FRICTION

POINTS TO REMEMBER

1) The force that always opposes the relative motion between two surfaces in contact and parallel to the surfaces, opposite to the direction of motion is called frictional force.

2) The frictional force is of three types. They are

1. Static friction 2.Dynamic friction 3.Rolling friction

Laws of friction: i) The frictional force is independent of the area of contact.

ii) The frictional force is directly proportional to the normal reaction.

3) Normal reaction is the resultant contact force acting on a body placed on a rigid surface perpendicular to the plane of contact. It is equal to mg on a horizontal surface and $mg\cos\theta$ on an inclined plane Coefficient of static friction $\mu_s = \tan\phi$

4)Angle of repose is defined as the angle of inclination of a plane with respect to horizontal for which the body will be in limiting equilibrium on the inclined plane .If α is the angle of repose $\mu_s = \tan\alpha$ • When a body slides down an inclined plane of angle of inclination (θ)greater than the angle of response (α) i.e., $\theta > \alpha$,the acceleration of the body $a = g(\sin\theta - \mu_k \cos\theta)$,

5) **The coefficient of friction** is, $\mu = \frac{f_s}{N} = \tan\theta$ where θ is the angle of repose.

6) **Acceleration of a body on a rough horizontal surface**, $a = \frac{P - \mu_k mg}{m}$

7) $a = g\sin\theta$ for a smooth inclined plane.

8) $a = g(\sin\theta - \mu_k \cos\theta)$ for a body sliding down rough inclined plane.

9) For a body moving up a rough inclined plane under the action of a force F,

$a = \frac{F - mg(\sin\theta + \mu_k \cos\theta)}{m}$

10) When a body is to be moved up a rough inclined plane with uniform velocity the force to be applied F=mg($\sin\theta$+$\mu_k \cos\theta$)

11) Pulling force $F = \frac{W\sin\phi}{\cos(\theta - \phi)}$ and pushing force $F = \frac{W\sin\phi}{\cos(\theta + \phi)}$ where W is the weight of body, ϕ is the angle of friction and θ is the angle made by F with the horizontal

LONG ANSWER QUESTIONS:

****1. Define coefficient of static and dynamic friction. Mention the laws of static and dynamic friction. Derive an expression for the accelerations of the body sliding on a horizontal plane having coefficients of friction μ_k .**

A. The ratio of limiting friction F_l to normal reaction N is called Coefficient of static friction

i.e. $F_s \ \alpha \ N$

$\mu_s = \frac{f_s}{N}$

The ratio of dynamic frictional froce to the normal reaction. Coefficient of dynamic friction $f_k \alpha N,\ f_k = \mu_k N$

$\mu_k = \frac{f_k}{N}$

Laws of static friction

1. The Limiting friction depends on nature of surface in contact and on their surface roughness. It is independent of area of contact
2. The Limiting friction is tangential to the surface in contact and opposite to the motion of the body.
3. Magnitude of limiting friction is directly proportional to normal reaction,

$f_s \alpha N \;\; or \;\; f_s = \mu_s N$

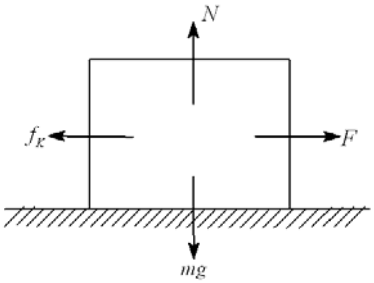
Laws of dynamic or kinetic friction :

1. The kinetic friction depends on nature of surface and independent of area of contact for smaller relative speeds.
2. The kinetic friction is tangential and opposite to motion of the bodies.
3. Dynamic friction is directly proportional to normal reaction

$F_k \propto N$

$\Rightarrow F_k = \mu_k N$

Motion of a body on rough horizontal surface:



Consider a horizontal force F is applied on a block of mass 'm', which is kept on a rough horizontal surface. The coefficient of kinetic friction between the block and the surface is μ_k.

The kinetic friction acting on the body is opposite to applied force and tangential at contact surface.

From laws of friction:

Kinetic frictional force $f_k = \mu_k N$ and $N = mg$

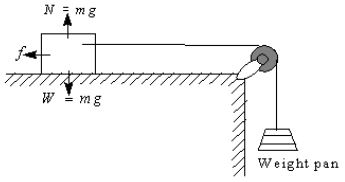
Net force acting on the body

$F_{net} = F - f_k = ma$

$a = \dfrac{F - \mu_k mg}{m}$

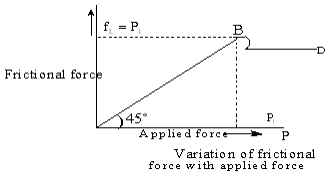
2. Describe with an experiment how static friction is explained. Distinguish static friction and dynamic friction.

A.



Consider a body be placed on a rough horizontal surface of a table. As shown the body is connected to a weight pan by a thread through smooth pulley. When weight in the pan is small, the block does not slide, since the tension in the thread is balanced by frictional

force on block, this frictional force is called static friction. Static friction is self adjusting; as the weight on the pan is gradually increased, static friction also increases to a maximum limit. After the limit, the body slides with an acceleration. The maximum value of static friction is called limiting friction.



Variation of frictional force with applied force

If a graph is drawn with applied force (weight) on X-axis and frictional force on Y-axis, the graph is as shown.

Starting from zero applied force, static friction increases up to the maximum value. After the limit, body starts sliding suddenly reducing frictional force by small amount (BC). After motion has started, the kinetic frictional force remains constant. This is represented by line PQ.

It is clear for diagram that, BE > PQ. Limiting friction > Kinetic friction

I.e. $f_s > f_k$ hence $\mu_s > \mu_k$

3. Derive an equation for the acceleration of a body sliding down an inclined plane without friction. Find also the time taken by the body to slide down the smooth inclined plane through a length '*l*' starting from rest.

A. Consider a body of mass 'm' is placed on a smooth inclined plane of inclination θ is as shown in figure.

The forces acting on the body are

(i) The weight of the body (mg) acts vertically downwards

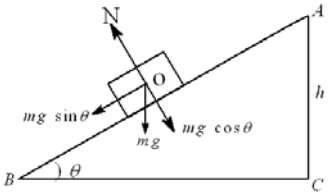
(ii) The normal reaction (N), acts perpendicular to the inclined plane

The weight of the body 'mg' can be resolved in to two components. They are mg $\cos\theta$,and mg $\sin\theta$. The component mg $\cos\theta$ acts perpendicular to the inclined plane and is balanced by normal reaction (N=mg $\cos\theta$) and mg $\sin\theta$ acts parallel to the inclined plane and it is un balanced force. The net down ward force on the body along the plane is given by

$$F_{net} = mg \sin\theta$$

$$ma = mg \sin\theta \quad (\because F_{net} = ma)$$

$$a = g\sin\theta$$



Consider the body starts from rest from the top of the plane and acquires a velocity V after travelling through displacement '*l*' along the plane, then

From the equation of motion,

$v^2 - u^2 = 2as$, as u = 0, $s = l$ and $a = g\sin\theta$

$v^2 = 2(g\sin\theta)l$

$V = \sqrt{2gl\sin\theta}$ $\quad (\because l\sin\theta = h)$

$V = \sqrt{2gh}$

$\rightarrow$Hence the velocity acquired by the body when it reaches the foot of the plane is independent of the inclination of the plane and depends only on the vertical height (h) through which the body is falling.

$\rightarrow$ If 't' time taken to travel the distance 'l 'with initial velocity u = 0, at the top of the plane, from $V = u + at$,

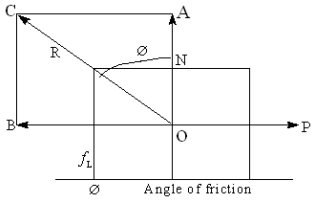
We get

$$t = \frac{V}{g\sin\theta} \qquad t = \frac{\sqrt{2\lg\sin\theta}}{g\sin\theta} \qquad t = \sqrt{\frac{2l}{g\sin\theta}}$$

4. **Define angle of friction and angle of repose.**

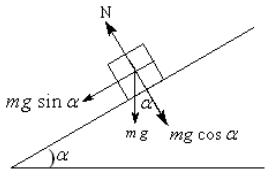
Show that angle of friction is equal to angle of repose for a rough inclined plane.

A. The angle made by the resultant of the normal reaction and the limiting frictional force With normal reaction is called angle of friction (ϕ).

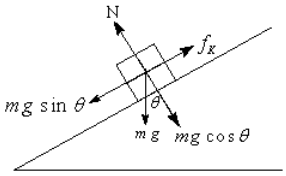


From the figure $\tan\phi = \frac{f_s}{N}$ and $\mu_s = \frac{f_s}{N}$. Hence $\mu_s = \tan\phi$.

The angle of repose (α) is defined as the angle of inclination of a rough inclined plane with horizontal for which the body will be just ready to slide down.



<u>To show that the angle of repose is equal to the angle of friction :</u>



When the body tends to move down the rough inclined plane, the friction on the block is called limiting or static friction f_s .Hence

$f_s = mg\sin\alpha$ and $N = mg\cos\alpha$

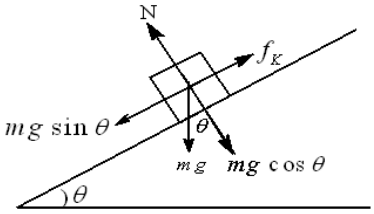
$\therefore$ Coefficient of static friction $\mu_s = \frac{mg\sin\alpha}{mg\cos\alpha} = \tan\alpha$

. $\mu_s = \tan\alpha$ Here α is the angle of repose.

Hence the angle of repose of a rough inclined plane is equal to the angle of friction of horizontal plane

5. Obtain an expression for the acceleration of a body down a rough inclined plane. Find the time taken by the body to slide down the inclined plane through a length '*l*' starting from rest. (June 2003, March 2002, 2003, and 2007)

A. Consider a body of mass 'm' is placed on a rough inclined plane of inclination θ as shown in figure.



The forces acting on the body are

(i) The weight of the body (mg) acts vertically downwards

(ii) The normal reaction (N), acts perpendicular to the inclined plane

(iii) The kinetic friction (f_k) acting up the inclined plane

The weight of the body 'mg' can be resolved into two components.They are mg $\cos\theta$ and mg sin θ The component mg $\cos\theta$ acts perpendicular to the inclined plane and is balanced by normal reaction (N=mg $\cos\theta$) and the component mgsin θ acts parallel to the inclined plane and it is unbalanced force.

The net downward force acting on the body is

$F_{net} = mg\sin\theta - f_k$

$F_{net} = mg\sin\theta - \mu_k N \qquad (\because f_k = \mu_k N)$

$F_{net} = mg\sin\theta - \mu_k mg\cos\theta \qquad (\because \text{N=mg}\cos\theta)$

$ma = mg(\sin\theta - \mu_k\cos\theta) \qquad (\because F_{net} = ma)$

$$\boxed{a = g(\sin\theta - \mu_k\cos\theta)}$$

Consider body starts from rest from the top of the plane and acquires a velocity V after travelling through displacement 'l' along the plane, then

From the equation of motion, $v^2 - u^2 = 2as$,

$$v^2 - o^2 = 2g(\sin\theta - \mu_k\cos\theta)s$$

$$\boxed{V = \sqrt{2gl(\sin\theta - \mu_k\cos\theta)}}(\because s = l)$$

The time taken by the body to slide down is given by $S = ut + \frac{1}{2}at^2$

Here, $S = l$, $u = 0$

$$\therefore l = 0(t) + \frac{1}{2}g(\sin\theta - \mu_k\cos\theta)t^2$$

$$\boxed{\therefore t = \sqrt{\frac{2l}{g(\sin\theta - \mu_k\cos\theta)}}}$$

Friction: It is the force which always opposes the relative motion between two surfaces in contact. It acts parallel to the surfaces and opposite to the direction of motion.

Laws of friction: i) The frictional force is independent of the area of contact.
ii) The frictional force is directly proportional to the normal reaction.

Static friction : Coefficient of static friction is defined as the ratio of limiting friction to normal reaction N.

$i.e., F_l \propto N$

$$\mu_s = \frac{f_s}{N}$$

Dynamic friction: Coefficient of dynamic friction is defined as ratio of dynamic frictional force to the normal reaction $f_k \alpha N$. $\mu_k = \frac{f_k}{N}$.

Rolling friction: The rolling friction is directly proportional to the normal reaction.

Coefficient of rolling friction, $\mu_r = \frac{f_r}{N}$

Angle of repose: It is defined as the angle of inclination of a plane with the horizontal for which the body will be in limiting equilibrium on the inclined plane. If α is the angle of repose $\mu_s = \tan\alpha$

Angle of friction: The angle made by the resultant of the normal reaction and the limiting friction with normal reaction is called angle of friction.

SHORT ANSWERS QUESTIONS

1. Explain the terms dynamic limiting frictions and rolling friction. (June 2002, 2003)

A. **Limiting friction: The** maximum frictional force developed between the bodies at rest is called limiting friction.

Dynamic or kinetic friction: When applied force is equals to or greater than limiting friction then the body will move. Frictional force between moving bodies is called dynamic or kinetic friction. Kinetic friction is always less than limiting friction.

Rolling friction : When one body rolls on the surface of the other body, (like a ball, a drum or a cylinder) then the existed friction in between that body and the surface of the other body is called rolling friction.

2. Write a note on the causes of friction.

A 1) Frictional force is caused due to the roughness of two surfaces in contact and interlocking of the irregularities between the surfaces.
2) Nature of surfaces.
3) Impurities between surfaces.
4) Shape of surfaces.

3. Explain advantages and disadvantages of friction. (March 2005, 2006)

A. **Advantages of friction:**
1) Safe walking on the floor is possible due to friction.
2) Nails, screws etc. are driven into walls; wooden surfaces etc. due to friction.
3) Writing with pens, pencils, holding objects with hands etc is possible due to friction.
4) A match stick is lightened due to friction.

Disadvantages of friction:
1) Due to friction wear and tear of engine occurs.
2) Due to friction some energy gets converted into heat which goes as waste
3) The large amount of power loss in engines due to friction.

4. Mention the methods used to decrease friction? (July 2001 March 2001, 2003, 2009)

A. 1. By polishing the surface in contact, friction can be reduced.
2 Wheels of vehicles are provided with ball bearings to reduce the friction here ball bearing convert sliding friction to rolling friction.
3. The lubricants form a thin layer between surfaces in contact. It reduces the friction. In light vehicles or machines, oils like “three in one” are used as lubricants. In heavy machines grease is used. In addition to this they guard the mechanical parts from over heating.
4. Automobiles and aero planes are streamlined to reduce the friction due to air.

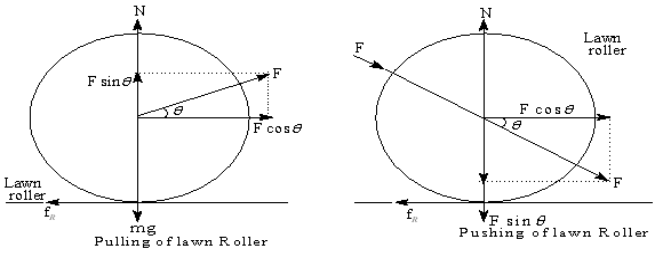
5. Sate the laws of rolling friction?

A. 1) The smaller the area of contact, a lesser will be the rolling friction.
2) The larger the radius of the rolling body, the lesser will be the rolling friction.
3) The rolling friction is directly proportional to the normal reaction.

i.e., $F_r \alpha N \quad or \quad \mu_r = \frac{F_r}{N}$

6. Why pulling the lawn roller is preferred than pushing the lawn roller? (March 2006)

A. **Pulling of lawn roller** :In the case of Pulling a lawn roller the vertical component of the applied force, $F \sin\theta$ is acting in the upward direction and reduces the normal reaction. The normal reactions becomes $N = mg - F\sin\theta$



The frictional force while pulling

$f_R = \mu_R N$

$f_R = \mu_R (mg - F\sin\theta)$ ---------- (1)

Hence less frictional force develops and hence less force is required to pull the lawn roller. In the case of Pushing the lawn roller, the vertical component of the applied force, $F\sin\theta$, is acting in the direction of weight, and increases the normal reaction. Then normal reaction becomes $N = mg + F\sin\theta$

The frictional force while pushing

$f_R^1 = \mu_R N$

$f_R^1 = \mu_R (mg + F\sin\theta)$ ---------- (2)

As $f_R < f_R^1$, less force is required to pull the lawn roller than pushing it.

Hence Pulling is easier than pushing

7. Show that velocity at the bottom of the smooth inclined plane is independent of slope of the inclined

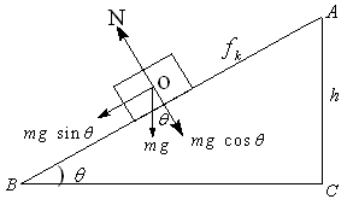
A. Consider a smooth inclined smooth plane of inclination 'θ', height 'h' and length 'L'. A block is released from the top of the inclined plane

Weight of the block is resolved into components. They are

1) Component along the incline $mg\sin\theta$, and

2) Component normal to the incline $mg\cos\theta$,

Only $mg\sin\theta$ creates acceleration of the block along the incline.



From the equation of motion

$v^2 - u^2 = 2as$

Acceleration of the block along the inclined plane

$a = \frac{mg\sin\theta}{m} = g\sin\theta$

Distance covered by the block on reaching the bottom s = L

$\therefore$ From $v^2 - u^2 = 2as$

$v^2 = 2g\sin\theta L$ (Since body released at the top, u = 0)

$= 2gL\sin\theta$

But $L\sin\theta = h$

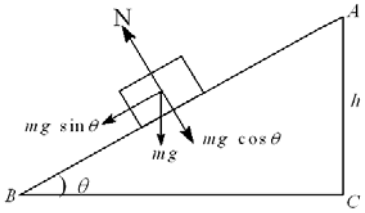
$\therefore\ v^2 = 2gh$

Or $v = \sqrt{2gh}$

$\therefore$ Velocity of the block at the bottom of the inclined plane depends only on height at which the block is released but not on inclination.

8. Find an expression for the time taken by a body projected up along a rough inclined plane of length '*l* 'with an initial velocity u.

A. **Body sliding up the plane**



If one body is projected up on the rough surface with initial velocity 'u',
Kinetic frictional force acts downward and it suffers retardation the net force acting on body is

$F = (mg\sin\theta + f_k)$

Retardation of the body

$$a = \frac{F}{m} = \frac{mg\sin\theta + f_k}{m} = \frac{mg\sin\theta + \mu_k mg\cos\theta}{m}$$

$$\Rightarrow a = g(\sin\theta + \mu_k\cos\theta)$$

When body reaches the top of the plane its final velocity becomes '0'

$V = u + at$

$$0 = u - g(\sin\theta + \mu_k\cos\theta)t \Rightarrow t = \frac{u}{g(\sin\theta + \mu_k\cos\theta)}$$

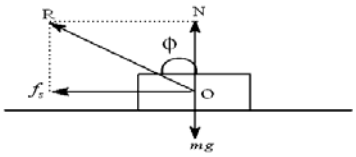
In this case, time of accent is not equal to time of descent.

9. **Define angle of friction. Obtain an expression for the coefficient of friction in terms of angle of friction?**

A. Angle of friction (ϕ):

The angle made by the resultant 'R' of normal reaction (N) and the static frictional force (f_s) with normal to the surface is called angle of friction.

Resolve the resultant force R into two perpendicular directions.



$R\cos\phi$ normal to the surface and $R\sin\phi$ along the surface, then the coefficient of static friction μ_s is defined as

$$\mu_s = \frac{f_s}{N} = \frac{R\sin\phi}{R\cos\phi} = \tan\phi$$

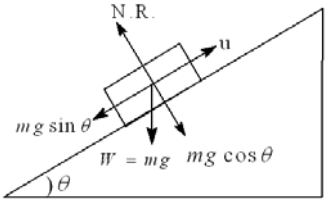
then $\tan\phi$ is called as the coefficient of static friction.

10. Find the time taken by a body projected up a smooth inclined plane with an initial velocity 'u' to travel through a distance *'l '*.

A. Consider a body is projected up along the smooth incline with an initial velocity u.

Initial velocity = u; Acceleration $a = -g\sin\theta$ (since body is moving up)

Final velocity at heighest point v = 0



1) Distance travelled up the plane S = ?

From $v^2 - u^2 = 2al$

$-u^2 = -2g\sin\theta.l$

or distance travelled upwards $l = \dfrac{u^2}{2g\sin\theta}$

2) Time taken to travel upwards t = ?

Initial velocity = u

Final velocity v = 0

Acceleration $a = -g\sin\theta$

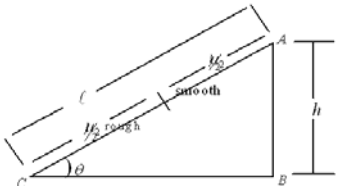
from $v = u + at$

$0 = u - g\sin\theta.t$ *or* $g\sin\theta t = u$

$\therefore$ Time taken to travel upwards $t = \dfrac{u}{g\sin\theta}$

11. The upper half of an inclined plane is smooth and the lower half rough. A block allowed to slide down from the top is brought to rest by friction just as it reaches the bottom. Find the ratio of the force of friction to the weight of the particle, assuming friction to be independent of velocity.

A.



Work done against friction (W) = $f.\dfrac{l}{2}$

P.E. at A = $mg(AB) = mgh$

P.E. at $A = mgl\sin\theta$ [$\because$ *In* Δ *ACB* $\sin\theta = \dfrac{h}{l}$]

$\therefore$ W = change in P.E.

$f.\dfrac{l}{2} = mgl\sin\theta$

$\dfrac{f}{mg} = 2\sin\theta$

$f = 2mg\sin\theta$

$\therefore \dfrac{f}{mg} = \dfrac{2mg\sin\theta}{mg} = 2\sin\theta = \dfrac{2h}{l}$

12. Explain Why

i) Roads should not be made very smooth

ii) It is not easy to write on unglazed or over a glazed paper.

iii) Tyres of motor cars and cycles should have a rough surface

iv) Brake shoes are made purposely rough

A. i) If roads are made very smooth the frictional force increases due to force of attraction between the molecules.
ii) Due to more friction it is difficult to write on unglazed and due to no friction it is difficult to write on a glazed surface.
iii) To have friction and to have grip on the road, the tyres of motor cars and cycles should have a rough surface.
iv) To have greater grip between the two surfaces in contact and to stop moving vehicle quickly brake shoes are made purposely rough.

VERY SHORT ANSWER QUESTIONS

1. Mention the factors on which frictional force depends.

A. Frictional force depends on
1) Nature of surfaces in contact
2) The force with which the surfaces are pressed
3) Nature of motion between the bodies

2. Can the coefficient of friction be greater than one?

A. Yes, In general, coefficient of friction is less than one. When the surface is heavily polished the adhesive forces between the molecules increases and then coefficient of friction will be greater than one.

3. What happens to the frictional force if the surface is moderately polished and heavily polished? (June 2002, 2003)

A. When surface is moderately polished frictional force reduces. When the surface is heavily polished the adhesive forces between the molecules increases and hence frictional force increases.

4. Why does the car with flattened tyre stop sooner than the one with inflated tyres?

A. Flattened tyre deforms more than that of the inflated tyre. Due to more deformation of the tyre the Area of contact between the surfaces increases Hence Rolling friction increases that are why the car with flattened tyre stops sooner.

5. A horse has to pull harder during the start of the motion than later. Explain? (March 2009)

A. For starting motion of the cart, the limiting friction is to be over come. Once Motion is set, the frictional force reduces therefore, the horse has to pull harder during starting of the cart.

6. Why the wheels of vehicles are fitted with ball bearings.

A. Wheels of vehicles are provided with ball bearings to reduce the friction here ball bearings convert sliding friction to rolling friction.

7. Why the coefficient of friction cannot be zero?

A. Friction between the surfaces can be reduced by polishing. If the surface are heavily polished due to the inter atomic forces the friction will increase. Even lubricants can reduce the friction but not to zero. So, friction between the surfaces cannot be reduced to zero.

8. **What is Normal reaction of the body when it is at rest on**

1) a Horizontal surface **2) an inclined surface**

A. 1) on horizontal surface Normal Reaction N = mg

2) on an inclined surface normal reaction N = $mg\cos\theta$

9. What happens to the coefficient of friction? If the weight of the body is doubled?

A. Coefficient of friction is Independent of mass of the body. Hence it remains same

10. Give examples of sliding and rolling friction.

A. Examples of sliding friction:
1) When brakes are applied suddenly during motion of a motor
2) A block is pulled on rough surface.
Examples of Rolling Friction:
1) Rotation of wheels of vehicles on roads.
2) Rotation of metallic balls in a bearing.

11. Why aeroplanes and automobiles are streamlined?

A. Automobiles and aeroplanes are streamlined to reduce the friction due to air.

12. Define coefficient of static friction. State its units and dimensions.

A. The ratio of limiting friction to the Normal Reaction is called coefficient of static friction.
i. e $\mu_s = \frac{f_s}{N}$ It has no units and dimensions

13. **Define coefficient of dynamic friction. State its units and dimentions.**

A. The ratio of kinetic friction to the Normal reaction is called coefficient of dynamic friction.
i.e. $\mu_K = \frac{f_K}{N}$. It has no units and dimensions.

14. What is the direction of dynamic friction between two bodies?

A. The dynamic friction is tangential between the two bodies and opposite to their motion.

15. What is the direction of static friction between two bodies?

A. Static frictional force is tangential between the surfaces in contact opposing them to move relative to each other.

16 What is the force required to move a body up a rough inclined plane with uniform velocity?

A. When a body is to be moved up a rough inclined plane with uniform velocity the force to be applied is

$F = mg\left[\sin\theta + \mu_k \cos\theta\right]$

ASSES YOUR SELF QUESTIONS

1. When a paper boat is kept on a flowing water surface will there be friction between the paper boat and the water surface?

Ans: Yes, if there were no friction, it cannot have been carried away by the water flow.

2. When a running athlete wants to stop immediately which force does he seek

Ans: Kinetic friction

3. Is friction a reaction of the applied force?

Ans: No

4. Does friction also occur as action-reaction pair?

Ans: Yes

5. In general, if static friction is the cause of rolling friction. How is it less than static friction?

Ans: Rolling starts before the static friction reaches its maximum value.

6. Guess the effect of temperature on coefficient of friction?

Ans: The coefficient of friction may decrease with increase in temperature.

7. What is the effect of lubrication on coefficient of friction?

Ans: Coefficient of friction decreases with lubrication.

8. What is the influence of normal reaction on the area of contact?

Ans: Area of contact increases with normal reaction.

9. How do you make a body slide down a smooth inclined plane with constant velocity?

Ans: By applying an external force F= $mg\sin\theta$ up the plane, while the body is in motion downwards.

10. How can you move the body up the smooth plane without acceleration?

Ans: By Appling an external force F= $mg\sin\theta$ up the plane, after giving an initial velocity up the plane

11. Is it possible to slide a body down a rough plane without retardation?

Ans: Yes, at By applying a suitable force up the plane.

12. How can you move a cylinder, such that it slides up a rough inclined plane without rolling?

Ans: By Appling a pulling force F= $mg(\sin\theta + \mu_k \cos\theta)$ such that the line of action of the force is parallel to the plane and passes through the centre of mass of the cylinder.

13. If a table is to be taken from one corner of a room to another corner which is better-pulling or pushing?

Ans: Pulling

14. If the angle of friction is ϕ , what will be the minimum pulling force required?

Ans: mg $\sin\phi$ where m is the mass of the body pulled on the rough horizontal surface

15. To pull a body from one place to another place, at what angle the body is to be pulled to apply least pulling force?

Ans: The angle made by the pulling force with the horizontal must be equal to the angle of friction.

Exercise 1

1. A box of mass 10kg is pulled on a horizontal rough surface by applying a horizontal force. If the coefficient of kinetic friction between the box and the horizontal surface is 0.25,find the force of friction exerted by the horizontal surface on the box.

Sol: m=10 kg,

$\mu_k = 0.25, g = 9.8ms^{-2}$

$f_k = \mu_k mg = 0.25 \times 10 \times 9.8 = 24.5N$

2. A force of 98N is required to pull a body of mass 100kg over the surface of ice. Calculate the coefficient of friction between the body and the surface of ice.

Sol: $m = 100kg,\ f_k = 98N, g = 9.8ms^{-2}$

$$\mu_k = \frac{f_k}{N} = \frac{f_k}{mg} = \frac{98}{100 \times 98} = 0.1$$

3. A block of mass 20kg is dragged along a rough horizontal surface through 1m with uniform speed. Calculate the work done if the coefficient of friction is 0.5

Sol: work done against friction = $f_k S = \mu_k mgS$

Here $\mu_k = 0.5, m = 20kg, g = 9.8ms^{-2}, S = 1m$

W $= 0.5 \times 20 \times 9.8 \times 1 = 98J$

4. When a car of mass 1000kg is moving with a velocity of $20\ ms^{-1}$ **on a rough horizontal road, its engine is switched off. How far does the car move before it comes to rest if the coefficient of kinetic friction between the road and tyres of the car is 0.75?(g=10ms^{--2}).**

Sol: When the engine of a car of mass m moving with a velocity v is switched off it comes to rest due to the force of friction, μ_k mg where μ_k the coefficient of kinetic friction is neglecting air resistance.

$$\therefore \text{Re}\,tardation \text{ of the car} = \frac{force \text{ of friction}}{mass \text{ of the car}} = \frac{\mu_k mg}{m} = \mu_k g$$

-a$=\mu_k g; a = -\mu_k g, \text{int}\,ial$ velocity u=v; final velocity,v=0

Substituting in

$$v^2 = u^2 = 2as$$

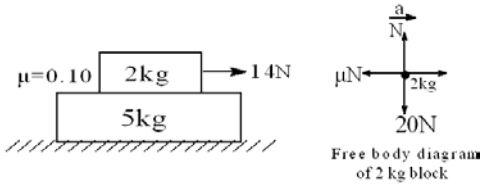
$$0\text{-}v^2 = 2(-\mu_k g)S$$

$$S = \frac{v^2}{2\mu_k g}$$

Here $v = 20ms^{-1}, \mu_k = 0.75, g = 10ms^{-2}$

$$S = \frac{(20)^2}{2 \times 0.75 \times 10} = \frac{400}{15} = 26.67m$$

5. A 2 kg block is placed over a 5kg block and both are placed on a smooth horizontal surface.The coefficient of friction between the blocks is 0.10.Find the acceleration of the two blocks if a horizontal force of 14N is applied to the upper block (g=10ms^{--2})



Sol: The forces on two kg block are

(a) gravitational force F= mg, 2g=210=20N,vertically downwards,

(b) Normal reaction N by the 5kg block, vertically upwards,

(c) force of friction f= μ N to the left and

(d) applied force 14N.

$\therefore N = 20N$

In the vertical direction, there is no acceleration.

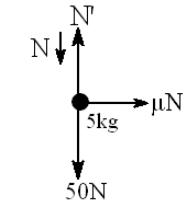
In the horizontal direction, the acceleration of the 2kg block is a.

$\therefore 14 - \mu N = 2\ a$

$14 - 0.10 \times 20 = 2\ a$ $(\because$ Resultant force = ma $)$

$14 - 2 = 2\ a;\ a = 6\ ms^2$ $(\because \mu = 0.10)$

The forces on 5kg block are (a) gravitational force
5 g = 5×10=50 N, Vertically downwards,



Free body diagram of 5 kg block

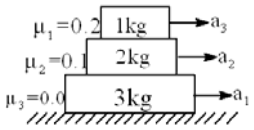
(b) Normal reaction N of the horizontal surface, vertically upwards, (c) force of friction f = μ N to the right by Newton's third law of motion and (d) normal reaction, N downwards by 2 kg block.

In the vertical direction, there is no acceleration.

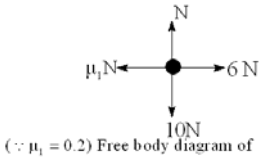
$\therefore$ N = 50 N.

6. Find the acceleration a_1, a_2, and a_3 of the three blocks shown in figure if a horizontal force of 6 N is applied on 1kg block. ($g = 10\ ms^{-2}$)

Sol:



When the force of 6 N is applied on the 1 kg block, the forces acting on it are
(a) Gravitational normal 1g = 1×10 N,
(b) Reaction N by the 2 kg block, vertically upwards,
(c) forces of friction f = μN to the left and
(d) applied force 6 N.
In the vertical direction, there is no acceleration.
$\therefore$ N= 10 N.



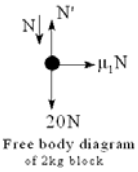
($\because \mu_1 = 0.2$) Free body diagram of 1kg block

In the horizontal direction, the acceleration of the 1 kg block is a.

$\therefore\ 6 - \mu_1 N = 1\ a$ $\qquad 6 - 0.2 \times 10 = 1\ a$

$6 - 2 = 1\ a;\ a = 4\ ms^{-2}$.

Consider the motion of 2 kg block. The forces on it are (a) gravitational force 2 g = 2 × 10 = 20 N, vertically downwards, (b) normal reaction N' by the 3 kg block, (c) force of friction f = μN to the right by Newton's third law of motion and (d) normal reaction N downwards by kg block.
In the vertical direction, there is no acceleration.



Free body diagram of 2kg block

N + 20 = N'

10+ 20 = N' ($\because$ N = 10 N)

N' = 30 N.

In the horizontal direction, force of the right is $\mu_1 N = 0.2 \times 10 = 2$ N. Limiting friction due to 3 kg block is 2 kg block moves with respect to 3 kg block is $\mu_2 N' = 0.1 \times 30 = 3$ N. As the force to the right 2 N is less than the limiting force of friction due to 3. kg, the 2 kg block does not move separately from 3 kg block. Both 2 kg and 3 kg blocks move together. The system off 2 kg and 3 kg blocks moves on the smooth horizontal force under the action of the force $\mu_1 N$ to the right.

$\therefore$ Acceleration of 2 kg and 3 kg blocks, $a' = \frac{\mu_1 N}{2+3} = \frac{0.2 \times 10}{5} = \frac{2}{5} = 0.4 \text{ ms}^{-2}$.

7. A block of mass 10 kg pushed by a force F on a horizontal rough plane moves with an acceleration of 5 ms^{-2}. **When force is doubled, its acceleration becomes** 18 ms^{-2}. **Find the coefficient of friction between the block and rough horizontal plane.**

($g = 10 \text{ ms}^{-2}$.)

Sol: On a rough horizontal plane, acceleration of a block of mass m is given by Eq.

$$a = \frac{P}{m} - \mu_k g. \qquad \text{..... (i)}$$

Initially, P = F; $a = 18 \text{ ms}^{-2}$

$$5 = \frac{F}{10} - \mu_k (10). \qquad \text{..... (ii)} \qquad (\because \text{ m = 10 kg})$$

When force is doubled i.e., P = 2F; $a = 18 \text{ ms}^{-2}$.

Substituting in Eq. (i)

$$18 = \frac{2F}{10} - \mu_k (10). \qquad \text{..... (iii)}$$

Substituting Eq. (ii) from Eq. (iii)

$$13 = \frac{F}{10}; \text{ F = 130 N.}$$

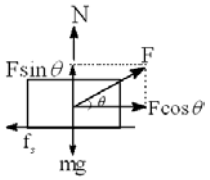
Substituting in Eq. (ii)

$$5 = \frac{F}{10} - \mu_k (10) \qquad \Rightarrow 5 = \frac{130}{10} - \mu_k (10)$$

$$10\mu_k = 13 - 5 = 8 \qquad \Rightarrow \mu_k = \frac{8}{10} = 0.8.$$

8. A pulling force making an angle θ to the horizontal is applied on a block of weight W is placed on a horizontal table. If the angle of friction is ϕ , find the magnitude of the force required to move the block.

Sol: A block of mass m is pulled by a force F making an angle θ to the horizontal. Resolving the force into horizontal and vertical components, the horizontal component is F cos θ and vertical component is F sin θ. From the free body diagram shown in the figure, in vertical direction.



$N + F \sin = mg$ .....(i) ($\because$ W = mg)

$N = W - F \sin$

and in horizontal direction

$f_s = F \cos \theta.$ .....(ii)

But $\mu_s = \frac{f_s}{N}$

$\mu_s = \frac{F \cos \theta}{W - F \sin \theta}$ .....(iii)

The angle of friction is ϕ

$\mu_s = \tan$ .....(iv)

From Eqs. (iii) and (iv)

$$\tan\phi = \frac{F \cos \theta}{W - F \sin \theta}$$

$$\Rightarrow W \tan \phi - F \sin\theta \tan\phi = F \cos\theta$$

$$\Rightarrow W \tan\phi - F \cos\theta + F \sin\theta \tan\phi$$

$$\Rightarrow W \frac{\sin\phi}{\cos\phi} = F \cos\theta + F \sin\theta \frac{\sin\phi}{\cos\phi}$$

$$W \sin\phi = F \cos\theta \cos\phi + F \sin\theta \sin\phi$$

$$= F \cos(\theta - \phi)$$

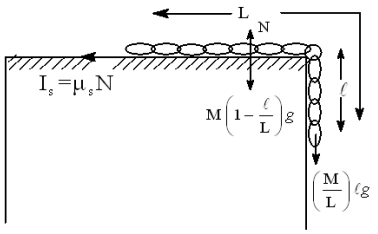
$$F = \frac{W \sin\phi}{\cos(\theta - \phi)}$$

Note: For pushing force it can be shown that $F = \frac{W \sin\phi}{\cos(\theta + \phi)}$

9. A uniform chain of length L hangs partly from a rough horizontal table. The chain is kept in equilibrium by friction. If the maximum length that can hang without slipping is ℓ find the coefficient of friction between the table and the chain.

Sol: Consider M be the mass of the chain of length L. The mass of the hanging part is $\left(\frac{M}{L}\right)\ell$.

The weight of the haning part is $\left(\frac{M}{L}\right)\ell g$. The weight of the remaining part on the table is $\left(M - \frac{M\ell}{L}\right)g = M\left(1 - \frac{\ell}{L}\right)g$. The limiting force of friction on the chain then becomes $\mu_s N = \mu_s M\left(1 - \frac{\ell}{L}\right)g$. When this is balanced by the weight of the hanging part, the resultant force becomes zero and the chain is in equilibrium.



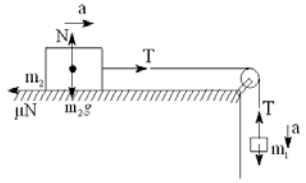
$$\mu_s\left(1-\frac{\ell}{L}\right)=\frac{\ell}{L}$$

$$\mu_s\left(\frac{L-\ell}{L}\right)=\frac{\ell}{L}$$

$$\mu_s=\frac{\ell}{L-\ell}.$$

10. Two objects of masses m_1 and m_2 are connected by a very light string passing over a clamped light smooth pulley. The object of mass m_2 is one rough horizontal table and the object of mass is hanging vertically. If the coefficient friction between the table and the object of mass m_2 is μ find the acceleration of the system.

Sol: Consider the acceleration of the system of two objects be a.
Considering the forces m_1 we can write the equation
for its acceleration 'a' as



$m_1 a = m_1 g - T.$ ..... (i)

Considering the forces on m_2,

$m_2 a = T - \mu N$

$m_2 a = T - \mu m_2 g$ ..... (ii) $(\because N = m_2 g)$

Adding Eqs. (i) and (ii), we get

$(m_1 + m_2)a = (m_1 - \mu m_2)g$

$$a=\left(\frac{m_1-\mu m_2}{m_1+m_2}\right)g.$$

11. A body of mass 100 g is sliding down an inclined plane of inclination 30°. What is the acceleration of the body if the coefficient of friction between the body and the inclined plane is 0.5?

Sol: $m = 100\ g = 0.1\ kg,\ \theta = 30^\circ,\ \mu = 0.5,\ g = 9.8\ ms^{-2}.$

Acceleration of the body is

$$a = g(\sin\theta - \mu\cos\theta)$$
$$= 9.8(\sin 30^\circ - 0.5\cos 30^\circ)$$
$$= 9.8(0.5 - 0.433) = 9.8\times 0.067 = 0.6566\ ms^{-2}.$$

12. A cube of weight 10 N rest on a rough inclined plane of slope 3 in 5. The coefficient of friction between the cube and the inclined plane is 0.6. Find the inclined plane is 0.6. Find the minimum force required to start the cube moving up the plane?

Sol: The minimum force required to start the motion of a body up the plane is

$$F = mg(\sin\theta + \mu\cos\theta).$$

Here, $mg = 10$ N; $\sin\theta=\frac{3}{5},\ \cos\theta=\frac{4}{5}, \mu=0.6$

$$F=10\left(\frac{3}{5}+0.6\times\frac{4}{5}\right)=10\left(\frac{5.4}{5}\right)=10.8\ \text{N}.$$

13. A body takes 2 times as much time to slide down a rough inclined plane as it takes to slide down an identical but smooth inclined plane. If the angle of inclination of the

inclined plane is 45° , what is the coefficient of friction between body and the rough plane?

Sol: Consider ℓ be the length of both the inclined planes and θ be the inclination. Then the times taken by a body sliding the smooth and rough inclined planes are

$$t_{smooth} = \sqrt{\frac{2\ell}{g \sin\theta}} \text{ and } t_{rough} = \sqrt{\frac{2\ell}{g(\sin\theta - \mu_k \cos\theta)}}.$$

If $t_{rough} = n\, t_{smooth}$

$$\sqrt{\frac{2\ell}{g(\sin\theta - \mu_k \cos\theta)}} = n\sqrt{\frac{2\ell}{g \sin\theta}}$$

$$\sin\theta - \mu_k \cos\theta = \frac{\sin\theta}{n^2}$$

$$\frac{\sin\theta}{\sin\theta} - \mu_k \frac{\cos\theta}{\sin\theta} = \frac{1}{n^2}$$

$$1 - \mu_k \cot\theta = \frac{1}{n^2}$$

$$\mu_k \cot\theta = 1 - \frac{1}{n^2}$$

$$\mu_k = \frac{1}{\cot\theta}\left(1 - \frac{1}{n^2}\right), \ \mu_k = \tan\theta\left(1 - \frac{1}{n^2}\right)$$

$$\mu_k = 1 - \frac{1}{n^2} \qquad (\because \theta = 45^\circ)$$

In this problem, n =2

$$\mu_k = 1 - \frac{1}{2^2} = \frac{3}{4}.$$

***14. When a horizontal force of 200 N is applied on a body, the acceleration produced is** $1.0\ ms^{-2}$. **When the force is 300 N, acceleration produced in the body is** $2.0\ ms^{-2}$. **Find the mass of the body.**

Sol: The acceleration of the body is

$$a = \frac{P}{m} - \mu_k g. \qquad \text{..... (i)}$$

Initially, P = 200 N, $a = 1.0\ ms^{-2}$.

$$1.0 = \frac{200}{m} - \mu_k g$$

When force, P= 300 N, $a = 2.0\ ms^{-2}$

$$2.0 = \frac{300}{m} - \mu_k g.$$

Substituting Eq. (ii) from Eq. (iii)

$$1.0 = \frac{100}{m}; \ m = 100 \text{ kg}.$$

15. An insect is crawling inside a hemispherical bowl of radius r. If the coefficient of friction between the bowl and insect is μ , find the maximum height to which the insect can crawl.

Sol: When the insect crawls inside a hemispherical bowl and reaches a point P, the forces on the insect are (i) the gravitational force, mg, vertically downwards, (ii) the normal reaction, N, normal to the surface of the bowl i.e., along the radius towards the centre and (iii) force of friction, μ N along the tangent.

Resolving mg into rectangular components

a) parallel to the tangent, mg $\sin\theta$ and

b) perpendicular to the tangent, mg $\cos\theta$.

Along the tangent, there is not acceleration.

$\therefore \mu N = mg \sin\theta$

Along the radius also there is no acceleration.

$N = mg \cos\theta$

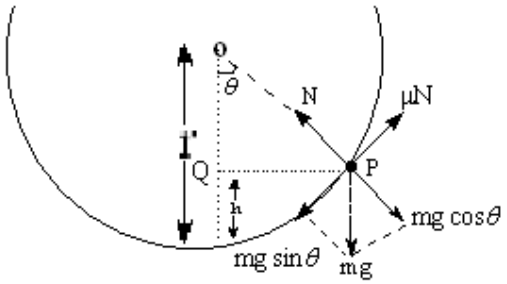
Hence $\mu mg \cos\theta = mg \sin\theta$

$\mu = \tan\theta$.

$\therefore \cos\theta = \frac{1}{\sqrt{\mu^2+1}}$ and in the Δ OPQ, $\cos\theta = \frac{r-h}{r}$.

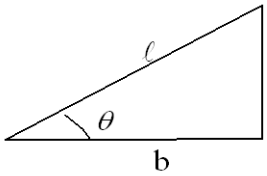
$$\frac{r-h}{r} = \frac{1}{\sqrt{\mu^2+1}}$$

$$1-\frac{h}{r} = \frac{1}{\sqrt{\mu^2+1}}; \frac{h}{r} = 1-\frac{1}{\sqrt{\mu^2+1}}; \; h = r\left(1-\frac{1}{\sqrt{\mu^2+1}}\right).$$



16. A body slides down a smooth inclined plane of base of length 'b'. If the time taken by the body to reach the bottom of the plane is to be minimum, what must be the angle of inclination?

Sol: If ℓ is the length of the inclined plane and θ is the angle of inclination of the inclined plane, the time taken by body starting from rest to reach its bottom is



$$t = \sqrt{\frac{2\ell}{g \sin\theta}}.$$

But $\cos\theta = \frac{b}{\ell}; \ell = \frac{b}{\cos\theta}$.

$$t = \sqrt{\frac{2b}{g\sin\theta\cos\theta}} = \sqrt{\frac{4b}{g \sin 2\theta}}.$$

If t is to be minimum, $\sin 2\theta$ must be maximum i.e., $\sin 2\theta = 1$. $2\theta = 90^\circ$.

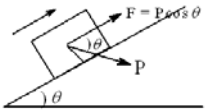
Hence $\theta = 45^\circ$.

17. A 30 kg block is to be moved up an inclined plane at an angle 30° to the horizontal with a velocity of $5\ ms^{-1}$. If the frictional force retarding the motion is 150 N find the horizontal force required to move the block up the plane. ($g = 10\ ms^{-2}$.)

Sol: The force required to body up an inclined plane is

F = mg sin θ + frictional force = 30(10) sin 30° + 150 = 300 N.

If P is the horizontal force,



F = P $\cos\theta$ $\qquad$ P = $\dfrac{F}{\cos\theta} = \dfrac{300}{\cos\theta} = \dfrac{300\times 2}{\sqrt{3}}$

$= 200\sqrt{3} = 346.4$ N.

EXERCISE PROBLEMS

1. A wooden block of 100 kg is about to be pushed on a floor of coefficient of friction 0.4.What is the magnitude of the force of friction on the wooden block when it is just pushed?

A. $m = 100kg$

$\mu = 0.4$

$f = \mu N = \mu(mg) = (0.4)(100)(9.8) = 392N$

2. A force of 147 N is required to just slide a sledge weighing 500 N over a surface of ice. Calculate the coefficient of friction between the surfaces in contact of the sledge and the ice.

A. w=mg=500N

$\mu = \frac{f}{N} = \frac{f}{mg} = \frac{147}{500} = 0.294$

3. When a car of mass 1200 kg is moving with a velocity of 15 ms^{-1} on a rough horizontal road, its engine is switched off.
How far does the car travel before it comes to rest if the coefficient of kinetic friction between the road and tyres of the car is 0.5? (g=10 ms^{-2}.)

A. m = 1200 kg

$u = 15ms^{-1}$

$g = 10ms^{-2}$

$\mu = 0.5$

We know $\mu = \frac{a}{g} \Rightarrow a = \mu g = 5ms^{-2}$ (or)

Kinetic energy =work done by frictional force

(Where a = retardation)

$\therefore s = \frac{v^2 - u^2}{2a} = \frac{225}{10} = 22.5m$

m=1200kg

v=15m/s

g=10m/s^2

$\mu = 0.5$

KE=W

$\frac{1}{2}mv^2 = \mu_k mgs$

$s = \frac{v^2}{2\mu_k g} = s = \frac{15^2}{2 \times 0.5 \times 10} = \frac{225}{10}$=22.5m

4. A train of mass 600 metric ton is pulled by a locomotive of mass 150 metric ton. The speed of the train is 54 km hr^{-1}. The locomotive pulls the train on a level track. The force of friction on the locomotive and the train 10 N per metric ton. Calculate the power of the locomotive.

A. $mass = 750 \times 10^3 kg$

speed $V = 54kmph = 15ms^{-1}$

$a = \frac{10}{1000} Nkg^{-1} = 10^{-2} ms^{-2}$

$\therefore$ Power = $(force)(velocity) = (ma)V$

$= (750 \times 10^3)(10^{-2})(15)$

= 112.5 KW

(or)

Mass of train = 600metrictons

Mass of locomotive = 150 metric tons

Total mass moving =600+150=750 metric tons

$= 750\times10^3 kg$

Total frictional force = 750×10 =7500N (10 N/metric ton)

Power P=F.V

P= $7500\times15 = 112500 = 112.5KW$

5. A car is driven round a curved path of radius 18 m without the danger of skidding. The coefficient of friction between the tyres of the car and the surface of the curved path is 0.2. What is the maximum speed in km/ph of the car for safe driving? ($g=10ms^{-2}$)

A. radius (r) = 18m

$\mu = 0.2$

for safe riding, maximum speed

$\left[\because \textit{frictional force} = \text{centripetal force}\right]$

$\mu mg = \frac{mv^2}{r}$

$V = \sqrt{\mu rg} = \sqrt{(0.2)(18)(10)}$

V $= 6ms^{-1} = 6\times\frac{18}{4} = \frac{108}{5} = 21.6kmph$

6. A body is sliding down a rough inclined plane which makes angle 30^0 with the horizontal. The coefficient of the friction between the surfaces of contact of the body and the plane is 0.25. Calculate the acceleration.

A. $\theta = 30^0,\ \mu_k = 0.25$

Acceleration (a) $= g(\sin\theta - \mu_k\cos\theta)$

$= 9.8\left[\sin 30^0 - (0.25)\cos 30^0\right]$

$= 9.8\left[\frac{1}{2} - \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right] = 9.8\left[\frac{4-\sqrt{3}}{8}\right] = 2.778ms^{-2}$

7. A body of mass 10 kg is on a rough inclined plane having an inclination of 30^0 with the horizontal. If coefficient of friction between the surface of contact of the body and the plane is 0.5 find the least force required to pull the body up.

A. least force required to pull the body up is

$F = mg\left[\sin\theta + \mu_k\cos\theta\right]$

$= 10(9.8)\left[\sin 30^0 + (0.5)\cos 30^0\right]$

$= 98\left[\frac{1}{2} + \frac{1}{2}\frac{\sqrt{3}}{2}\right] = 98\left[\frac{2+\sqrt{3}}{4}\right] = 98\left[\frac{2+1.732}{4}\right] = 91.4N$

$(\because \sqrt{3} = 1.732)$

8. A block of wood of mass 0.5 kg is placed on a plane making angle 30^0 with the horizontal. If the coefficient of friction between the surfaces of contact of the body and the plane is 0.2 what force is required to keep the body sliding down with uniform velocity?

A. m = 0.5 kg $\mu = 0.2$ $\theta = 30^0$

Force required for the body to slide down with uniform speed

$$F = mg\left[\sin\theta - \mu\cos\theta\right] = (0.5)(9.8)\left[\frac{1}{2} - \frac{(0.2)\sqrt{3}}{2}\right]$$

$$= \left[\frac{1-(0.2)\sqrt{3}}{2}\right]4.9 = \left[\frac{1-0.3464}{2}\right]4.9 = 1.6N \qquad (\because \sqrt{3} = 1.732)$$

9. A body of mass 'm' slides down a smooth inclined plane having an inclination of 45^0 with the horizontal. It takes 2 s to reach the bottom. If the body is placed on a similar plane having coefficient friction 0.5 what is the time taken for it to reach the bottom?

A. $\theta = 45^0, t_2 = 2\sec, t_1 = ? \mu_k = 0.5$

Time taken by the body to slide down on a rough inclined plane

$$t_1 = \sqrt{\frac{2l}{g\left(\sin\theta - \mu_k\cos\theta\right)}} \qquad \text{..... (1)}$$

time taken by the body to slide down a smooth inclined plane

$$t_2 = \sqrt{\frac{2l}{g\sin\theta}} \qquad \text{..... (2)}$$

From (1) and (2), $\theta = 45^0$ $= \frac{t_1}{t_2} = \sqrt{\frac{\sin 45}{\sin 45 - 0.5\cos 45}} = \sqrt{\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} - \frac{0.5}{\sqrt{2}}}} = \sqrt{\frac{1}{1-0.5}} = \sqrt{2}$

$$\frac{t_1}{t_2} = \sqrt{2} \Rightarrow \frac{t_1}{2} = \sqrt{2}$$

$$\therefore t_1 = 2\sqrt{2} = 2.828s$$

10. A wooden block is placed on an inclined plane. The block just begins to slide down when the angle of the inclination is increased to 45^0. What is the coefficient of the friction?

A. $\mu_k mg\cos\theta = mg\sin\theta$ (condition for the body just to slide down) $\theta = 45^0$

$$\Rightarrow \mu_k = \tan\theta = \tan 45^0 = 1$$

11. What is the magnitude of work done against the friction on a body of mass 'M' to travel through a distance S, if is the coefficient of friction?

A. Work done (W) = (force)(displacement) = fs= μNs

$= \mu(Mg)s = \mu Mgs$

12. Find the power of an engine which can draw a train of 400 metric tons up the inclined plane of 1in 98 at the rate of 10 ms^{-1}. The resistance due to friction acting on the train is 10 N per tone.

A. Given mass = $400\times10^3 kg$

$$\sin\theta = \frac{1}{98}$$

V = speed = $10ms^{-1}$

Retardation = $\frac{10}{1000} = 10^{-2}ms^{-2}$

Power = $mg\left[\sin\theta+\mu_k\cos\theta\right]\text{V}=mg\left[\sin\theta+\frac{a}{g}\cos\theta\right]\text{V}$

$=\left(400\times10^3\right)(9.8)\left[\frac{1}{98}+\frac{10^{-2}}{9.8}\right]10$

= 440000 W = 440 kW.

(or)

Given mass = $400\times10^3 kg$

$\sin\theta=\frac{1}{98}$

V = speed = $10ms^{-1}$

Retardation force

$f=400\times10=4000N$

$p=?$

$p=F.V$

$p=(mg\ \sin\theta+\text{f})\text{V}$

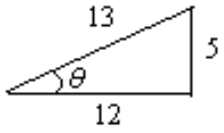
$p=(400\times10^3\times9.8\times\frac{1}{9.8}+4000).10$

$p=(40000+4000).10=440000w$

$p=440kw$

13. If the angle of inclination of the inclined plane is $\sin^{-1}\left(\frac{5}{13}\right)$ when the body just starts sliding find the angle of repose and coefficient of static friction between the body and the inclined plane

A. $\theta=\sin^{-1}\frac{5}{13}\Rightarrow\sin\theta=\frac{5}{13}$



$\mu=\tan\theta=\frac{5}{12}=0.4167$

and angle of repose $\theta=\tan^{-1}\mu=\tan^{-1}(0.4167)=42^0 43'=(22^0 24')$

14. A body of mass 60 kg is pushed up with just enough force to start it moving on a rough surface with $\mu_s=0.5$ and $\mu_k=0.4$ and the force continues to act afterwards. What is the acceleration of the body?

A. Force of static friction = $\mu_s mg=(0.5)(60)(9.8)N$

Force of kinetic friction = $\mu_k mg=(0.4)(60)(9.8)N$

Additional force needed for the body just to move F = $(\mu_s-\mu_k)mg$

ma= $(\mu_s-\mu_k)mg$

Acceleration of the body a = $(\mu_s-\mu_k)g=(0.1)(9.8)=0.98ms^{-2}$

15. A body takes $1\frac{1}{3}$ times as much time to slide down a rough inclined plane as it takes to slide down an identical but smooth inclined plane. If the angle of inclination is 45^0, find the coefficient of friction.

A. Time to slide down on a rough inclined plane $= \frac{4}{3}$ [time to slide down on a smooth inclined plane]

$$\sqrt{\frac{2l}{g(\sin\theta - \mu_k \cos\theta)}} = \frac{4}{3}\sqrt{\frac{2l}{g\sin\theta}}$$

$$\Rightarrow 16\left[\sin\theta - \mu_k \cos\theta\right] = 9\sin\theta$$

$$\Rightarrow 7\sin\theta = 16\mu_k \cos\theta$$

$$\Rightarrow \mu_k = \tan\theta\left(1 - \frac{1}{n^2}\right)$$

(or)

$$\Rightarrow \mu_k = \tan\theta\left(1 - \frac{1}{n^2}\right)$$

$$\theta = 45^0, n = 1\frac{1}{3} = \frac{4}{3}$$

$$\mu_k = \tan 45\left(1 - \frac{1}{\left(\frac{4}{3}\right)^2}\right)$$

$$\mu_k = 1\left(1 - \frac{9}{16}\right) = \frac{7}{16}$$

16. A body is sliding down an inclined plane have coefficient of friction 0.5. If the normal reaction is twice that of the resultant downward force along the incline, find the angle between the inclined plane and the horizontal.

A. $\mu = 0.5$

Given that, normal reaction = 2 (resultant downward force)

$$\Rightarrow mg\cos\theta = 2\left[mg\left(\sin\theta - \mu_k \cos\theta\right)\right] \Rightarrow \cos\theta = 2\sin\theta - 2(0.5)\cos\theta$$

$$\Rightarrow 2\sin\theta = 2\cos\theta \qquad \Rightarrow \tan\theta = 1$$

$$\therefore \theta = 45^0$$

17. A body of mass 5 kg rests on a rough horizontal surface of coefficient of friction 0.2. The body is pulled through a distance of 10 m by a horizontal force of 25N. Find the kinetic energy acquired by the body. (g = 10 ms^{-2}.)

A. Resultant acceleration of the body $a = \frac{25-10}{5} = 3ms^{-2}$

$$\therefore K.E. = work\ done = (ma)s$$

$$= 5(3)(10) = 150J$$

18. A particle is projected up along a rough inclined plane of inclination 45^0 with the horizontal. If the coefficient of friction is 0.5, calculate the retardation.

A. Given

$\theta = 45^0$

$\mu = 0.5$

Retardation

(a) $= g\left[\sin\theta + \mu_k \cos\theta\right] = g\left[\sin 45^0 + (0.5)\cos 45^0\right]$

$= g\left[\frac{1}{\sqrt{2}}+\frac{1}{2}\frac{1}{\sqrt{2}}\right]=\frac{g}{\sqrt{2}}\left[\frac{2+1}{2}\right]==\frac{3g}{2\sqrt{2}}$

19. A body is moving up an inclined plane of angle θ with an initial kinetic energy E. The coefficient of friction between the plane and the body is μ . What is the work done against friction before the body comes to rest? Express it in terms of E.

A. Given, $\frac{1}{2}mu^2 = E \Rightarrow u^2 = \frac{2E}{m}$

v = 0 as the body is coming to rest

$a = g(\sin\theta + \mu\cos\theta)$

distance travels $s = \frac{u^2}{2a} = \frac{2E}{2mg(\sin\theta+\mu\cos\theta)} = \frac{E}{mg(\sin\theta+\mu\cos\theta)}$

$\Rightarrow W = f.s$

$W = \mu(mg\cos\theta)\left[\frac{E}{mg(\sin\theta+\mu\cos\theta)}\right] \Rightarrow \frac{E\mu\cos\theta}{(\sin\theta+\mu\cos\theta)}$

20. A block of mass 2 kg rests on a rough inclined plane making an angle of 30^0 with the horizontal. If $\mu_s = 0.6$, what is the frictional force on the block?

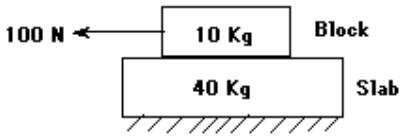
A. m = 2kg $\theta = 30^0$ $\mu_s = 0.6$

when block is at rest frictional force = $mg\sin\theta$

$mg\sin\theta = 2(9.8)\sin 30^0 = 9.8N$

21. A 40 kg slab rests on a frictionless floor. A 10 kg block rests on top of the slab. The coefficient kinetic friction between the block and the slab is 0.40. A horizontal force of 100 N is applied on the 10 kg block. Find the resulting acceleration of the slab. (g = 10 ms^{-2}.)

A. Given m = 10 kg



M = 40 kg ; $\mu_k = 0.4$

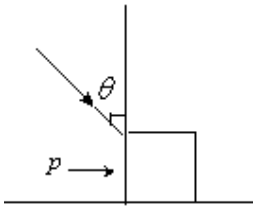
$\mu_k mg = Ma \Rightarrow a = \frac{\mu_k mg}{M} = \frac{(0.4)(10)(10)}{40} = 1ms^{-2}$

22. A block of mass m lying on a rough horizontal plane is acted upon by a horizontal force P and another force Q inclined at an angle θ to the vertical. If the block remains in equilibrium, find the coefficient of friction between the block and the horizontal plane.

A. From the figure,

Normal reaction $R = mg + Q\cos\theta$

$\therefore$ Force of friction $f = \mu R = \mu(mg + Q\cos\theta)$



Applied force along horizontal direction $F = P + Q\sin\theta$

If the body is in equilibrium, $\mu(mg + Q\cos\theta) = P + Q\sin\theta$

$$\therefore \mu = \frac{P + Q\sin\theta}{mg + Q\cos\theta}$$

23. A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of friction between the block and the surface is 0.5. If the acceleration of the truck is 6 ms^{-2} find the acceleration of the block.

A. mass of the block (m) = 1 kg

$\mu = 0.5$

Acceleration of the truck = $6ms^{-2}$

$a_{block} = a_{truck} - \mu_g$

If $g = 10ms^{-2}$, then $a_{block} = 6 - (0.5)(10) = 1ms^{-2}$

24. A block of mass 2 kg lying on ice when given a velocity of 6 ms^{-1} is stopped by friction in 5 s. Find the coefficient of friction between the block and ice. (g = 10 ms^{-2}.)

A. mass (m) = 2 kg

$u = 6ms^{-1}$

v = 0

t = 5s

Retardation (a) = $\frac{6}{5} = 1.2ms^{-2}$ $\therefore \mu = \frac{a}{g} = \frac{1.2}{10} = 0.12$

25. A block A of mass 3 kg and another block B of mass 2 kg are connected by a light inextensible string as shown in figure. If the coefficient of friction between the surface of the table and A is 0.5. What maximum mass C is to be placed on A so that the system is to be in equilibrium?

A. For the system to be in equilibrium, force of friction of A and C must be equal to the weight of B.

$\mu(M_A + M_C)g = M_B g = 5(3 + M_C)g = 2g = 1.5 + 0.5M_C = 2 = 0.5M_C = 0.5$

$$M_C = \frac{0.5}{0.5} = 1kg$$

