 67\text{cm}$

Let *m* be the total mass of all the particles

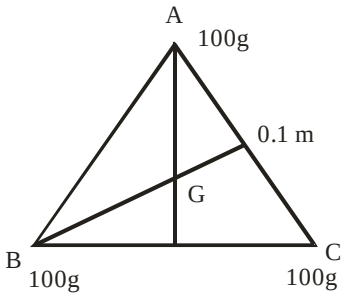
$m = 1 + 2 + 3 + \ldots\ldots\ldots\ldots + 100 = = \frac{100\times101}{2} = 50\times101\text{g}$

r = OC = 67 – 50 = 17 cm

M.I of the scale about the perpendicular bisector

= mr^2 = 50 × 101 × 17 × 17 g-cm^2 $= 0.146 kgm^2$

4. Three particles each of mass 100 g are placed at the vertices of an equilateral triangle of side length 10 cm. Find the moment of inertia of the system about an axis passing through the cetroid of the triangle and perpendicular to its plane.



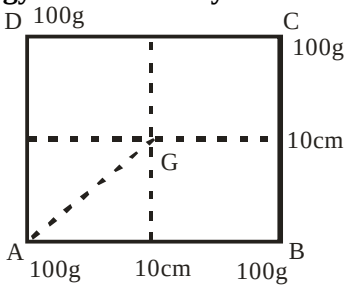
A. Let G be the position of the centroid of the triangle.

Then AG = BG = CG $= \frac{a}{\sqrt{3}} = \frac{0.1}{\sqrt{3}} m$

M.I of the system = 3 m r^2

$= 3 \times 0.1 \left(\frac{0.1}{\sqrt{3}}\right)^2 = 10^{-3} Kg - m^2$

5. Four particles each of mass 100 g are placed at the corners of a square of side 10 cm. Find the moment of inertia of the system about an axis passing through the centre of the square and perpendicular to its plane. Find also the radius of gyration of the system.



A.

AG = $5\sqrt{2}$ cm

M.I of thje system about an axis passing through G and perpendicular to the plane

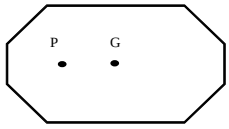
= 4 m r^2 = $4\times100\times(5\sqrt{2})^2$

M.I of the system = $4\times100\times25\times2 = 2\times10^4$ g-cm^2

But , $k^2 = \sqrt{\frac{I}{M}} = \sqrt{\frac{2\times10^4}{400}} = \sqrt{50}$ or k = $\sqrt{50} = 7.071$ cm

6. The radius of gyration of a body about an axis passing through its centre of mass is 24 cm. Calculate the radius of gyration of the body about a parallel axis passing through a point at a distance of 7 cm from its centre of mass.

A.



$I_G = m\, k_G^2 = m\times24^2$

$I_P = I_G + m(PG)^2$

$= m\times24^2 + m\times7^2 = m\times k_p^2$

Or $k_p^2 = 7^2 + 24^2 = 25^2$ or k_p = 25 cm

7. A bar magnet of length 5.0 cm and 1.2 cm is rotated about an axis passing through its center and perpendicular to its plane. Find its moment of inertia if the mass of the magnet is 200 g.

A. *l* = 5.0 cm, b = 1.2 cm, m = 200 g

$$I = \frac{m\left(l^2 + b^2\right)}{12} = \frac{200\left(5^2 + 1.2^2\right)}{12} = 200\times2.203 = 440.6\ g - cm^2 = 4.406\times10^{-5} Kg.m^2$$

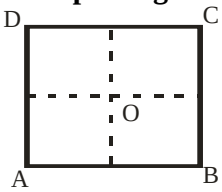
8. A thin square plate is of mass m and the length of each of its side is *l*. Find the moment of inertia of the thin square plate when it is rotating about an axis i) passing through its centre and perpendicular to its plane and ii) passing through one of its corners and perpendicular to its plane.

A. mass=m, length = breadth = *l*

$$I_0 = \frac{m\left(l^2 + l^2\right)}{12} = \frac{ml^2}{6}$$

$$I_A = I_0 + m\left(AO\right)^2 = I_0 + m\left(\frac{l}{\sqrt{2}}\right)^2 = \frac{ml^2}{6} + \frac{ml^2}{2} = \frac{2ml^2}{3}$$

9. Four thin uniform rods each of mass m and length *l* are arranged to form a square. Find the moment of inertia of the system about an axis passing through its centre and perpendicular to its plane. Find also its moment of inertia about an axis passing through one of its sides.



A. Let 'c' be the mid point of AB.

M.I of AB about an axis through its center and perpendicular to the length, $I = \frac{ml^2}{12}$

From parallel axis theorem, $I_0 = \frac{ml^2}{12} + \frac{ml^2}{4} = \frac{ml^2}{3}$

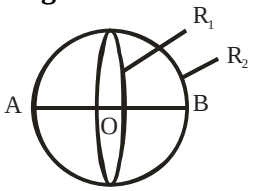
M.I of the system about 'O' , $I = 4I_0 = \frac{4ml^2}{3}$

M.I of the system about any rod AB is given by

$$I_{AB} = I_{AD} + I_{BC} + I_{DC} = \frac{ml^2}{3} + \frac{ml^2}{3} + ml^2 = \frac{5ml^2}{3}$$

10. Two circular rings of equal mass and radius are placed in such a way that their centres coincide but their planes are mutually perpendicular. Find the moment of inertia of the system when the system is rotated about the diameter of one of the rings.

A.



Let M be the mass and R be the radius of each ring. AOB is the line passing through centre O and perpendicular to the plane of ring R_1.

M.I of R_1 about AOB (I_1) = MR^2.

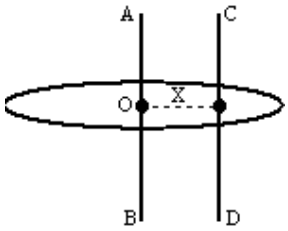
M.I of R_2 about AOB (I_2) = $\frac{MR^2}{2}$

Moment of inertia of the system = $I_1 + I_2 = MR^2 + \frac{MR^2}{2} = \frac{3}{2}MR^2$

11. The radius of gyration of a uniform disc about a line perpendicular to the disc is equal to its radius. Find the distance of the line from the centre of the disc.

A. Let m be the mass and R is the radius of the disc. AB is a line passing though the center O and perpendicular to the plane of the disc. CD is a parallel axis at a distance x from AB.

$I_{CD} = I_{AB} + Mx^2 = \frac{MR^2}{2} + Mx^2$

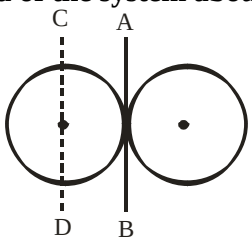


If K is the radius of gyration, K = R

$MK^2 = MR^2 = \frac{MR^2}{2} + Mx^2$

$Mx^2 = MR^2 - MR^2/2 = MR^2/2$ or $x = \frac{R}{\sqrt{2}}$

12. Two uniform circular discs, each of mass 1 kg and radius 20 cm, are kept in contact about the tangent passing through the point of contact. Find the moment of inertia of the system about the tangent passing through the point of contact.



A.

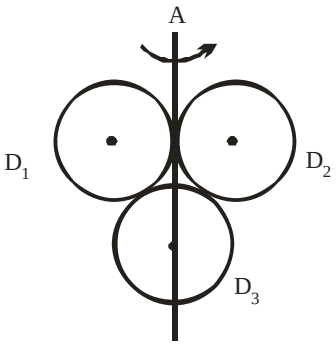
m = 1Kg, r = 20 cm = 0.2 m

MI of the disc about the diameter CD, $I_{CD} = \frac{mr^2}{4}$

$I_{AB} = I_{CD} + mr^2 = \frac{mr^2}{4} + mr^2 = \frac{5mr^2}{4}$

M.I of the system = $2 \times \frac{5mr^2}{4} = \frac{5}{2}mr^2 = \frac{5}{2} \times 1 \times \frac{2}{10} \times \frac{2}{10} = 0.1 \ \text{kg} - \text{m}^2$

13. Three uniform circular discs, each of mass 0.5 kg and radius 10 cm, are kept in contact with each other as shown in the figure. Find the moment of inertia of the system when it rotates about the axis AB.



A. m = 0.5 Kg, r = 10 cm = 0.1 m

Let I_1, I_2 and I_3 be the moments of inertia of discs D_1, D_2 and D_3 respectively, about AB

$$I_1 = I_2 = \frac{mr^2}{4} + mr^2 = \frac{5mr^2}{4}; \quad I_3 = \frac{mr^2}{4}$$

M.I of the system = $I_1 + I_2 + I_3$

$$= 2\times\frac{5mr^2}{4} + \frac{mr^2}{4} = \frac{11mr^2}{4}$$

$$= 11\times\frac{0.5\times0.1\times0.1}{4} = 1.375\times10^{-2}\,Kg.m^2$$

14. Four spheres each diameter 2a and mass m are placed with their centers on the four corners of a square of the side b. Calculate the moment of inertia of the system about any side of the square.

A. . Let S_1, S_2, S_3 and S_4 be the spheres each of radius a.

Let I_1, I_2, I_3 and I_4 be the moments of inertia of spheres S_1, S_2, S_3 and S_4 respectively about AB

$$I_1 = I_2 = \frac{2ma^2}{5}$$

$$I_3 = I_4 = \frac{2ma^2}{5} + mb^2$$

M.I of the whole system = $I_1 + I_2 + I_3 + I_4$

$$= \left(2\times\frac{2ma^2}{5}\right) + 2\left(\frac{2ma^2}{5} + mb^2\right) = \frac{8ma^2}{5} + 2mb^2$$

15. Determine the kinetic energy of a circular disc rotating with a speed of 60 rpm about an axis passing through a point on its circumference and perpendicular to its plane. The circular disc has a mass of 5 kg and radius 1 m.

A. m = 5 kg; r = 1 m and $\omega = 2\pi\times\frac{60}{60} = 2\pi$ rad s^{-1}

MI of the disc about an axis passing through a point on its circumference and perpendicular to its plane is given by $I_p = I_c + mr^2 = \frac{mr^2}{2} + mr^2 = \frac{3mr^2}{2}$

$$\therefore \text{K.E} = \frac{1}{2}\ I\omega^2 = \frac{1}{2}\times\frac{3}{2}\times mr^2\times\omega^2 = \frac{3}{4}\times5\times1^2\times4\pi^2 = 15\pi^2 = 148.1\ \text{J}$$

ASSESS YOURSELF

1) Does the moment of inertia of a rigid body change if its angular velocity of rotation is changed?

A) No. Moment of inertia of a rigid body it is independent of ω .

2) A steel sphere and a wooden sphere have the same mass. Which will have greater moment of inertia about the diameter?

A) $I = mr^2$. For a Wooden sphere radius is more and hence moment of inertia is greater.

3) A steel sphere and a wooden sphere have the same radius. Which will have smaller moment of inertia about the diameter?

A) $I = mr^2$. For same radius wooden sphere has smaller and hence its moment of inertia is smaller