## ii. Common roots and Signs of the Quadratic Expressions

## EXERCISE – ii

### Very Short Answer Questions

**1.** **If the quadratic equations $ax^2+2bx+c=0$ and $ax^2+2cx+b=0\ (b\neq c)$ have a common root then show that a + 4b + 4c = 0**

**Sol:** Let $\alpha$ be the common root of $ax^2+2bx+c=0$ and $ax^2+2cx+b=0$

$\therefore a\alpha^2+2b\alpha+c=0$

$a\alpha^2+2c\alpha+b=0$

Subtracting $2\alpha(b-c)+c-b=0$

$2\alpha(b-c)=(b-c)$

$\alpha=\frac{1}{2}$

Sub $\alpha=\frac{1}{2}$ in $a\alpha^2+2b\alpha+c=0$

$\therefore a\left(\frac{1}{4}\right)+2b\left(\frac{1}{2}\right)+c=0$

a + 4b + 4c = 0

**2.** **If $x^2-6x+5=0$ and $x^2-12x+p=0$ have a common root then find p**

**Sol:** Let $\alpha$ be the common root of $x^2-6x+5=0$ and $x^2-12x+p=0$

$\therefore \alpha^2-6\alpha+5=0\rightarrow(1)$

$\alpha^2-12\alpha+p=0\rightarrow(2)$

From (1) $(\alpha-1)(\alpha-5)=0$

$\therefore \alpha=1$ pr $\alpha=5$

Substituting $\alpha=1$ in (2) we have

$1-12+p=0\Rightarrow p=11$

Substituting $\alpha=5$ in (2) we have

$5^2-(2)(5)+p=0\Rightarrow p=11$

**3.** **If $x^2-6x+5=0$ and $x^2-3ax+35=0$ have a common root then find a**

**Sol:** Let $\alpha$ be the common root of $x^2+6x+5=0$ and $x^2-3ax+35=0$

$\therefore \alpha^2-6\alpha+5=0\rightarrow(1)$

$\alpha^2-3a\alpha+35=0\rightarrow(2)$

From (1) we have

$(\alpha-1)(\alpha-5)=0$

$\therefore \alpha=1$ or $\alpha=5$

If $\alpha=1$ then

$1^2-3a(1)+35=0\Rightarrow a=12$

If $\alpha=5$ then

$5^2-3a(5)+35=0\Rightarrow a=4$

**4.** **If equations $x^2+ax+b=0$ and $x^2+cx+d=0$ have a common root and the first equation has equal roots then prove that 2(b + d) = ac**

**Sol:** Let $\alpha$ be the common root of $x^2+ax+b=0$ and $x^2+cx+d=0$ since $x^2+ax+b=0$ has equal roots

$\alpha,\alpha$ be the roots of $x^2+ax+b=0$

$\therefore \alpha+\alpha=-a\Rightarrow\alpha=-\frac{a}{2}$

$\alpha\times\alpha=b\Rightarrow\alpha^2=b$

$\alpha$ is also a root of $x^2+cx+d=0$

$\therefore \alpha^2+c\alpha+d=0$

$b+c\left(-\frac{a}{2}\right)+d=0$

$b+d=\frac{ac}{2}$

2 ( b +d ) = ac

**5. Discuss the signs of the following quadratic expressions when x is real**

**i)** $x^2 - 5x + 4$

here $b^2 - 4ac = (-5)^2 - 4(1)(4)$

= a > 0

$\therefore$ Roots are real

Coefficient of $x^2 = 1 > 0$

Roots of $x^2 - 5x + 4 = 0$

Are 1, 4

By theorem 2

If $\alpha < x < \beta$ then $ax^2 + bx + c$ and a have opposite sign here $\alpha = 1$, $\beta = 4$

Coefficient of $x^2 > 0$

$\therefore 1 < x < 4 \Rightarrow x^2 - 5x + 4 < 0$

If x = 1 or x = 4 then

$x^2 - 5x + 4 = 0$

$\therefore x < \alpha$ or $x < \beta$ then $x < \beta$ and a have same sign

$\therefore x < 1$ or $x < 4$ then

$x^2 - 5x + 4 > 0$

**ii)** $x^2 - x + 3$

Here $b^2 - 4ac = (-1)^2 - 4(1)(3)$

= 1 – 12 = - 11 < 0

There fore roots of the equation are imaginary coefficient of $x^2$ is 1 > 0

Hence $\forall x \in R$ $ax^2 + bx + c$ and a have same sign

$\therefore x^2 - x + 3 > 0 \ \forall x \in R$

**6. For what values of x the following expressions are positive**

**i)** $x^2-5x+6$ **ii)** $3x^2-4x+4$ **iii)** $4x-5x^2+2$ **iv)** $x^2-5x+14$

**Sol:** $x^2-5x+6=0$ Have real roots 2 and 3

$\therefore$ When x < 2 or x > 3 $x^2-5x+6>0$

$\therefore x^2-5x+6>0$ is possible when $x\in(-\alpha,2)\cup(3,\alpha)$

**ii)** $3x^2+4x+4$

$\Delta=b^2-4ac=16-(3)(4)=-32<0$

$\therefore$ Roots of $3x^2+4x+4=0$ are imaginary

Hence $3x^2+4x+4>0\ \ \forall x\in R$ because coefficient of $x^2=3>0$

**iii)** $4x-5x^2+2$

$\Delta=b^2-4ac=(4)^2-4(-5)(2)$

= 16 + 40 =56

Roots of $4x-5x^2+2=0$

$5x^2-4x-2=0$

$x=\dfrac{4\pm\sqrt{16-4(5)(-2)}}{10}$

$x=\dfrac{4\pm\sqrt{56}}{10}$

If $\dfrac{4-\sqrt{56}}{10}<x\ \dfrac{4+\sqrt{56}}{10}$

Then $4x-5x^2+2>0$

$\because \alpha<x<\beta\Rightarrow ax^2+bx+c$ and a have opposite sign

**iv)** $x^2-5x+14$

Here $\Delta=b^2-4ac$

= 25 – 4(1) (14) = -31 < 0

Roots of $= x^2 - 5x + 14 = 0$

Are imaginary

$\therefore x^2 - 5x + 14 > 0 \ \ \forall x \in R$

**7. For what values of x the following expressions are negative**

**i)** $x^2 - 7x + 10$ ii) $15 + 4x + 3x^2$ iii) $2x^2 + 5x - 3$ iv) $x^2 - 5x - 6$

**i)** $x^2 - 7x + 10$

Here $\Delta = b^2 - 4ac = 49 - 40 = 9$

$\therefore$ Roots are real coefficient of $x^2 = 1 > 0$

$\therefore$ Roots of the equation $x^2 - 7x + 10 = 0$ are 2, 5

If $\alpha < x < \beta$ then $ax^2 + bx + c$ and a have opposite sign

$\therefore 2 < x < 5 \Rightarrow x^2 - 7x + 10 < 0$

**ii)** $15 + 4x - 3x^2$

a = -3, b = 4, c = 15

$\Delta = b^2 = 4ac = 16 - 4(-3)(15)$

= 196 > 0

$\therefore$ Roots are real roots of $15 + 4x - 3x^2 = 0$

$\Rightarrow 3x^2 - 4x - 15 = 0$

$3x^2 - 9x + 5x - 15 = 0$

x = -5/3, x = 3

Coefficient of $x^2 = -3 < 0$

$\therefore 15 + 4x - 3x^2 < 0$ When

$x < -\frac{5}{3}$ and x > 3

$\therefore x < \alpha$ or $x > \beta \Rightarrow ax^2 + bx + c$ and a have same sign

**iii)** $2x^2+5x-3$

a = 2, b = 5, c = - 3

$\Delta = b^2-4ac = 25+24 = 49$

$\therefore$ Roots are real

$2x^2+5x-3=0 \Rightarrow 2x^2+6x-x-3=0$

(2x-1)(x + 3) = 0 $\Rightarrow$

$\therefore$ Roots of $2x^2+5x-3=0$

Are $-3, \frac{1}{2}$

Coefficient of $x^2 = 2 > 0$

$\therefore 2x^2+5x-3<0$ When

$-3 < x < \frac{1}{2}$

$\therefore \alpha < x < \beta \Rightarrow ax^2+bx+c$ and a have opposite sign

**iv)** $x^2-5x-6$

a = 1, b = - 5, c = -6

$\Delta = b^2-4ac = 25+24 = 49$

$\therefore$ Roots are real

$x^2-5x-6=0 \Rightarrow (x-6)(x+1)=0$

x = 6; x = -1

Coefficient of $x^2 = 1$

$\therefore x^2-5x-6<0 \Rightarrow 6 < x < -1$

$\therefore \alpha < x < \beta \Rightarrow ax^2+bx+c$ and a have opposite sign

**8. Find the changes in sign of the following expressions and find their extreme values**

**i)** $x^2-5x+6$

Here a = 1, b = - 5, c = 6

$\Delta = b^2-4ac = 25-24 = 1$

Hence roots are real

$x^2-5x+6=0 \Rightarrow x=2,3$

$\therefore 2<x<3 \Rightarrow x^2-5x+6<0$

$x=2 \ \ or \ x=3 \Rightarrow x^2-5x+6=0$

$x<2 \ or \ \ x>3 \Rightarrow x^2-5x+6>0$

Here a = 1 > 0

$\therefore x^2-5x+6$ has absolute minimum value

$\therefore$ absolute minimum value $= \dfrac{4ac-b^2}{4a} = \dfrac{24-25}{4} = -\dfrac{1}{4}$

**ii)** $15+4x-3x^2$

a = -3, b =4, c = 15

$\Delta = b^2-4ac = 16-4(-3)(15)$

= 196 > 0

$\therefore$ Roots are real roots of $15+4x-3x^2=0$ are $-\dfrac{5}{3}$ and 3

{Refer to prob (ii) of 7}

$\therefore -\dfrac{5}{3} < x < 3 \Rightarrow 15+4x-3x^2>0$

$x<-\dfrac{5}{3} \ or \ \ x>3 \Rightarrow 15+4x-3x^2<0$

$x<-\dfrac{5}{3} \ or \ \ x>3 \Rightarrow 15+4x-3x^2=0$

Here a = - 3 < 0

$\therefore$ Absolute maximum value

$$= \frac{4ac - b^2}{4a} = \frac{-180 - 16}{4x - 3}$$

$$= \frac{196}{12} = \frac{49}{3}$$

**9. Find the maximum or minimum values of following expressions as x varies over R**

**i)** $x^2 - x + 7$

Here a = 1, b = -1, c = 7

a > 0 $\therefore$ absolute minimum $= \frac{4ac - b^2}{4a}$

$$= \frac{28 - 1}{4} = \frac{27}{4}$$

**ii)** $12x - x^2 - 32$

a = -1, b = 12, c = -32

a < 0 hence absolute maximum value $= \frac{4ac - b^2}{4a} = \frac{49 - 1)(-32) - 144}{-4} = 4$

**iii)** $2x + 5 - 3x^2$

Here a = -3, b = 2, c = 5

a < 0 hence maximum value $= \frac{4ac - b^2}{4a}$

$$= \frac{-60 - 4}{-12} = +\frac{16}{3}$$

**iv)** $ax^2 + bx + a \quad (a, b \in R)$

a > 0 minimum value $= \frac{4a^2 - b^2}{4a}$

a < 0 maximum value $= \frac{4a^2 - b^2}{4a}$

## SHORT ANSWER QUESTIONS:

**Determine the range of the following expressions**

**1.** $\dfrac{x^2+x+1}{x^2-x+1}$

**Sol**: let $\dfrac{x^2+x+1}{x^2-x+1}=y$

$\therefore x^2+x+1=x^2y-xy+y$

$x^2(y-1)-x(y+1)+y-1=0$

x is real

$\therefore b^2-4ac\geq 0$

$(y+1)^2-4(y-1)(y-1)\geq 0$

$-3y^2+10y-3\geq 0$

$3y^2-10y+3\leq 0$

$(3y-1)(y-3)\leq 0 \Rightarrow \dfrac{1}{3}\leq y\leq 3$

**ii)** $\dfrac{x+2}{2x^2+3x+6}$

Let $\dfrac{x+2}{2x^2+3x+6}=y$

$x+2=2x^2y+3xy+6y$

$2x^2y+x(3y-1)+6y-2=0$

Since x is real $b^2-4ac\geq 0$

$\therefore (3y-1)^2-4(2y)(6y-2)\geq 0$

$(9y^2-6y+1)-(48y^2-16y)\geq 0$

$-39y^2+10y+1\geq 0$

$39y^2-10y-1\leq 0$

$13y(3y-1)+1(3y-1)\le 0$

$(13y+1)(3y-1)\le 0$

$\therefore -\frac{1}{13}\le y\le \frac{1}{3}$

**iii)** $\frac{(x-1)(x+2)}{x+3}$

Let $\frac{(x-1)(x+2)}{x+3}=y$

$\therefore x^2+x-2=xy+3y$

$x^2+x(1-y)-(2+3y)=0$

Since x is real

$b^2-4ac\ge 0$

$(1-y)^2+4(2+3y)\ge 0$

$y^2-2y+1+8+12y\ge 0$

$y^2-10y+7\ge 0$

$(y-1)(y-9)\ge 0$

$y\le 1$ or $y\ge 9$

$\therefore y\in(-\infty,-1]\cup[9,\alpha)$

**iv)** $\frac{2x^2-6x+5}{x^2-3x+2}$

Let $\frac{2x^2-6x+5}{x^2-3x+2}=y$

$2x^2-6x+5=x^2y-3xy+2y$

$x^2(2-y)-6x+3xy+5-2y=0$

$x^2(2-y)+3x(y-2)+5-2y=0$

Since x is real $b^2-4ac\ge 0$

$\therefore 9(y-2)^2-4(2-y)(5-2y)\geq 0$

$9y^2-36y+36-4\{10-9y+2y^2\}\geq 0$

$y^2-4\geq 0\Rightarrow (y+2)(y-2)\geq 0$

$\therefore y\in(-\infty,-2]\cup[2,\infty)$

**2. Prove that $\frac{1}{3x+1}+\frac{1}{x+1}-\frac{1}{(3x+1)(x+1)}$ does not lie between 1 and 4 if x is real**

**Sol:** Let $\frac{1}{3x+1}+\frac{1}{x+1}-\frac{1}{(3x+1)(x+1)}=y$

$\frac{x+1+3x+1-1}{(3x+1)(x+1)}=y$

$\frac{4x+1}{3x^2+4x+1}=y$

$4x+1=3x^2y+4xy+y$

$3x^2y+4x(y-1)+y-1=0$

Since x is real

$b^2-4ac\geq 0$

$\{4(y-1)\}^2-4(3)(y)(y-1)\geq 0$

$16y^2-32y+16-12y^2+12y\geq 0$

$4y^2-20y+16\geq 0$

$y^2-5y+4\geq 0$

$(y-1)(y-4)\geq 0$

$\therefore y\in(\infty,1]\cup[4,\infty)$

Hence y does not lie between 1 and 4

**3. If x is real then prove that $\frac{x}{x^2-5x+9}$ lies between $-\frac{1}{11}$ and 1**

**Sol:** Let $\frac{x}{x^2-5x+9}=y$

$x=x^2y-5xy+9y$

$x^2y-(5y+1)x+9y=0$

Since x is real

$b^2-4ac\geq 0$

$(5y+1)^2-4y(9y)\geq 0$

$25y^2+10y+1-36y^2\geq 0$

$-11y^2+10y+1\geq 0$

$11y^2-10y-1\leq 0$

$11y^2-11y+y-1\leq 0$

$(11y+1)(y-1)\leq 0$

$\therefore -\frac{1}{11}\leq y\leq 1$

Hence y lies between $-\frac{1}{11}$ and 1

**4. If the expression $\frac{x-p}{x^2-3x+2}$ takes all real values $\forall x\in R$ then find the bounds of p**

**Sol:** Let $\frac{x-p}{x^2-3x+2}=y$

$x-p=x^2y-3xy+2y$

$x^2y-x(3y+1)+2y+p=0$

Since x is real

$b^2-4ac\geq 0$

$(3y+1)^2 - 4y(2y+p) \geq 0$

$y^2 + 6y - 4py + 1 \geq 0$

$y^2 + y(6-4p) + 1 \geq 0$

Since coefficient of $y^2$ and expression have same sign $\forall y \in R$

$(6-4p)^2 - 4 \leq 0$

$36 + 16p^2 - 48p - 4 \leq 0$

$16p^2 - 48p + 32 \leq 0$

$p^2 - 3p + 2 \leq 0$

$(p-1)(p-2) \leq 0$

$p \in (1,2)$

{$\therefore$ for p – 1, 2, given expression is not defined}

**5.** **If $c^2 \neq ab$ and the roots of $(c^2-ab)x^2 - 2(a^2-bc)x + (b^2-ac) = 0$ are equal then show that $a^3+b^3+c^3 = 3abc$ (or) a = 0**

**Sol:** Given equation is

$(c^2-ab)x^2 - 2(a^2-bc)x + (b^2-ac) = 0$

Roots are equal

$\therefore 4(a^2-bc)^2 - 4(c^2-ab)(b^2-ac) = 0$

$a^4 + b^2c^2 - 2a^2bc - b^2c^2 + ac^3 + ab^3 - a^2bc = 0$

$a^4 + ab^3 + ac^3 - 3a^2bc = 0$

$a(a^3+b^3+c^3-3abc) = 0$

$\therefore a = 0$ or $a^3 + b^2 + c^3 - 3abc = 0$

## III) QUADRATIC INEQUATIONS

## EXERCISE -- 1(c)

**1. Solve the in equations by algebraic method**

**i)** $15x^2 + 4x - 4 \le 0$

$15x^2 + 10x - 6x - 4 \le 0$

$5x(3x+2) - 2(3x+2) < 0$

$(3x+2)(5x-1) \le 0$

$\therefore -2/3 \le x \le 2/5$

**ii)** $x^2 - 2x + 1 < 0$

$(x-1)^2 < 0$

Square of any real number cannot be negative

Hence $x \in \phi$

**iii)** $2 - 3x - 2x^2 \ge 0$

$2x^2 + 3x - 2 \le 0$

$2x^2 + 4x - x - 2 \le 0$

$2x(x+2) - (x+2) \le 0$

$(x+2)(2x-1) \le 0$

$\therefore -2 \le x \le \frac{1}{2}$

**iv)** $x^2 - 4x - 21 \ge 0$

$x^2 - 7x + 3x - 21 \ge 0$

$x(x-7) + 3(x-7) \ge 0$

$(x+3)(x-7) \ge 0$

$x \le -3$ or $x \ge 7$

$\therefore x \in (-\infty, -3] \cup [7, \infty)$

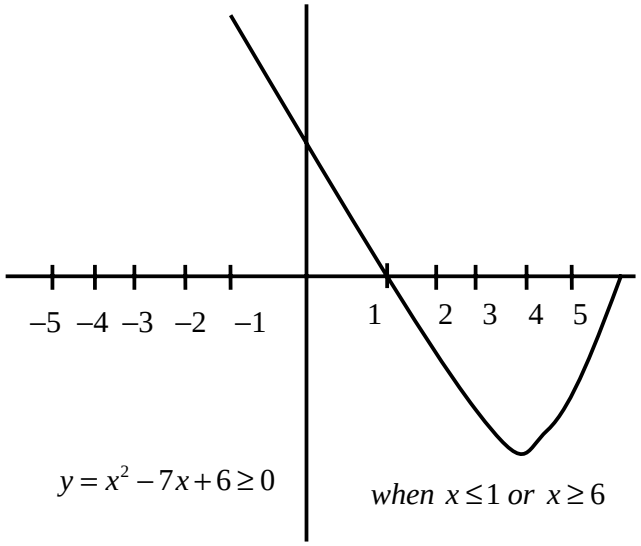
**2. Solve the following in equations by graphical method**

**i)** $x^2 - 7x + 6 > 0$

let $y = x^2 - 7x + 6$

The values of y for some selected values of x are given below

| x | -5 | -4 | -3 | -2 | -1 | 0 | 1 |
|---|---|---|---|---|---|---|---|
| y | 66 | 50 | 36 | 24 | 14 | 6 | 0 |
| x | 2 | 3 | 4 | 5 | 6 | | |
| y | -8 | -6 | -6 | -4 | 0 | | |



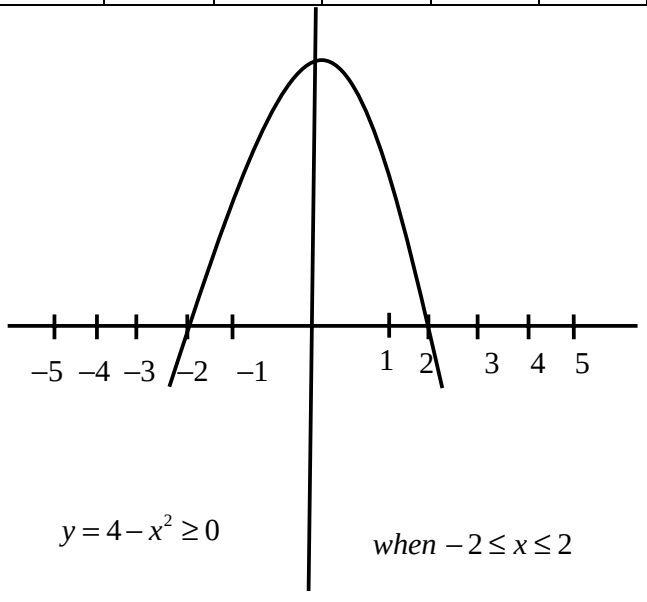

$y = x^2 - 7x + 6 \geq 0$ *when* $x \leq 1$ *or* $x \geq 6$

Hence solution set $= (-\infty, -1] \cup [6, \infty)$

**ii)** $4 - x^2 \geq 0$

Let $y = 4 - x^2$

| x | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| y | -12 | -5 | 0 | 3 | 4 | 3 | 0 | -5 | -12 |



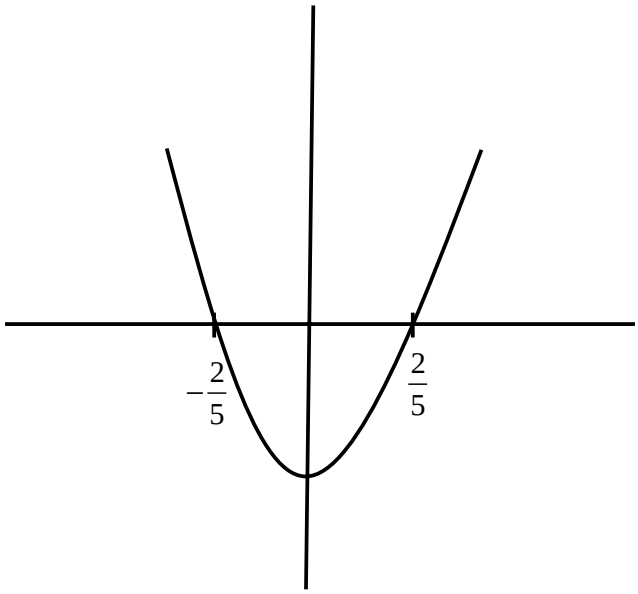

$y = 4 - x^2 \geq 0$ *when* $-2 \leq x \leq 2$

Hence solution set $x \in [-2, 2]$

**iii)** $15x^2 + 4x - 4 \leq 0$

let $y = 15x^2 + 4x - 4$

| x | -3 | -2 | -1 | $-\frac{2}{3}$ | 0 | $\frac{2}{5}$ | 1 |
|---|---|---|---|---|---|---|---|
| y | 89 | 48 | 7 | 0 | -4 | 0 | 15 |




$y = 15x^2 + 4x - 4 \leq 0$

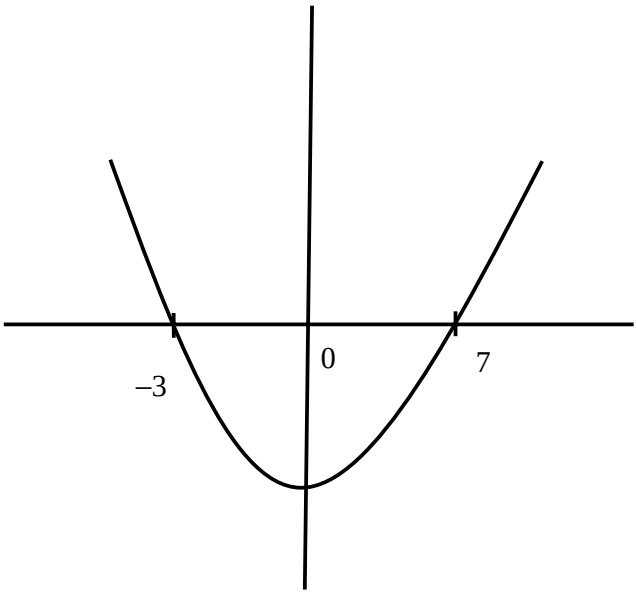
$\Rightarrow -\frac{2}{3} \leq x \leq \frac{2}{5}$

Hence $x \in \left[-\frac{2}{3}, \frac{2}{5}\right]$

**iv)** $x^2 - 4x - 21 \geq 0$

$y = x^2 - 4x - 21$

| x | -3 | -2 | -1 | 0 | 1 | 2 | 7 |
|---|---|---|---|---|---|---|---|
| y | 0 | -9 | -16 | -21 | -24 | -25 | 0 |




$y = x^2 - 4x - 21 \geq 0$

When $x \leq -3$ or $x \geq 7$

**2. Solve the following in equations**

$\sqrt{3x-8} < -2$

$3x - 8 \geq 0 \Rightarrow x \geq \frac{8}{3} \rightarrow (1)$

And $\sqrt{3x-8} < -2$

Here $\sqrt{3x-8} \geq 0$

But given $\sqrt{3x-8} < -2$

$\therefore$ No solution

**ii)** $\sqrt{-x^2+6x-5} > 8-2x$

$\Leftrightarrow -x^2 + 6x - 5 \geq 0$

And i) 8 – 2x < 0 or $-x^2 + 6x - 5 > (8-2x)^2$ and $8 - 2x \geq 0$

We have $x^2 - 6x + 5 \leq 0$

$(x-1)(x-5) \leq 0 \Rightarrow 1 \leq x \leq 5$

And $2x - 8 > 0 \Rightarrow x > 4$

**Or** $-x^2+6x-5>64+4x^2-32x$

$0>5x^2-38x+69$

$5x^2-38x+69<0$

And x < 4

$5x^2-23x-15x+69<0$

$x(5x-23)-3(5x-23)<0$

$(x-3)(5x-23)<0$

$3<x<\frac{23}{5}$

$1\le x\le 5$ and x > 4

Or $3<x<\frac{23}{5}$ and x < 4

$\therefore 3<x\le 5$

**iii) Solve the in equation** $\sqrt{x+2}>\sqrt{8-x^2}$

When a, $b\in R$ and $a\ge 0$, $b\ge 0$ then $\sqrt{a}>\sqrt{b}\Leftrightarrow a>b\ge 0$

$\therefore \sqrt{x+2}>\sqrt{8-x^2}$

$\Leftrightarrow x+2>(8-x^2)\ge 0$

$x>-2,\ |x|<2\sqrt{2}$

$x^2+x-6>0$

$x\in(-\infty,-3)\cup(2,\infty)$

$\therefore$ Solution

$\therefore x\in(2,2\sqrt{2})$

**iv) Solve the in equation**

$\sqrt{(x-3)(2-x)} < \sqrt{4x^2+12x+11}$

$(x-3)(2-x) \geq 0$ and $(x-3)(2-x) < 4x^2+12x+11$

$2 \leq x \leq 3$ and $5x-6-x^2 < 4x^2+12x+11$

$2 \leq x \leq 3$ and $0 < 5x^2+7x+17$

$2 \leq x \leq 3$ and $5x^2+7x+17 \geq 0$

$5x^2+7x+17 > o \quad \forall x \in R$

$\therefore 7^2-4(5)(17) < 0$

$\therefore$ Solution set [2, 3]

**v) Solve the in equation**

$$\frac{\sqrt{6+x-x^2}}{2x+5} \geq \frac{\sqrt{6+x-x^2}}{x+4}$$

$6+x-x^2=0$ , $2x+5 \neq 0$, and $x+4 \neq 0$

(or) $6+x-x^2 > 0$ and $\frac{1}{2x+5} \geq \frac{1}{x+4}$

$6+x-x^2-0 \Rightarrow x=-2$ or x = 3

$6+x-x^2 > 0 \Rightarrow -2 < x < 3$

$\frac{1}{2x+5} \geq \frac{1}{x+4} \Rightarrow 2x+5 \leq x+4$

$x \leq -1$

$\therefore \{-2,3\} \cup [-2,-1]$

$[-2,-1] \cup \{3\}$

**vi) Solve the in equation**

$\sqrt{x^2-3x-10} > 8-x$

Sol: $x^2-3x-10 \geq 0$

**and i) 8 – x < 0**

**or (ii)** $x^2-3x-10 > (8-x)^2$ **and** $(8-x) \geq 0$

$(x-5)(x+2) \geq 0$

$\Rightarrow x \in [\alpha, -2] \cup [5, \infty]$

$8-x < 0 \Rightarrow x > 8$

**ii)** $x^2-3x-10 > 64 + x^2 - 16x$

$13x > 74 \Rightarrow x > \frac{74}{13}$

And $8-x \geq 0 \Rightarrow x \leq 8$

$x \in \left(\frac{74}{13}, \infty\right)$

$x \in (-\infty, 8)$

$(8, \infty) \cup \left\{\left(\frac{74}{13}, \infty\right) \cap (-\infty, 8)\right\}$

$(8, \infty) \cup \left\{\left(\frac{74}{13}, \infty\right) = \left(\frac{74}{13}, \infty\right)\right\}$

## SOLVE THE FOLLOWING EQUATIONS

**1.** $9\left(x^2 + \frac{1}{x^2}\right) - 27\left(x + \frac{1}{x}\right) + 8 = 0$

Let $x + \frac{1}{x} = a$

$x^2 + \frac{1}{x^2} + 2 = a^2 \Rightarrow x^2 + \frac{1}{x^2} = a^2 - 2$

$$9\left(x^2+\frac{1}{x^2}\right)-27\left(x+\frac{1}{x}\right)+8=0$$

$$9(a^2-2)-27a+8=0$$

$$9a^2-27a-10=0$$

$$9a^2+3a-30a-10=0$$

$$3a(3a+1)-10(3a+1)=0$$

$$(3a+1)(3a-10)=0$$

$$a=-\frac{1}{3},\ a=\frac{10}{3}$$

**Case i)**: suppose $a=-\frac{1}{3}$

$$x+\frac{1}{x}=-\frac{1}{3}\Rightarrow\frac{x^2+1}{x}=-\frac{1}{3}$$

$$3x2+3=-x\Rightarrow 3x^2+x+3=0$$

$$x=-\frac{1\pm\sqrt{1-4(3)(3)}}{2\times 3}$$

$$x=-\frac{1\pm i\sqrt{35}}{6}$$

**Case ii)** : $a=\frac{10}{3}$

$$x+\frac{1}{x}=\frac{10}{3}\Rightarrow 3x^2+3=10x$$

$$3x^2-10x+3=0$$

$$3x^2-9x-x+3=0$$

$$3x(x-3)-1(x-3)=0$$

$$(x-3)\ (3x-1)=0$$

$$x=3,\ x=\frac{1}{3}$$

**2.** $8\sqrt{\frac{x}{x+3}}-\sqrt{\frac{x+3}{x}}=2$

$\sqrt{\frac{x}{x+3}}=a$

$8a-\frac{1}{a}=2\Rightarrow 8a^2-2a-1=0$

$8a^2-4a+2a-1=0$

(4a +1)(2a – 1) = 0

$a=-\frac{1}{4},\ a=\frac{1}{2}$

**Case i)** $a=-\frac{1}{4}$

$\sqrt{\frac{x}{x+3}}=-\frac{1}{4}\Rightarrow 16x=x+3$

$15x=3\Rightarrow x=\frac{1}{5}$ Does not satisfy the equation

**Case ii)** : $a=\frac{1}{2}$

$\sqrt{\frac{x}{x+3}}=\frac{1}{2}\Rightarrow 4x=4x=x+3$

$3x=3\Rightarrow x=1$ is only solution

**3.** $\sqrt{\frac{4x-1}{4x+1}}=\sqrt{\frac{4x+1}{4x-1}}=\frac{8}{3}$

$\sqrt{\frac{4x-1}{4x+1}}=a$

$a-\frac{1}{a}=\frac{8}{3}\Rightarrow 3a^2-8a-3=0$

$3a^2-9a+a-3=0$

(3a +1)(a -3) = 0

$a = -\frac{1}{3}$, a = 3

**Case i)** $a = -\frac{1}{3}$

$$\sqrt{\frac{4x-1}{4x+1}} = -\frac{1}{3} \Rightarrow 36x - 9 = 4x + 1$$

$$32x = 10 \Rightarrow x = \frac{10}{32} = \frac{5}{16}$$

**Case ii)** a = 3

$$\sqrt{\frac{4x-1}{4x+1}} = 3 \Rightarrow \frac{4x-1}{4x+1} = 9$$

$$4x - 1 = 36x + 9 \Rightarrow 32x = -10$$

$x = -\frac{5}{16}$ is only solution

$x = \frac{5}{16}$ Does not satisfy the equation

**3.** $\sqrt{3x^2+1} + \frac{4}{\sqrt{3x^2+1}} = 5$

Let $\sqrt{3x^2+1} = a$

$$a + \frac{4}{a} = 5 \Rightarrow a^2 - 5a + 4 = 0$$

$$(a-4)(a-1) = 0 \Rightarrow a = 4; a = 1$$

**Case i)** a = 4

$$\sqrt{3x^2+1} = 4 \Rightarrow 3x^2 + 1 = 16$$

$$\Rightarrow x^2 = 5 \Rightarrow x = \pm\sqrt{5}$$

**Case ii)** a = 1

$$\sqrt{3x^2+1} = 1 \Rightarrow 3x^2 + 1 = 1 \Rightarrow x = 0$$

**5.** $2\left(x^2+\frac{1}{x^2}\right)-3\left(x+\frac{1}{x}\right)=1$

This is same model as (1) try your self

**6.** **x (x + 2 ) (x + 3) (x + 5) = 72**

**X (x +5) (x + 2) (x +3) = 72**

$(x^2+5x)(x^2+5x+6)=72$

Let $x^2+5x=a$

$a(a+6)=72\Rightarrow a^2+6a-72=0$

(a +120 (a – 6) = 0

a = - 12, a = 6

**case i)** a = - 12

$x^2+5x=-12\Rightarrow x^2+5x+12=0$

$x=\frac{5=\sqrt{25-48}}{2}=\frac{5\pm i\sqrt{23}}{2}$

**Case ii)** a = 6

$x^2+5x=6\Rightarrow x^2+5x-6=0$

$x^2+6x-x-6=0$

$(x+6)(x-1)=0$

$\therefore x=-6$ , x = 1

**7.** **x (x -1) (x + 2) (x – 3) = - 8**

$(x^2-x)\{x^2-x-6\}=-8$

Let $x^2-x=a$

$a(a-6)=-8\Rightarrow a^2-6a+8=0$

(a – 4) (a – 2) = 0

a – 4 or a = 2

**Case i)** : a = 4

$$x^2 - x = 4 \Rightarrow x^2 - x - 4 = 0$$

$$x = \frac{1 \pm \sqrt{1+16}}{2} = \frac{1 \pm \sqrt{17}}{2}$$

**Case ii)** a = 2

$$x^2 - x = 2 \Rightarrow x^2 - x - 2 = 0$$

$$x^2 - 2x + x - 2 = 0$$

X (x – 2) + (x – 2) = 0

(x + 1)( x – 2) = 0

X = 2; x = -1

**8.** **(x – 1) (x + 1) (2x +3) (2x – 1) = 3**

(x – 1) (2x + 3) (x +1) 2x – 1) = 3

$$(2x^2 + 3x - 2x - 3)\{2x^2 - x + 2x - 1\} = 3$$

$$(2x^2 + x - 3)(2x^2 + x - 1) = 3$$

$$2x^2 + x = a$$

$$(a-3)(a-1) = 3 \Rightarrow a^2 - 4a + 3 = 3$$

$$a^2 - 4a = 0 \Rightarrow a = 0, 4$$

If $2x^2 + x = a \Rightarrow 2x^2 + x = 0$

$$x(2x+1) = 0 \Rightarrow x = 0; -\frac{1}{2}$$

If $2x^2 + x = 4 \Rightarrow 2x^2 + x - 4 = 0$

$$x = \frac{-1 \pm \sqrt{1 - 4(2)(-4)}}{4} = \frac{-1 \pm \sqrt{33}}{4}$$

**9. Find the range of the values of x which satisfy** $\frac{(x+1)(x-3)}{x-2} \geq 0$

**Sol:** $\frac{(x+1)(x-3)}{x-2} \geq 0$

**Case i)** $(x+1)(x-3) \geq 0$

And $(x - 2) > 0$

$\therefore x \geq 3$

**Case ii)** $(x+1)(x-3) \leq 0$ and $x - 2 < 0$

$-1 \leq x \leq 3$ and $x < 2$

$\therefore -1 \leq x < 2$

$\therefore$ solution set $[-1, 2) \cup [3, \infty)$