ROTATORY MOTION

HORIZONTAL AND VERTICAL CIRCULAR MOTION

POINTS TO REMEMBER

1. **Translatory motion**: Every point in the body follows the path of its preceding one with same velocity including the centre of mass.
2. **Rotatory motion**: Every point move with different velocity with respect to the axis of rotation. The particle on the axis of rotation will have zero velocity.
3. **Circular Motion: If** the motion of a particle about a fixed point is a circle, then it is called circular motion.
4. **Axis of rotation**: During rotational motion, all the particles of a rigid body move in different circles and the center of all these circles lie on a line called axis of rotation.
5. **Radius vector:** - Any instant of time, the line joining the position of rotating particle with the centre of rotation is called radius vector at that instant.
6. **Radian definition:** - The angle subtended at the centre of a circle by an arc of length equal to its radius is called radian.

   $2\pi \ radians = 360^0$

   1 Radian = $57\ ^0 18^1$
7. **Angular displacement (θ):** The angle through which the radius vector of a rotating particle rotates in a given time interval is called angular displacement.

   Unit: radian Dimension: $\left[M^\circ L^\circ T^0\right]$
8. **Angular velocity (ω):** The rate of change of angular displacement of a rotating particle is called angular velocity.

   $\text{Angular velocity}\left(\omega\right) = \frac{d\theta}{dt}$)

   Unit: rad sec^{-1} Dimensions: $\left[M^\circ L^\circ T^{-1}\right]$
9. **Angular acceleration (α) :** The rate of change of angular velocity of a rotating particle is called angular acceleration.

   $\text{Angular acceleration } \left(\alpha\right) = \frac{d\omega}{dt}$

   Unit : rad sec^{-2} Dimension**:** $\left[M^\circ L^\circ T^{-2}\right]$
10. **Centripetal force**: A real force which is acquired by a body to perform circular motion and which acts towards the centre of the circle is called a centripetal force.

$$\text{Centre petal force} = \frac{mr^2}{r} = mr\omega^2$$

11. **Centrifugal force:** A pseudo force which acts radially outwards from the centre of the circular path is called centrifugal force. It is equal in magnitude to the centripetal force.

$$\text{Centrifugal force} = \frac{mv^2}{r} = mr\omega^2$$

12. If a body of mass m is rotated in a vertical circle of radius r tension in the string at any point of the vertical circle is given by $T = \frac{mv^2}{r} + mg\cos\theta$

Velocity at any point $v = \sqrt{gr(3+2\cos\theta)}$

13. Tension will be maximum at the lowest point $T_{max} = \frac{mv^2}{r} + mg$

Tension will be minimum at the highest point $T_{min} = \frac{mv^2}{r} - mg$

14. For a body of mass m is rotated in a vertical circle of radius r, the velocity at the lowest point $v = \sqrt{5gr}$ and at the highest point the velocity is $v = \sqrt{gr}$

Translatory motion	**Rotatory motion**
v = u + at	$\omega_2 = \omega_1 + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\omega_2^2 = \omega_1^2 + 2\alpha\theta$
$s = \frac{u+v}{2}$	$\theta = \frac{\omega_1 + \omega_2}{2}$
$s_n = u + \frac{a}{2}(2n-1)$	$\theta_n = \omega_1 + \frac{\alpha}{2}(2n-1)$

Short Answer Questions:

1. Explain the terms: Circular motion, angular displacement, angular velocity and angular acceleration.

A. **Circular Motion: If** the motion of a particle about a fixed point is a circle, then it is called circular motion.

Angular displacement (θ) : The angle through which the radius vector of a rotating particle rotates in a given time interval is called angular displacement.

Angular velocity (ω): The rate of change of angular displacement of a rotating particle is called angular velocity.

Angular velocity $(\omega) = \frac{d\theta}{dt}$

Angular acceleration (α): The rate of change of angular velocity of a rotating particle is called angular acceleration.

Angular acceleration $(\alpha) = \frac{d\omega}{dt}$

2. a) Define angular velocity (b) State its units and dimensions c) State whether it is a scalar or a vector. If it is a vector given its direction.

A. a) The rate of change of angular displacement of a rotating body is called angular velocity angular velocity $(\omega) = \frac{d\theta}{dt}$

b) **Unit:** rad sec^{-1}

Dimensions: $\left[M^{\circ}L^{\circ}T^{-1}\right]$

c) Angular velocity is a vector quantity whose direction is perpendicular to the plane of rotation and it can be known from right hand thumb rule.

3. a) Define angular acceleration. b) State its units and dimension.
c) State wither it is a vector (or) scalar. If it is a vector give its direction.

A. a) The rate of change of angular velocity of a rotation body is called angular acceleration.

Angular acceleration $(\alpha) = \frac{d\omega}{dt}$

b) **Unit :** rad sec^{-2}

Dimension: $\left[M^{\circ}L^{\circ}T^{-2}\right]$

c) Angular acceleration is a vector quantity whose direction is along the direction of change in the angular velocity. If the angular velocity increases, the direction is along the direction of angular velocity. If the angular velocity decreases, the direction is opposite to that of angular velocity.

4. Derive $v = r\omega$

A. Consider a particle moving in a circular path of radius 'r ' with a constant angular velocity ω. Let the particle moves from A to B in a small time interval δt and $\delta\theta$ be a small angular displacement. The instantaneous angular velocity 'ω' at any instant of time t is given by

$$\omega = \lim_{\delta t \to 0} \frac{\delta\theta}{\delta t} = \frac{d\theta}{dt}$$

Now, the linear velocity of the particle 'v', is given by

$$v = \lim_{\delta t \to 0} \frac{AB}{\delta t} = \lim_{\delta t \to 0} \frac{r\delta\theta}{\delta t}$$

$$= r\left[\lim_{\delta t \to 0} \frac{\delta\theta}{\delta t}\right] = r\frac{d\theta}{dt}$$

$$\therefore v = r\omega$$

Vectorially $\bar{v} = \bar{\omega} \times \bar{r}$

5. What is uniform circular motion? Obtain an expression for the acceleration of a particle performing uniform circular motion.

A. A particle is said to be in uniform circular motion if its angular velocity is uniform (or) linear speed is constant throughout its motion along the circle. Ex: motion of the moon around the earth.

Centripetal acceleration: Consider a particle moving in a circular path of radius 'r ' with centre at 'O' in the anti clock wise direction. At any instant of time 't' the position vector rotates through an angle θ from its initial position along X-axis so that $\theta = \omega t$.

$$\therefore \bar{r} = (r\cos\omega t)\hat{i} + (r\sin\omega t)\hat{j}$$

(or) $$\bar{r} = r\left[(\cos\omega t)\hat{i} + (\sin\omega t)\hat{j}\right]$$

But $\bar{v} = \frac{dr}{dt} = r\omega\left[-(\sin\omega t)\hat{i} + (\cos\omega t)\hat{j}\right]$

Now acceleration $\bar{a}$ is given by

$$\bar{a} = \frac{d\bar{v}}{dt} = r\omega^2\left[-(\cos\omega t)\hat{i} - (\sin\omega t)\hat{j}\right]$$

$$= -r\omega^2\left[(\cos\omega t)\hat{i} + (\sin\omega t)\hat{j}\right]$$

The direction of acceleration is given by $-\left[(\cos\omega t)\hat{i} + (\sin\omega t)\hat{j}\right]$ which is the unit vector opposite to $\bar{r}$. Hence the centripetal (or) normal acceleration is towards the centre of the circular path.

6. Define centripetal force. Explain its importance with suitable examples.

A. A real force which is acquired by a body to perform circular motion and which acts towards the centre of the circle is called a centripetal force.

$$\text{Centre petal force} = \frac{mr^2}{r} = mr\omega^2$$

Importance: Centripetal force is a net (or) unbalanced force towards the centre of the circle with out which the particle moves along the tangent to the circle.

Examples:

1) The gravitational force between the moon and the earth supplies the necessary centripetal force for the moon to perform a uniform circular motion.

2) While taking a turn on the road, the friction between the road and tyres of a car provides the necessary centripetal force.

3) When a stone tied to a thread is rotated in a circular path, the tension in the thread supplies the necessary centripetal force.

4) While taking a turn in air for an aero plane, the lift provided by its banked wings supplies the necessary centripetal force.

7. Define centrifugal force. Explain its importance with suitable examples.

A. A pseudo force which acts radially outwards from the centre of the circular path is called centrifugal force. It is equal in magnitude to the centripetal force.

$$\text{Centrifugal force} = \frac{mv^2}{r} = mr\omega^2$$

Importance : Centrifugal force is a pseudo (or) fictious force felt. when we are in a non inertial frame.

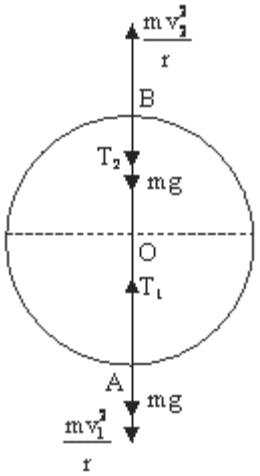
Examples: 1) Due the spin motion of the earth, the centrifugal force is maximum at the equator and minimum at the poles. Hence the weight of the body is maximum at the poles and minimum at the equator.

2) Centrifuge is a device in which centrifugal force separates lighter and heavy particles. A centrifuge is used to separate cream from milk (or) to separate substance of different masses.

3) A centrifugal governor controls the speed of an engine by using spinning masses that respond to centrifugal force produced by the engine.

8. Deduce the relation for the tension on the string of an object in vertical circular motion when it is at the lowest point and (ii) highest point of the circle.

A. Let a body of mass m is tied to a string and whirled in a vertical circle of radius ' r'.



At the lowest point (A)

Let T_1 be the tension and v_1 be the velocity when the body is at the lowest point of the vertical circle. The forces on the body are

i) Tension T_1 in the upward direction

ii) Weight mg of the body acting downwards.

The resultant of these supplies the necessary centripetal force

$$\therefore T_1 - mg = \frac{mv_1^2}{r} \text{ (Or) } T_1 = \frac{mv_1^2}{r} + mg$$

At the highest point (B)

Let T_2 be the tension and v_2 be the velocity of the body when it is at the highest point of the vertical circle. The forces on the body are :

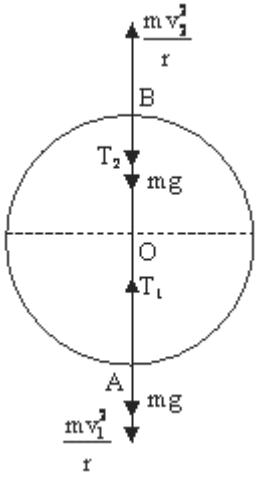
i) Tension T_1 acting down wards

ii) Weight of the body mg downwards.

$$\therefore T_2 + mg = \frac{mv_2^2}{r} \qquad \text{Or} \qquad T_2 = \frac{mv_2^2}{r} - mg$$

9. Show that the difference in the tensions is 6mg where mg is the weight of a block performing vertical circular motion under gravity.

A. Let a body of mass m is tied to a string and whirled in a vertical circle of radius ' r'. When the body is at the highest point of the vertical circle



Tension $T_2 = \frac{mv_2^2}{r} - mg$ and Velocity $v_2 = \sqrt{rg}$

At the lowest point of the vertical circle

Tension $T_1 = \frac{mv_1^2}{r} + mg$ and Velocity $v_1 = \sqrt{5rg}$

$\therefore$ Now the difference in tensions is given by

$$T_1 - T_2 = \frac{m}{r}\left(v_1^2 - v_2^2\right) + 2mg = \frac{m}{r}(5rg - rg) + 2mg$$

$$\therefore T_1 - T_2 = 6mg$$

Very Short Answer Questions

1. What is 'axis of rotation'?

A. During rotational motion, all the particles of a rigid body move in different circles and the center of all these circles lie on a line called axis of rotation. The axis of rotation may lie in or outside of the body. For the earth moving around itself the axis passes through the center of the earth where as for the motion of the earth around the sun the axis passes through the center of the sun.

2. Which of the following are vectors?

i) Angular displacement, ii) Angular Velocity, iii) Angular acceleration IV) Moment of inertia.

A. Angular displacement, angular velocity and angular acceleration are pseudo vectors Moment of inertia is a scalar.

3. Give the vector form of the relation between v and ω

A. $\bar{V} = \bar{\omega} \times \bar{r}$

4. A rigid body is moving in a vertical plane. If its velocity about the lowest point of the

Circle is $\sqrt{5gr}$. What is its velocity when it reaches horizontal position?

A. (PE + KE) at the lowest point = (PE + KE) at the horizontal point

$$0+\frac{1}{2}m\left(\sqrt{5gr}\right)^2=\frac{1}{2}mv^2+mgr \Rightarrow v=\sqrt{3gr}$$

Exercise 1

1. Find the average angular velocity of the second's hand of a watch if the second's hand of a watch completes one revolution in 1 minute

.Sol : $n=1, t=1$ minute = 60 s.

Angular velocity $(\omega)=\frac{2\pi n}{t}=\frac{2\pi\times1}{60}=\frac{\pi}{30}rad\ s^{-1}$

2. Find the average angular velocity of the spinning motion of the earth.

Sol: For one complete rotation of the earth t = 24 hour =86400 seconds

Angular velocity $(\omega)=\frac{2\pi n}{t}=\frac{2\pi\times1}{86,400}=\frac{\pi}{43,200}rad\ s^{-1}.$

3. When a motor cyclist takes a U-turn in 4 seconds what is the average angular velocity of the motor cyclist?

Sol: Angular displacement $(\theta)=\pi\ rad$ and $t=4s.$

Average angular velocity $(\omega)=\frac{\theta}{t}=\frac{\pi}{4}=0.7855\,rad\ s^{-1}.$

4. When the angular displacement of a particle is given by $\theta=t^3+t^2+t+1$ find its (i) angular velocity and (ii) angular acceleration at t = 2 second.

Sol: The angular displacement, $\theta=t^3+t^2+t+1$ and t = 2 s

Angular velocity, $\omega=\frac{d\theta}{dt}=3t^2+2t+1=\ 17\ \ rad\ s^{-1}$

Angular acceleration, $\alpha=\frac{d\omega}{dt}=6t+2.=14\,rad\ s^{-2}.$

5. A car is moving in a circular path with a uniform speed v. Find the magnitude of change in its velocity when the car rotates through an angle θ.

Sol: The magnitude of change in velocity = $\sqrt{v^2+u^2-2uv\cos\theta}$

$$=2v\sin\left(\frac{\theta}{2}\right). \qquad (\because u=v)$$

6. What is the linear velocity of a person at the equator of the earth due to its spinning motion? (Radius of the earth = 6400 km.)

Sol : Angular velocity of the earth , $\omega=\frac{2\pi n}{t}=\frac{2\pi\times1}{24\times60\times60}=\frac{\pi}{43,200}rad\ s^{-1}.$

$R=6400\,km=6.4\times10^6\,m$

The linear velocity, $v=R\omega=6.4\times10^6\times\frac{\pi}{43,200}=465.5\,m/s$

7. A ball of 200 g is on one end of a string of length 20 cm. It is revolved in a horizontal circle at an angular frequency of 6 rpm. Find (i) the angular velocity, (ii) the linear velocity, (iii) the centripetal acceleration, (iv) the centripetal force and (v) the tension on the string.

Sol: $m = 0.2\,kg$; $r = 0.2m$; $n = 6$; $t = 1$ minute = 60 s.

(i) Angular velocity, $\omega = \frac{2\pi n}{t} = \frac{2\pi \times 6}{60} = \frac{\pi}{5} = 0.6284\ rad\ s^{-1}$.

(ii) Linear velocity, $v = r\omega = 0.2 \times 0.6284 = 0.1257\,m\,s^{-1}$.

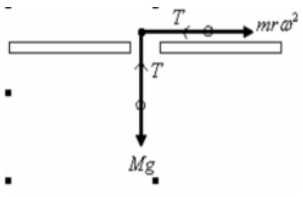
(iii) Centripetal acceleration, $a_c = r\omega^2 = 0.2 \times (0.6284)^2 = 0.0790\,m\,s^{-2}$.

(iv) Centripetal force, $F_c = mr\omega^2 = 0.2 \times 0.2 \times (0.6284)^2 = 0.0158\,N$.

(v) The tension on the string = centripetal force = 0.0158 N.

8. Considering figure, (a) with what angular speed $'\omega'$ must 'm' with a radius 'r' rotate on a frictionless table so that 'M' does not move? (b) If m = 1.0 kg, M = 10.00 kg and r = 0.5 m, find ω.

Sol : (a) The tension on the string (Mg) supplies the necessary centripetal force to the body of mass m revolving in a circular orbit of radius r.



$T = mr\omega^2$ (or) $m\ r\omega^{\ 2} = M\ g$

Angular speed = $\omega = \sqrt{\frac{Mg}{mr}}$.

b) m = 1.0 k.g ; M = 10.0 kg ; r = 0.5 m ; $g = 9.8\,m\,s^{-2}$.

$\omega = \sqrt{\frac{Mg}{mr}} = \sqrt{\frac{10.0 \times 9.8}{1.0 \times 0.5}} = \sqrt{196} = 14\,rad\,s^{-1}$.

9. A stone tied to the end of a string is whirled in a horizontal circle. The mass of the stone is 1.0 kg and the string is 0.50 m long. If the stone revolves at a constant speed of 10 times in 15.71 s, (a) what is the tension on the string ? (b) What would happen to the tension on the string if the mass was doubled and all the other quantities stayed the same? (c) What would happen to the tension on the string if the period was doubled and all the other quantities remain the same?

Sol : m =1 kg; r = 0.5 m; n = 10; t = 15.71 s.

The angular velocity, $\omega = \frac{2\pi n}{t} = \frac{2\pi \times 10}{15.71} = \frac{31.42 \times 2}{15.71} = 4\,rad\,s^{-1}$.

a) The tension on the string = $mr\omega^2 = 1.0 \times 0.5 \times (4.0)^2 = 8\,N$.

b) Keeping r and ω. constant, if m is doubled, the tension on the string becomes double.

i.e. T= 16 N.

c) Keeping m and r constant, if time period is doubled ,

Before doubling, the tension = $Tension = mr\omega^2 = m\ r\left(\frac{2\pi}{T}\right)^2 = 8N$

After doubling, the tension = $Tension = mr\omega^2 = m\ r\left(\frac{2\pi}{2T}\right)^2 = \frac{8}{4} = 2\,N$

Here T is the time period.

10. A stone of mass 2.0 kg is tied to the end of a string of 2 m length. It is whirled in a horizontal circle. If the breaking tension of the string is 400N, calculate the maximum velocity of the stone.

Sol : $m = 2.0\,kg; r = 2\,m;$ tension = 400 N.

At the breaking tension, let v be the maximum velocity of the stone .

$$400 = \frac{mv^2}{r} \qquad \Rightarrow 400 = \frac{2\times v^2}{2} \Rightarrow \quad v = 20\,m\,s^{-1}.$$

11. A mass m is revolving in a vertical circle at the end of a string of length r. Calculate the difference in kinetic energies at the top and bottom of the circle.

Sol: If the velocities of the body at the top and bottom are and respectively.

But, $v^2 = u^2 + 2as \Rightarrow {v_B}^2 = {v_T}^2 - 2g\times 2r$

$u = v_l, v = v_h, a = -g, s = 2r$ Since the body goes from the bottom to the top of the circle.

$v_T^2 - v_B^2 = 4gr$

Change in kinetic energy = $= \frac{1}{2}m\left(v_T^2 - v_B^2\right) = \frac{1}{2}m\times 4gr = 2mgr$

12. A pendulum bob of mass m is held out in the horizontal position and then released from rest. If the string is of length l, what is the velocity of the bob and tension on the string when the bob reaches the lowest position?

Sol : Consider the lowest position as reference position

i) From the law of conservation of energy,

Total energy of the bob in the horizontal = Total energy of the bob at the lowest position.

(KE + PE) in the horizontal position = (KE + PE) at the lowest position.

$0 + mgl = \frac{1}{2}mv^2 + 0 \;\Rightarrow v = \sqrt{2gl}.$

ii) The tension in the string at the lowest position

$= \frac{mv^2}{r} + mg = \frac{m(2gl)}{l} + mg = 3mg$.

13. A body is allowed to slide down a frictionless track from rest position at its top under gravity. The track ends in a circular loop of diameter D. What is the minimum height of the inclined track in terms of D so that it may complete successfully the loop?

Sol : Consider the end of the track is taken as reference position

From the law of conservation of energy,

Total energy of the body at the top of the inclined track = Total energy of the body at the end of the track.

(KE + PE) at the top = (KE + PE) at the bottom

$0 + mgh = \frac{1}{2}mv^2 + 0 \Rightarrow v = \sqrt{2gh}$ Where h is the height of the inclined track.

$\sqrt{5rg} = \sqrt{2gh} \Rightarrow h = \frac{5r}{2} = \frac{5D}{4} \quad \left(\because r = \frac{D}{2}\right)$

To have a complete the loop,

14. A 300 g ball on a 60 cm long string is swung in a vertical circle whose lowest point is 200 cm above the floor. The tension in the string when the ball is at the very bottom of the circle is 5.10 N. A very sharp knife is suddenly inserted to cut the string directly below the point of support. How far does the ball travel before if hits the floor?

Sol : M = 0.3 kg; r = 0.6 m ; h = 2 m ;
tension = 5.1 N; $g = 9.8\,ms^{-2}$.

Tension at the bottom = $\frac{Mv_l^2}{r} + Mg \Rightarrow 5.10 = 0.300\left(\frac{v_l^2}{0.60} + 9.8\right)$

$\Rightarrow v_l = 2.078\,m\,s^{-1}$.

This velocity is in the horizontal direction and hence the ball becomes a horizontal projectile.

Range of the horizontal projectile

$$R = v_l t = v_l\sqrt{\frac{2h}{g}} = 2.078\sqrt{\frac{2\times 2}{9.8}} = 1.328\,m$$

Exercise 1

1. A car covers a semi circular turn in 10 seconds. What is the average angular velocity of the car ?

A.. Angular displacement = π radian
Time taken = 10s

Angular velocity $\omega = \frac{\text{angular displacement}}{\text{time}} = \frac{\pi}{10} = 0.3412\,rad\,s^{-1}$

2. The angular displacement of a particle is $\theta = 3t^3 + 9t^2 + 6t + 4$ where θ **is in radian and t is in second. Find its i) angular velocity and ii) angular acceleration at t = 1 s.**

A. Angular displacement $\theta = 3t^3 + 9t^2 + 6t + 4$

(i) Angular velocity $\omega = \frac{d\theta}{dt} = 9t^2 + 18t + 6$

At t = 1, $\omega = 91^2 + 18\ 1 + 6 = 33$ rad s^{-1}

Angular acceleration

At t = 1, a = 18 ´ (1) + 18 = 36 rad s^{-2}

3. What is the angular displacement of the seconds hand of clock in 45 second?

A. Angular displacement of seconds hand in 60 s = 2π rad

Angular displacement in 45 s = $\frac{45}{60}\times 2\pi = \frac{3\pi}{2}$ rad

4. To simulate the acceleration of large rockets, astronauts are spun at the end of a long rotating beam of radius 9.8 m. What angular velocity is required to generate a centripetal acceleration 8 times the acceleration due to gravity?

A. Centripetal acceleration = $r\omega^2$

r = 9.8 m and $r\omega^2 = 8g$ or $\omega = 2\sqrt{2} = 2.828$ rad s^{-1}

5. A 0.5 kg mass is rotated in a horizontal cirlce of radius 20 cm. Calculate the centripetal force acting on it, if its angular speed of revolution is 0.6 rad S^{-1}

A. m = 0.5 kg, r = 20 x 10^{-2}m, ω = 0.6 rad s^{-1}

Centripetal force = mr^2 = 0.5 x $\frac{20}{100}$ x 0.36 = 0.036 N

6. A certain string breaks under a tension of 45 kg wt. A mass of 100 g is attached to this string of length 50cm and is whirled in a horizontal circle. Find the maximum number of revolutions possible per second without breaking the string.

A. m = 100 g = 0.1 kg, l = 50 cm = 0.5 m

Tension in the string = = 0.1 x 0.5 x ω^2 where ' ω ' is the angular velocity of the mass

But, ml ω^2 = 45 g

0.1 x 0.5 x $4\pi^2$ x n^2 = 45g or $n^2 = \frac{45g}{0.1\times0.5\times4\pi^2} = \frac{45\times100}{1\times5\times4}$ or n = 15 rev/s

7. If the centripetal force acting on a body revolving along a circular path of radius 25 m is 200 N find its kinetic energy.(May2009)

A. Centripetal force = $\frac{\mathbf{mv^2}}{\mathbf{r}}$ and r = 25 m

mv^2 = 200 × r = 200 × 25

$K.E = \frac{1}{2}mv^2 = \frac{200\times25}{2} = 2500$ J

8. In a motor cycle stunt called 'well of death' a rider drives round the inner wall of a hollow cylindrical chamber. If the diameter of the cylindrical chamber is 25 m, what should be the minimum angular speed of the rider to prevent him from sliding down ? $(\mu = 0.8)$

A.. Radius of the chamber = $\frac{25}{2}$m

Normal reaction on the rider = R = $\frac{mv^2}{r} = mr\omega^2$ Where is the minimum angular speed required to prevent the rider from sliding down

Frictional force = μR = μmr^2

$\therefore \mu mr\omega^2 = mg$ or $\omega^2 = \frac{g}{\mu r} = \frac{9.8\times2}{0.8\times25} = 0.98 = 0.49\times2$ or

$\omega = 0.7\sqrt{2}$ rad s^{-1}

9. A motor cyclist rides in vertical circles in a hollow sphere of radius 5 m. Find the minimum speed required so that he does not lose contact with sphere at the highest point.

A. r = 5m, g = 9.8 ms^{-2}

Critical speed at the highest point = $\sqrt{rg} = \sqrt{5\times9.8} = 7ms^{-1}$

10. A bucket containing water is tied to end of a rope 0.5 m long and rotated about the other end in a vertical circle. Find the minimum number of rotations the bucket can make per minute so that the water in the bucket does not spill.

A. The weight of water should provide the necessary centripetal force.

ie., $mr\omega^2 = mg \Rightarrow mr\left(\frac{2\pi n}{t}\right)^2 = mg \Rightarrow n = \sqrt{\frac{gt^2}{4\pi^2 r}} =$ = 42.42 rpm

11. A coin is kept a distance of 10 cm from the centre of a circular turn table. If the coefficient of static friction between the table and the coin is 0.8 find the frequency of rotation of the disc at which the coin will just begin to slip.

A. For the coin just not to slip, the required condition is mr $\omega^2 = \mu$ mg

or $m r \times 4\pi^2 n^2 = \mu g$

or $n^2 = \frac{\mu g}{4\pi^2 r} = \frac{0.8 \times 9.8}{4\pi^2 \times 0.1} = 2$ or $n = \sqrt{2}$ rev/s = 60 × 1.414 = 84.84 rev/min

12. A stone of mass 0.5 kg is tied to one end of a string of length 1.2 m and whirled in vertical circular path. Find the maximum and minimum tension in the string, if the speed of the stone at the lowest point is 8 ms^{-1}

A. m = 0.5 Kg, l = 1.2 m

$V_1 = 8$ ms^{-1}

$V_2^2 - V_1^2 = 2$ g s Or $V_2^2 - 64 = 2(-9.8)\ 2.4$ Or $V_2^2 = 16.96$

$$T_{max} = \frac{mv_1^2}{l} + mg = \frac{0.5 \times 64}{1.2} + 0.5 \times 9.8 = \frac{32}{1.2} + 4.9 = 31.57N$$

$$T_{min} = \frac{mv_2^2}{l} - mg = \frac{0.5 \times 16.96}{1.2} - 0.5 \times 9.8 = \frac{8.48}{1.2} - 4.9 = 2.166N$$

13. A stone of mass 1 kg is tied to one end of a string of length 50 cm. It is whirled in a vertical circle. If it can withstand a maximum tension of 58.8 N what is the velocity of the stone at the top of the vertical circle?

A. m = 1 Kg, l = 50 cm = 0.5 m

$T_{max} = T_1 = mg + \frac{mv_1^2}{l}$

where V_1 is the velocity at the bottom most point of the vertical circle

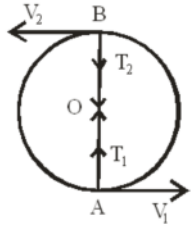
$58.8 = 1 \times 9.8 + \frac{1 \times v_1^2}{0.5}$ or $v_1 = \frac{7}{\sqrt{2}} ms^{-1}$

Let V_2 be the velocity at the top most point

$V_2^2 - V_1^2 = 2g \times 2l$ or $V_2^2 - 49/2 = 4 \times 9.8 \times ½$

$V_2 = \sqrt{4.9} = 2.214$ ms^{-1}

14. A stone of mass 2 kg is tied to one end of a string of length 50 cm. It is whirled in a vertical circle. If the velocity of the stone at the top of the circle is 5.0m s^{-1} calculate the tension in the string at a) the top of the circle and b) the bottom of the circle.



A. m = 2 Kg, l = 50 cm = 0.5 m

V_2 = 50 ms^{-1}

$V_2^2 - V_1^2 = 2g\,2\,l$ or $V_1^2 = 25 + 19.6 = 44.6$

a) $T_2 = \frac{mv_2^2}{l} - mg = \frac{2\times 5^2}{0.5} - 2\times 9.8 = 100 - 19.6 = 80.4N$

b) $T_1 = \frac{mv_1^2}{l} + mg = \frac{2\times 44.6}{0.5} + 2\times 9.8 = 178 + 19.6 = 198.0N$

15. The angular frequency of a fan increases from 30 rpm to 60 rpm in 3.142 s. A dust particle is present on the blades of the fan at a distance of 20 cm from the axis of rotation. Find i) the average angular acceleration ii) the radial acceleration of the dust particle at the end of 3.142 s, iii) the tangential acceleration of the dust particle at the end of 3.142 s and iv) the net acceleration of dust particle at the end of 3.142 s.

A. . ω_0 = 30/60 rps $= \frac{1}{2}\times 2\pi = \pi rads^{-1}$

ω = 60/60 rps = 2π rad s^{-1}

t = 3.142 s = π s

Distance of dust particle from centre r = 20 cm = 0.2 m

i) Let α be the angular acceleration

Then $\omega = \omega_0 + \alpha t$ or $2\pi = \pi + (\alpha \times t)$ or α = 1 rad s^{-2}

ii) Radial acceleration $a_r = \frac{v^2}{r} = r\omega^2$

where ω is the angular velocity at the end of 3.142 s = π s

This is equal to 2p rad s^{-1}

$\therefore a_r = r\omega^2 = 0.2\times(2\pi)^2 = 0.8\pi^2 = 7.902$ ms^{-2}

iii) Tangential acceleration $a_t = r\alpha = 0.2\times 1 = 0.2\,ms^{-2}$

iv) Radial acceleration $a = \sqrt{a_r^2 + a_t^2} = \sqrt{7.902^2 + 0.2^2} = 7.903\,m\,s^{-2}$

ASSESS YOURSELF

1) A body is in pure rotation. The linear speed v of a particle of the body, the distance r of the particle from the axis and the angular velocity ω of the body are related as $\omega = \frac{v}{r}$ Does this mean that the angular velocity is inversely proportional to the distance of the particle from the axis ?

A) No, In **pure rotation the** angular velocity (ω) of a body is independent of distance of particle from the axis of rotation i.e. all particles in a rotating body move with same angular velocity.

2) What is the magnitude of $\vec{\omega} . \vec{v}$?

A) Zero, angular velocity vector, linear velocity vector are perpendicular to each other

3) What is the magnitude of $\vec{\omega} . \vec{r}$?

A) Zero, because angular velocity vector and radius vector are perpendicular to each other.

4) According to Bohr's theory, an electron revolves round the nucleus in a circular orbit in an atom with constant angular velocity. Which supplies the necessary centripetal force ?

A) Electrostatic force of attraction between the positively charged nucleus and revolving electron of negative charge acting towards the nucleus supplies **the necessary** centripetal force.

5) Is the kinetic energy of a particle performing uniform circular motion constant?

A) Yes, because in uniform circular motion magnitude of linear velocity (v) remains constant.

6) Which physical quantities are constant in uniform circular motion?

A) Angular velocity, magnitude of linear velocity, kinetic energy and angular momentum remains constant.

7) Which physical quantities are variable in uniform circular motion?

A) The direction of linear velocity, linear momentum, centripetal acceleration and centripetal force variable.

8) A body moves along a circular path with variable angular velocity. Does the body have only centripetal acceleration?

A) **A body moving along a circular path with variable angular velocity** possesses both centripetal acceleration $a_c = \frac{v^2}{r}$ along the radius towards the centre and tangential acceleration $a_t = r\alpha$ along the tangent. Hence the resultant acceleration is given by $a = \sqrt{a_c^{\,2} + a_t^{\,2}} = \sqrt{\left(\frac{v^2}{r}\right)^2 + (r\alpha)^2}$

9) The driver of a car traveling with a velocity v suddenly sees a broad wall ahead of him at a distance a. Is it better for him to break or turn sharply in a circle?

A) Breaking requires less friction than turning sharply in a circle .Hence braking is preferred

10) An artificial satellite moving round the earth in a circular orbit possesses a changing acceleration. Is it true?

A) Yes, The acceleration is constant in magnitude but changes its direction.

11) A bottle of soda water is grasped by the neck and rotated in a vertical circle. Near which portion of the bottle do the bubbles collect?

A) Near the neck

12) When a motor cyclist moves in a vertical circle around a spherical globe without falling down at the top most point which balances the gravitational force?

A) Centrifugal force.