Dynamics ------- Points to remember

1. According to Newton's first Law of motion every body continue to be its state of restore of uniform motion in a straight unless compelled by an external force to change that state.
2. According to Newton's law of motion the rate of change of momentum of a body is directly proportional to the external force applied and takes place in the same direction in which the external force is acting.
3. According to Newton's law of motion to every action ,there is always an equal and opposite reaction.
4. When two objects connected by a cable ,one object having horizontal motion and the other having vertical motion –Tension T= $\frac{m_1 m_2}{(m_1+m_2)} g$.
5. In the case of Atwood's machine tension T= $\frac{2m_1 m_2}{(m_1+m_2)} g$.
6. Block placed in contact with each other on a frictional Horizontal surface Common acceleration a= $a = \frac{F}{m_1+m_2}$ and the magnitude of contact force $f_1 = f_2 = F\left[\frac{m_2}{m_1+m_2}\right]$.
7. When the lift moves up with an acceleration 'a' then reaction force R=mg$\left(1+\frac{a}{g}\right)$.
8. When the lift moves downwards with an acceleration 'a'then reaction force $R = mg\left(1-\frac{a}{g}\right)$
9. When the lift is stationary or moves with uniform velocity then acceleration is zero and net force is zero.
10. When the resultant force acting on a system is zero , the total momentum of the system remains Constant. This is called law of conservation of linear momentum
11. Work done is measured by the dot product of force $\vec{F}$ and displacement $\vec{S}$
12. Rate of doing work is called power
13. Capacity to do work is called energy
14. The energy possessed by a body by virtue of its state is called potential energy.
15. The energy possessed by a body by virtue of its motion is called kinetic energy
16. According to work energy theorem work done =change in kinetic energy.
17. According to law of conservation of energy total energy of a system is always its constant.

Long Answer Questions:

1. **State Newton second law of motion. Hence derive the equation of motion F = ma from it. A body is moving along a circular path such that its speed always remains constant. Should there be force acting on the body?**

Second Law: - The rate of change of momentum of a body is directly proportional to external force and along the direction of force.

Significance: 1) Proves F = ma 2) Define unit of force

Consider a body of mass 'm' traveling with an initial velocity 'u'. Let an external force acts on the body for a time (t), changes its velocity to v.

$\therefore$ Rate of change of momentum of the body = $\frac{mv-mu}{t}$

According to second Law $F\alpha\frac{mv-mu}{t} \Rightarrow F\alpha\frac{m(v-u)}{t}$

$\Rightarrow F\alpha ma$

$\Rightarrow \boxed{F = kma}$

If F = 1 , m = 1 and a = 1, then k = 1 $\boxed{\Rightarrow F = ma}$

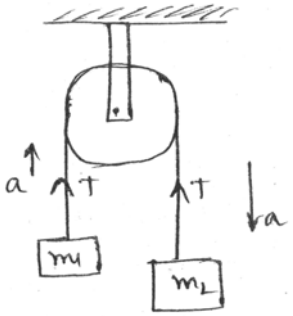
i.e., if the force acting on the body of unit mass produces unit acceleration, then it is called unit force

If the body is moving along a circular path its speed always remains constant, its velocity changes at every point and resultant force acts on the body.

2. Two unequal masses are connected by a very light string over a clamped light smooth pulley. Find the acceleration of the system and the tension in the string. ?

Two Objects of unequal masses suspended by a cable passing over a pulley : (ATWOOD'S MACHINE)

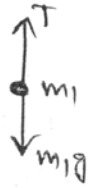
1) Two objects of unequal masses m_1 and m_2 suspended by a cable that passes over a mass less, frictionless pulley, as shown in the fig. is called At woods machine. It is used to determine free fall acceleration in the laboratory.

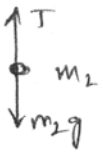


2) As $m_2 > m_1$, m_1 will be accelerating upwards and the mass 'm_2' will be accelerating downwards with the same acceleration (a) .

3) From the free body diagram of mass 'm_1'

$T - m_1 g = m_1 a$ _____ (1)





4) From the free body diagram of mass m_2

$m_2 g - T = m_2 a$ _____ (2)

Add (1) and (2)

$\boxed{a = \frac{(m_2 - m_1)}{(m_1 + m_2)} g}$ _____ (3) Substituting (3) in (1)

$\boxed{T = \frac{2m_1 m_2 g}{(m_1 + m_2)}}$

$\Rightarrow \boxed{g = \frac{(m_1 + m_2) a}{(m_2 - m_1)}}$

3) **Discuss how the apparent weight of a person in a lift changes, when it is**
a) Moving up with acceleration

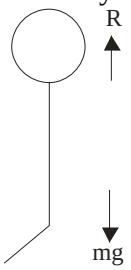
b) moving down with acceleration and

c) moving with uniform velocity.

Apparent Weight :- If a body is moving along the gravity (or) opposite to the gravity, the weight of the body appears to change, then the resultant weight of the body is called apparent weight (w') .

Apparent Weight of a Man In a lift:-

Consider a man of mass (m) on the floor of the lift. Then the gravitational force (mg) will act vertically downwards and the floor exerts a reaction force (R) on the person.



DYNAMICS

When the lift is stationary:

In this case R - mg = 0

$\Rightarrow \boxed{R = mg}$

Here the apparent weight an acceleration (a) :-

In this case R > mg

$\Rightarrow$ R-mg = ma

$\Rightarrow$ R = mg +ma

$\Rightarrow$ R = m(g + a)

$$\Rightarrow \boxed{R = mg\left(1+\frac{a}{g}\right)}$$

Thus man appears to be heavier by $\left(\frac{a}{g}\right)$ times his true weight (mg)

When the lift moves down with an acceleration (a) : -

In this case mg > R

$\Rightarrow$ mg –R =ma

$\Rightarrow$ R=mg-ma

$\Rightarrow$ R= m(g-a)

$$\Rightarrow \boxed{R = mg\left(1-\frac{a}{g}\right)}$$

Thus the man appears to be lighter than his true weight 9mg) by a factor $\left(\frac{a}{g}\right)$

When the lift falls freely under the action of gravity

In this case a = +g

R = m(g-g) = 0

Thus the man feels weightlessness.

4) a) Define kinetic energy. Deduce an expression for K.E. of the body

b) Obtain the relation between Kinetic Energy and momentum of the body can the moment of a body change without changing its Kinetic Energy.

Kinetic Energy: The energy possessed by a body by virtue of its motion is called kinetic energy (K.E)

Example: (i) A moving train (ii) Flowing water (iii) Blowing wind.

Expression for Kinetic energy:

***** Consider a bullet of mass m is moving with a velocity v strikes a wooden block and penetrate through a distance x before coming to rest.

*** As** the bullet strikes the block with a velocity 'v' ,

Here: U = v, S = x, V = 0, A = ?

We know that $a = \frac{v^2 - u^2}{2s}$

$$a = \frac{-v^2}{2x}$$

The resistance force offered by the block.

$$F = \frac{-mv^2}{2x} \quad (\because F = ma)$$

According to Newton's III law the force with wh ich the bullet strikes the wooden block is given by .

$$F = \frac{mv^2}{2x}$$

$\therefore$ work done by the bullet (W) = FS

$$W = \frac{mv^2}{2x} \times x$$

$$\boxed{W = \frac{1}{2}mv^2}$$

This work done is stored in the form of kinetic energy.

$$\boxed{K.E = \frac{1}{2}mv^2}$$

<u>Relation between Kinetic Energy and momentum</u>:

We know that the Kinetic Energy of a body is given by

$$K.E = \frac{1}{2}mv^2$$

$$K.E = \left(\frac{1}{2}mv^2\right)\frac{m}{m}$$

$$K.E = \frac{(mv)^2}{2m}$$

$$\boxed{K.E = \frac{p^2}{2m}}$$

Yes, in the uniform circular motion, the speed is constant; hence the K.E is constant but momentum changes due to change in the direction of linear velocity.

5) State Law of conservation of energy and verity in the case of freely falling body. What are the conditions under which the law of conservation of energy is applicable?

<u>Law of Conservation of energy</u>:

The total energy in the universe always remains constant i.e., the energy neither be created nor destroyed but it can be converted from one form to another form.

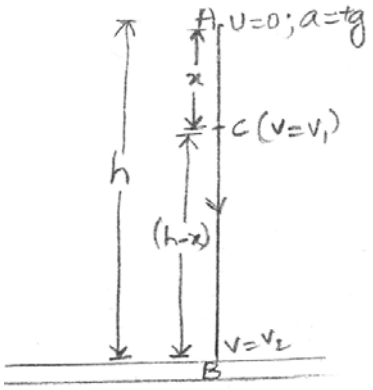
Ex: i) In the battery chemical energy is converted in to electrical energy.

ii) In the electric bulb electrical energy is converted into light and heat energy.

<u>Law of conservation of energy in case of freely falling body:</u>

***** Consider a body of mass m is falling freely from a height h on the surface of the earth. As the body is falling freely v = 0 ; a = +g

Since the body is freely falling height, Potential Energy decreases and velocity, Kinetics Energy increases



Kinetics Energy $= \frac{1}{2}m(0) = 0$ $\therefore$ E = mgh _____(1)

P.E = mgh

At 'C' P.E = mg(h-x) = mgh –mgx

K.E = $\frac{1}{2}mv_1^2 = \frac{1}{2}m(2gx) = mgx$

$\therefore$ E = mgh _____(2)

At 'B' P.E =mg(0) = 0

K.E = $\frac{1}{2}mv_2^2 = \frac{1}{2}m(2gh) = mgh$

$\therefore$ E = mgh _____(3)

From (1) ,(2) & (3) it is proved that total energy is constant in the case of freely falling body Under the action of gravitational force only, the total energy the system constant.

6) State and prove Work – Energy. What are the conditions under which it is applicable?

Work Energy theorem:

Statement: The work **done by the constant force on a body is equal to change in its Kinetics Energy this is called work energy theorem.**

Proof:

Consider a body of mass m is traveling with an initial velocity u let an external force F acts on the body for a time t causes a displacement S and let the velocity is changed to v. Let a is the acceleration of the body.

$\therefore$ The work done by the constant force on a body is given by

W = FS

W =mas

$W = m\left(\frac{v-u}{t}\right)\left(\frac{v+u}{2}\right)t$ ($\because$ S= avg , velocity ×time)

$W = \frac{1}{2}m(v^2 - u^2)$

$$\boxed{W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2}$$

-Hence the work energy theorem is proved.

The theorem is applicable not only for a single particle, but also for a system. When it is applied to a system of two (or) more particles, change in the Kinetics Energy of the system is equal to work done on the system by the external as well as the internal forces this theorem is applicable for a system under the action of variable forces, conservative and non conservative forces.

7) State the law of conservation of energy and verify it in case of a body projected vertically upwards.

(or)

Can we apply the laws of conservation of energy to a system if the internal forces doing work are non – conservative?

Statement: - "Energy can be neither created nor destroyed but it can be converted from one form to another form (or) the total energy of a closed system always remains constant" is called law of conservation of energy.

Proof: In the case of a body projected vertically

Upwards:

Suppose a body of mass 'm' is projected vertically upwards with an initial velocity 'u' form the ground at 'A' as shown in figure .

The total mechanical energy (E) = KE +PE

Let A, B,C be the points on the ground, at heights 'h', and 'H' respectively.

At 'A' = P.E. of the body = mg ×0=0 [$\because$ h = 0]

Kinetic energy K.E = $\frac{1}{2}mv^2$

Total mechanical energy (EA) at 'A' = PE+KE =0+ $\frac{1}{2}mv^2$

$\boxed{\therefore EA = \frac{1}{2}mv^2}$ _____(1)

At 'B' : When the body reaches the point 'B' at a height 'h' velocity of the body VB at 'B' can be determine using the equation.

$v^2 - u^2 = 2as \quad \mu = \mu$

$v^2_B - u^2 = 2(-g)h \quad v = v_B = ?$

$v^2_B = u^2 - 2gh \quad a = -g$

KE = $\frac{1}{2}mv^2_B$ s = h

= $\frac{1}{2}m\left(u^2 - 2gh\right) = \frac{1}{2}mu^2 = mgh$

Potential energy (P.E) = mgh.

Total mechanical energy (E_B) at B = P.E +K.E

$E_B = mgh + \frac{1}{2}mv^2 - mgh$

$\boxed{E_B = \frac{1}{2}mv^2}$

At 'C'

When the body reaches the maximum height (H) above the ground, its final velocity becomes zero maximum height (H) = $\frac{u^2}{2g}$

Potential Energy of the body = mgH = $mg\left(\frac{u^2}{2g}\right)$

Potential Energy = $\frac{1}{2}mu^2$

Kinetics Energy of the body = 0

Total mechanical energy at 'C' (Ec) = P.E +K.E

$Ec = \frac{1}{2}mu^2 + 0$ $\boxed{Ec = \frac{1}{2}mu^2}$ _____(3)

From the above three equations the total mechanical energy of the system remains constant. Hence law of conservation of energy is proved incase of vertically projected body.

Reason:- No, we cannot apply the law of conservation energy to a system. If the inter forces doing work. Some internal forces of the system are non – conservative the mechanical energy of

the system is not constant. In law of conservation of energy the total mechanical energy of the system constant. The air resistance is a non – conservation, then total mechanical energy is not constant. Similarly if friction is present between parts of a system. The total mechanical energy does not remain constant

8. A ball is projected vertically upwards from ground with an initial velocity of 9.8 ms^{-1}**. ; g = 9.8** ms^{-2}**.**

Total mechanical energy of the ball on the ground = $\frac{1}{2}mu^2$

The total mechanical energy at the maximum height position = mgh.
By the law of conservation of energy

$$mgh = \frac{1}{2}mu^2$$

$$h = \frac{u^2}{2g} = \frac{9.8\times9.8}{2\times9.8} = 4.9m.$$

9. Calculate the velocity of the bob of a simple pendulum at its mean position if it is able to rise to a vertical height of 10 cm.
Solution:
h = 10 × 10^{-2} m ; g = 9.8 ms^{-2}. Taking the mean position as reference position for potential energy, the total mechanical energy of the bob at its mean position = $\frac{1}{2}mu^2$.

When the bob is able to rise to a vertical height h its velocity at that point is zero. So, its kinetic energy is zero and it has only potential energy. The total mechanical energy of the bob at that position = mgh.
By the law of conservation of energy

$$\frac{1}{2}mu^2 = mgh$$

$$u = \sqrt{2gh}$$

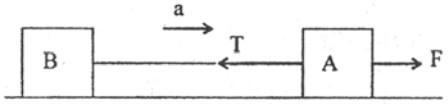
$$u = \sqrt{2\times9.8\times10\times10^{-2}} = 1.4ms^{-1}.$$

Example -18: Two identical blocks A and B each of mass M are connected to each other through a light string. The system is placed on a smooth horizontal floor. When constant force F is applied horizontally on the block A, find the tension in the string.
Solution:

The acceleration of the system of two blocks A and B = $\frac{\text{Force}}{\text{Total mass}}$

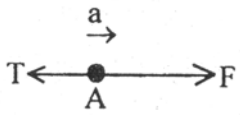
$$a = \frac{F}{M+M} = \frac{F}{2M}$$

If we consider the free body diagram of A, the forces acting on it are (i) the applied force F and (ii) the tension T on the string as shown in the following Fig 4.21.
The resultant force = F-T
Ma = F-T



$$m\left(\frac{F}{2M}\right) = F - T \qquad \left(\because a = \frac{F}{2M}\right)$$

$$\frac{F}{2} = F - T$$



$T = F - \frac{F}{2} = \frac{F}{2}$.

10. A car starts from rest and accelerates uniformly with $2 ms^{-2}$. At t = 5s, a stone is dropped out of the window (1m high) of the car. What are the (i) velocity and (ii) acceleration of the stone at t = 5.1 s ?

Solution:

i) Initial velocity of the car, u = 0; a = 2 ms^{-2} ; t = 5 s. Final velocity of the car after 5s,

$v = u + at = 0 + 2 \times 5 = 10 ms^{-1}$.

This velocity of the car is the horizontal direction. So, it is written as $V_x = 10 ms^{-1}$. When the stone is dropped out of the window it becomes a freely falling body with the initial vertical velocity, u = 0 ; a = +g = $+9.8 ms^{-2}$, time interval between the dropping of the stone and the time at which is velocity is to be determined , t =5.1 – 5=0.1 s. Final vertical velocity of the stone at t = 5.1 s is

$v = u + at = 0 + 9.8 \times 0.1 = 0.98 ms^{-1}$

As this velocity is in vertical direction it can be written as $v_y = 0.98 ms^{-1}$. The resultant velocity of the stone has magnitude

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{10^2 + (0.98)^2} = 10.05 ms^{-1}$

$\tan\theta = \frac{v_y}{v_x} = \frac{0.98}{10} = 0.098$; $\theta = 5°36'$

ii) The acceleration of the stone at t = 5.1 s is a +g = 9.8 ms^{-2}.

11. A particle of mass 1g moves on a straight line. The variation of its velocity with time is shown in Fig Find the force acting on the particle at t = 1 s, 4s and 7s.

Solution:

Mass of the particle, m = 1 g = 1 × 10^{-3} kg. To find the force acting on the particle we have to determine its acceleration from is velocity – time graph. Acceleration is equal to slope of velocity - time graph.

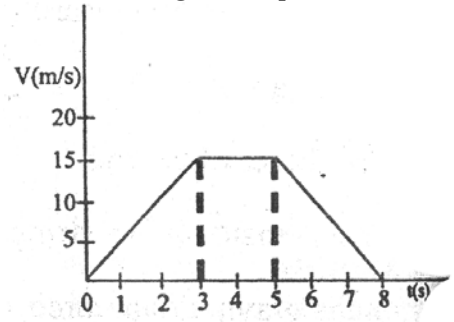
i) At t = 1s, acceleration of the particle is equal to the slope of the straight line from 0 to 3 s as 1 s is between 0 and 3 s.

Acceleration of the particle at t = 1 s is

$a = \frac{v_f - v_i}{t_f - t_i} = \frac{15 - 0}{3 - 0} = 5 ms^{-2}$

Where v_f is the final velocity at $t_f = 3s, v_1$ is the initial velocity at $t_i = 0s$.

∴ Force acting on the particle = ma = $1 \times 10^{-3} \times 5 = 5 \times 10^{-3} N$.



ii) At t = 4s, acceleration is given by

a = slope of the straight line from t = 3 to 5 s

$= \frac{v_f - v_i}{t_r - t_i} = \frac{15-15}{5-3} = \frac{0}{2} = 0 \text{ ms}^{-2}$.

$\therefore$ Force acting on the particle = ma = $1\times10^{-3} \times0=0$ N.

iii) At t = 7s , acceleration is given by

a = slope of the straight line from t = 5 to 8 s

$= \frac{v_f - v_i}{t_f - t_i} = \frac{0-15}{8-5} = \frac{15}{3} = -5 \text{ ms}^{-2}$.

$\therefore$ Force acting on the particle = ma =$1\times10^{-3} \times-5\times10^{-3}$ N.

Negative sign indicates that the force is in the opposite direction of the motion of the particle.

12. Two objects of masses m_1 and m_2 are connected by a very light string going over a clamped light smooth pulley as shown in Fig. 4.24 (a) . The object of mass m_2 is on a smooth inclined plane of angle of inclination θ and the object of mass m_1 is hanging vertically. Find the acceleration of the system and the tension in the string.

Solution: The two objects form a system since both the objects have same acceleration a. The forces on m_1 are (a) the tension in the string, T vertically upward , (b) the gravitational force, m_1g vertically downward .

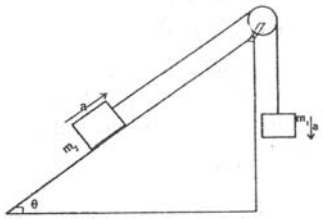


Fig. m_2 on inclined plane and m_1 hanging vertically

The free body diagram of m_1 is shown in Fig. 4.24 (b). The acceleration of m_1 is downwards. So, m_1g is greater than the tension in the string.

The resultant downward force on $m_1 = m_1g - T$.

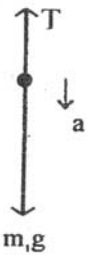


Fig. (b) Free body diagram of m_1

$m_1a = m_1g - T$ ……(4.17)

The forces on m_2 are a) the tension in the string, T up along the plane

b) the gravitational force, m_2g vertically downward

c) the normal reaction, N perpendicular to the plane.

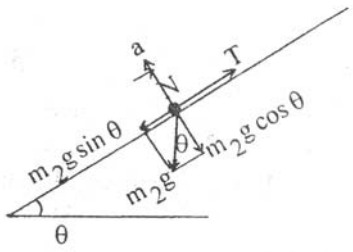


Fig. (c) Free body diagram of m_2

Resolve $m_2 g$ into two rectangular components

i) Parallel to the plane equal to $m_2 g \sin\theta$,

ii) perpendicular to the plane equal to $m_2 g \cos\theta$.

The free body diagram of m_2 is shown in Fig.4.24 (c). As the acceleration of m_2 is along the plane up, T is to be greater than $m_2 g \sin\theta$.

$\therefore$ The resultant force up the plane = T- $m_2 g \sin\theta$.

$m_2 a = T - m_2 g \sin\theta.$ ……..(4,18)

Adding Eqs. (4.17) and (4.18)

$(m_1 + m_2)a = g(m_1 - m_2 \sin\theta)$

$\therefore$ Acceleration of the system , $\boxed{a = \dfrac{(m_1 - m_2 \sin\theta)g}{m_1 + m_2}}$ ….(4.19)

The tension in the string T can be obtained from Eq. (4.17).

$$T = m_1(g - a) = m_1 g\left(1 - \frac{(m_1 - m_2 \sin\theta)}{m_1 + m_2} g\right) = \frac{m_1 m_2 g(1 + \sin\theta)}{m_1 + m_2} \quad \text{……(4.20)}$$

13. Two blocks of masses 80 kg and 30 kg are connected by a light string passing over a smooth pulley and rests on two inclined planes. Which of the masses will move down the plane? Calculate the acceleration of the system and tension in the string. $(g = 10\,ms^{-2})$

Solution*:* i) To decide which of the masses will move down the plane, we have to find the component of the gravitational force on each block parallel to the plane. The component of gravitational force on 80 kg block parallel to the plane,

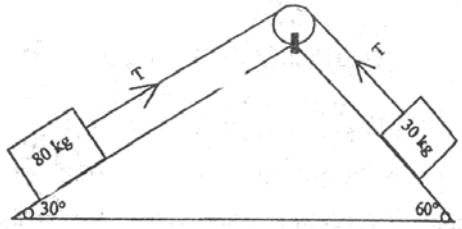


Fig.4.25(a) Blocks on inclined planes

$F_1 = mg \sin\theta = 80 \times 10 \times \sin 30° = 400\text{N}$.

The component of gravitational force on 30 kg block parallel to the plane,

$F_2 = mg \sin\theta = 30 \times 10 \times \sin 60° = 150\sqrt{3}\text{N}$.

As $F_1 > F_2$, the 80 kg block will move down the plane with acceleration a .

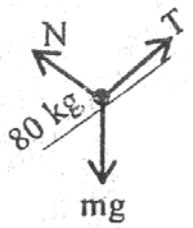


Fig.4.25(b) Free body diagram of 80 kg mass

ii) Forces on the 80 kg block are
Tension on the string, T, up the plane,
B) Gravitational force, mg on the block vertically downwards,
C) Normal reaction, N of the plane.
Free body diagram of the block is shown in Fig 4.25 (b).

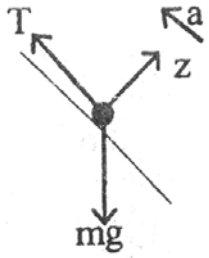


Fig.4.25(c) Free body diagram of 30 kg mass :

Resolving mg = 80 g into two rectangular components (i) parallel to the plane,
$F_1 = mg\sin\theta = 80\times10\times\sin 30° = 400\text{N},$

(ii) Perpendicular to the plane- $mg\cos\theta = 80\times10\times\cos 30° = 400\sqrt{3}$ N

$F_1 - T = ma$ $\quad 100 - T = 80\,a.$ $\quad$ ……..(I)

Forces on the 30 kg block are a) tension on the string, T , up the plane , b) gravitational force, mg on the block vertically downwards, c) normal reaction , N of the plane.
Its free body diagram is shown in Fig. 4.25 (c).
Resolving mg = 30 g into two rectangular components (I) parallel to the plane,
$F_2 = mg\sin\theta = 30\times10\times\sin 60° = 150\sqrt{3}\text{N},$
(ii) perpendicular to the plane = $mg\cos\theta = 30\times10\times\cos 60° = 150$ N .

$T - F_2 = ma$ $\quad T - 150\sqrt{3} = 30\,a.$ $\quad$ ……..(ii)

Adding Esq. (I) and (ii)
400 -150 $\sqrt{3}$ = 110 a
A = 1.274 ms^{-2}
Substituting a = 1.274 ms^{-2} in Eqn . (i)
400-T =80×1.274 $\quad$ T = 298.1 N.

14. Three blocks of masses M, m_1 and m_2 are arranged as in Fig . 4.26 (a) . Find the acceleration of M, m_1 and m_2 assuming that the pulleys and the threads are mass less and neglecting friction.
Solution:

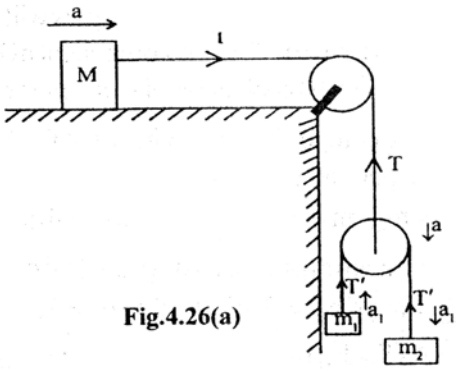


Fig.4.26(a)

Let a_1 be the acceleration of the masses m_1 and m_2 relative to M and the acceleration of M relative to the table be a .

Assuming that $m_2 > m_1$, acceleration of m_2 downwards relative to the table is $a + a_1$ and that of m_1 downwards is $a - a_1$.

T and T' are the tensions in the threads as shown in Fig . 4.26 (b) . By using the free body diagram of Fig. 4.26 (b) of the pulley hanging vertically downwards we find that .

$$T = 2T'; \; T' = \frac{T}{2}.$$



Fig.4.26(b) Free body diagram of pulley hanging vertically

For the block of mass M, T = Ma and $T' = \frac{Ma}{2}$

For the block of mass m_1 from its free body diagram (Fig. 4.26(c)).

$$m_1 g - T' = m_1 (a - a_1)$$

$$g - \frac{T'}{m_1} = a - a_1 \qquad \text{.....(I)}$$

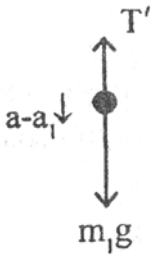


Fig.4.26(c) Free body diagram of m_1

For the block of mass m_2 using free body diagram (Fig. 4.26 (d)) .

$$m_2 g - T' = m_2 (a + a_1)$$

$$g - \frac{T'}{m_2} = a + a_1 \qquad \text{..... (ii)}$$

Adding Eqns . (I) and (ii)

$$2g - T'\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = 2a$$

$$2g - \frac{Ma}{2}\left(\frac{m_1 + m_2}{m_1 m_2}\right) = 2a \qquad \left(\because T' = \frac{Ma}{2}\right)$$

$$2g = 2a + \frac{Ma}{2}\left(\frac{m_1 + m_2}{m_1 m_2}\right)$$

$$2g = \left[\frac{4m_1 m_2 + M(m_1 + m_2)}{2m_1 m_2}\right]a$$

Acceleration of M , $a = \dfrac{4m_1 m_2 g}{4m_1 m_2 + M(m_1 + m_2)}$

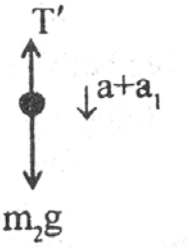


Fig.4.26(d) free body diagram of m_2

From Eq. (i)

Acceleration of $m_1, a - a_1 = g - \dfrac{T'}{m_1} = g - \dfrac{Ma}{2m_1} \qquad \left(\because T' = \dfrac{Ma}{2}\right)$

$$= g - \frac{M}{2m_1} \times \frac{4m_1 m_2 g}{4m_1 m_2 + M(m_1 + m_2)}$$

$$= \left(\frac{4m_1 m_2 + M(m_1 - m_2)}{4m_1 m_2 + M(m_1 + m_2)}\right)g$$

From Eq. (ii)

Acceleration of m_2, $a + a_1 = g - \dfrac{T'}{m_2} = g - \dfrac{Ma}{2m_2}$

$$= g - \frac{M}{2m_2} \times \frac{4m_1 m_2 g}{4m_1 m_2 + M(m_1 + m_2)}$$

$$= \left(\frac{4m_1 m_2 + M(m_2 - m_1)}{4m_1 m_2 + M(m_1 + m_2)}\right)g$$

15. Show that the velocity of a rocket at any instant, when its mass is m given by $v = v_0 + u \log_e\left(\dfrac{m_0}{m}\right)$ where v_0 and m_0 are the velocity and mass of the rocket at t = 0 and u is the velocity of the exhaust gases relative to the rocket. (It is an example of variable mass).

Solution Consider a rocket moving vertically upwards from the surface of the earth. Let its mass be m_0 and velocity with respect to the earth be V_0 at time t = 0. At time t = t let the mass of the rocket be m and velocity v. Suppose that in the time interval between an d t + dt, mass dm of the fuel get burnt. The exhaust gases from the fuel burnt move downwards with a velocity v_g with respect to the earth and the velocity of the rocket increases to v +dv. Applying the principle of conservation of linear momentum, the linear momentum of the rocket at time t = vector sum of the momentum of the rocket and the exhaust gases at time t + dt

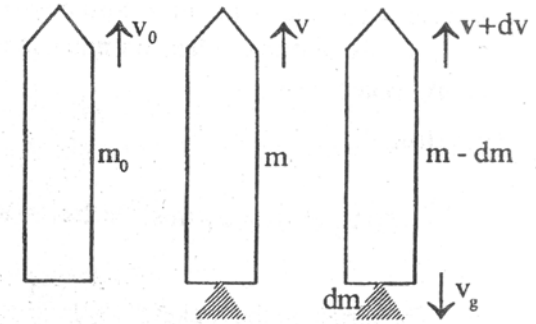


Fig.4.27(a) Fig.4.27(b) Fig.4.27(c)

Rocket propulsion

$$mv = (m - dm)(v + dv) + dm(-v_g). \qquad \text{.....(i)}$$

As the velocity of the exhaust gases is in the opposite direction of the velocity of the rocket, v_g is taken.

$$mv = mv + mdv - vdm - dmdv - v_g dm$$

$$o = mdv - vdm - v_g dm \quad (\because \text{dm dv is negligibly small})$$

$$mdv = (v + v_g)dm$$

Here $v + v_g$ represents the relative velocity of the exhaust gases with respect to the rocket in the downward direction denoted by u .

$$\therefore mdv = udm \qquad \text{.....(ii)}$$

$$dv = -u\frac{dm}{m} \qquad \text{.....(iii)}$$

The negative sign shown that as the mass of the rocket decreases, its velocity increases.

Integrating Eq. (iii) between the limits m = m, m = m_0 and $v = v_0$, v = v at time t = t, t = t + dt

respectively $\int_{v_0}^{v} dv = -u\int_{m_0}^{m}\frac{dm}{m}$

$$[v]_{v_0}^{v} = -u[\log_e m]_{m_0}^{m}$$

$$v - v_0 = -u[\log_e m - \log_e m_0] \quad = u\log_e\frac{m_0}{m}$$

$$v = v_0 + u\log_e\frac{m_0}{m}$$

Is the expression for the velocity of the rocket at any instant of time.

16. In the above example, find the thrust on the rocket.

Solution Dividing both sides of eq. (ii) by dt, we get

$$m\frac{dv}{dt} = u\frac{dm}{dt}$$

$$F = u\frac{dm}{dt} \qquad \left(\because F = ma = m\frac{dv}{dt}\right)$$

Is the expression for the thrust on the rocket? Here $\frac{dm}{dt}$ is the mass of the fuel consumed per unit time.

17. Fuel is consumed at the rate of $50\,kg\,s^{-1}$ in a rocket. Find the thrust on the rocket if the velocity of the exhaust gases is 2 $km\,s^{-1}$. Also calculate the velocity of the rocket at the instant, when its mass is reduced to 1/10th of its initial mass if its initial velocity is zero.

Solution

$\frac{dm}{dt} = 50\,kg\,s^{-1}$; u = 2kms^{-1} = $2\times10^3\,ms^{-1}$, initial velocity , $u_0 = 0$.

(i) The thrust on the rocket, $F = u\frac{dm}{dt} = 2\times10^3\times50 = 1\times10^5\,N$.

(ii) The velocity of the rocket, $v = v_0 + u\log_e\frac{m_0}{m}$

$= 0 + 2\times10^3\log_e\left(\frac{m_0}{m_0}\times10\right)$ $\qquad\qquad$ $\left(m = \frac{m_0}{10}\right)$

$= 2\times10^3\,log_e\,10 = 2\times10^3\times2.303\,log_{10}\,10 = 4.606\times10^3\,\frac{m}{s}$.

18. A hunter fires 50 g bullets from a machine gun. If each bullet moves with a velocity of $150\,ms^{-1}$ how many such bullets are to be fired into a tiger of mass 60 kg coming towards him with a velocity of $10\,ms^{-1}$ in order to stop the tiger?

Solution: Let n be the number of bullets each of mass m fired with a velocity v towards a tiger of mass M moving with a velocity V. When the tiger stops, applying the principle of conservation of linear momentum

$n(mv) + MV = 0$ $\qquad$ ($\because$ Final momentum = 0)

$n = \frac{-MV}{mv}$

Hence m = 50 g = 50×10^{-3} kg ; v = $150\,ms^{-1}$, M = 60 kg , V = $-10\,ms^{-1}$

$n = \frac{-60(-10)}{50\times10^{-3}\times150} = 80$

19. Two billiard balls each of mass 0.05 kg moving towards each other with a speed of 5 ms^{-1} collide and rebound with the same speed. What is the impulse imparted to each ball due to the other?

Solution: Let A and B be the two balls moving towards each other.

Initial momentum of the ball A = mv = 0.05×5=0.25 kg ms^{-1}. Final momentum of the ball $A = mv = 0.05\times(-5) = -0.25\,kg\,ms^{-1}$.

Impulse on A = Final momentum – Initial momentum

$= -0.25 - 0.25 = -0.5\,kg\ ms^{-1}$

An equal impulse will be received by the ball B .

20. A lorry carries a tank of water. If the lorry moves with a uniform acceleration 'a' in the horizontal direction, determine the angle made by the surface of water in the tank with the horizontal.

Solution: Let the lorry move towards the left. The lorry moving with uniform acceleration a is a non – inertial frame of reference. An observer in this frame considers a pseudo force on the water in tank. If the lorry moves towards the left as shown in Fig. (4.28) the pseudo force on the water of mass m is ma to the right is represented by $\overrightarrow{OA}$. The gravitational force mg is represented by $\overrightarrow{OB}$. The resultant force is represented by $\overrightarrow{OC}$. For equilibrium, the water surface in the tank must be perpendicular to the resultant $\overrightarrow{OC}$. So, the liquid surface tilts, being raised at the rear. If θ is the angle made by the liquid surface with the horizontal, $\theta = \angle BOC$.

$\tan\theta = \frac{BC}{OB} = \frac{a}{g}$ i.e., $\theta = \tan^{-1}(a/g)$.

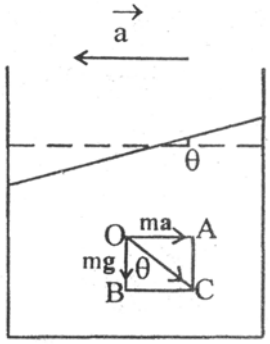


Fig.4.27

21. An electric pump on the ground floor of a building takes 15 minutes to fill a tank of volume 30m^3 with water . If the tank is 40 m above the ground and the efficiency of the pump is 30% find the electric power consumed by the pump in filling the tank. (Density of water = $1\times10^3\,\text{kg m}^{-3}$)

Solution: Volume of the tank = $30\,\text{m}^3$; density of water = 1×10^3 kgm^{-3}

Mass of the water to be filled = (Volume of the tank) × density of water

= $30\times10^3 = 3\times10^4$ kg height , h = 40m ; time taken , t = 15 minutes = 15×60 = 900 s ;

Efficiency of a pump $\eta = \dfrac{\text{Output power}}{\text{Input power}}$

Input power = $\dfrac{\text{mgh}}{\eta t}$ $\left(\because \text{output power} = \dfrac{\text{mgh}}{\text{t}}\right)$

= $\dfrac{3\times10^4\times9.8\times40}{0.30\times900} = 43.56\,\text{kW}.$

22. The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1m , what is the speed with which the bob reaches the lower most point, given that it dissipates 10% of the initial energy against air resistance?

Solution: When the bob is in the horizontal position its potential energy = mgh . When it reaches the lowest position, its kinetic energy = $\dfrac{1}{2}\text{mv}^2$. As part of the initial energy is dissipated against air resistance, by the principle of conservation of energy potential energy in the horizontal position = Kinetic energy at the lower most point + Energy dissipated $\text{mgh} = \dfrac{1}{2}\text{mv}^2 + \left(\dfrac{10}{100}\right)$ initial energy

$\text{mgh} = \dfrac{1}{2}\text{mv}^2 + 0.1\text{mgh}$

$0.9\text{mgh} = \dfrac{1}{2}\text{mv}^2$

$\text{v} = \sqrt{2\times0.9\times\text{g}\times\text{h}}$ $= \sqrt{2\times0.9\times9.8\times1}$ $(\because\ \text{h} = l\ ) = 4.2\ \text{ms}^{-1}$

23. A machine gun fires 240 bullets per minute with a velocity of 500 ms^{-1} . If the mass of each of the bullet is $5\times10^{-2}\,kg$, find the power of the gun.

Solution: The power of the gun = $\dfrac{n\left(\dfrac{1}{2}mv^2\right)}{t}$ where n is the number of bullets fired by the gun in a time interval t, m is the mass of the bullet and v is its velocity.

Here, n = 240 , t = 1 minute =60s ; m = 50×10^{-2} kg; v=500ms^{-1}.

$$\therefore P = \frac{240\left(\frac{1}{2}\times 5\times 10^{-2}\times (500)^2\right)}{60} = 2500\text{W}..$$

Example -33: The momentum of a body is double. By what percentage does its kinetic energy increase?

Solution: From Eq. (4.43)

$$K = \frac{p^2}{2m}$$

When the momentum, p of a body changes from p_1 and p_2 its kinetic energy, K changes from K_1 to K_2.

$$\frac{K_1}{K_2} = \frac{p_1^2}{p_2^2}$$

Let $K_1 = K, p_1 = p$ then $p_2 = 2P, K_2 = ?$

$$\frac{K}{K_2} = \frac{p^2}{(2p)^2}$$

$K_2 = 4K$.

Increase in kinetic energy = $K_2 - K_1$

% increase in kinetic energy = $\frac{\text{Increase in kinetic energy}}{\text{Initial Kinetic energy}}\times 100$

$= \frac{K_2 - K_1}{K_1}\times 100 = \frac{4K - K}{K}\times 100 = 300\%$.

24. A car and truck have same momentum. Which will have more kinetic energy?

Solution: From Eq. (4.43)

$$K = \frac{p^2}{2m}$$

If p is constant, K is inversely proportional to the mass of the body. Hence, the kinetic energy of the car is more than that of the truck.

Example -35: A lorry and a car moving with same kinetic energy are brought to rest by the application of brakes, which provide equal retarding forces. Which of them will come to rest in shorter time? Which will come to rest in less distance?

Solution: Initial kinetic energy of each vehicle is K and when they come to rest their final kinetic energy is zero. The change in kinetic energy of both the car and lorry is K. By work – energy theorem FS = change in kinetic energy

FS =K

$$S = \frac{K}{F}$$

Where F is the retarding force. As K and F are same for both the vehicles, they come to rest after travelling the same distance. To find the time in which each vehicle comes to rest , we have to consider the momentum, $p = \sqrt{2m\times K}$ since $K = \frac{p^2}{2m}$. The initial momentum of the car is less than that of the lorry as K is same for both but mass of the car is less than of the lorry. Final momentum is zero for both the vehicles. So, change in momentum is less for the car. By Newton's second law,

$$F = \frac{\Delta p}{\Delta t} \qquad \Delta t = \frac{\Delta p}{F}.$$

Time is directly proportional to the change in momentum since F is same. Hence, the car stops in less time.

VERY SHORT ANSWER QUESTIONS:

1. **What is inertia? What gives the measure of inertia?**

A Inertia: The resistance of the body to change its state of rest or state of uniform motion in a straight line is called inertia of the body.

$$\text{Acceleration} = \frac{\text{Force}}{\text{Mass}}$$

$\therefore$ The more is the mass less is the acceleration and more is the inertia. The mass of a body is a quantitative measure of its inertia.

2. **According to Newton's third law, every force is accompanied by an equal and opposite force. How can a movement ever take place?**

A Because both action and reaction are taking place on different bodies.

3. **When a bullet is fired from a gun, the gives a kick in the backward direction? Explain.**

A Due to law of conservation of momentum,

Recoil of the gun $V = \frac{mu}{M}$ where M = mass of the gun

M = mass of the bullet, U = Velocity of the bullet.

4. **Why does a heavy rifle not recoil as strongly as a light rifle using the same cartridge?**

A.: Recoil of the gun $V = \frac{mu}{M}$

Due to heavy mass of rifle the recoil is less.

5. **If a bomb at rest explodes into two pieces the pieces must travel in opposite directions. Explain.**

A.: According to law of conservation of Δ momentum

$MU = m_1v_1 + m_2v_2$

Initially the bomb is at rest U = 0

$\therefore\ m_1v_1 + m_2v_2$

$\boxed{m_1v_1 = -m_2v_2}$ (Negative sign indicates the pieces must travel in opposite direction.)

6. **What is impulse? State its SI units and dimensional formula**.

A: **Impulse**: The product of force and time of action of force is called impulse.

$I = F(\Delta t)$.

Unit: NS or Kg ms^{-1}.

7. **State two applications of impulse.**

A. 1.Shock absorbers are used in vehicles to reduce the magnitude of impulsive force when they travel on uneven roads.

2. While catching a ball, a cricket player lowers his hands to reduce the impulsive force and save himself from getting hurt.

8. **State two examples of Newton's third law of motion**.

A. 1. When we walk on the road we push the road backward and the road applies an equal and opposite force on us so that we can move forward.

2. The moon revolves round the earth due to gravitational force of the earth on the moon.

9. **Define force. What are the basic forces in nature?**

A. The force is one which changes or tends to change the state of rest or motion of a body. Basic forces: 1. Gravitational force 2. Electromagnetic force

3. Nuclear force 4. Weak interaction force.

10. State the conditions under which a force does no work.

A. i) When the displacement is zero
ii) When the displacement is perpendicular to the direction of the force
iii) When the body moves under the action of a conservative force over a closed path.

11. State the relation between the kinetic energy and momentum of a body.

A. Kinetic energy $K = \frac{1}{2}m\theta^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2$

$K = \frac{p^2}{2m}$

Where p is the momentum of the body.

12. State the sign of work done by a force in the following a) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket b) work done by gravitational force in the above case.

A. a) To lift a bucket out of a well, force equal to the weight of the bucket has to be applied by the man along the vertical in upward direction. Since the displacement is also in upward direction $\theta = 0$ and hence the work done is positive.
b) The angle between the gravitational force and the displacement is 180°. Therefore work done by the gravitational force is negative.

13. State the sign of work done by a force in the following.
a) Work done by friction on a body sliding down an inclined plane.
b) Work done by gravitational force in the above case.

A. a) Friction always acts in a direction, opposite to the direction of motion. Hence work done is negative.
b) The work done is positive.

14. State the sign of work done by a force in the following.
a) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity.
b) Work done by resistive force of air on a vibrating pendulum in bringing it to rest.

A. a) The applied force and the displacement are in same direction. Work is positive.
b) The direction of resistive force is opposite to the direction of motion of the pendulum. Hence work done is negative.

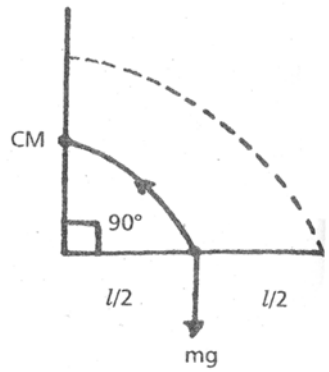
15. Which physical quantity does the area between F –S graph and S – axis represent?

A. Work done.

16. Which physical quantity does the area between F – T graph and T – axis represent?

A. Impulse.

17. A metre scale of mass m is lying horizontal on the floor of a room. What is the work done to be done to make it vertical?



A. $\theta = 90°$

Work done W = mgh $= mg\frac{l}{2}$

18. State if each of the following statements is true or false. Give reasons for your answer.
i) Total energy of a system is always conserved no matter, what internal and external forces on the body are present?
ii) The work done by earth gravitational force in keeping the moon in its orbit for its one revolution is zero.

A. i) False
ii) True, because gravitational force is a conservative force.

ASSESS YOURSELF

1. When a horse pulls a cart, which force helps the horse to move forward?

A. The force exerted by the ground on the horse.

2. A stone is suspended by a thread from a rigid support. Another thread is attached at the bottom of the stone. If the lower thread is pulled suddenly which thread upper or lower does break? If the lower thread is pulled slowly which thread upper or lower does break?

A. Lower thread since the applied force is not transmitted to the upper thread due to inertia. Upper thread since the force applied will be transmitted to the upper thread and the total force on it is the sum of the applied force and the weight of the stone.

3, Why does a loaded truck start more slowly than an empty truck?

A. An inertia is more for loaded truck.

4. Which is greater, the attraction of the earth on a 1 kg body or the attraction of the 1kg body on the earth?

A. Both are equal.

5. While jumping out of a moving bus, what must be done for a safe jump ?

A. A passenger of the bus should jump out running in the direction of motion of the bus because the body of the passenger tends to move forward by inertia of motion .

6. The particles of mud fly off tangentially from the wheel of a moving vehicle. Explain.

A. When the particles of mud fly from the wheel they continue to move with uniform velocity along a straight line which is the tangent to the wheel. This is due to inertia of motion.

7. While dusting a carpet, we give a sudden jerk or beat it with a stick. Why?

A. Because inertia of rest keeps the dust in its position and the dust gets removed as the carpet moves away.

8. When a person is in a lift accelerated upwards or downwards, do force of gravity and normal reaction on the person form action and reaction pair?

A. No because they are not equal and opposite.

9. What is the resultant force on a drop of rain falling down with constant speed?

A. Zero

10. In a tug of war competition, two men pull the ends of the rope with forces equal in magnitude and opposite in direction. What is the tension in the rope?

A. F.

11. One end of a rope is fastened to the ceiling of a room and the other is pulled by a force 2F. What is the tension in the rope at the lower end?

A. 2F

12. A rope possess over a pulley which is sufficiently high. Two monkeys of equal mass climb the rope from opposite ends, one of them climbing quickly than the other, relative to the rope. Assuming that the pulley and the rope are weightless and the rope is in extensible, which of monkeys will reach the top first?

A. Both will reach the top at the same time because the length of the two segments of the rope on the two sides of the pulley is always equal.

13. What is the net force on a piece of wood floating on water?

A. Zero because the weight of the wooden piece is balanced by the upward thrust or buoyancy of the liquid.

14. An astronaut accidentally gets thrown out of his small spaceship accelerating in interstellar space at a constant rate. What is the acceleration of the astronaut, the instant after he is outside the spaceship?

A. Zero because the external force on him becomes zero.

15. When a bullet is fired from a gun, the bullet moves forward where as the gun moves backward. Why?

A. By the principle of conservation of momentum, the final momentum of gun =- (final momentum of bullet). Since the initial momentum of bullet gun system is zero. So, the direction of motion of the gun is in the opposite direction of motion of bullet.

16. If a nucleus at rest disintegrates into two smaller nuclei, the products should travel in opposite directions. Why?

A. If a nucleus at rest disintegrates into two smaller nuclei, the products should travel in opposite directions since they obey the law of conservation of linear momentum.

17. What causes collision?

A. Impulse.

18. When a body of mass m is lifted up through a height h what is the work done by gravitational force?

A. –mgh

19. An object of mass m is moved up a smooth inclined plane of inclination θ through a length l of the inclined plane and height h. What is the work done?

A. Mgh or mg$l$ sinθ

20. Is the work done by an external agency against an opposing force positive or negative?

A. Positive

21. If a person A walks up stairs in 1 minute and another person B walks up the same stair case in half a minute, whose power is greater?

A. B

22. A man rowing a boat up stream is at rest with respect to the bank. Is he doing work ?

A. No, the man applies force for rowing the boat upstream. But he is at rest with respect to the bank i.e., displacement is zero and work done is zero.

23. A person carrying a bucket of water walks on a horizontal road with uniform velocity . What is work done by him if the road is smooth ?

A. Zero, because the force required to hold the bucket and his displacement are perpendicular to each other.

24. When is potential energy of a body said to be positive or negative ?

A. Potential energy is positive if two bodies are held at some distance against the force of repulsion. It is negative if two bodies are held at some distance against the force of attraction.

25. When an electron is taken from k- shell to L – shell does potential energy increase or decrease?

A. Increases.

26. Is the work done by friction on a body sliding down an inclined plane positive or negative?

A. Negative, since friction acts in a direction opposite to the direction of motion.

27. When a conservative force does positive work on a body, what is the change in its potential energy?

A. Its potential energy decreases since du = -Fdx (or) $U_f - U_i = -dw$ where U_f and U_i are the final and initial potential energies respectively.

28. Does work done by a body against friction always result in a loss of its kinetic or potential energy?

A. The work done by a body against friction always results in the loss of only kinetic energy.

29. Does the absolute value of potential energy have any physical significance?

A. No, it is difference of potential energies that matters.

30. If the potential energy at infinity is taken to be zero for forces which fall of to zero at large distances, what is the potential energy of a planet satellite system and an electron system ?

A. If the force is attractive, the electron potential energy is negative and if the force is repulsive the potential energy is positive. So the positive energy of the planet satellite system is negative and that of electron – electron system is positive.

31. Work done in the motion of a body over a closed loop is zero for every force in nature. Is it true?

A. No, because all forces in nature are not conservative.

Exercise 2

1 A force of 20 N acts on a body of mass 5kg at rest. What is the acceleration of the body ? What is its velocity after 5 seconds if the same force acts? After 5 seconds if the force ceases to act how will the body move?

Sol: i) F = 20 N, M = 5 kg

$$a = \frac{F}{M} = \frac{20}{5} = 4\,ms^{-2}$$

ii) U = 0 , a = 4 ms^{-2} , t = 5 s

V = u + at = 0 + 4 × 5 =20 ms^{-1}.

iii) When the force ceases to act, the body continues to move with uniform velocity $20\,ms^{-1}$ along a straight line by Newton's first law. (In the absence of friction)

2. A uniform force acts on a body of mass 2 kg at rest. If the body makes a displacement of 20 m in 4 seconds what is the force acting on the body? If the force acts only for 5 seconds, how far will the body move in 3 seconds after force has ceased to act?

Sol: i) M = 2 kg , u = 0 , S = 20 m, t = 4s.

We can find the acceleration of the body using the kinematical equation

$S = ut + \frac{1}{2}at^2 \qquad 20 = 0 + \frac{1}{2}(a)4^2$

$20 = 8a;\ a = \frac{20}{8} = 2.5\ ms^{-2}$.

F = Ma = 2(2.5) = 5 N .

ii) After 5 seconds , final velocity can be found by the kinematical equation v = u + at = 0 + 2.5 (5) = 12.5 ms^{-1}.

As the force ceases to act after 5 seconds, it will continue to move with uniform velocity. U = 12.5 ms^{-1} , a = 0 , t = 3 s.

$S = ut + \frac{1}{2at^2} = 12.5(3) + 0 = 37.5\,m$.

3. A body of mass 1 kg is moving with velocity 30 ms^{-1} due north. It is acted on by a force on 10 N due east for 4 seconds. Find the velocity of the body after the force ceases to act.

Sol: M = 1 kg, f = 10 N due east ; t = 4 s ; initial velocity due east u = 0.

$a = \frac{F}{M} = \frac{10}{1} = 10\,ms^{-2}$ due east.

Final velocity due east at the end of 4 seconds,

V = u + at =0 + 10(4) = 40 ms^{-1}.

The body has initial velocity 30 ms^{-1} due north. At the end of 4 seconds, It also has velocity of 40 ms^{-1} due east. The resultant of the initial velocity 30 ms^{-1} due north and velocity 40 ms^{-1} due east can be obtained by parallelogram law of vectors.

Resultant velocity = $\sqrt{30^2 + (40)^2 + 2(30)(40)\cos 90°} = 50 ms^{-1}$.

Single the angle between north and east Is 90°.

Direction of resultant velocity : $\tan\alpha = \frac{30}{40} = \frac{3}{4}$

The resultant velocity makes an angle $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$ with the east towards north.

4. A boy of mass 40 kg climbs up a rope with an acceleration of $2\,ms^{-2}$. What is the tension in the rope?

Sol: m = 40 kg ; a = 2 ms^{-2} ; g = 9.8 ms^{-2} .

The tension in the rope , T =m(g+a) = 40(9.8+2) = 40(11.8) = 472 N.

5. A man of mass 70 kg stands on a weighing machine in a lift which is moving (a) upwards with a uniform speed of 10 ms^{-1}, (b) downwards with a uniform acceleration of 2.8 ms^{-2}, (c) upwards with a uniform acceleration of 2.2 ms^{-2}. What would be the reading on the scale in each case?

Sol: m = 70 kg; g = 9.8 ms^{-2} .

Apparent weight or normal reaction is the reading on the scale of the weighing machine.

a)V = 10 ms^{-1}; a = 0.

Reading on the scale = N = mg = 70 (9.8) = 686 N.

b)A = 2.8 ms^{-2} .

Reading on the scale = m (g-a) = 70 (9.8 – 2.8) = 490 N.

c) A = 2.2 ms^{-2} .

Reading on the scale = m (g + a) = 70 (9.8 - 2.2) =840 N.

6. Two masses of 3 kg and 4 kg are connected at the two ends of a light inextensible string that passes over a frictionless pulley. Find the acceleration of the masses and the tension in the string, when the masses are released.

Sol $M_1 = 4\text{kg};\ M_2 = 3\text{kg}\ ; g = 9.8\text{ms}^{-2}$

(i) The acceleration $a = \dfrac{(M_1 - M_2)g}{M_1 + M_2} = \dfrac{(4-3)9.8}{4+3} = \dfrac{9.8}{7} = 1.4\text{ms}^{-2}$

(ii) The tension $T = \dfrac{2M_1M_2g}{M_1 + M_2} = \dfrac{2(4)(3)(9.8)}{4+3} = 33.6\,\text{N}$

7. Two blocks of masses 5 kg and 2 kg are kept in contact with each other on a frictionless horizontal surface. If a force of 14 N is applied on the larger block what is the acceleration of the system? What is the contact force between the two blocks?

Sol. $M_1 = 5\text{kg};\ M_2 = 2\text{kg}; F = 14\,\text{N}$.

(i) $a = \dfrac{F}{M_1 + M_2} = \dfrac{14}{5+2} = 2\ \text{ms}^{-2}$.

(ii) $f = \dfrac{M_2F}{M_1 + M_2} = \left(\dfrac{2}{5+2}\right)14 = 4\,\text{N}$

8. A force of 5 N acts on a body for 2 ms. Calculate the impulse. If the mass of the body is 5g, calculate the change of velocity.

Sol. F = 5 N ; t = 2 ms = 2×10^{-3} s ; M = 5g = 5×10^{-3} kg.

a) Impulse = Ft = $5(2\times10^{-3}) = 1\times10^{-2}$ Ns.

b) Impulse = m (v – u)

$1\times10^{-2} = 5\times10^{-3}(v-u)$

$\therefore$ The change in velocity of the body, (v –u) = $\dfrac{1\times10^{-2}}{5\times10^{-3}} = 2\,\text{ms}^{-1}$.

9. A shell of mass 0.01 kg is fired by a gun of mass 10 kg. If the muzzle speed of the shell is 50 ms^{-1}, what is the recoil speed of the gun ?

Sol. Mass of the shell; its velocity; mass of the gum. By the law of conservation of momentum,

The recoil speed of the gun $-v_2 = \dfrac{M_1V_1}{M_2} = \dfrac{0.01\times50}{10} = 0.05\,\text{ms}^{-1}$

10 A man of mass 70 kg ascends a flight of 36 steps each 20 cm high. What is the work he does against gravity?

Sol. m = 70 kg ; S = 36 × 0.20 = 7.2 m ; g = 9.8 ms^{-2} ;

W = Fs = mg s = 70 × 9.8 ×7.2 = 4932 J.

11. A force of 10 N acts on a body of mass 1.0 kg at rest. Find the work done in 4 seconds.

Sol. F = 10 N ; m = 1.0 kg ; u = 0 , t = 4 s ; acceleration $a = \dfrac{F}{m} = \dfrac{10}{1} = 10\text{ms}^{-2}$.

Its displacement $S = ut + \dfrac{1}{2}at^2 = 0 + \dfrac{1}{2}(10)4^2 = 80\text{m}$.

The work done W = FS = maS = 1.0 × 10 × 80 = 800 J .

12. 6000 litres of water is to be filled in a tank at a height of 10 m. If the time taken by a motor to fill the water is 30 minutes find the power of the motor. (Density of water = 1000 kg/m^3.)

Sol. Volume of water = 6000 litres = 6000 × 1000 cm^3 = 6× 106$(10h^{-2}m)^3$ = $6m^3$;

Mass of water = volume × density = 6× 1000 = 6000 kg ; S = 10 m ;

Time taken = 30 minutes = 30× 60 s = 1800 s.

The power of the motor $P = \frac{W}{t} = \frac{FS}{t} = \frac{mgS}{t} = \frac{6000 \times 9.8 \times 10}{1800} = 326.7W$

13. An automobile is moving at 72 kmph and is exerting attractive force of 4000 N. What is the power of its engine?

Sol. F = 4000 N ; $v = 72 \times \frac{5}{18} = 20\,ms^{-1}$.

The power of its engine = Fv = 4000(20) = 80,000 W = 80 kw.

14. A body of mass 10 kg moves with a velocity of $10\,ms^{-1}$. What is its kinetic energy?

Sol. m = 10 kg ; v = 10 ms^{-1}.

Its kinetic energy $10\,K = \frac{1}{2}mv^2 = \frac{1}{2} \times 10 \times 10^2 = 500J$

15. A vehicle of mass 1000 kg climbs up a hill of 200 m high. Find the potential energy gained by it.

Sol. m = 1000 kg ; h = 200 m ; g = 9.8 ms^{-2}.

The potential energy u = mgh = 1000 × 9.8 × 200 = 1.96 × 10^6 J.

EXERCISE PROBLEMS

1. A force of 20 N acts on a body of mass 10 kg at rest. What is its acceleration? What is its velocity after 5 seconds if the same force acts?

Sol: Force (F) = 20 N

Mass (m) = 10 kg

Initial velocity (v) = 0 A =?

Acceleration 9a) = $\frac{F}{m}$

$a = \frac{20}{10} = 2\,m/s^2$

Velocity after 5 seconds v =?

A = $2\,m/s^2$, $t = 5\,sec$

V = 0 + 2 × 5 = 10 m/s

2. A force of 10 N inclined to the horizontal at an angle of 60° acts on a body of mass 2 kg. If the body can move in horizontal direction only find its acceleration.

Sol: Force (F) = 10N

$\theta = 60°$

Horizontal component of force , $F_x = F\cos\theta$

Mass 9m) = 2kg

A = ?

$a = \frac{F_x}{m} = \frac{F\cos\theta}{m}$

$= \frac{10 \times \cos 60}{2}$ $= 10 \times \frac{1}{2} \times \frac{1}{4} = 2.5\,m/s^2$

3. A constant retarding force of 20N acts on a body of mass 5kg moving initially with a speed of 10 ms^{-1}. How long does the body take to stop?

Sol: Force (F) =-20 N

Mass (m) = 5kg

U = 10 m/s^2, V = 0 T = ?

$a = \frac{F}{m} = \frac{-20}{5}$

$\boxed{a = -4m/s^2}$

From the equation v = u + at

0 = 10- 4× t

4t = 10

$t = \frac{10}{4} = 2.5\text{sec}.$

4. The driver of a three wheeler moving with a speed of $10\,ms^{-1}$ sees a child standing in the middle of the road at a distance of 10m. He applies brakes and brings the vehicle to rest just in front of the child. If the mass of the three wheeler is 400 kg and the driver is 50 kg find the brake force.

Sol: u = m/s S = 10 V = 0

M = 400 + 50 = 450 kg F = ?

We know (a) = $\frac{v^2 - u^2}{2s}$

= $\frac{0^2 - 10^2}{2\times 10} = \frac{-100}{20}$ = -5m/s^2

Breaking force (F) = ma = 450×5=2250 N.

5. A bullet of mass 0.005 kg moving with a speed of 200 ms^{-1} enters a heavy wooden block and is stopped after a distance of 50 cm. What is the average force exerted by the block on the bullet?

Sol: m = 0.005 kg, u = 200 m/s

S = 50 cm = 0.5 m and v = 0 F = ?

$F = ma = m\left(\frac{v^2 - u^2}{2s}\right)$

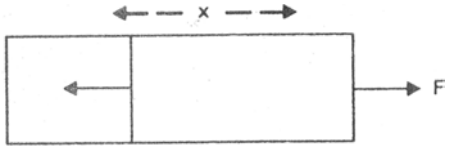
$= 0.005\left(\frac{0^2 - 4\times 10^4}{2\times 0.5}\right)$

= 0.005×-4×10^4=-200 N

$\boxed{F = -200N}$

6. A uniform rope of length L, resting on a frictionless horizontal table is pulled at one end by a force F. What is the tension in the rope at a distance x from the end where the force is applied ?

Sol: Let m be the mass of the rope $\frac{m}{L}$ = Mass per unit length



Mass of the rope of length ×= $\frac{m}{L}\times$

Acceleration a = $\frac{F}{m}$, Tension = T

F – T = ma = $\frac{m}{L}.x.\frac{F}{m} = \frac{Fx}{L}$

$T = F - \frac{Fx}{L} = F\left(1 - \frac{x}{L}\right)$

Tension $T = F\left(1-\frac{x}{L}\right)$

7. Two bodies of masses 1 kg and 2 kg are connected by a very light string passed over a clamped light smooth pulley. If the system is released from rest, find the acceleration of the two masses and the tension in the string.

Sol: $m_1 = 1\text{kg}, m_2 = 2\text{kg}$

Acceleration of two masses (a) $= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g = \frac{1-2}{1+2}\times 9.8 = 3.266\,\text{m/s}^2$

Tension in the string $T = \frac{2m_1m_2g}{m_1+m_2} = \frac{2\times1\times2\times9.8}{1+2} = 13.07$

8. Two blocks of masses 2 kg and 4kg are connected by a light string passing over a light smooth pulley clamped to the edge of a horizontal table. The 2 kg block is on the smooth horizontal table and the other block is hanging vertically. (I) Find the acceleration of the system if it is released from rest. (ii) Find the tension in the string.

Sol: Hanging mass $(m_1) = 4\text{kg}$

Mass on table $(m_2) = 2\text{kg}$

(I) Acceleration (a) $= \frac{m_1 g}{m_1 + m_2}$

$= \frac{4\times9.8}{4+2} = \frac{4\times9.8}{6} = \frac{19.6}{3} = \boxed{6.533\,\text{m/s}^2}$

ii) Tension in the string $T = \frac{m_1m_2g}{m_1+m_2} = \frac{4\times2\times9.8}{4+2}$

$= \frac{4}{3}\times9.8 = \boxed{13.07\,\text{N}}$

9. A block of mass 3 kg which is on a smooth inclined plane making an angle of 30° to the horizontal is connected by a cord passing over a light frictionless pulley to a second block of mass 2 kg hanging vertically. What is the acceleration of each block and what is the tension of the cord?

Sol: Mass of the block = 3kg

Angle of inclination (θ) = 30°

Hanging mass = 2 kg

For 2 kg mass:

$2\times9.8 - T = 2a$

19.6-T =2a _____(1)

For 3 kg mass:

T – 3 ×9.8×sin 30 = 3a

$T - 3\times9.8\times\frac{1}{2} = 3a$

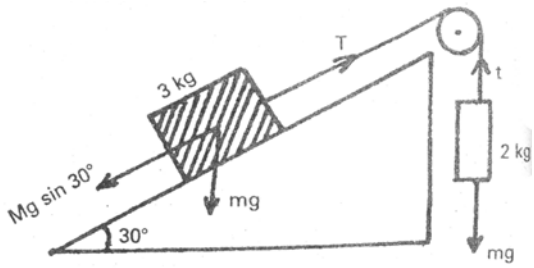
T – 14.7 = 3a _____(2)

Solving the equation

$+(2) \Rightarrow 19.6 - 14.7 = 5a$

5a = 4.9

$a = \frac{4.9}{5} = \boxed{0.98\ \text{m/s}^2}$



$A = 0.98\ m^2 \Rightarrow T = ?$

$19.6 - T = 2 \times 0.98$ $\qquad$ $19.6 - 1.96 = T$

$\boxed{T = 17.64\ N}$

10. Two blocks of masses 3 kg and 1 kg are kept in contact with each other on a frictionless horizontal surface. If a force of 10 N is applied on the larger block what is the contact force between the two blocks?

Sol: $M_1 = 3kg$

$M_2 = 1kg$

Force (F) = 10 N

Acceleration of system $a = \dfrac{F}{M_1 + M_2}$

$a = \dfrac{10}{3+1} = \dfrac{10}{4} = \dfrac{5}{2}$

$\boxed{a = 2.5\,m/s^2}$

Contact force (F) = $\dfrac{M_2 F}{M_1 + M_2} = \dfrac{1 \times 10}{3+1} = \dfrac{10}{4}$

$\boxed{F = 2.5N}$

11. A horizontal force of 500 N pulls two masses 10 kg and 20 kg connected by a light string and placed on a frictionless horizontal table (I) What is the tension in the string if the force is applied on the 20 kg mass ? (ii) What is the tension in the string if the force is applied on the 10 kg mass?

Sol: F = 500 N , $m_1 = 10kg,\ m_2 = 20kg$

$T = m_1 a$

$F - T = m_2 a$

(i) Acceleration (a) = $\dfrac{F}{m_1 + m_2} = \dfrac{500}{10+20} = \dfrac{500}{30} = \dfrac{50}{3}\,m/s^2$

$\therefore 500 - T = 20 \times \dfrac{50}{3}$

$\Rightarrow T = 500 - \dfrac{20 \times 50}{3} = \dfrac{1500 - 1000}{3}$

$= \dfrac{500}{3} = 166.66 = 166.7N$

(ii) Tension when the force is on 10 kg

$F - T = m_1 a \Rightarrow T = F - m/a = 500 - 10 \times \dfrac{50}{3}$

$= \dfrac{1500 - 500}{3} = \dfrac{1000}{3}$ $\qquad$ $= \boxed{333.3\,N}$

12. A stream of water flowing horizontally with a speed of $15ms^{-1}$ gushes out of a tube of cross – sectional area $10^{-2}m^2$ and hits a vertical wall near by. What is the force exerted on the wall by the impact of water, assuming that it does not rebound? (Density of water = 1000 kg m^{-3})

Sol: Speed of water (V) = 15 m/s Area of cross – section (A) = $10^{-2}m^2$
Volume of water striking per second = AV = $15\times10^{-2} = 0.15m^3/s$
Density of water (p) = 1000 kg m^{-3}
Mass of water striking the wall per second
Avp = 0.15×1000=150 kg/s
Initial momentum of water before striking = 150×15 = 2250 kg m/s
Momentum of water after striking the wall is zero.
Force = - (-2250) kg m/s = 2250 N

13. A man of mass 75 kg stands on a weighting machine in a lift which is moving (a) upwards with a uniform acceleration $1.2\,ms^{-2}$ (b) downwards with a uniform acceleration of $1.8\,ms^{-2}$ (c) upwards with a uniform speed of $5\,ms^{-1}$ and (d) freely falling. What would be the readings on the scale of the weighting machine in each case?

Sol: Mass of man = 75 kg
a) Upward acceleration (a) = 1.2 m/s^2.
Reading on the scale = M(g+a)
= 75 (9.8 +1.2) = 75×11= 825 N
b) Downward uniform acceleration (a) = 1.8 m/s^{-2}
Reading on the scale = M (g – a)
= 75 (9.8 – 1.8) = 75×8 = $\boxed{600\,N}$

(c) If the body moves uniform speed its acceleration is zero.
Reading on the scale = M(g +a) =Mg
= 75× 9.8 = 735 N
(d) In case of freely falling body a = g
Reading on the scale = M (g – a) = M (g – g) = 0

14. A boy of mass 40 kg stands on a weighting machine in an accelerated lift. The reading on the scale of the weighing machine is 300 N . Find the magnitude and direction of acceleration.

Sol: Mass (M) = 40 kg
Reading on scale (N) = 300 N
When the left is accelerated downward with uniform acceleration 'a'.
N = M(g – a)
300 = 40 (9.8 – a)

$$\Rightarrow 9.8 - a = \frac{300}{40} = 7.5$$

$$\Rightarrow a = 9.8 - 7.5$$

$$\Rightarrow a = 2.3\ m/s^2 \text{ downwards}$$

15. A person of mass 60 kg stands on a weighing machine in a lift which is moving (a) upwards with a uniform retardation of 2.8 ms^{-2} (b) downwards with a uniform retardation of 2.2 ms^{-2}. Find the reading shown by the weighing machine in each case.

Sol: Mass of the person (M) = 60 kg.
(a) When the lift moves upward with a uniform radiation
(a) = -2.8 m/s^2.
R = M(g – a) = 60 (9.8 – 2.8) = 60×7 = 420 N
(b) When the lift moves downward with uniform retardation
A = - 2.2 m/s^2.
R = M (g +a) = 60 (9.8 +2.2)= 60× 12 = 720 N

16. An elevator of mass 4000 kg is moving upwards and the tension in the supporting cable is 40, 000 N. Find the upward acceleration. How far does it rise in a time of 10 s starting from rest?

Sol: M = 4000 kg , N = 40, 000 N

Upward acceleration 'a' = ?

N =M (g +a)

$$g + a = \frac{N}{M} = \frac{40000}{4000}$$

A = 10 – g = 10-9.8 = 0.2 m/s^2.

$$\boxed{a = 0.2 m/s^2}$$

u = 0, t = 10 sec $\Rightarrow$ S =?

$$S = 0 \times 10 + \frac{1}{2} \times 0.2 \times 10^2$$

S = 0+0.1×100 $\boxed{S = 10 m}$

17. How can you lower a 100 kg body from the roof of a house using a cord with a breaking strength of 80 kg weight without breaking the rope? (1kg weight = g N)

Sol: Tension in the string (T) = 80 kg . wt = 80× 9.8 N

M = 100 kg

Resultant force : F = mg –T

Ma = mg –T

100a = 100g – 80 g

100a = 20 g

$$a = \frac{20 \times 9.8}{100}$$

$$\boxed{a = 1.96 m/s^2}$$

18. Ten one – rupee coins are put on top of each other on a table. Each coin has a mass of m kg. Find the magnitude and direction of (a) the force on the 6^{th} coin (counted from the bottom) due to all the coins above it, (b) the force on the 6^{th} coin by the 7^{th} coin on it and (c) the reaction of the 5^{th} coin on the 6^{th} coin.

Sol: Mass of coin = m

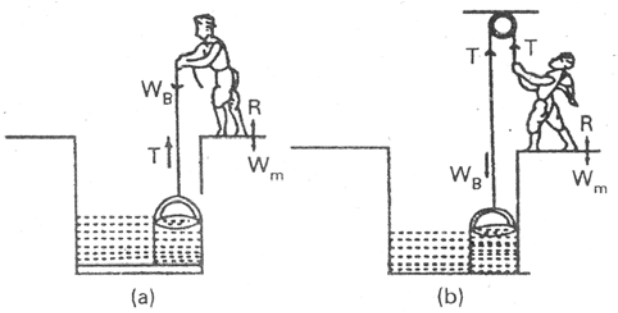
a) Force on the 6^{th} coin: Sum of the weight of the four coins above it.

F = 4×mg = 4 mg N (in downward direction)

b) Force on 6^{th} coin by 7^{th} coin = the 7^{th} coin supports the weights of three coins above it = 4 mg N (in downward direction).

c) 5^{th} coin experiences force equal to the weights of the five coins above it. Hence reaction of the 5^{th} coin on the 6^{th} coin = 5 mg N (In upward direction)

19. A man wants to draw a bucketful of water in two different ways. As shown in figure (a) he draws the bucket directly and as shown in figure (b) he uses a pulley. The weight of the man is 50 kg and the bucket with full of water weights 25 kg. Find the action on the floor by the man in the two cases. (Take g = 10 ms^{-2})



Sol: Mass of the man (M_1) = 50 kg

Mass of water with bucket = 25 kg

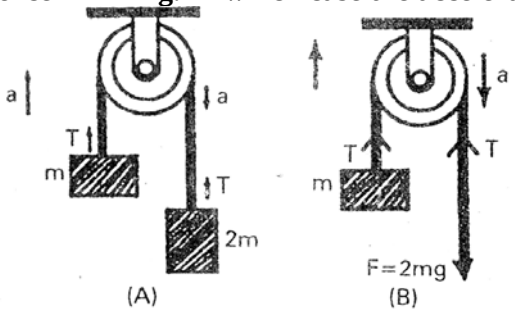
(a) From fig (a) the effective force (action) exerted by the man on the floor

$R = W_m + W_w = M_1g + M_2g = 50\times10 + 25\times10 = 750$ N

(b) When pulley is used: Force existed by the man of floor (R) = $W_m - T = W_m - W_B$

$R = 50\times10 - 25\times10 = 500 - 250 = \boxed{250\,N}$

20. The pulley arrangements of Figs. (A) and (B), are identical. The mass of the rope is negligible. In (A) the mass m is lifted up by attaching a mass 2 m to the other end of the rope. In (B) m is lifted up by pulling the other end of the rope with a constant downward force F = 2 mg. In which case the acceleration of ' m' is more ?



Sol: From fig :

$m_1 = m, m_2 = 2m$

Case A : Acceleration in downward direction

a) $\left(\frac{m_2 - m_1}{m_1 + m_2}\right)g = \left(\frac{2m - m}{2m + m}\right)g$

$= \frac{m}{3m}\times g = \boxed{\frac{g}{3}}$

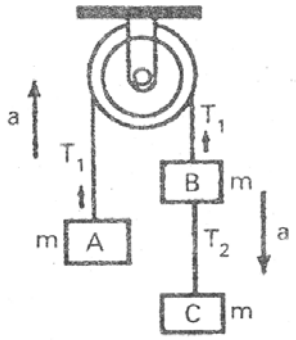
Case B: Resultant Force = 2mg – mg Upward Force (F) = 2 mg

$\therefore F_R = mg$

$\Rightarrow$ Acceleration a = $\frac{F_R}{m} = \frac{mg}{m} = \boxed{g}$

The acceleration is move in case B.

21. Three equal weights of mass m each are hanging on a string passing over a fixed pulley as shown in figure. What are the tension in the string connecting weights A to B and B to C?



Sol: From Fig. A = B = C =m

Acceleration of the system

$$(a)=\left(\frac{m_2-m_1}{m_1+m_2}\right)g$$

$$\boxed{\begin{matrix}\because m_2=m_B+m_c\\ m_1=m_A\end{matrix}}=\left(\frac{m_B+m_C-m_A}{m_A+m_B+m_C}\right)g=\left(\frac{2m-m}{3m}\right)g=\frac{mg}{3}=\boxed{\frac{g}{3}}$$

Tension in the string connecting A $\rightarrow$ B

$T_1-m_g=m_a \qquad T_1=m_g+m_a$

$$=m_g+m\frac{g}{3}=\frac{4mg}{3}$$

$$\boxed{T_1=\frac{4mg}{3}}$$

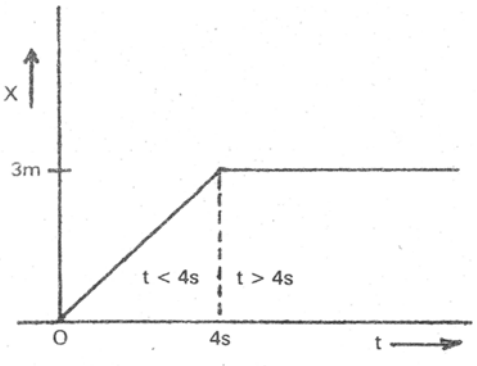
Tension in the string connecting B $\rightarrow$C

$m_g-T_2=m_a$

$\Rightarrow T_2=m_g-m_a$

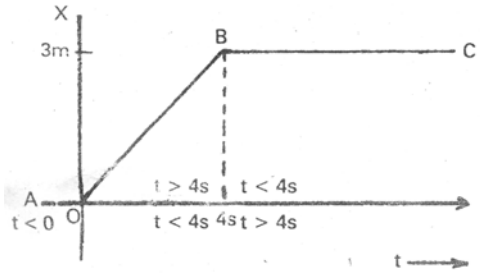
$$=\ m_g=m\frac{g}{3}=\boxed{\frac{2mg}{3}}$$

22. The figure given below shows the position – time graph of a particle of mass 8 kg. What is the force on the particle for t < 0, 0 <t<4s and t>4s? The motion may be considered one dimensional.



Sol: Mass of the body (m) = 8 kg

i)T < 0 $\Rightarrow$'AO' represents in graph that shows the particle is always zero distance means body is at rest . Hence the force on the particle is 0



$0 < t < 4s \Rightarrow$ 'OB' represents in graph. Which shown the particle moves with uniform velocity.

$\Rightarrow$ acceleration is zero .

$T > 4s \Rightarrow$ The graph BC represents the body is always at rest.

Hence the force on the particle is zero.

23. A hammer of mass 1 kg strikes on the head of a nail with a velocity of $2\,ms^{-1}$. It drives the nail 0.01 m into a wooden block. Find the force applied by the hammer and the time of impact.

Sol: M = 1 kg

Initial velocity of the nail (V) = 0

Distance (S) = 0.01 m

Force applied by the hammer F = ma

$$F = m\left(\frac{v^2 - u^2}{2S}\right) = 1\left(\frac{0^2 - 2^2}{2\times 0.01}\right) = 1\times\frac{-4}{0.02} = \frac{-2}{0.01}$$

$$\boxed{F = -200\,N}$$

Time of impact (t) = ?

From the equation $v = u + at \Rightarrow 0 = 2 + (-200)t$

$$\Rightarrow t = \frac{-2}{-200} = 0.01 sec$$

24. A force of 10 N acts on a body for 2 micro second. Calculate the impulse. If the mass of the body is 1g, calculate the change of velocity.

Sol: Force (F) = 10 N

Time (t) = $2\mu c = 2\times 10^{-6}$ sec

Mass (m) = 1g = 1×10^{-3} kg

Impulse I = F × t

$I = 10\times 2\times 10^{-6}$

$I = 2\times 10^{-5}$ N.s

Change of velocity

$$(v - u) = at = \frac{F}{m}\times t = \frac{10}{1\times 10^{-3}}\times 2\times 10^{-6}$$

$2\times 10^{-2}\,m/s$

25. A batsman deflects a ball by an angle of 60° without changing its initial speed of 20 ms^{-1}. What is the impulse imparted to the ball if its mass is 0.15 kg?

Sol: Velocity of ball (u) = 20 m/s

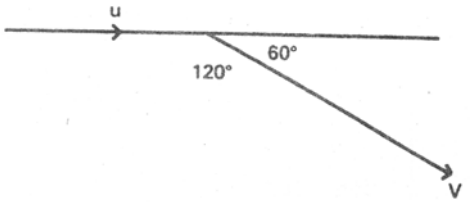
Mass of hall = 0.15 kg

Deflecting angle (θ)=60°

Angle between two velocities (θ) = 120°

Change in velocity = $2u \sin \theta/2$

$$= 2\times 20\sin\frac{120}{2} = 2\times 20\times \sin 60$$



Impulse (1) = m× change in velocity = $0.15\times2\times20\times\frac{\sqrt{3}}{2}$

$= \frac{0.15\times40\times1.732}{2}$ $= \boxed{5.196\,\text{kg m/s}}$

26. A shell of mass 0.02 kg is fired … gun of mass 100 kg. If the muzzle speed of the shell is 60 ms^{-1}, what is the recoil speed of the gun?

Sol: Mass (m) = 0.02 kg

M = 100 kg

Recoil speed = $(v_2) = ?$

$V_1 = 60\ m/s$

According to law of conservation of momentum

$mv_1 + mv_2 = 0$

$v_2 = \frac{-mv_1}{m} = \frac{-0.02\times60}{100} = 0.012\ m/s$

∴ Recoil speed of the gun = 0.012 m/s.

27. A machine gun is mounted on a flat rail road car. The gun is firing bullets at the rate of 10 bullets per second each of mass 10 g. The bullets come out with velocity 500 ms^{-1} relative to the car. Calculate the acceleration of the car at the instant when its mass is 200 kg. Also calculate the force at the instant.

Sol: Mass of bullet (m) = 10 gr = 10×10^{-3} kg

U = 0

$\frac{n}{t} = 10$ bullets per second

Velocity of bullet (v) = 500 m/s

Mass of the car (m) = 200 kg

A = ? , F = ?

Impulse $I = F_0t$

But I = mv – mu

nmv = Mat

$a = \frac{n}{t}.\frac{mv}{M} = \frac{10\times10^{-2}\times500}{200}$

$a = \frac{5}{20} = 0.25\,m/s$

Force F = ?

F = ma = 200 × 0.25

$\boxed{F = 50\,N}$

28. A 3000 kg rocket is fired. If the exhaust speed is 500ms^{-1}, how much gas must be ejected per second to supply the thrust needed, (a) to overcome the weight of the rocket, (b) to give the rocket an initial acceleration of 19.6 ms^{-2} ?

Sol: m = 3000 kg, u = 500 m/s

$\frac{dm}{dt} = ?$

a) $F = ma = m.\ \frac{du}{dt} = u\frac{dm}{dt}$

$\frac{dm}{dt} = \frac{ma(=g)}{u} = \frac{3000\times 9.8}{500}$ $= 58.8\ kg/sec$

b) $A = 19.6\ m/s^2 \Rightarrow \frac{dm}{dt} = ?$

Upward $\Rightarrow m(g+a)$

$u\frac{dm}{dt} = m(g+a) \Rightarrow \frac{dm}{dt} = \frac{m}{u}(g+a)$

$\frac{dm}{dt} = \frac{3000}{500}\times 29.4$

$\frac{dm}{dt} = 6\times 29.4 = 17.64\ kg/sec$

29. A rocket consumes 20 kg fuel per second. The exhaust gases escape at a speed of 1000 ms^{-1} relative to the rocket. Calculate the up thrust received by the rocket. Also calculate the velocity acquired by the rocket, when it's mass reduces to 1/100 of its initial mass.

Sol: $\frac{dm}{dt} = 20\,kg/sec$; u = 1000 m/s , $V_0 = 0$

Up thrust (F) = u. $\frac{dm}{dt}$

$= 1000\times 20 = 2\times 10^4$

Initial mass of Rocket = m_0.

Final mass of Rocket = $\frac{m_0}{100}$

Velocity of Rocket = $40 + 4\times 2.303\times \log\frac{m_0}{m}$

$= 0 + 1000\times 2.303\times \log\frac{m_0}{m_0/100} = 2.303 \times \log 100 = 2.303\times 2 = 4.606$ m/s

= $\boxed{4.606\,km/s}$

30. Sand drops from a stationary hopper at the rate of 5 Kgs^{-1} onto a conveyor belt moving with a constant speed of $2\,ms^{-1}$. What is the force required keeping the belt moving and what is the power delivered by the motor moving the belt?

Sol: $\frac{m}{t} = 5\,kg/s;\ v = 2\,m/s$

Force (F) = $\frac{mv}{t} = 5\times 2 = 10\,N$

Power (p) = $\frac{W}{t} = \frac{F.S}{t} = F.V$

P = 10 × 2 = 20 watt.

31. A particle of mass m is moving in a horizontal circle of radius r under a centripetal force equal to $\frac{-k}{r^2}$, where k is a constant. What is the total energy of the particle?

Sol: Mass of the body = m; Circular path radius = r

Centripetal force (F) = $\frac{mv^2}{r}$

But $F = \frac{-k}{r^2}$

$\frac{mv^2}{r} = \frac{-k}{r^2}$

$\Rightarrow \boxed{mv^2 = \frac{-k}{r^2}}$

Kinetic Energy of particle = $\frac{1}{2}mv^2 = \frac{-k}{2r}$

Potential energy of particle = work done to bring the object from infinity distance to the distance:

$\therefore PE = \int_{\infty}^{r} F.dr = \int_{\infty}^{r} \frac{-k}{r^2}$

$= -k\left[r^{-1}\right]_{\infty}^{r} = +k\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{k}{r}$

Total energy = KE +PE

$= \frac{-k}{2r} + \frac{k}{r} = \frac{-k+2k}{2r} = \frac{k}{2r}$

32. A ball of mass 2 kg is suspended by a thread of length 1 m and of negligible mass from a rigid support. The ball is pulled aside through an angle 60° and then released from rest. What is the change in its potential energy when it makes an angle (i) 60° and (ii) 30° assuming that the potential energy of the bob is zero in the mean position? Find also the velocity of the bob when it reaches the mean position. (g = $10\,ms^{-2}$)

Sol: m = 2 kg ; length = 1m

(i) At $\theta = 60° \Rightarrow$ P.E = ?

P.E = mgl1-cosθ]

= 2×10×1 (1-cos60°)

$= 20 \times \frac{1}{2} = 10\,j$

(ii) At $\theta = 30°$

P.E = mg [1-cosθ]

$= 2 \times 10 \times 1[1 - \cos 30°]$

$= 20 \times \left[1 - \frac{\sqrt{3}}{2}\right]$

$= 20 \times 0.134 = \boxed{2.680\,j}$

At mean position K.E. $_{Max}$

$\frac{1}{2}mv^2 = 2 \times 10 \times 1(1 - \cos 60°)$

$v^2 = 2 \times 10 \times \frac{1}{2}$

$v^2 = 10$

$v = \sqrt{10} = 3.162$

33. A ball bounces to 80% of its original height. What fraction of its mechanical energy is lost in each bounce? Where does this energy go?

Sol: Let 'h' be the initial height

$h_1 = h$

$h_2 = 80\% \Rightarrow h = \frac{80}{100}h = \frac{8}{10}h = \frac{4h}{5}$

Fraction of mechanical energy = $\frac{mgh - mgh_2}{mgh_1}$

$= \frac{h_1 - h_2}{h_1} = \frac{h - \frac{4h}{5}}{h} = \frac{1}{5}$

This energy converts as heat energy.

34. A bullet of mass 25 g moving with a velocity of 500 ms^{-1} enters into a wooden block and comes out of it with a velocity of 100 ms^{-1}. Find the work done by the bullet while passing through the wooden block.

Sol: Mass (M) = 25gr = 25×10^{-3} kg
Initial velocity (u) = 500 m/s
Final velocity 9v) = 100 m/s
According to Work energy theorem

$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}\times25\times10^{-3}\left(100^2 - 500^2\right) = 12.5\times10^{-3}\times10^4\left(1-25\right)$

=2.5×-2.4= -300J
Work done = $\boxed{-300\,J}$

35. An automobile is moving at 100 kmph and is exerting attractive force of 3920 N. What horse power must the engine develop, if 20% of the power developed is waster?

Sol: Velocity = 100 kmph = $100\times\frac{5}{18} = \frac{250}{9}\,m/s$
Force = 3920 N
Wastage = 3920 N
Wastage of power = 20 %
Used power = 80 %
80% p = F.V

$\frac{80\times p}{100} = 3920\times\frac{250}{9}$

$p = 49\times\frac{250}{9}\times\frac{100}{80}$

$= \frac{49\times25}{9}\times10^3 = 182.3$ hp

36. Calculate the power of an engine which can pull a mass of 500 metric ton up an incline rising 1 in 100 with a velocity of $10\,ms^{-1}$**.**

Sol: $\sin\theta = \frac{1}{100}$
Velocity = 10 m /s
Mass (m) = 500 metric ton
= 500×10^3 kg
Force applied (F) = P sin θ = $mg\sin\theta$

$= 500\times10^3\times9.8\times\frac{1}{100} = 5\times9.8\times10^2$ N

Power (p) = FV
= $49\times10^3\times10 = 49\times10^3$ =490 kw.

37. Find the work done in lifting a stone of mass 10 kg and specific gravity 3 from the bed of a lake to a height of 6 m in water.

Sol: Mass (m) = 10 kg

Specific gravity = 3

$$S = 3 \Rightarrow \frac{m}{m - m_w} = \frac{10}{10 - m_w}$$

$$30 - 3m_w = 10$$

$$3m_w = 20$$

$$m_w = \frac{20}{3}$$

Mass of stone in water $\left(m_w\right) = \frac{20}{3}$

Work done (W) = $m_w gh$

$= \frac{20}{3} \times 9.8 \times 6 = 40 \times 9.8$

$\boxed{W = 392\,J}$

38. A massive box is dragged along a horizontal floor by a rope. The rope makes an angle of 60° with the horizontal. Find the work done it the tension in the rope is 200 N and the box is moved through a distance of 20 m .

Sol: Tension (T) = 200 N

Distance (S)=20 m

$\theta = 60°$

Work done (W) = $FS\cos\theta = 200 \times \cos 60° \times 20 = 200 \times \frac{1}{2} \times 20 = 2000$ J

39. A rod of length 2 m and mass 0.5 kg is fixed at one end and allowed to hang vertically from a rigid support. Find the work done is raising the other end of the rod until it makes an angle of 60° with the vertical. (g = 10 ms^{-1})

Sol: Length of rod (l) = 2m

Mass (m) = 0.5 kg

$\theta = 60°$

$g = 100\,m/s^2$

Work done (W) = $mg\frac{l}{2}\left(1 - \cos\theta\right) = 0.5 \times 10 \times \frac{2}{2}\left(1 - \cos 60°\right)$

$= 5\left(1 - \frac{1}{2}\right) = \frac{5}{2} = 2.5\,J$

$\boxed{W = 2.5\ J}$

40. A cyclotron accelerates a proton to a final speed of $3 \times 10^7\,ms^{-1}$ which in initially at rest . Find how much work is done on the proton by the electrical force of the cyclotron. Mass of the proton is 1.67×10^{-27} kg.

Sol: Mass of proton (m) = $1.67 \times 10^{-27}\,kg$.

Initial velocity of proton (u) = 0

Final velocity of proton (v) = $3 \times 10^{-1}\ m/s^{-1}$

Work done by the force on the proton = KE

Work done = $\frac{1}{2}mv^2 = \frac{1}{2} \times 1.67 \times 10^{-27} \times \left(3 \times 10^7\right)^2 = \frac{1}{2} \times 1.67 \times 10^{-27} \times 9 \times 10^{14}$

W = 7.515×10^{-3} J

41. Find the power of an electric motor; if it lifts 200 kg of water in 5 minutes from a well of 120 m depth.

Sol: Mass of water (m) = 200 kg
Depth of well = height lifted = 120 m
Time (t) =5×60 sec = 300 sec

Power (p) = $\frac{mgh}{t} = \frac{200 \times 9.8 \times 120}{5 \times 60}$

$\boxed{p = 784W}$

42. A ball of mass 50 g falls from rest vertically downwards through a distance of 40 m and hits the ground. Find the kinetic energy and final velocity of the ball before it hits the ground. (g = 9.8 ms^{-2})

Sol: Mass of ball (m) = 50 gm = $50 \times 10^{-3} kg$
Height of fall (h) = 40 m
G = 9.8 m/s^2
According to law of conservation of energy loss in P.E = gain in K.E

$mgh = \frac{1}{2}mv^2$

$v = \sqrt{2gh}$ and $K.E = mgh$

K.E = $50 \times 10^{-3} \times 9.8 \times 40$ = 19.6 J

$v = \sqrt{2 \times 9.8 \times 40} = 28\,m/s$

43. A man carried a load of 50 kg through a height of 40 m in 25s. If the power of the man is 1568 watt, find his mass.

Sol: Mass of the man = M
Load = 50 kg
Total Mass = (M+50) kg
Power (p) = 1568 watts
H = 40 m, t = 25 sec

Power (p) = $\frac{mgh}{t} = \frac{(M+50)gh}{t}$

$M+50 = \frac{p \times t}{gh} = \frac{1568 \times 25}{9.8 \times 40} = 100$

$\boxed{M = 50\,kg}$

44. A machine gun fires 240 bullets per minute with certain velocity . If the mass of each bullets 10^{-2} kg and the power of the gun is 7.2 k.W, find the velocity which each bullet is fire.

Sol: Mass of each bullet (m) = 10^{-2} kg
Power = 7.2 kW = 7.2 ×10^3 W

$\frac{n}{t} = 240$ bullet / min = $\frac{240}{60}$ bullets / sec.

Power p = $\frac{1}{2}\frac{nmv^2}{t}$

$v^2 = \frac{2pt}{nm} = \frac{2 \times 7.2 \times 10^3 \times 60}{240 \times 10^{-2}}$

$v^2 = 36 \times 10^4$

V = 600 m/s

45. The power of the motor placed near a water pond is 3 k W. Find how much of water can be lifted to a height of 10 m in one minute. (g = 10 ms^{-2})

Sol: Power (p) = 3kW = 3×10^3 watt

Height (h) = 10 m, m = ?

Time (t) = 1 minute = 60 sec.

Power (p) = $\frac{mgh}{t}$

$m = \frac{pt}{gh} = \frac{3\times10^3\times60}{10\times10}$ = 1800 kg

46. If v = 3i +4j +5k is the instantaneous velocity of a body of mass 1.50 kg. calculate its kinetic energy.

Sol: V = 3i +4 j+ 5k

$v = \sqrt{50}\ m/s$

Mass (m) = 1.5 k.g

K.E = ?

K.E. of the body = $\frac{1}{2}mv^2 = \frac{1}{2}\times1.5\times\sqrt{50^2}$

K.E = 37.5 J

47. A man standing on the edge of a roof of a 20 m tall building projects a ball of mass 100 g vertically up with a speed of 10 ms^{-1} and simultaneously throws another ball of same mass vertically down with the same speed. Find the kinetic energy of each ball when they reach the ground

Sol: Height of building (h) = 20 m

Mass (m) = 100 gr = 10^{-1} kg

Speed (u) = 10 m/s

K.E = ?

From the equation $v^2 - u^2 = 2as$

$v^2 = 10^2 + 2\times10\times20$

$v^2 = 500\ m/s$

K.E. of each ball = $\frac{1}{2}mv^2 = \frac{1}{2}\times10^{-1}\times500 = 25$ J

K.E. of balls = 25 J, 25 J

48. A cart of mass 250 kg is taken along a polished inclined surface to the top of a platform which is at a height of 2 m above the ground level. Find the work done by the external force in moving the cart to the top.

Sol: M = 250 kg; h = 2m

W =?

W = mgh = 250 × 9.8 × 2

= 4900 J = 4.9 kJ

49. An engine of power 1.5 MW applies a force for 6 minutes on a train moving with a velocity of 10 ms^{-1}**. If there is no friction and the velocity attained is 25** ms^{-1}**, find the mass of the train.**

Sol: power = 1.5 MW = 1.5 × 10^6W

Time = 6 min = 6×60 = 360 sec.

U = 10 m/s ; v = 25 m/s

Mass of the train (m) = ?

W = p × t

$$\Rightarrow \frac{1}{2} m\left(v^2 - u^2\right) = p \times t$$

$$m = \frac{2pt}{v^2 - u^2} = \frac{2 \times 1.5 \times 10^6 \times 360}{625 - 100}$$

$$\boxed{m = 2.05 \times 10^6 \text{ kg}}$$