CENTRE OF MASS

POINTS TO REMEMBER

1. If each point on a body experiences same displacement so that the motion of any particle represents the motion of the whole body is called translator motion.
2. The centre of mass is that fixed point of a system of particles or a rigid body with in the boundaries of the system where the entire mass of the system of the body is supposed to be concentrated.
3. The point inside a body where the total weight of body appears to be concentrated is called centre of gravity
4. In a uniform gravitational field centre of mass and centre of gravity of a system coincide. i.e. For small objects c.m. and c.g. coincide but for large or extended objects like hills, buildings they do not coincide.
5. Internal forces, however strong they are, can not produce acceleration in centre of mass.
6. If no external force acts on a system, the acceleration of centre of mass is zero, i.e. the velocity and momentum of the centre of mass remains constant, though velocity and momentum of individual particles vary.
7. The algebraic sum of moments of masses of all the particles about the centre of mass is always zero.

8. $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + ... + m_nx_n}{m_1 + m_2 + ... + m_n}$

9. $y_{cm} = \dfrac{m_1y_1 + m_2y_2 + ... + m_ny_n}{m_1 + m_2 + ... + m_n}$

10. $z_{cm} = \dfrac{m_1z_1 + m_2z_2 + ... + m_nz_n}{m_1 + m_2 + ... + m_n}$

11. $v_{cm} = \dfrac{m_1v_1 + m_2v_2 + ... + m_nv_n}{m_1 + m_2 + ... + m_n}$ $M_{cm} = \sum m_1x_1$

12. $a_{cm} = \dfrac{m_1a_1 + m_2a_2 + ... + m_na_n}{m_1 + m_2 + ... + m_n}$ $a_{cm} = \dfrac{1}{M}(\sum m_1a_1)$

13. If a shell explodes in mid air as it moves, the fragments of the shell move in different parabolic paths, but the centre of mass of the shell continues to move in the same parabolic path as the shell, as a single piece, would have moved.
14. **Pappu's theorems**

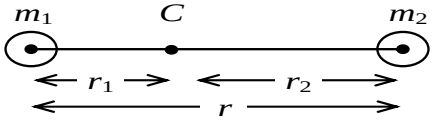
First theorem: volume, V = S $(2\pi X_{cm})$

Where, V - volume generated by the revolution of closed plane area about an axis
S - Area of the plane
$2\pi x_{cm}$ - Circumference of the circle described by center of mass of the plane

Second theorem:

$X_{com} = \dfrac{2R}{\pi}$, Where X_{cm} Position of CM, R - Radius of half circular bent wire.

15. Two particles of masses m_1, m_2 are separated by a distance r, and then distance of centre of mass is



$r_1 = \dfrac{m_2r}{m_1 + m_2}$ and $r_2 = \dfrac{m_1r}{m_1 + m_2}$

SHORT ANSWER QUESTIONS:

1. Mention the characteristics of centre of mass

[March-2011, March-2009, March-2008, June - 2003, March-2003]

Ans: 1.There need not be any mass at the centre of mass (Ex.: Hollow sphere)
2. Internal forces can not change the position of centre of mass.
3. If the line of action of force passes through the centre of mass of a body, then if undergoes translatory motion.
4. If no external force acts on a system, the acceleration of centre of mass is zero, the velocity and momentum of the centre of mass remains constant, though velocity and momentum of individual particles vary.
5. The algebraic sum of moments of masses of all the particles about the centre of mass is always zero.
6. The position of center of mass depends on the shape of the body and distribution of mass.
7. The location of the centre of mass is independent of the reference frame used to locate it.
8. The centre of mass of a system of particles depends only on the masses of the particles and their relative positions.

2. Show that the momentum of a system of particles is equal to the sum of momenta of individual particles comprising the system.

Ans: Consider the motion of a system of n particles of masses m_1, m2 ----- m_n with position vectors $\vec{r}_1, \vec{r}_2 ---\vec{r}_n$ respectively. At an instant of time't' the position vector of centre of mass of the system is

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ---+ m_n\vec{r}_n}{m_1 + m_2 + ---+ m_n}$$

The velocity of the Center of mass of the system is $\frac{d\vec{r}_{cm}}{dt} = \vec{V}_{cm}$

$$\therefore \vec{V}_{cm} = \frac{m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + ----+ m_n\frac{dr_n}{dt}}{m_1 + m_2 + ---+ m_n} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2 + ---+ m_n\vec{V}_n}{M}$$

$$\therefore \vec{V}_{cm} = \frac{(m_1\vec{V}_1 + m_2\vec{V}_2 + --m_n\vec{V}_n)}{M}$$

Where V_1, V2 ----- V_n are the velocities of system of particles of masses m_1, m2----- m_n

And, mass of the system $M=m_1+m_2+---------+m_n$.

Or $$V_{cm} = \frac{p_1 + p_2 + -----+ p_n}{M}$$

$$V_{cm} = \frac{\sum p_n}{M}$$

$$\therefore MV_{cm} = \sum p_n$$

Where $\sum p_n$ is the total momentum of 'n' particles. Hence the total linear momentum of the system of particles is equal to the sum of momenta of individual particles comprising the system.

3. Distinguish between centre of mass and centre of gravity

Ans:

Centre of mass	**Centre of Gravity**

1.The centre of mass of a body is an imaginary point at which the entire mass of the body is concentrated	1. The centre of gravity of a body is the point fixed relative to the body, through which the weight of the body always acts.
2. If refers to the mass of the body	2. If refers to the weight acting on all particles of the body.
3.For small and regular bodies centre of mass and centre of gravity will coincide	3. For larger bodies centre of gravity and centre of mass do not coincide.
4. It does not depend on acceleration due to gravity	4. It depends on acceleration due to gravity

4. Show that a system of particles moves under the influence of an external force as if the force is applied at its centre of mass.

Ans: Let V1, V2----V_n are the velocities of system of the particles of mass m_1, m2, -----m_n and M= m_1+m_2+-----+m_n. The velocity of centre of mass of the system is given by

$$\overrightarrow{V_{cm}} = \frac{1}{M}\left(m_1\vec{v}_1 + m_2\vec{v}_2 + - - - + m_n\vec{v}_n\right)$$

When an external force is applied on the system, the centre of mass moves with an acceleration $\overrightarrow{a_{cm}}$ which is given by

$$\overrightarrow{a_{cm}} = \frac{d\overrightarrow{V_{cm}}}{dt} = \frac{1}{M}(m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + - - - + m_n\frac{d\vec{v}_n}{dt})$$

$$\overrightarrow{a_{cm}} = \frac{1}{M}\left(m_1\vec{a}_1 + m_2\vec{a}_2 + - - - + m_n\vec{a}_n\right)$$

But, $\overrightarrow{a_{cm}} = \frac{1}{M}\left[\vec{F}_1 + \vec{F}_2 + - - - - - + \vec{F}_n\right]$

$$\therefore M\overrightarrow{a_{cm}} = \sum\vec{F}_n = \vec{F}_{ext}$$

Hence the system of particles moves under the influence of an external force as if the force is applied at its centre of mass

5. Explain about the Centre of mass of Earth - moon system and its rotation around the sun.

Ans: Though we say that moon revolves around the earth it is proper to mention that the earth –moon system rotates about the common centre of mass. Since the mass of the earth is about 81 times that of the moon, the centre of mass of the earth-moon system is relatively very near to the centre of the earth.

The interaction of the earth and moon does not influence the motion of the centre of mass of the earth - moon system. The gravitational force attraction of the sun is the only external force that acts on the earth-moon system, and the centre of mass of the earth-moon system moves in an elliptical path around the sun.

6. State the theorems of Pappu's to locate centre of mass of bodies whose formulae for surface and Volumes are known.

Ans: Pappu's theorems are used to locate position of centre of mass of bodies whose formulae for surface areas and volumes are known.

I law: The Volume 'V' generated as a result of the revolution of a closed plane area about an axis

(such that every point moves perpendicular to the plane) equals to the area S of the plane times the circumference $\left(2\pi x_c\right)$ of the circle described by the centre of mass of the plane. i.e, V=S$\left(2\pi x_c\right)$

II law : The surface area S generated by revolution of a plane curved line about an axis such that every point moves perpendicular to the line of the curve is equal to the product of length of the line and the distance through which the centre of mass moves.

$S = 4\pi r^2 = (\pi R)\, X\, (2\pi x_c)$

7. Find the center of mass of a system of masses 1kg,2kg,3kg and 4kg located at (0,0),(1,0),(1,1) and (0,1).

Ans. $X_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4} = \dfrac{1(0) + 2(1) + 3(1) + 4(0)}{1+2+3+4} = \dfrac{1}{2}m$

$Y_{cm} = \dfrac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4} = \dfrac{1(0) + 2(0) + 3(1) + 4(1)}{1+2+3+4} = 0.7\ m$

VERY SHORT ANSWERS QUESTIONS

1. Under what conditions centre of mass and centre of gravity of a body coincide? (May2007, March2007, June2005)

Ans: For small and regular bodies centre of mass and centre of gravity coincide because the acceleration due to gravity is same on all particles of the body.

2. Is it necessary that any mass should be present at the Centre of mass of a system?

Ans: No, In the case of uniform circular ring the centre of mass lies at the centre of the ring where there is no mass.

3. In a diagram, the weight of an extended body is shown to act from the Centre of mass. Does it mean that the other particles are not attracted by the earth? What is the relation between the weight of the body and that of all other particles?

Ans: No. All the particles are attracted by the earth. The weight of the body is equal to sum of the weight of all other particles.

4. In which of the following cases the centre of mass of a body coincides with its geometric centre of body?

a) The density continuously decreases from right to left

b) The density decreases from left to right up to the Centre and then increase.

Ans: a) If the density of the body continuously decreases from right to left, the mass distribution becomes non-uniform. Hence centre of mass and geometric centre will not coincide.

b) If the density decreases from left to right up to the Centre and then increase, the mass distribution will be same on both sides .Hence Centre of mass and geometric Centre coincide.

5. Are the following expressions valid for homogeneous pieces of masses?

a) $X_{cm} = \dfrac{A_1x_1 + A_2x_2 + ----A_nx_n}{A_1 + A_2 + ---A_n}$, **where A_1,A_2,-------A_n are the areas of the homogeneous pieces and x_1,x_2,x_3,---------x_n are their positions respectively.**

b) $X_{cm} = \dfrac{V_1x_1 + V_2x_2 + ----V_nx_n}{V_1 + V_2 + ---V_n}$,**where v_1,v_2,v_3,---v_n are the volumes of the homogeneous pieces and x_1,x_2,---x_n are there positions respectively**

Ans a). Mass = volume x density = Length x Area of cross - section x density = ℓ A ρ

Mass α Area only with ℓ and ρ as constant and the expression

$X_{cm} = \frac{A_1x_1 + A_2x_2 + ----A_nx_n}{A_1 + A_2 + ----a_n}$ is valid with the pieces of same length

b) As mass α volume for homogeneous pieces the given expression

$X_{cm} = \frac{V_1x_1 + V_2x_2 + ----V_nx_n}{V_1 + V_2 + ---V_n}$ is valid

6. A boy stands at one end of a boat that is stationary relative to the shore. He starts to walk towards the opposite end of the boat away from the same. Does the boat move? If it moves in what direction does it move?

Ans: Here boy and boat together are considered as single system. The boat moves in a direction opposite to that of the motion of the boy, but the position of the centre of mass of the system remains the same,.

7. A uniform wire is bent into the form of a rectangle with length L and width W. If two of the sides coincides with x and y axes. What are the coordinates of centre of mass**?**

Ans: The centre of mass of the bent wire lies at the geometric centre of the rectangular frame. The coordinates of centre of mass are $\left[\frac{L}{2}, \frac{W}{2}\right]$

8. Choose a large bound book

a). It is placed on the table, locate its centre of mass

b).If it is made to stand vertically on the table will the centre of mass remains same.

c). If it is placed on the table and open it such that half the number of papers are to one side and half to the other, where will the centre of mass be now approximately? If 3/4th of the papers are to one side, to which side the center of mass will shift?

Ans: Let the length, breadth and thickness of a large bound book are ℓ,b and t respectively.

a).When the book is placed on the table, the centre of mass is located at a height $\frac{t}{2}$ from the surface of the table.

b) No. When the book is made to stand vertically on the table the centre of mass is located at a height $\ell/2$ from the surface of the table.

c). When the book is open

i). With half the number of papers are to one side and other half of the other side, then the centre of mass of the system is at a height of $\frac{t}{4}$ from the surface of the table.

ii). If $\frac{3}{4}$ th of the papers are to one side, then the centre of mass of the system shifts to the side where there are $\frac{3}{4}$ th of the papers.

9. Can we change the period of oscillation of a swing by sitting and standing on the plank of swing while swinging? Explain.

Ans: $T \alpha \sqrt{l}$. The time period of oscillation of a swing depends on the length of the swing. The length of the swing is the length between the point of suspension and the center of

gravity of the plank. By sitting and standing on the plank of swing while swinging, the length and there by time period changes.

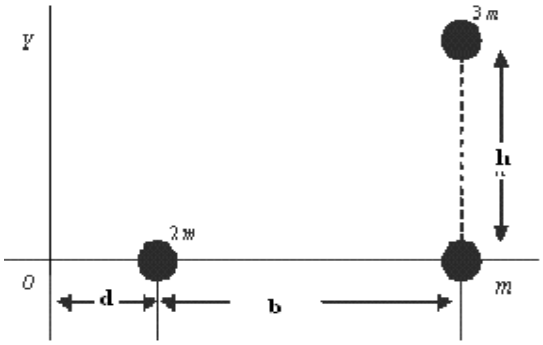
10. Define centre of mass and centre of gravity. (May2008)

Ans. **Centre of mass:** The centre of mass of a body is an imaginary point at which the entire mass of the body is concentrate

Centre of gravity: The centre of gravity of a body is the point fixed relative to the body, through which the weight of the body always acts.

Exercise 1

1. A system consists of three particles located at the corners of a right triangle as shown in figure; Find the centre of mass of the system.



Ans: $x_c \frac{\sum m_i x_i}{M} = \frac{2md + m(b+d) + 3m(d+b)}{6m} = d + (2/3)b$

$$y_c \frac{\sum m_i y_i}{M} = \frac{2m(0) + m(0) + 3mh}{6m} = h/2$$

$$r_c = x_c i + y_c j + z_c k = \left(d + \frac{2}{3}b\right)i + \frac{h}{2}j.$$

2. The position vector of three particles of mass $m_1 = 1kg$, $m_2 = 2kg$, $m_3 = 4kg$ are $r_1 = (i + 4j + k)m, r_2 = (i + j + k)m$ and $r_3 = (2i - j - 2k)m$ respectively . Find the position vector of their centre of mass.(March 2010)

Ans: The position vector of centre of mass of the system of particles is given by

$$r_c = \frac{m_1 r_1 + m_2 r_2 + m_3 r_3}{m_1 + m_2 + m_3}$$

$$\therefore r_c = \frac{1(i+4j+k)m + 2(i+j+k)m + 4(2i-j-2k)m}{1+2+3+4} = \frac{(11i+2j-5k)m}{7} = \frac{1}{7}(11i+2j-5k)m$$

3. Two 3 kg masses have velocities $V_1 = 2i + 3j\ m/s$ and $v_2 = 4i - 6j\ m/s$ Find a) Velocity of centre of mass , b) The total momentum of centre of mass, c) The velocity of centre of mass after 5 seconds of application of a constant force $F = 24iN$, d) Position of centre of mass after 5 seconds if it is at the origin at .

Ans: a) Velocity of centre of mass, $V_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

$m_1 = m_2 = 3kg, v_1 = 2i + 3j\ m/s$ and $v_2 = 4i - 6j\ m/s$

$$V_c = \frac{3(2i+3j) + 3(4i-6j)}{6} = 3i - 1.5j$$

b) Momentum of the system = (Mass of the system) x (Velocity of centre of mass)

$Mv_c = 6(3i - 1.5j) = 18i - 9j\ kgms^{-1}$

c) The velocity of the centre of mass after 5 seconds of application of the force $F = 24i\ N$

Acceleration of the centre of mass $a_c = \frac{F}{M} = \frac{24i}{6} = 4i\ ms^{-1}$

The velocity of centre of mass before the force is applied is V_c

$v_o = v_c$ And from the equations $v_c^{\,1} = v_c + a_c.t$

$v_o^{\,1} = (3i - 1.5j) + 4i \times 5 = (3i - 1.5j + 20i) = (23i - 1.5j)ms^{-1}$

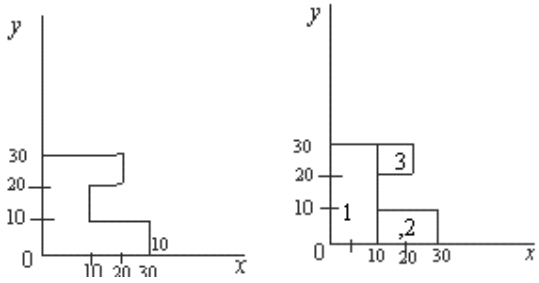
d) From the equation of the position vector $r = r_o + v_o t + \frac{1}{2}at^2$

where $r_o = 0$ (origin at t = 0); $v_0 = v_c a = a_0$ *and* $t = 5s$

$r = (3i - 1.5j).5 + \frac{1}{2}4i.25 = (15i - 75j + 50i)m = (65i - 7.5j)m$

The coordinates of the centre of mass after 5 seconds of application of the force F are $(65m, -7.5m)$

4. A uniform piece of metal sheet is cut in to the form as shown in the figure locate the centre of the mass of the piece



Sol: Let the mass per unit area be σ. The metal sheet is divided in to three parts 1,2 and 3. The mass of the first part is 300σ , mass of the second is 200σ and mass of the third is 100σ

The coordinates of centre of mass of part 1 are (5,15); that of part 2 are (20,5); and that of part 3 are (15,25).

$$X_{cm} = \frac{(300\sigma)5 + (200\sigma) + (100\sigma)15}{600\sigma} \approx 11.7$$

$$Y_{cm} = \frac{(300\sigma)15 + (200\sigma)(5) + (100\sigma)(25)}{600\sigma} \approx 13.3$$

In vector notation $r_{cm} = 11.7i + 13.3j$

Exercise 2

1. Obtain the centre of mass of a system of 4 particles 1 kg, 2 kg, 3 kg and 4 kg respectively located at the corners of square of 1 m side. Take origin at 1 kg, 2 kg and 3 kg are on X and Y–axis

Sol. Co-ordinates of the particles of masses 1 kg, 2 kg, 3 kg and 4 kg are (0, 0),(1, 0),(0, 1) and (1, 1) m

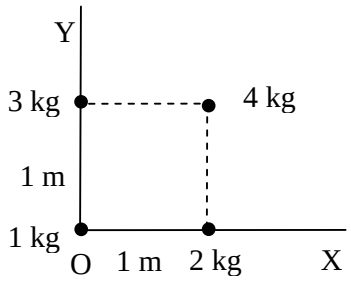
$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{1(0) + 2(1) + 3(0) + 4(1)}{1 + 2 + 3 + 4} = 0.6\ \text{m}$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$$

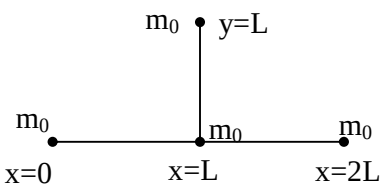
$$= \frac{1(0) + 2(0) + 3(1) + 4(1)}{1 + 2 + 3 + 4} = 0.7\ \text{m}$$

$(X_{cm}, Y_{cm}) = (0.6\ \text{m}, 0.7\ \text{m})$



2. Find the centre of mass of the system

Sol. Co-ordinates of the particles of masses are (0, 0) (L, 0) (2L,0) (L, L)



$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4} = \frac{m_0(0) + m_0(L) + m_0(2L) + m_0(L)}{m_0 + m_0 + m_0 + m_0} = \frac{4m_0L}{4m_0} = L$$

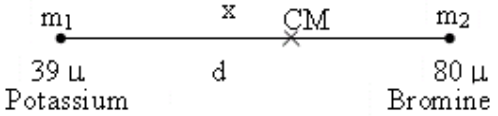
$$Y_{cm}=\frac{m_1y_1+m_2y_2+m_3y_3+m_4y_4}{m_1+m_2+m_3+m_4}=\frac{m_0(0)+m_0(0)+m_0(0)+m_0(L)}{m_0+m_0+m_0+m_0}=\frac{m_0L}{4m_0}=\frac{L}{4}$$

$$\therefore (X_{cm}, Y_{cm})=\left(L,\frac{L}{4}\right)$$

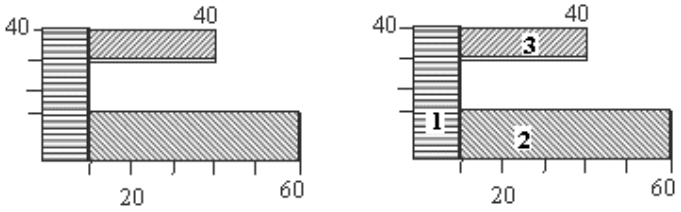
3. If the distance between the centers of atoms of potassium and bromine in KBr (Potassium Bromide) molecule is 0.282×10^{-9} m. Find the centre of mass of this two particle system from Potassium. (Mass of bromine = 80 u and mass Potassium = 39 U).

Sol: $X_{cm}=\frac{m_2}{m_1+m_2}d=\frac{80}{39+80}\times0.282\times10^{-9}$ m

=0.1895 nm.



4. A uniform sheet of steel is cut into the shape as shown. Compute the x and y coordinates of the centre of mass of the piece?



Sol: Let the mass per unit area of the sheet be λ. The metal sheet is divided in to three parts 1,2 and 3. The mass of the first part is 400 λ , mass of the second is 500 λ and mass of the third is 300 λ. The coordinates of centre of mass of part 1 are (5, 20); that of part 2 are (35, 5); and that of part 3 are (25, 35)

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$$X_{cm}=\frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}=\frac{400\lambda\times5+500\lambda\times35+300\times25}{400\lambda+500\lambda+300\lambda}=\frac{(20+175+75)100\lambda}{1200\lambda}=22.5$$

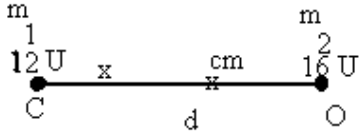
$$Y_{cm}=\frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3}=\frac{400\lambda\times20+500\lambda\times5+300\lambda\times35}{400\lambda+500\lambda+300\lambda}=\frac{(80+25+105)100\lambda}{1200\lambda}=17.5$$

$$(X_{cm}, Y_{cm})=(22.5, 17.5)$$

5. Find the position of centre of mass of carbon monoxide (CO) relative to carbon atom if the distance between the centre of the carbon and oxygen atoms is 1.130 × 10–10 m.

Sol: The masses of Carbon and Oxygen atoms are m_1 and m_2 with their distance of separation equal to'd'.

Let'x' be the distance od CM of the system from the Carbon atom, then



$$X=\frac{m_2d}{m_1+m_2}=\frac{16\times1.130\times10^{-10}}{12+16}=6.46\times10^{-11}\text{ m}$$

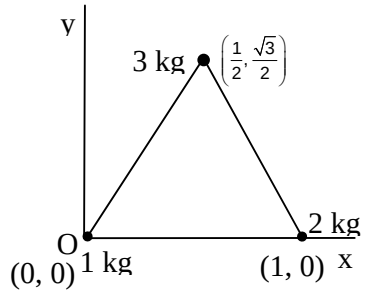
6. Find the position of centre of mass of the system of 3 objects of masses 1 kg, 2 kg and 3 kg located at the corners of an equilateral triangle of side 1 m. Take 1 kg mass object at the origin and 2 kg is along x–axis.

Sol: The masses and their positions are 1 kg (0, 0), 2 kg (1,0) and 3 kg $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{1(0) + 2(1) + 3\left(\frac{1}{2}\right)}{1+2+3} = \frac{7}{2}$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{1(0) + 2(0) + 3\left(\frac{\sqrt{3}}{2}\right)}{1+2+3} = \frac{\sqrt{3}}{4}$$

$$(X_{cm}, Y_{cm}) = \left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$$



7. A body of mass 4 kg is attached to another body of mass 2kg with a mass less rod. If 4 kg mass is at $\left(2\hat{i}+5\hat{j}\right)$ and 2 kg mass is at $\left(4\hat{i}+2\hat{j}\right)$m, find the centre of mass of that system.

Sol: $\vec{x}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \frac{(4+4)\hat{i} + (10+2)\hat{j}}{3} = \frac{1}{3}\left[8\hat{i} + 12\hat{j}\right]$

8. Find the (i) Instantaneous position and(ii) Velocity of the CM of a system of particles of masses 1 kg and 3 kg which are at $(2\hat{i}+5\hat{j}+13\hat{k})$m and $(-6\hat{i}+4\hat{j}-2\hat{k})$m possessing velocities $(10\hat{i}-7\hat{j}-3\hat{k})ms^{-1}$ and $(7\hat{i}+9\hat{j}+6\hat{k})ms^{-1}$ respectively.

Sol: The instantaneous position of the centre of mass is given as

i) $\vec{x}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$

$= \frac{1}{(1+3)}\left[1(2\hat{i}+5\hat{j}+13\hat{k}) + 3(-6\hat{i}+4\hat{j}-2\hat{k})\right] = = \frac{1}{4}\left[-16\hat{i}+17\hat{j}+7\hat{k}\right]$m

ii) The velocity of the centre of mass of the system

$\vec{V} = \frac{1}{m_1 + m_2}[m_1\vec{v}_1 + m_2\vec{v}_2]$

$= \frac{1}{(1+3)}[1(10\hat{i}-7\hat{j}-3\hat{k}) + 3(7\hat{i}+9\hat{j}+6\hat{k})] = \frac{1}{4}(31\hat{i}+20\hat{j}+15\hat{k})\ ms^{-1}$

9. If the centre of mass of three particles of masses 1 kg, 2 kg, 3 kg is at (2, 2, 2), then where should a fourth particle of mass 4 kg will be placed so that the combined centre of mass be at (0, 0, 0).

Sol: $x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$

$\Rightarrow 0 = \frac{1\times2 + 2\times2 + 3\times2 + 4\times x_4}{1+2+3+4} \Rightarrow x_4 = -3$

Similarly y_4 and z_4 -3 and -3

Hence the position of the fourth particle is at = $(x_4, y_4, z_4) = (-3, -3, -3)$

10. Two spheres of masses 4 kg and 8 kg are moving with velocities 2 ms^{-1} and 3 ms^{-1} away from each other along the same line. Find the velocity of the centre of mass.

Sol: $V_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{4(-2) + 8(3)}{(4+8)} = \frac{4}{3}$

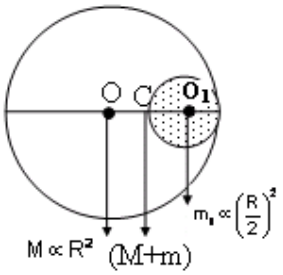
The velocity centre of mass is 4/3 ms^{-1} towards the second sphere.

11. Two bodies of masses 10 kg and 20 kg are located in the X-Y plane at (0, 1) and (1, 0). Find the position of their centre of mass.

Sol: $\vec{x}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = = \frac{10(0) + 20(1)}{(10 + 20)} = \frac{2}{3}$

$Y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{10(1) + 20(0)}{(10 + 20)} = \frac{1}{3}$

12. A uniform disc of radius R/2 is put over another uniform disc of radius R of the same thickness and density. The peripheries of the two discs touch each other. Locate the centre of mass of the system.



Sol: $M \propto R^2$

$M(OC) = m(CO_1)$

$R^2(X) = \left(\frac{R}{2}\right)^2\left(\frac{3R}{4} - x\right) \Rightarrow 4x = \frac{3R}{4} - x \Rightarrow x = \frac{3R}{20}$

The new centre of mass the system is at a distance of $\frac{3R}{20}$ from the centre of the bigger disc towards the smaller disc.

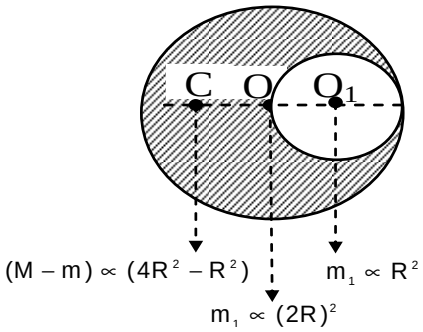
13. A disc of radius R is cut out from a larger disc of radius 2R in such a way that the edge of the hole touches the edge of the disc. Locate the centre of mass of the remaining portion.

Sol: Mass = $\pi R^2 t d$ Or $M \propto R^2$

$(M - m)CO = m(OO_1) \Rightarrow (4R^2 - R^2)x = R^2 \times R$

$\therefore x = \frac{R}{3}$

The centre of mass the remaining portion is at a distance of $\frac{R}{3}$ from the centre of the larger disc away from the centre of the hole.



14. A circular disc of diameter d and a square plate of side'd' are placed as shown in the figure. Locate the centre of mass of the combination if both the objects are having same mass per unit area.

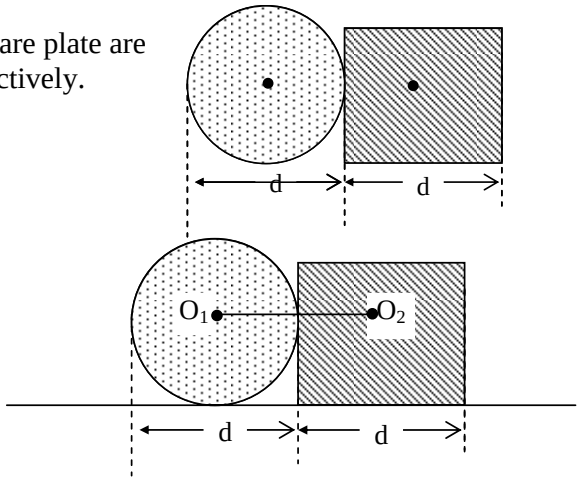
Sol: Let the masses of the circular disc and the square plate are m_1 and m_2 and their areas are A_1 and A_2 respectively.

$\frac{m_1}{A_1} = \frac{m_2}{A_2} \Rightarrow \frac{m_1}{m_2} = \frac{A_1}{A_2} = \frac{\pi\left(\frac{d}{2}\right)^2}{d^2} = \frac{\pi}{4}$

The centre of mass 'C' lies on O_1O_2

$m_1 (O_1C) = m_2 (O_2C)$

$m_1x = m_2 (d - x) \Rightarrow \frac{d - x}{x} = \frac{m_1}{m_2} = \frac{\pi}{4}$



$$\therefore x = \frac{4d}{\pi + 4}$$

15. The separation between the centers of hydrogen and chlorine atoms in HCl molecule is about 0.13 nm. Locate the centre of mass of the molecule with respect to the centre of Hydrogen atom. The atomic masses of H and Cl are 1 and 35.4 respectively

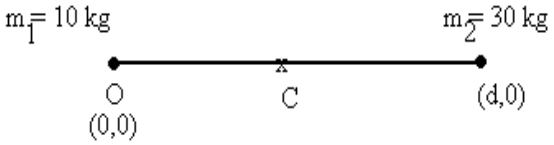
Sol: $x = \frac{m_2}{m_1 + m_2} d = \frac{(35.4)(0.13)}{1 + 35.4} = 0.126\,nm$

16. Two blocks of masses 10 kg and 30 kg are placed on the x–axis. The first mass is moved on the axes by a distance of 2 cm. By what distance should second mass be moved to keep the positions of the centre of mass unchanged.

Sol: $X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

$$\therefore X_{CM} = \left(\frac{m_2}{m_1 + m_2}\right) d = \left(\frac{30}{10 + 30}\right) d = \frac{3}{4} d$$

$$10\left(\frac{3}{4} d - 2\right) = \left(\frac{1}{4} d - x\right) 30 \Rightarrow x = \frac{2}{3}\,cm$$



17. Two blocks of masses 10 kg and 30 kg are placed along a vertical line. If the first block is raised through a height of 7 cm, by what distance should the second mass be moved to raise the centre of mass by 1 cm.

Sol: **Let** C be the CM of the system

(i) $10 d_1 = 30(d - d_1)$

C_1 is the new CM

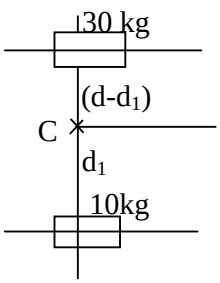
(ii) $10[d_1 - 7 + 1] = 30\left[(d - d_1) - 1 - y\right]$

$10 d_1 - 60 = 30(d - d_1) - 30 - 30y$

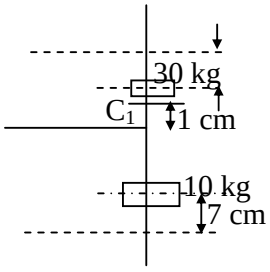
From equation (1)

$-60 = -30 - 30y$ Or $-30 = -30y$ Or $y = 1$ cm

Hence 30 kg mass is moved down by 1 cm.



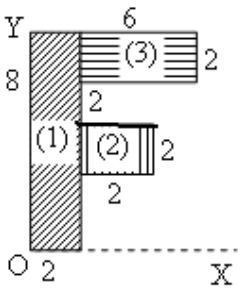
Fig(i)



Fig(ii)

8. Find the centre of mass of the letter F which was cut from a uniform metal sheet

Sol: Let the mass per unit area of the sheet be λ. The metal sheet is divided in to three parts 1,2 and 3. The mass of the first part is 16 λ , mass of the second is 4 λ and mass of the third is 8 λ. The coordinates of centre of mass of part 1 are(1,4); that of part 2 are (3, 3); and that of part 3 are (4, 7).



Position of CM $X_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{16\lambda + 12\lambda + 32\lambda}{28\lambda} = \frac{15}{7}$

$$Y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{64\lambda + 12\lambda + 56\lambda}{28\lambda} = \frac{33}{7}$$

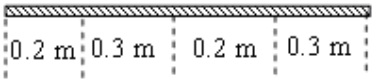
9. A 1 m long rod having a constant cross-sectional area is made of four materials. The first part 0.2 m long is made of iron, the next part 0.3 m long is of lead,

followed by aluminium of 0.2 m long and the remaining part is made of copper. Find the centre of mass of the rod. The densities of iron, lead, aluminium and copper are 7.9×10^3 kgm^{-3}, 11.4×10^3 kgm^{-3}, 2.7×10^3 kgm^{-3} and 8.9×10^3 kgm^{-3} respectively.

Sol: length mass = length × area of cross-section × density

1 part (iron)	0.2 m	$m_1 = 0.2\,A \times 7.9 \times 10^3 = 1.58\,A \times 10^3$ kg
2 part (lead)	0.3 m	$m_2 = 0.3\,A \times 11.4 \times 10^3 = 3.42\,A \times 10^3$ kg
3 part (aluminum)	0.2 m	$m_3 = 0.2\,A \times 2.7 \times 10^3 = 0.54\,A \times 10^3$ kg
4 part (copper)	0.3 m	$m_4 = 0.3\,A \times 8.9 \times 10^3 = 2.67\,A \times 10^3$ kg

Total mass = 8.21 A × 10^3 kg



Taking the free end of iron rod as origin co-ordinates of m_1,m_2,m_3 and m_4 are 0.1 m, 0.35 m , 0.6 m and 0.7 m respectively.

$$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{1.58\,A \times 10^3 \times 0.1 + 3.42\,A \times 10^3 \times 0.35 + 0.54\,A \times 10^3 \times 0.6 + 2.67\,A \times 10^3 \times 0.7}{8.21\,A \times 10^3}$$

$$= \frac{3.548 \times 10^3\,A}{8.21\,A \times 10^3} = 0.432\ \text{m}$$

The centre of mass is at a distance of 0.432 m from the free end of iron rod.

10. A cylindrical rod whose length is 1.2 m is made of three materials. The first part is of length 0.5 m and is made of iron; the second part is of length 0.3 m and of copper. The last part is of 0.4 m and of aluminum. If the densities of iron, aluminum and copper are 7.9×10^3 kgm^{-3}, 2.7×10^3 kgm^{-3} and 8.9×10^3 kgm^{-3} respectively. Find the centre of mass of the system.

Sol:

iron	**copper**	aluminium
0.5 m	0.3 m	0.4 m

Mass = volume × density = length × area of cross-section × density

Mass of part 1 $= m_1 = 0.5\,A \times 7.9 \times 10^3$ kg $= 3.95\,A \times 10^3$ kg

Mass or part 2 $= m_2 = 0.3\,A \times 8.9 \times 10^3$ kg $= 2.67\,A \times 10^3$ kg

Mass of part 3 $= m_3 = 0.4\,A \times 2.7 \times 10^3$ kg $= 1.08\,A \times 10^3$ kg

Total Mass $= m_1 + m_2 + m_3 = 7.70 \times 10^3\,A$ kg

Positions of the masses m_1, m_2 and m_3 are given by

$x_1 = 0.25$ m

$x_2 = 0.5 + 0.15 = 0.65$ m

$x_3 = 0.8 + 0.2 = 1$ m

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{3.803 \times 10^3\,A}{7.7 \times 10^3\,A} = 0.4939\ \text{m}$$

11. An object located at the origin has mass M. It explodes into three pieces having masses $\frac{M}{4}, \frac{M}{3}$ and $\frac{5M}{12}$. The pieces move on a horizontal frictionless plane (call it x-y plane). The fragment with mass $\frac{M}{4}$ moves with velocity 5.0 ms^{-1} at 37° to the x–

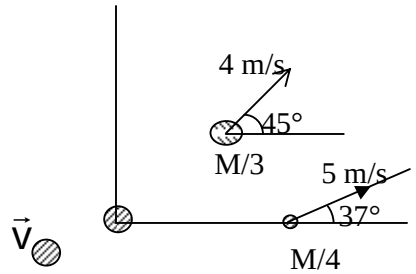
axis. The fragment of mass $\frac{M}{3}$ moves with velocity 4.0 ms^{-1} in the direction making an angle 45° above to the x–axis.

a) What are the velocity components of the third piece?

b) What can be said about the motion of the centre of mass of the systems after the explosion?

Sol: Since object is initially at rest, the momentum before explosion is zero,. Explosion is an internal force and the position of CM does not change.

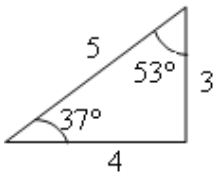
$$M\vec{V}_{CM} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 = 0$$



$$\vec{v}_3 = -\frac{1}{m_3}\left[m_1\vec{v}_1 + m_2\vec{v}_2\right] = -\frac{m_1}{m_3}\vec{v_1} - \frac{m_2}{m_3}\vec{v_2} = -\frac{\frac{M}{4}}{\frac{5M}{12}}\vec{v}_1 - \frac{\frac{M}{3}}{\frac{5M}{12}}\vec{v}_2$$

$$\vec{v}_3 = -\frac{3}{5}\vec{v}_1 - \frac{4}{5}\vec{v}_2$$

$$\vec{v}_3 = -\frac{3}{5}\left[v_1\cos 37°\hat{i} + v_1\sin 37°\hat{j}\right] - \frac{4}{5}\left[v_2\cos 45°\hat{i} + v_2\sin 45°\hat{j}\right]$$



$$= \left(-\frac{3}{5}v_1\cos 37° - \frac{4}{5}v_2\cos 45°\right)\hat{i} + \left(-\frac{3}{5}v_1\sin 37° - \frac{4}{5}v_2\sin 45°\right)\hat{j}$$

$$V_{3x} = -\frac{3}{5}\times 5\cos 37° - \frac{4}{5}\times 4\times\frac{1}{\sqrt{2}}$$

$$= -\frac{3}{5}\times 5\times\frac{4}{5} - \frac{4}{5}\times 4\times 0.7 \qquad \left(\because \cos 53° = \frac{3}{5}\right)$$

$$-4.64\ \text{ms}^{-1}$$

$$= -2.4 - 2.24 = -4.64\ \text{ms}^{-1}$$

$$V_{3y} = -\frac{3}{5}\times 5\times\frac{3}{5} - \frac{4}{5}\times 4\times\frac{1}{\sqrt{2}} = 4.062m^{-1}$$

ASSESS YOURSELF QUESTIONS

1. A carpet of length x is lying on the floor. It is folded to half of its length backwards over itself. What is the new position of the centre of mass of the carpet?

Ans: 3/4 x from the left end.

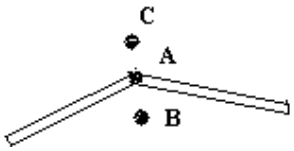
2. Can you change the centre of the mass of your body by pulling your hair up? (March 2009)

Ans: No, the pulling force is an internal force which can not change the position of centre of mass.

3. While a high jumper clears the hurdle-bar, is it possible that the high jumper's body passes over the bar and his centre of mass passes under it?

Ans: Yes, as the high jumper clears the bar his body bends into a curve and his centre of mass now lies out side of his curved body and may lie below the bar.

4. Guess the possible position of centre of mass of the body shown here.



Ans: The position of centre of mass is at B

5. A projectile is launched into the air suddenly explodes. Where the does the centre of the mass of the fragments touch the ground?

Ans. The centre of mass of the fragments touches the ground at a point as that of the unexploded would.

6. A bomb moving under the influence of gravity explodes in mid air. What will the centre of mass of the system?

Ans: Parabola, because internal forces doesn't effect the centre of mass and hence it follows the same path.