COLLISIONS

POINTS TO REMEMBER:

1. According to the law of conservation of linear momentum, in the absence of external force, the total momentum before collision is equal the total momentum after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2.$$

2. The interaction between two or more bodies causing the exchange of momentum is called collision.
3. Collisions are classified into two types namely i) Elastic collision and .ii) Inelastic collision.
4. The collision in which both momentum and KE are conserved is called elastic collision

Eg. Atomic collisions, collision between fundamental particles.

5. The collision in which only momentum is conserved is called inelastic collision. Kinetic Energy is not conserved. Eg. Collision of a bullet with a sand bag

6. In case of elastic head on collision, the relative velocity of approach is equal to the relative velocity of separation.
7. When m_1, m_2 are moving with velocities u_1, u_2 and v_1, v_2 before and after elastic collisions, then

a) $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$

b) $v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$

8. A body of mass m_1 collides head on with another body of mass m_2 at rest. The collision is perfectly elastic. Then

   a) Fraction of kinetic energy lost by the first body is $\frac{4m_1m_2}{(m_1+m_2)^2}$.

   b) Fraction of kinetic energy retained by first body is $\left(\frac{m_1-m_2}{m_1+m_2}\right)^2$.

   c) If $m_1 = m_2$, kinetic energy transferred is 100%.

9. Two bodies of equal masses suffering one dimensional elastic collision, exchange their velocities after collision. i.e., if $m_1 = m_2$ then $v_1 = u_2$ and $v_2 = u_1$.
10. If $m_1 = m_2$ and the second body is at rest $(u_2 = 0)$, then the first body comes to rest and the second body moves with the velocity of the first body after collision.
11. When a very light body strikes another very massive one(if $m_2 >> m_1$) at rest, the velocity of the lighter body is almost reversed and the massive body remains at rest.
12. When a massive body strikes a lighter one (if $m_1 >> m_2$) at rest, the velocity of the massive body remains practically unaffected where as the lighter one begins to move with a velocity nearly double that of the massive one
13. The coefficient of restitution between two bodies in a collision is defined as the ratio of the relative velocity of separation after collision to the relative velocity of their approach before their collision.

$$e = \frac{\text{relative velocity of separation}}{\text{relative velocity of approach}} = \frac{v_2 - v_1}{u_1 - u_2}$$

e = 1 for a perfect elastic collision

e = 0 for a perfect inelastic collision

For any other collision ‘e’ lies between 0 and 1.

14. The value of coefficient of restitution (e) is independent of the masses and velocities of the colliding bodies. It depends on the nature of their colliding surfaces.
Eg. The value of e is 0.94 for two glass balls 0.2 for two lead balls.

15. Let two bodies of masses m_1 and m_2 traveling with velocity u_1 and u_2 collide each other and after collision their velocities are v_1 and v_2. If 'e' is the co-efficient of restitution, then

$$v_1 = \left(\frac{m_1 - e m_2}{m_1 + m_2} \right) u_1 + \left(\frac{(1+e) m_2}{m_1 + m_2} \right) u_2$$

$$v_2 = \left(\frac{(1+e) m_1}{m_1 + m_2} \right) u_1 + \left(\frac{m_2 - e m_1}{m_1 + m_2} \right) u_2$$

Loss of K.E. = $\frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2 (1 - e^2)$

16. A body freely falls from a height h_1 on to a hard floor and rebounds to a height.

Then the coefficient of restitution $e = \sqrt{\frac{h_2}{h_1}}$

For perfectly elastic collision $h_2 = h_1$

For perfectly inelastic collision $h_2 = 0$

For other collision $h_2 < h_1$

For any collision h_2 cannot be greater than h_1

17. In the case of perfectly inelastic collision ,we have

i) Two bodies stick together and move with common velocity $(v_2 = v_1 = v)$ $v = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$

ii) Loss in K. E. due to collision= $\frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2$

iii) If the bodies are moving in opposite directions. Loss in K. E. = $\frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2$

18. A body of mass m_1 moving with velocity u_1 collides with another body of mass m_2 at rest. If the collision is perfectly elastic collision.

i) $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right)$ ii) $v_2 = \frac{2 m_1 u_1}{m_1 + m_2}$

iii) Height reached after 'n' th bounce : $h_n = (e^2)^n h$

19. Velocity after 'n' th bounce : $v_n = e^n \sqrt{2gh}$

LONG ANSWER QUESTIONS:

1. What are elastic and inelastic collisions? Derive expressions for the final velocities of the two bodies which undergo one dimensional elastic collisions? (June 2003)

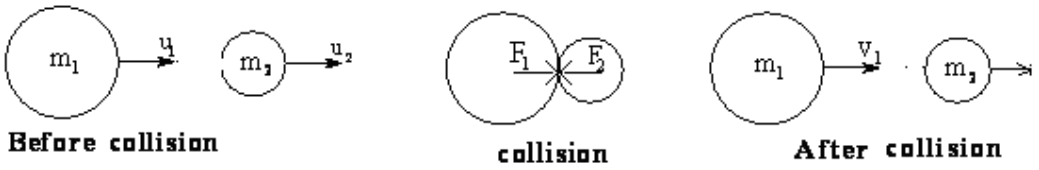
Ans. Elastic collision:

The collision in which both momentum and kinetic energy of the system are conserved is known as elastic collision. E.g.: The collision between nuclei and fundamental particles.

In-elastic collisions: The collision in which kinetic energy of the system is not conserved but momentum alone is conserved is known as in-elastic collision. E.g.: Collision between two vehicles.

One dimensional elastic collision: If the velocities of the bodies involved in collision are along the same straight line before and after collision, then such collisions are called one dimensional collision.

Consider two smooth and non - rotating spheres of masses m_1 and m_2 moving with velocities u_1 and u_2 collide elastically. Let t be the time of collision. Let the two spheres move with velocities V_1 and V_2 after the collision along the same direction. Assume $m_1 > m_2$ and $u_1 > u_2$



Before collision collision After collision

Applying the law of conservation of linear momentum,

Momentum of the system before collision = Momentum of the system after the collision

$m_1u_1 + m_2u_2 = m1v1 + m_2v_2$

Or

$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$

$m_1(u_1 - v_1) = m_2(v_2 - u_2)$ ----------(1)

Applying the law of conservation kinetic energy,

Kinetic energy of the system before collision = Kinetic energy of the system after the collision

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ Or $\frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_2u_2^2$

$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$ .....(2)

From equations (1) and (2) ,

$$\frac{m_1(u_1^2 - v_1^2)}{m_1(u_1 - v_1)} = \frac{m_2(v_2^2 - u_2^2)}{m_2(v_2 - u_2)}$$

$u_1 + v_1 = v_2 + u_2$

$u_1 - u_2 = v_2 - v_1 ----(3)$

$\therefore$ *Relative velocity of approach = Relative velocity of separation*

From equation (3) we get, $v_2 = u_1 - u_2 + v_1$ ..... (4)

From equation (1) and (4)

$m_1(u_1 - v_1) = m_2(u_1 - u_2 + v_1 - u_2)$

$m_1u_1 - m_1v_1 = m_2u_1 - m_2u_2 + m_2v_1 - m_2u_2$

$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1 - 2m_2u_2$

$m_1u_1 - m_2u_1 + 2m_2u_2 = m_1v_1 + m_2v_1$

$(m_1 - m_2)u_1 + 2m_2u_2 = v_1(m_1 + m_2)$

$$\boxed{v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2}$$ ..... (5)

Again, from equation (3), we get $v_1 = v_2 + u_2 - u_1$

Using the equation (1),

$$m_1(u_1 - v_2 - u_2 + u_1) = m_2(v_2 - u_2)$$

$$m_1u_1 - m_1v_2 - m_1u_2 + m_1u_1 = m_2v_2 - m_2u_2$$

$$2m_1u_1 - m_1u_2 + m_2u_2 = m_1v_2 + m_2v_2$$

$$2m_1u_1 + (m_2 - m_1)u_2 = (m_1 + m_2)v_2$$

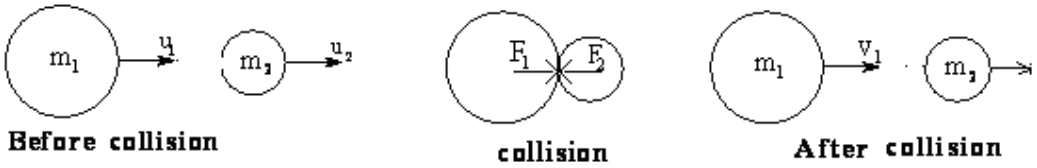
$$\boxed{v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2} \quad \text{..... (6)}$$

SHORT ANSWER QUESTIONS:

1. Show that in the case of one dimensional elastic collision, the relative velocity of approach of two colliding bodies before collision is equal to the relative velocity of separation after collision. (March 2007, March 2008, June2006, March 2001, March 2003, March 2004, March 2005)

Ans : **One dimensional elastic collision:** If the velocities of the bodies involved in collision are along the same straight line before and after collision, then such collisions are called one dimensional collision.

Consider two smooth and non - rotating spheres of masses m_1 and m_2 moving with velocities u_1 and u_2 collide elastically. Let t be the time of collision. Let the two spheres move with velocities V_1 and V_2 after the collision along the same direction. Assume $m_1 > m_2$ and $u_1 > u_2$



Before collision collision After collision

Applying the law of conservation of linear momentum,

Momentum of the system before collision = Momentum of the system after the collision

$m_1u_1 + m_2u_2 = m1v1 + m_2v_2$ Or

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \text{ ----------(1)}$$

Kinetic energy of the system before collision = Kinetic energy of the system after the collision

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{Or} \quad \frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_2u_2^2$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \text{ .....(2)}$$

From equations (1) and (2) ,

$$\frac{m_1(u_1^2 - v_1^2)}{m_1(u_1 - v_1)} = \frac{m_2(v_2^2 - u_2^2)}{m_2(v_2 - u_2)}$$

$$u_1 + v_1 = v_2 + u_2$$

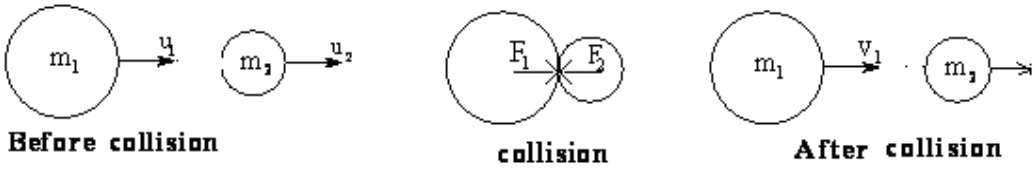
$$u_1 - u_2 = v_2 - v_1 ----(3)$$

$\therefore$ *Relative velocity of approach = Relative velocity of separation*

2. Obtain the final velocities of two bodies undergoing elastic head on collision.

Ans: **One dimensional elastic collision:** If the velocities of the bodies involved in collision are along the same straight line before and after collision, then such collisions are called one dimensional collision.

Consider two smooth and non - rotating spheres of masses m_1 and m_2 moving with velocities u_1 and u_2 collide elastically. Let t be the time of collision. Let the two spheres move with velocities V_1 and V_2 after the collision along the same direction. Assume $m_1 > m_2$ and $u_1 > u_2$



Before collision **collision** **After collision**

Applying the law of conservation of linear momentum,

Momentum of the system before collision = Momentum of the system after the collision

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ Or

$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$

$m_1(u_1 - v_1) = m_2(v_2 - u_2)$ ----------(1) .....

Applying the law of conservation kinetic energy,

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$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ Or $\frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_2u_2^2$

$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$ .....(2)

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$u_1 + v_1 = v_2 + u_2$

$u_1 - u_2 = v_2 - v_1 ----(3)$

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From equation (3) we get, $v_2 = u_1 - u_2 + v_1$ ..... (4)

From equation (1) and (4)

$m_1(u_1 - v_1) = m_2(u_1 - u_2 + v_1 - u_2)$

$m_1u_1 - m_1v_1 = m_2u_1 - m_2u_2 + m_2v_1 - m_2u_2$

$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1 - 2m_2u_2$

$m_1u_1 - m_2u_1 + 2m_2u_2 = m_1v_1 + m_2v_1$

$(m_1 - m_2)u_1 + 2m_2u_2 = v_1(m_1 + m_2)$

$$\boxed{v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2}$$ ..... (5)

Again, from equation (3), we get $v_1 = v_2 + u_2 - u_1$

Using the equation (1),

$m_1(u_1 - v_2 - u_2 + u_1) = m_2(v_2 - u_2)$

$m_1u_1 - m_1v_2 - m_1u_2 + m_1u_1 = m_2v_2 - m_2u_2$

$2m_1u_1 - m_1u_2 + m_2u_2 = m_1v_2 + m_2v_2$

$2m_1u_1 + (m_2 - m_1)u_2 = (m_1 + m_2)v_2$

$$\boxed{v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2}$$ ..... (6)

3. Show that the two spheres of equal masses moving along the (x-axis) straight line exchange their velocities after a head-on elastic collision.
(March 2011, June 2005, March 2000)

Ans : Consider two smooth and non-rotating spheres of masses m_1 and m_2 which are moving along the line joining their centers. Let u_1 and u_2 are velocities of the spheres before collision and v1 and v_2 be their final velocities after collision. Assume that m1 > m_2 and $u_1 > u_2$.

$$\therefore v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \text{ ..... (1)}$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 \text{ ..... (2)}$$

If the masses of the two spheres are equal i.e. If $m_1 = m_2$

From equation (1) we get $\boxed{v_1 = u_2}$

From equation (2) we get $\boxed{v_2 = u_1}$

Hence in head-on collision between two spheres of equal masses they interchange their velocities after collision.

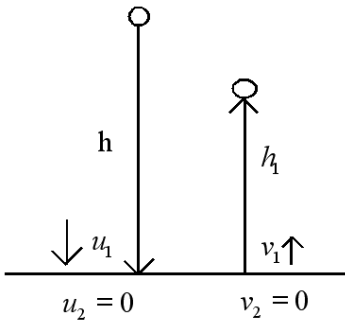
4. Derive an expression for the height attained by a freely falling body after 'n' number of rebounds from the floor.

Ans: Consider a body falling freely from a height 'h' on to the floor acquires a velocity u_1 at the floor.

$$\therefore u_1 = \sqrt{2gh} \quad \text{..... (1)}$$

The initial and the final velocities of the floor are zero. i.e., $u_2 = 0$ and $v_2 = 0$

Let v_1 be the velocity of the body with which it rebounds to a height h_1 and $v_1 = \sqrt{2gh_1}$



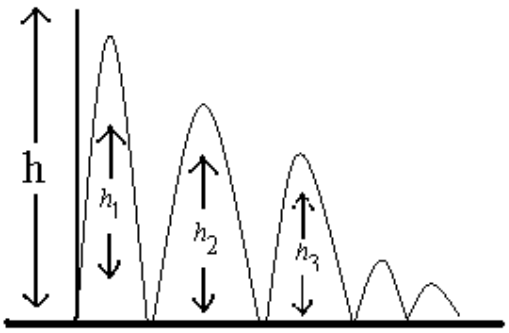
Coefficient of restitution $e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{0 - \left(-\sqrt{2gh_1}\right)}{\sqrt{2gh} - 0} = \sqrt{\frac{2gh_1}{2gh}} = \sqrt{\frac{h_1}{h}} \Rightarrow h_1 = e^2 h$

If the body rebounds for the second time height reached by the body is $h_2 = (e^2)^2 h$

Hence the maximum height reached by the body after 'n' rebounds is $h_n = e^{2n} h$

5. Find the total distance traveled by a freely falling body, after successive rebounds from the ground before its comes to stop. Assume that 'e' is the coefficient of restitution between the body and ground.

Ans: The total distance traveled by a freely falling body, after successive rebounds from the ground before coming to stop is given by



$$d = h + 2h_1 + 2h_2 + 2h_3 + \ldots\ldots\ldots = h + 2(he^2) + 2(he^4) + 2(he^6) + \ldots\ldots$$

$$= h + 2he^2(1 + e^2 + e^4 + \ldots\ldots\infty) = h + 2he^2\left(\frac{1}{1-e^2}\right) = h\left(1 + \frac{2e^2}{1-e^2}\right) = h\left(\frac{1-e^2+2e^2}{1-e^2}\right)$$

$$\therefore d = h\left(\frac{1+e^2}{1-e^2}\right)$$

VERY SHORT ANSWER QUESTIONS:

1. What is a Collision? Mention the types of Collisions.

Ans: **Collision:** A strong interaction between two bodies that occurs for a very short interval of time during which redistribution of momenta occur neglecting the effect of other forces is called a collision.

Types of collisions:

(i) Elastic collision (ii) Inelastic collision

2. Which physical quantity remains constant?

(i) in elastic collision (ii) in an inelastic collision

Ans: **(i)** In elastic collision both momentum and kinetic energy remains constant. **(ii)** In an inelastic collision momentum alone remains constant.

3. Define coefficient of restitution? What are its practical values? (March 2010, March 2009, March 2001, March 2002)

Ans : **Coefficient of restitution :** It is defined as ratio of relative velocity of separation ($v_2 - v_1$) after the collision to the relative velocity of approach ($u_1 - u_2$) before collision.

$$\text{Coefficient of restitution}\,(e) = \frac{v_2 - v_1}{u_1 - u_2}$$

e = 1 for a perfect elastic collision

e = 0 for a perfect inelastic collision

4. Distinguish between elastic and inelastic collisions. (June 2010)

Ans :

Elastic collision	***Inelastic collision.***
a) In this collision both momentum and K.E. are conserved	a) In this collision momentum alone is conserved but K.E. is not conserved.

(b) Collision between atoms, molecules are considered as elastic collisions. (c) Coefficient of restitution e = 1 (d) Relative velocity before and after Collisions are equal.	(b) Collision between two vehicles can be considered as inelastic collision. (c) Coefficient of restitution e = 0 (d) Relative velocity before and after collisions are unequal.

5. State the values of coefficient of restitution for elastic and inelastic collisions.

Ans: e = 1 for a perfect elastic collision
e = 0 for a perfect inelastic collision

6. A body freely falling from a height strikes the ground with a velocity 100 ms^{-1}. With what velocity it rebounds, if the coefficient of restitution between the body and the ground is 0.6?

$u_1 = 100 ms^{-1}$ $u_2 = 0$ $v_1 = -v_1$ $v_2 = 0$

$$e = \frac{v_2 - v_1}{u_1 - u_2} \quad \Rightarrow \; 0.6 = \frac{-v_1}{100}$$

$$v_1 = 0.6 \times 100 = -60 \; ms^{-1}$$

7. A body freely falling from a certain height 'h' after striking a smooth floor rebounds and rises to a height $\frac{h}{4}$. What is the coefficient of restitution between the floor and the body?

Ans : $h_1 = h$, $h_2 = \frac{h}{4}$

$$e = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{h/4}{h}} = \sqrt{\frac{1}{4}} = 0.5$$

8. What is the total displacement of a freely falling body, after successive rebounds from the ground, before it comes to stop ? Assume that 'e' is the coefficient of restitution between the body and the ground.?

Ans: The total displacement of a freely falling body, after successive rebounds from the ground, before it comes to stop is 'h'.

9. A ball of mass 0.1 kg is dropped down from height 1m above the ground. If it rebounds and rises to a height of 0.81 m**. Find the coefficient of restitution between the ground and the ball. (March2009)**

Ans : $h_1 = 1m$, $h_2 = 0.81m$

$$e = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{0.81}{1}} = 0.9$$

10. A sphere of mass 3 kg moving with a velocity of 2 ms^{-1} collides head-on with another sphere of mass 4 kg moving in opposite direction with a velocity of 2 ms^{-1}. After collision if they stick together and move with a velocity v. Then, find common velocity of the spheres. . ? (May 2007)

Sol : m_1 = 3 kg ; m_2 = 4 kg ; u_1 = 2 ms^{-1} ; u_2 = –2 ms^{-1}

Common velocity, $v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{(3\times 2)+(4\times -2)}{3+4} = -\frac{2}{7}\ ms^{-1}$

Negative sign indicates that the bodies after the collision move in the direction of second body.

Exercise 1:

1. **A sphere of mass $50x10^{-3}$ kg moving with a velocity of 2 ms^{-1} hits another sphere which is at rest. Assuming the collision to be head-on collision and if they stick together after collision and move in the same direction with a velocity of 0.5 ms^{-1}, find the mass of the second sphere.?**

Ans. $m_1=50\times10^{-3}kg$ $m_2=?$, $u_1=2ms^{-1}$ $u_2=0$, $v = v_1 = v_2 = 0.5ms^{-1}$

$$v=\frac{m_1u_1+m_2u_2}{m_1+m_2}$$

$$0.5=\frac{50\times10^{-3}\times2+m_2(0)}{50\times10^{-3}+m_2} \quad \Rightarrow 0.5(50\times10^{-3})+0.5m_2=100\times10^{-3} \Rightarrow \quad m_2=150\times10^{-3}kg$$

2. **If a ball of mass 0.4 Kg moving with a velocity of 2 ms^{-1} collides elastically with another ball of mass 0.6 Kg which is at rest, find their velocities after collision. (March 2006)**

Ans. $m_1=0.4Kg$ $m_2=0.6Kg$, $u_1=2ms^{-1}$ $u_2=0$, $v_1=?$ $v_2=?$

$$v_1=\left(\frac{m_1-m_2}{m_1+m_2}\right)u_1+\frac{2m_2u_2}{m_1+m_2}$$

$$v_1=\left(\frac{0.4-0.6}{0.4+0.6}\right)3+\frac{2(0.6)(0)}{0.4+0.6} \quad \Rightarrow \quad v_1=-0.6m/s$$

$$v_2=\left(\frac{2m_1}{m_1+m_2}\right)u_1+\left(\frac{m_2-m_1}{m_1+m_2}\right)u_2$$

$$v_2=\left(\frac{2\times0.4}{0.4+0.6}\right)(3)+\left(\frac{0.6-0.4}{0.4+0.6}\right)(0) \quad \Rightarrow v_2=2.4ms^{-1}$$

3. **A bullet of mass 2 g traveling with a velocity of 500 ms^{-1} is fired into a block of wood of mass 1kg suspended from a string of 1 m on length. If the bullet penetrates the block of wood and comes out with a velocity of 100 ms^{-1} , find the vertical height through which the block of wood will rise (assuming the value of g = 10 ms^{-2}.**

Ans. $m_1=2g=0.002$ kg ; $m_2=1kg$

$u_1=500m/s$; $u_2=0$

$v_1=100ms^{-1}$; $v_2=?$

L= 1m ; $g=10ms^{-2}$

$m_1u_1+m_2u_2=m_1v_1+m_2v_2$

$0.002\times500+1\times0=0.002\times100+1\times v_2$

$v_2=0.8m/s$

Let h be the h height to which the block rises. According to the law of conservation of energy

$$(m_1+m_2)gh=\frac{1}{2}(m_1+m_2)v_2^{\ 2} \Rightarrow h=\frac{v_2^2}{2g}$$

$$h=\frac{(0.8)^2}{2\times10}=0.032m$$

4. A ball of mass 0.2 kg is dropped down vertically from a height of 1 m above the ground. If it rebounds to a height of 0.64 m, find the coefficient of restitution between the surface of the ball and ground. (March 2009)

Ans. Coefficient of restitution $e = \sqrt{\frac{h_2}{h_1}}$

$h_1 = 1\text{ m}, h_2 = 0.64\text{ m}$; $e = \sqrt{\frac{0.64}{1}} = 0.8.$

5. A sphere of mass 2 kg moving with a velocity collides elastically with another sphere which is at rest. If after the collision the first sphere move in the same direction with its velocity reduced to one third of its initial velocity, find the mass of the second sphere.?

Ans. $m_1 = 2\text{kg}, u_1 = u_1, \; u_2 = 0, \; v_1 = \frac{1}{3}u_1$

In elastic collision,

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \quad \Rightarrow v_1 = \left(\frac{2 - m_2}{2 + m_2}\right)u_1 + \left(\frac{2m}{2 + m_2}\right)\times 0$$

$$\frac{u_1}{3} = \left(\frac{2 - m_2}{2 + m_2}\right)u_1 \Rightarrow m_2 = 1kg$$

6. Two bodies of masses m_1 and m_2 are moving with velocities 1ms^{-1} and 3ms^{-1} respectively in opposite directions. If the bodies undergo one dimensional elastic collision, the body mass m_1 comes to rest. Find the ratio of m_1 and m_2 .?

Ans. $u_1 = 1\text{ ms}^{-1}, \; u_2 = -3\text{ ms}^{-1}, \; v_1 = 0$

In elastic collision,

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \quad \Rightarrow 0 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)1 + \left(\frac{2m_2}{m_1 + m_2}\right)(-3)$$

Solving , $\frac{m_1}{m_2} = \frac{7}{1}.$

7. A steel ball is allowed to fall freely from a height of 36 cm onto a smooth floor. Find the height to which it rises after rebound from the floor. Coefficient of restitution between the steel ball and the floor = 0.60.

Ans. $e = 0.60, \; h_1 = 36\text{ cm} = 36\times 10^{-2}\text{m}.$

$$e = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{h_2}{36\times 10^{-2}}} \Rightarrow h_2 = 36\times 10^{-2}\times 0.36 = 0.13m$$

8. Two perfectly elastic spheres of masses 2 kg and 3 kg moving in opposite direction with velocities 8ms^{-1} and 6ms^{-1} respectively collide with each other. Find their velocities after the impact.

Ans. $m_1 = 2\text{ kg}, \; m_2 = 3\text{kg}, \; u_1 = 8\text{ m/s}, \; u_2 = -6\text{m/s}, \quad v_1 = ?, \quad v_2 = ?$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_1 = \left(\frac{2-3}{2+3}\right)8 + \left(\frac{2X3}{2+3}\right)(-6) \qquad \Rightarrow v_1 = -8.8 ms^{-1}$$

$$v_2 = \left(\frac{2m_1}{m_1+m_2}\right)u_1 + \left(\frac{m_2-m_1}{m_1+m_2}\right)u_2$$

$$\mathrm{v}_2 = \left(\frac{2\times 2}{2+3}\right)8 + \left(\frac{3\text{-}2}{2+3}\right)(-6)$$

$$v_2 = 5.2\, ms^{-1}$$

9. A body of mass 1 kg makes an elastic collision with another body at rest and continues to move in the original direction after collision with a velocity equal to $\frac{1}{5}$ th of its original velocity. Find the mass of the second body.?

Ans. $\mathrm{m}_1 = 1\ \mathrm{kg}\ ,\ \mathrm{u}_1 = \mathrm{u},\ \mathrm{u}_2 = 0, \mathrm{v}_1 = \frac{\mathrm{u}}{5}\ m_2 = m$

$$v_1 = \left(\frac{m_1-m_2}{m_1+m_2}\right)u_1 + \left(\frac{2m_2}{m_1+m_2}\right)u_2 \Rightarrow \frac{u}{5} = \left(\frac{1-m}{1+m}\right)u$$

$$m = \frac{2}{3} Kg$$

Exercise

1. A 2 Kg mass moving with a velocity of 10 ms^{-1} collides with another 6 kg mass moving in opposite direction with a velocity of 20 ms^{-1}. During collision, they stick together. Find their common velocity and the momentum. ?

Sol: $m_1 = 2\,kg\,;\, m_2 = 6\,kg\,;\, u_1 = 10\,ms^{-1}\,;\, u_2 = 20\,ms^{-1}$

Common velocity, $v = \frac{m_1u_1 + m_2u_2}{m_1+m_2}$

$\therefore v = \frac{20-120}{9} = \frac{-100}{8} = -12.5\ ms^{-1}$.

Negative sign indicates that the bodies after the collision move in the direction of second body. Final momentum, $P = (m_1+m_2)v$ = (2 + 6) (–12.5) = – 100 kg ms^{-1}.

2. A particle experiences a perfectly elastic collision with a stationary particle. Determine the ratio of their masses, if after this head-on collision the particles fly apart in opposite directions with equal velocities. ?

Sol. $v_1 = \left(\frac{m_1-m_2}{m_1+m_2}\right)u_1 + \left(\frac{2m_2}{m_1+m_2}\right)u_2$ *and* $v_2 = \left(\frac{2m_1}{m_1+m_2}\right)u_1 + \left(\frac{m_2-m_1}{m_1+m_2}\right)u_2$

Second body is stationary, $u_2 = 0$

For 1st body, $-v = \left(\frac{m_1-m_2}{m_1+m_2}\right)u_1$

And, for 2nd body, $v = \left(\frac{2m_1}{m_1+m_2}\right)u_1$

Solving the above, $\frac{m_1}{m_2} = \frac{1}{3}$

3. A U^{238} nucleus emits an α-particle and it in turn converted into Th^{234} nucleus. If the velocity of the -particles is 1.4×10^{7} ms^{-1}. Calculate the velocity of recoil of remaining Th^{234} nucleus. ?

Sol : Mass of $_{92}U^{238}$ nucleus, M = 238 a.m.u. ; Initial velocity, U = 0.

Mass of Thorium nucleus, $m_1 = 234$ *a.m.u.* ; Recoil velocity of Thorium nucleus, v_1 =?

Mass of α-particle, m_2 = 4 a.m.u

Velocity of -particle, v_2 = 1.4 x 10^7 ms^{-1}.

From the law of conservation of linear momentum, $MU = m_1v_1 + m_2v_2$

$\therefore 238\times0 = (234\times v_1) + (4\times1.4\times10^{7})$

$\therefore\ v_1 = \dfrac{-4\times1.4\times10^{7}}{234} = -2.393\times10^{5}\ ms^{-1}$

$\therefore$ Recoil velocity of Th^{234} nucleus, $v_2 = 2.393\times10^{5}\ ms^{-1}$

4. A neutron of mass 1.67×10^{-27} kg moving with a velocity 10^6 ms^{-1} collides with a duteron of mass 3.34×10^{-27} kg at rest. After collision, if both moves as a single particle, then find its velocity.?

Sol : $m_1 = 1.67\times10^{-27}\ kg$; m_2 = 3.34 x 10^{-27} kg.

$u_1 = 10^{6}\ ms^{-1}$; u_2 = 0

Common velocity after collision, v =?

$Common\ velocity,\ v = \dfrac{m_1u_1 + m_2u_2}{m_1 + m_2}$

$\therefore v = \dfrac{(1.67\times10^{-27}\times10^{6}) + (0)}{(1.67\times10^{-27} + 3.34\times10^{-27})} = \dfrac{1.67\times10^{-27}\times10^{6}}{5.01X10^{-27}}$

$\therefore v = 3.33\times10^{5}\ ms^{-1}$

5. Two balls each of mass 0.06 kg moving in opposite directions each with a velocity of 8 ms^{-1} collide and move back with equal velocities. Find the change in momentum of each ball due to collision.?

Sol : , $m_1 = m_2 = 0.06\ kg = m\ (say)$

$u_1 = u_2 = 8\ ms^{-1} = u$

$v_1 = v_2 = -8\ ms^{-1} = v$

Change in momentum of each ball, $\Delta p = ?$

$Change\ in\ momentum,\ \Delta p = p_f - p_i = mv - mu = m(v-u)$

$\therefore \Delta p = 0.06(-8-8) = -0.06\times16 = -0.96\ kg\ ms^{-1}$

Negative sign gives the direction of change in momentum of each ball.

6. A ball of mass 0.4 kg moving with a uniform speed of 2 ms^{-1} strikes normally a wall and rebounds. Assuming the collision to be elastic and the time of contact of the ball with the walls as 0.4, find the force exerted on the ball.?

Sol : m = 0.4 kg

u = 2 ms^{-1} ; v = – u = – 2ms^{-1}

t = 0.4 sec ; F = ?

$$F = \frac{mv - mu}{t} = \frac{m(v-u)}{t} == \frac{0.4(-2-2)}{0.4}$$

$$\therefore F = -4\,N$$

Negative sign indicates that the force exerted on the ball is opposite to its direction of motion.

7. A sphere of mass 2 kg moving with a velocity of 40 ms^{-1} hits another sphere which is at rest. After the collision, if they move together with a velocity of 25 ms^{-1} in one direction, then find the mass of the second sphere. ? (June 2002)

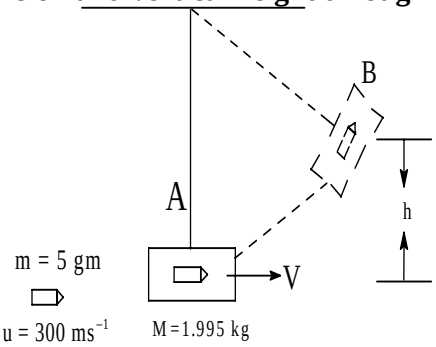
Sol : $m_1 = 2kg$; $u_1 = 40\ ms^{-1}$; $u_2 = 0$; $m_2 = ?$

Common velocity after collision, $v = 25\ ms^{-1}$

$$Common\ velocity,\ v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

$$25 = \frac{(2\times 40) + (m_2 \times 0)}{2 + m_2} \Rightarrow\ m_2 = 1.2\ kg$$

8. A bullet of mass 5 g moving with a velocity of 300 ms^{-1} strikes a plank of wood of mass 1.995 kg suspended by a thread and gets embedded in it. Find the velocity of the plank along with bullet, immediately after collision and vertical height through which the combined mass rises.?



Sol : Mass of bullet, m = 5 g = 0.005 kg

Initial velocity of bullet, u = 300 ms^{-1}

Mass of plank, M = 1.995 kg

Initial velocity of plank, U = 0

(a)From the law of conservation of momentum,

$$mu = (M+m)v$$

$$\therefore (0.005\times 300) = (1.995 + 0.005)v$$

$\therefore$ Common velocity,

$$v = \frac{0.005\times 300}{2} = 0.75\ ms^{-1}$$

(b) From the law of conservation of energy, $TE_A = TE_B$

$$\frac{1}{2}(M+m)v^2 = (M+m)gh \quad \Rightarrow\ h = \frac{v^2}{2g} \quad \Rightarrow h = \frac{0.75\times 0.75}{2\times 10} = 0.028\ m$$

9. A sphere of mass 3 kg moving with a velocity of 2 ms^{-1} collides head-on with another sphere of mass 4 kg moving in opposite direction with a velocity of 1 ms^{-1}. After collision if they stick together and move with a velocity v. Then, find (i) the total initial momentum and (ii) velocity v. (March 2002)

Sol : $m_1 = 3$ kg ; $m_2 = 4$ kg ; $u_1 = 2\ ms^{-1}$; $u_2 = -1\ ms^{-1}$

(i) Total initial momentum, $= m_1u_1 + m_2u_2 = (3\times 2) + (4\times(-1)) = 6 - 4 = 2$ kg ms^{-1}.

(ii) Common velocity, $v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{(3\times 2) + (4\times -1)}{3+4} = \frac{2}{7}\ ms^{-1}$

10. A bomb at rest explodes into two pieces of masses 1 kg and 2 kg. If the velocity of the lighter piece is 200 ms^{-1}, find the velocity of the heavier piece.?

Sol : $m_1 = 1$ kg ; $m_2 = 2$ kg

$v_1 = 200\ ms^{-1}$; $v_2 = ?$

$\therefore$ Mass of bomb, $M = m_1 + m_2 = 3\ kg$ and initial velocity U = 0.

From the law of conservation of momentum, $(3\times 0) = (1\times 200) + 2v_2$

$v_2 = -100\ ms^{-1}$

Negative sign indicates that heavier fragment flies off in the direction opposite to that of lighter fragment**.**

11. A ball strikes the floor vertically and rebounds with a co-efficient of restitution 0.6. If the mass of the ball is 0.01 kg and its velocity is 5 ms^{-1} just before it hits the ground, then what is the velocity and momentum of the ball as it leaves the floor?

Sol : m = 0.01 kg ; $u_1 = 5\ ms^{-1}$; e = 0.6.

Coefficient of restitution, $e = \dfrac{v_2 - v_1}{u_1 - u_2}$

But , $u_2 = v_2 = 0$

$\therefore 0.6 = \dfrac{0 - v_1}{5 - 0} \Rightarrow v_1 = -3\ ms^{-1}$

Negative sign indicates that the ball bounces up in opposite direction.

Momentum of the ball, $p = mv_1 \Rightarrow = 0.01\times 3 = 0.03\ kgms^{-1}$

SELFASSESMENT QUESTIONS & ANSWERS

1. Which physical quantity is conserved in any type of collision?

Ans. As no external force acts on the system Linear momentum is always conserved.

2. When two bodies undergo perfectly inelastic collision what is their relative velocity?

Ans. In an inelastic collision, after collision the two bodies stick together and hence relative velocity is zero.

3. When two bodies A and B collide, A exerts a force F on B. What is the force applied by B on A?

Ans. According to Newton's third law of motion action and reaction are equal and opposite. Hence the force applied by B on A is -F

4. What is the nature of the forces involved in an elastic collision?

Ans. Conservative in nature.

5. What is the nature of the forces involved in an inelastic collision?

Ans. Some or all of the forces involved are non conservative in nature.

6. Is total energy conserved in a collision?

Ans. Yes, total energy is always conserved in both elastic and inelastic collisions.

7. In which type of collision mechanical energy is transformed into other forms of energy?

Ans. In the Inelastic collision the loss of energy is converted into heat energy.

8. A ball hits the ground and rebounds after collision. In this process, momentum of which bodies is conserved?

Ans. Total momentum of the earth and the ball is conserved.

9. A small sphere dropped from a height of 10 cm on striking the floor, rebounds and rises to a height of 15 cm is it possible? Explain your answer.

Ans. $e = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{15}{10}} > 1.$ Since the maximum value of e is 1, it is not possible

10. A small sphere freely falling from a certain height after striking the floor rebounds and raises it to same height. When is it possible?

Ans. $e = \sqrt{\frac{h_1}{h_1}} = 1$ .This is possible in a perfectly elastic collision.