KINEMATICS

POINTS TO REMEMBER

1. The acceleration of a freely falling body is called acceleration due to gravity.
2. The height travelled by a vertically projected body before its final velocity becomes zero is called maximum height
3. The time of ascent of a vertically projected body is equal to its time of descent
4. The velocity with which a body is projected vertically upwards is equal to the velocity with which it reaches the point of projection
5. A projectile is a body projected with an initial velocity directed at angle other than 90^0 with the horizontal and moves under only the force of gravity and no other force acts on it.
6. **The equation of motion for a projectile is given by**

   $$y = (\tan\theta)\,x - \left[\frac{g}{2u^2\cos^2\theta}\right]x^2$$

   Where θ is the angle of projection X and Y are the coordinates of the body after certain time 't'.
7. **Time of ascent**, $t_a = \frac{u\sin\theta}{g}$

   The time of descent, $t_d = \frac{u\sin\theta}{g}$

   Time of flight T= $\frac{2u\sin\theta}{g}$

   The maximum height, H= $\frac{u^2\sin^2\theta}{2g}$

   Range R= $\frac{u^2\sin 2\theta}{g}$.
8. A body projected horizontal from a height 'h' above the ground also follows parabolic path. After t seconds of projection with horizontal velocity and the coordinates of the body x = ut,

   $y = \frac{1}{2}gt^2 \;\; where\, t = \frac{x}{u}.$

   $Hence\;\; y = \left(\frac{g}{2u^2}\right)x^2.$

SHORT ANSWER TYPE QUESTIONS

1. **If the average velocity of an object is zero in same time interval, what can you say about the displacement of the object for that interval?**

Sol: **Average velocity =** $\frac{Toatal\ displacement}{Total\ time}$

$$\bar{V}_{av} = \frac{s_2 - s_1}{t_2 - t_1}$$

$$\bar{V}_{av} = 0\ means$$

$$s_2 - s_1 = 0$$

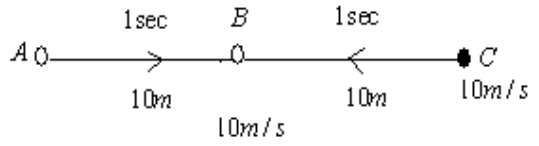
$$s_2 = s_1$$

$$\therefore s = constant$$

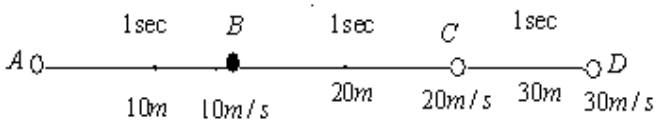
That means displacement of the body is constant.

2. **Can the instantaneous Velocity of an object during a time interval over be greatest in magnitude than average velocity over the entire interval? Can it be less?**

Sol: **if a body is moving with uniform velocity, then instantaneous velocity =average velocity**



If a body is moving with uniform acceleration, then instantaneous velocity> Average velocity



3. **Give examples for the particles where a) the velocity is in opposite direction to the acceleration b) the velocity of the particle is zero but its acceleration is not zero**

Sol: **a)** When a body thrown vertically upwards, the velocity of particle is in opposite direction to the 'a'

b) At the highest point of a vertically projected body, v=0 and 'a' acts downwards**.**

4. **Can equations of kinematics be used in a situation where the acceleration arises in time/ can they be used when acceleration is zero?**

Sol: **The equations of kinematics i.e**

$1. v = u + at$

$2. s = ut + \frac{1}{2}at^2$

$3. v^2 - u^2 = 2as$

$4. s_n = u + \frac{a}{2}(2n-1)$

These equation hold uniformly accelerated motion.

Acceleration is zero means that the body is either in the state of rest or moves with velocity.

Yes they can be used when acceleration is zero

5. **A particle moves rectilinearly with uniform acceleration. Its velocity time t 0 is v_1 and at time $t = t$ is v_2 .The average velocity of the particle in this time interval is $\frac{v_1 + v_2}{2}$. Is this statement true or false?**

Sol : *Given* $v_{ave} = \frac{v_1 + v_2}{2}$

$v_{ave} = \frac{s_1 + s_2}{t_1 + t_2}$ *By definition*

$= \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}$

$= \frac{(v_1 + v_2)t}{2t}$ *if* $t_1 = t_2 = t$, *i.e time intervals are equal.*

$v_{ave} = \frac{v_1 + v_2}{2}$

$\therefore$ **The statement is false.**

6. **Show that time of ascent of a vertically projected body is equal to time of descent?**

Sol: Time of ascent is defined as the time taken by the body to reach the maximum height.
Let us consider that the body is projected vertically upwards with initial velocity u, and let the time taken to reach the maximum height be t_a, substitute the value v = 0, t = t_a in v= u + at, then 0=u-g$t_a \Rightarrow t_a = u/g$.

7. **Show that the velocity with which the body is projected vertically upwards is equal to the velocity with which it reaches the point of projection.**

Sol: Consider a body is projected vertically upwards with an initial velocity 'u'.
Let is travels to a height S = H and at highest point velocity becomes zero.
During this case;

$$H = \frac{u^2}{2g}$$

$$u^2 = 2gh$$

$$u = \sqrt{2gh} \ ...(1)$$

A vertically projected body after reaching maximum height becomes a freely body and let it touches the point of projection with a velocity 'v'.
During this case;

$u = 0$

$a = +g$

$V = V$

$S = H$

$Substitute\ these\ values\ in\ v^2 - u^2 = 2as$

$v^2 - o^2 = 2gh$

$v = \sqrt{2gh}$ .....(2)

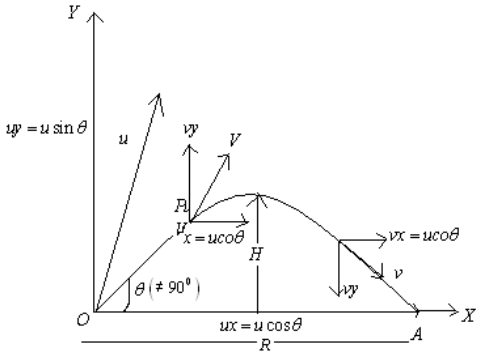
$From\ 1\&2$

$u = v$

Hence; the initial velocity =final velocity.

8. **Show that the trajectory of an object projected at certain angle with the horizontal is a parabola.**

Sol:



1) As shown in the figure considers a body is projected into the air with an initial velocity 'u' making an angle $\theta[\theta \neq 90]$ with the horizontal ox. Then the body remains in the air for certain time and then it travels and touches the ground at 'A'
2) To explain the motion of the body resolve the initial velocity into two components. They are horizontal component $u_x = u\cos\theta$ & vertical component $uy = u\sin\theta$.

3) The body does not undergo acceleration along horizontal direction but undergoes acceleration due to gravity in vertical direction.
4) Let at any time‘t’ the horizontal distance traveled is ’x’ and vertical distance traveled ‘y’.
There fore;

X=horizontal velocity x time

$$\Rightarrow X = (u\cos\theta)t$$

$$t = \frac{x}{u\cos\theta} \rightarrow (1)$$

$$In\ vertical\ direction: -$$

$$S = Y$$

$$u_y = u\sin\theta$$

$$t = t$$

$$a = -g$$

$$Substitute\ \ these\ \ values\ in\ s = ut + \frac{1}{2}at^2$$

$$Then\ \ Y = (Tan\theta)X - \left(\frac{g}{2u^2\cos^2\theta}\right)X^2$$

Let $Tan\theta = A;\ and\ \frac{g}{2u^2\cos^2\theta} = B$

$$Y = AX - BX^2$$

The above equation represents presents parabola. Hence the path of the projectile is parabola.

9. **Explain the terms average velocity and instantaneous velocity. When are they equal?**

Sol: **Average velocity:** The average velocity of the particle is defined as the ratio of total displacement to the total time.
Average velocity is does the path followed by the particle between the initial and final positions and that gives the result of the motion.
Instantaneous velocity: The velocity of a particle at a particular instant of time is known as instantaneous velocity.

$$V_x = \underset{\Delta x\to 0}{Lim}\frac{\Delta x}{\Delta t}$$

The instantaneous velocity may be positive or negative in straight line motion.
In uniform motion the instantaneous velocity of a body is equal to the average velocity**.**

10. Show that the maximum height and range of projectile are $\frac{u^2\sin^2\theta}{2g}$ and $\frac{u^2\sin^2 2\theta}{g}$ respectively where the terms have their regular meanings.

Sol: The distance traveled by the projectile along vertical direction units its vertical component velocity becomes zero called maximum height.
In vertical direction initial velocity $= u\sin\theta$

Final velocity= 0
Distance(S) = H = maximum height reached;
Acceleration a = -g
From the equation $v^2 - u^2 = 2as$

$$u^2\sin^2\theta = 2gH \Rightarrow H = \frac{u^2\sin^2\theta}{2g}.$$

11. **Show that the trajectory of an object thrown horizontally from certain height is a parabola.**

Sol: 1.Consider a body is projected horizontally from a top of the tower of height 'h' with an initial velocity. The angle of projection is zero

2. The body does not undergo acceleration in horizontal direction. It undergoes acceleration due to gravity in vertical direction.

Let at any time't' the horizontal distance travelled is X and the horizontal velocity X time

$\Rightarrow X = ut$

$\Rightarrow t = \frac{X}{t} \rightarrow (1)$

$In\ vertical\ direction: -$

$S = Y;\ a = g;\ t = t; uy = 0$

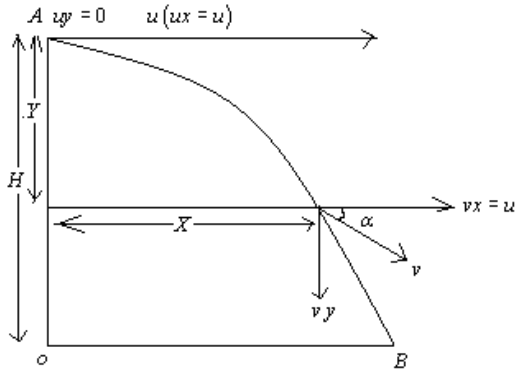
$Substitute\ in\ s = ut + \frac{1}{2}at^2$

$Y = \frac{1}{2}g\left(\frac{X^2}{u^2}\right)$

$Y = \left(\frac{g}{2u^2}\right)X^2$

$Let\ \frac{g}{2u^2} = A$

$Y = AX^2$

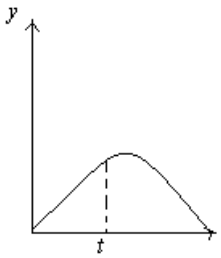


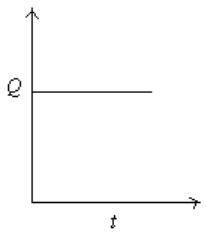
The above equation represents parabola. Hence the path of the horizontal projectile is parabola.

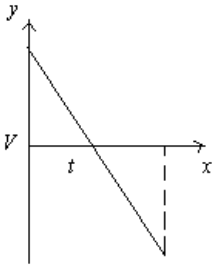
12. **Can the velocity of an object be in a direction other than the direction of acceleration of the object? Explain.**

Sol: Yes. The velocity of an object can be in a direction other than the acceleration direction. In the case of the upward motion of a projectile, the angle between velocity and acceleration is 180^0

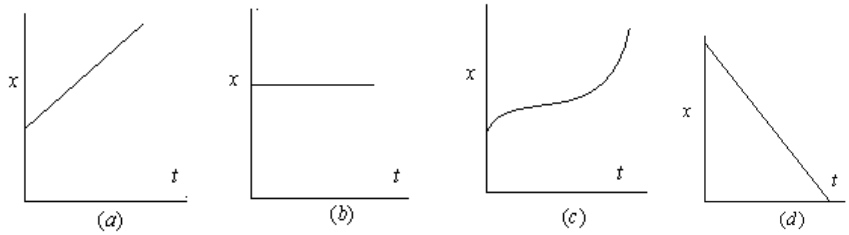
13. **A stone is thrown up in the air. It rises to a height h and then returns to the thrown. For the time stone is in air, sketch the following graphs. Y versus t, v versus t, a versus t.**

Sol: 1) 

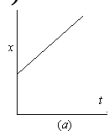
2) 

3) 

14. **The figure below shows four graphs of x versus time. Which graph shows a constant, positive, non-zero velocity?**



Sol: a)

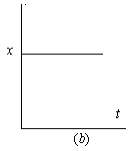


Body moving with constant velocity having some initial displacement.

$V = \frac{x}{t} = Constant$

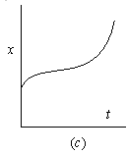
$\therefore x \propto t$

b)



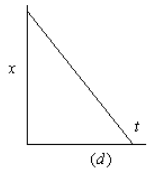
A body at rest.

c)



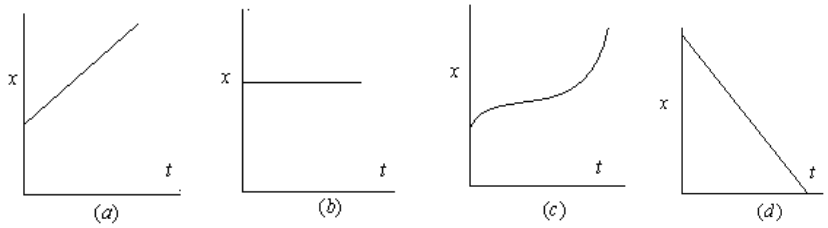
A body with some initial displacement and moving with non-uniform

d)



The body has some initial displacement and gradually the displacement becomes zero as time increases negative velocity.

15. **If the below four graph have ordinate axis indicating velocity v and abscissa time I which graph shows (a) constant positive acceleration**
(b) Constant and negative acceleration
(c) A changing acceleration that is always positive
(d) A constant velocity?



16. **Show that for a projectile launched at an angle of 45^0 the maximum height of the projectile is one `quarter of the range.**

Sol: Range of the projectile $R = \frac{u^2 \sin 2\theta}{g}$

When $\theta = 45^0 \Rightarrow \frac{u^2 \sin 2\times 45}{g} = \frac{u^2 \sin 90}{g} = \frac{u^2}{g}$

$\therefore R = \frac{u^2}{g}$

Maximum height H= $\frac{u^2 \sin^2 \theta}{2g}$

When $\theta = 45^0 \Rightarrow H = \dfrac{u^2.(\sin 45)^2}{2g} = \dfrac{u^2.(1/2)}{2g} = \dfrac{u^2}{4g}$

$$H = \frac{1}{4}.\frac{u^2}{g} = \frac{R}{4}$$

$$\therefore R = \frac{u^2}{R}$$

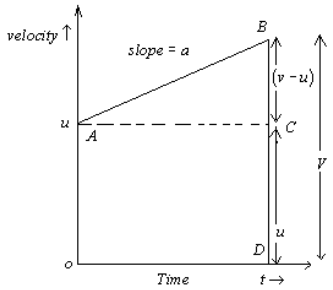
$$\therefore H = \frac{R}{4}$$

17. **A bird holds a walnut between its bills takes it high above ground. While flying parallel to the ground it lets the nut go off. (a) What is the trajectory of the nut with respect to the bird and (b) as seen by an observer on the ground?**

Sol: (a) With respect to the bird the trajectory or the path of the walnut is a straight line.
(b) If the incident is observed by an observer on the ground it is a horizontal parabola.

18. **Derive the equation $S = ut + \dfrac{1}{2}at^2$ using graphical method where the terms have their usual meaning.**

Sol:



1) Consider a body moving with an initial velocity ‘u’. Let it is travelling with an uniform acceleration ‘a’.
2) Let ‘V’ be its velocity after‘t’ seconds and‘s’ is the distance travelled at the same time.
3) A graph is plotted with time along X axis and velocity along Y-axis and Vt curve (straight line AB) is shown in the figure.
4) Slope of the curve will give acceleration

From the graph slope

$$= \frac{BC}{AC}$$

$$\Rightarrow a = \frac{v-u}{t}$$

$$\Rightarrow v - u = at$$

$$\Rightarrow v = u + at$$

5) Area under the curve gives the distance travelled by the body.

$$\Rightarrow S = Area\ of\ \ rectangle OACD + Area\ of\ \ \Delta ABC$$

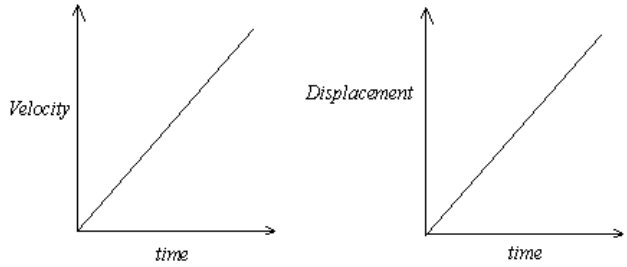
$$\Rightarrow S = (OA \times CD) + \frac{1}{2}(AC \times BC)$$

$$\Rightarrow S = ut + \frac{1}{2} \times t \times (v-u)$$

$$\Rightarrow S + ut + \frac{1}{2}at^2$$

19. **Represent the motion of a body starting from rest and moving with uniform acceleration both in terms of velocity-time and displacement-time.**

Sol: graphical representation of a body moving with uniform acceleration



VERY SHORT ANSWER QUESTIONS

1. Give two examples of motion of big objects where the object can be treated as a particle and where it cannot be?

Sol: The rectilinear motion of any body is treated a as particle, also if the septum consisting of no. of particles, then also the system considered as a body.
Ex:1) motion earth around sun 2) motion of billiard balls.

2. The state of motion is relative. Explain?

Sol: Rest and motion are relative.
A man in a moving train is at rest w.r.t co-passenger but he is in motion w.r.t a person on the ground.

3. How is average velocity different from instantaneous velocity?

Sol: The instantaneous velocity gives the velocity of a particle at particular instant whereas average velocity gives the resultant motion in uniform motion average and instantaneous velocity is same.

4. If instantaneous velocity does not change from instant to instant will the average velocities differ from interval to interval?

Sol: In variable motion the average velocity depends on the interval of which only the velocity is calculated.
In uniform motion the average velocity is a constant value and is same for all the intervals of time during which it is calculated.

5. Can an object have (i) a constant even though its speed is changing (ii) a constant speed even though its velocity is changing?

Sol: i) No it is not possible
ii) An object moving along a curved path or circular path with constant speed has varying velocity.

6. Give an example of a case where the velocity of an object is zero but its acceleration is not zero?

Sol: At the initial point of a freely falling body and at the highest point of a vertically projected body. Where the velocity of a body is zero, acceleration due to gravity acts towards earth.

7. Give an example of a motion for which both the acceleration and velocity are negative.

Sol: A body thrown vertically upwards.

8. "Speed of a particle can be negative." Is this statement correct? If not why?

Sol: Speed is a scalar quantity and is always positive.

9. What is the acceleration of a projectile at the top of its trajectory?

Sol: The acceleration of a projectile at the top of its trajectory is vertically downwards.

10. Can a body in free fall in equilibrium? Explain?

Sol: No; upward thrust due to air and downward force due to gravity mg are acting on freely falling body. The body experiences a resultant force is the downward direction.

11. if the trajectory of a body is parabolic in one reference frame, can it be parabolic in another reference frame that moves at constant velocity with respect to the first reference frame? If the trajectory can be other than parabolic, what else it can be?

Sol: No. the trajectory is a vertical straight line.

12. Name the situation where the speed of an object is constant while the velocity is not?

Sol: An object moving along a curved path or a body in circular motion. The magnitude is constant but direction changes continuously.

ASSESS YOUR SELF

1. Under what conditions is the magnitude of the average velocity of a particle moving in one dimension smaller than the average speed over same time interval?

Sol: If the particle moves along a line with out changing the direction the magnitude of average velocity and average speed are the same. When change in the direction occurs displacement would be smaller than the distance, hence average velocity would be smaller than the average speed.

2. Is it possible that the average velocity for some interval may be zero although the average velocity for a shorter interval included in the first interval is not zero?

Sol: Yes. If a particle moves along a straight line with constant acceleration 1st in one direction say in +x direction for some interval of time Δt_1 then it reverses its direction and moves fro another time interval Δt_2 and reaches the same point then for total time interval $\Delta t_1 + \Delta t_2$ average velocity is zero $\left(\because \frac{\Delta x}{\Delta t} = \frac{O}{\Delta t} = 0 \right)$ displacement is zero, but for 1st and 2nd time interval the average velocity is not zero.

Is $\frac{\Delta x}{\Delta t}$. The average velocity of the particle during a tie interval Δt is equal to the slope of the straight line joining initial and final points on the position – time graph.

3. The acceleration dur to gravity is always down word i.e., along the –ve y direction can be choose this direction as the positive direction for the acceleration due to gravity?

Sol: Yes, the direction can be taken +ve for the 'g' when the case is free fall of a body.

4. Can an object accelerate if its speed is constant?

Sol: Yes; an object moving along a curved path with constant speed has varying velocity because its direction of velocity changes from point to point along the trajectory.

5. Can an object accelerate if its velocity is constant?

Sol: No; if the velocity is constant, there is no a change in the velocity hence acceleration is zero.

6. Can a particle have a constant velocity and varying speed?

Sol: No; if velocity is constant, the magnitude of the instantaneous velocity, i.e., speed is constant.

7. A body is dropped freely from the window of a railway car. Will the time of free fall be equal it.

i) the car is stationary

ii) the car moves with a constant velocity v

iii) the car moves with an acceleration a

Sol: In all these case the body will have same time of descent. The motion of the car only affects the magnitude of the horizontal components of the velocity and the acceleration of the body but does not affect the nature of its motion along the vertical direction

Exercise 1

1. A particle is at x = +5m at t = 0, x = -7m at t = 6s and x = +2m at t = 10s. Find the average velocity of the particle during the intervals (a) t = 0 to t = 6s, (b) t = 6s to t = 10s, (c) t = 0 to t = 10s.

Sol: Average velocity $V = \frac{x_2 - x_1}{t_2 - t_1}$

a) Between the times t = 0 to t = 6s, the average velocity is

$x_1 = +5m$, $t_1 = 0$, $x_2 = -7m$, $t_2 = 6s$.

$\therefore \overline{V_1} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-7-5}{6-0} = -2ms^{-1}$

b) In between the times $t_2 = 6s$ to $t_3 = 10s$, the average velocity is

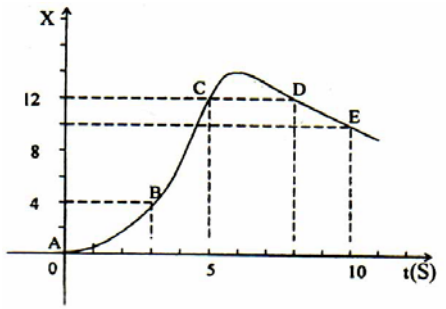
$\therefore \overline{V_2} = \frac{x_3 - x_2}{t_3 - t_2} = \frac{2-(-7)}{10-6} = \frac{9}{4} = 2.25ms^{-1}$.

c) In between times $t_1 = 0$ to $t_3 = 10s$, the average. Velocity is

$\therefore \overline{V_3} = \frac{x_3 - x_1}{t_3 - t_1} = \frac{2-5}{10-0} = -0.3ms^{-1}$

2. Figure shows the motion of a particle along a straight line. Find the average velocity of the particle during the intervals (a) A to E; (b) B to E; (c) C to E; (d) D to E; (e) C to D

Sol: a) since particle moves from A to E, A is the initial point and E is the final point. The slope of the line drawn from A to E will give the average velocity during that interval of time.



The displacement Δx is $x_E - x_A = 10cm - 0\,cm = +10cm$.

The time interval $\Delta t_{EA} = t_E - t_A = 10s$

$\therefore$ During this interval average velocity $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{+10cm}{10s} = +1cms^{-1}$

b) For the interval B to E, the displacement $\Delta x = x_E - x_B = 10cm - 4\,cm = 6cm$

and $\Delta t = t_E - t_B = 10s - 3s = 7s$.

$\therefore$ Average velocity $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{6cm}{7s} = +0.857\,cms^{-1} = 0.86\,cms^{-1}$

c) For the interval C to E, the displacement $\Delta x = x_E - x_C = 10cm - 12cm = -2cm$.

And $\Delta t = t_E - t_C = 10s - 5s = 5s$ $\quad\therefore \bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{-2cm}{5s} = -0.4cms^{-1}$

d) For the interval D to E, the displacement $\Delta x = x_E - x_D = 10cm - 12cm = -2\,cm$.

And the time interval $\Delta t = t_E - t_D = 10s - 8s = 2\,s$

$\therefore \bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{-2cm}{2s} = -1cms^{-1}$

e) For the interval C to D, the displacement $\Delta x = x_D - x_C = 12cm - 12cm = 0$.

And the time interval $\Delta t = t_D - t_C = 8s - 5s = 3s$

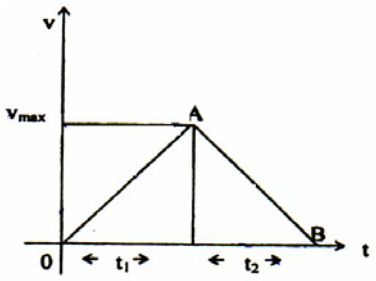
$\therefore$ The average velocity $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{0m}{3s} = 0\,ms^{-1}$

3. A car accelerates from rest at a constant rate α for some times, after which it decelerates at a constant rate β to come to rest. If the total time elapsed is 1s evaluate

a) the maximum velocity reached and b) the total distance travelled graphically.

Sol: a) The total time is $t = t_1 + t_2$ .....(1)

Where $t_1 \rightarrow$ accelerating time, $t_2 \rightarrow$ decelerating time



From graph $t = t_1 + t_2 = \dfrac{v_{max}}{\alpha} + \dfrac{v_{max}}{\beta}$

$t = v_{max}\left(\dfrac{\alpha+\beta}{\alpha\beta}\right)$

$\therefore v_{max}\dfrac{\alpha\beta t}{\alpha+\beta}$

b) Total travelled distance = displacement

= area under the v-t graph = area of ΔOAB

$$= \frac{1}{2}(base)(height) = \frac{1}{2} t\, v_{\max}$$

$$= \frac{1}{2} t \left[\frac{\alpha\beta t}{\alpha+\beta} \right] = \frac{1}{2} \left[\frac{\alpha\beta t^2}{\alpha+\beta} \right]$$

4. A particle moving along a straight line with initial velocity u, and acceleration continues its motion for seconds. What is the distance covered by it n the last n^{th} second?

Sol: S_n = Distance covered in n seconds – Distance covered in (n-1) seconds.

$$= \left[un + \frac{1}{2} an^2 \right] - \left[u(n-1) + \frac{1}{2} a(n-1)^2 \right]$$

$$= un + \frac{1}{2} an^2 - un - u - \frac{1}{2} an^2 + an - \frac{1}{2} a$$

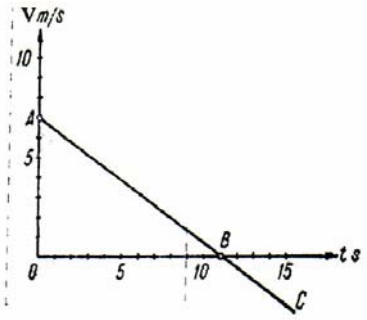
$$= u + a\left[n - \frac{1}{2} \right]$$

So the distance covered by the body in n^{th} second

$$S_n = u + a\left[n - \frac{1}{2} \right].$$

5. Velocity –time graph for the motion of a certain body is shown in figure. Explain the nature of this motion. Find the initial velocity and acceleration and write the equation for the variation of displacement with time. What happens to the moving body at point B? How will the body move after this moment?

Sol: On fig $A \to B$ is retarding and $B \to C$ is accelerating motion.



Initial velocity at A is u = $7ms^{-1}$.

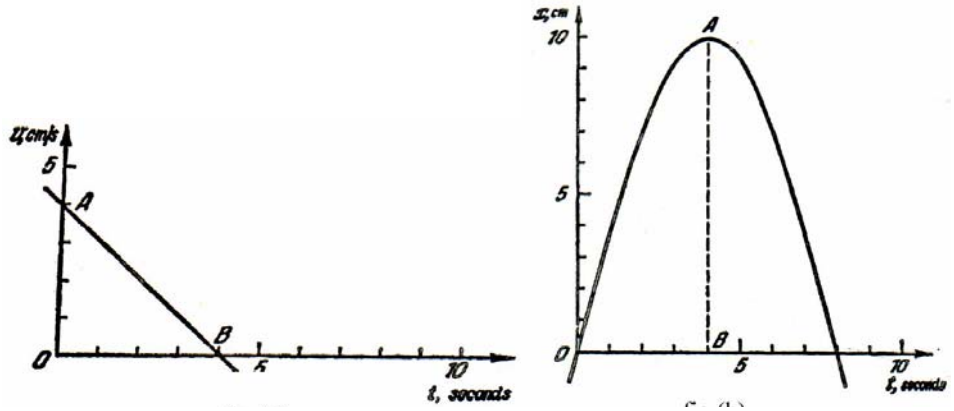
the acceleration $a \frac{dv}{t} = \frac{v-u}{t} = \frac{0-7}{11} = \frac{-7}{11} m/s^2$

The equation of motion for this body which gives variation of displacement with time is

$$s = 7t - \frac{1}{2} 0.64t^2 = 7t - 0.32t^2$$

6. The graphs is fig. a and b show the velocity of a body and the change in its coordinate with time. The origin of time reading coincides on the graph. Are the motions shown in the graphs represented by OAB the same?

Sol:



In graph (a), the initial velocity u $= 4ms^{-1}$ $(0A = 4ms^{-1})$ and acceleration a = slope of the line $= \frac{OA}{OB} = \frac{4ms^{-1}}{4s} = 1ms^{-2}$ .First the velocity of the body decreases. At t = 4s it is zero and then accelerates (i.e., increases in magnitude).

(b) Before the body momentarily, it travels a distance of 10m as shown in (b) travels uniformly accelerated motion.

In graph (a), the distance travelled by the body before it stops it equal to the area of triangle OAB and is equal to $\frac{1}{2} \times 4 \times 4 = 8m$.

Therefore, the graphs show different motions.

For the body travelled by the graph (b) it is having uniform acceleration. So the distance travelled is given by the product of average velocity and time, the distance covered is 10m. Suppose the body starts with initial velocity u then average velocity is $\frac{u+0}{2}$.

So the distance travelled during time T is $\left(\frac{u}{2}\right)T = h = 10m$

$\therefore u = \frac{2h}{T} = \frac{2 \times 10m}{4s} = \frac{20}{4} ms^{-1} = 5ms^{-1}$.

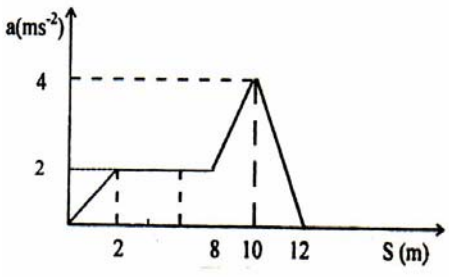
$v^2 - u^2 = 2$ as

$\Rightarrow (0)^2 - (5)^2 = 2a(10)$ = - 25 = 20a $(\because s = h = 10m)$

$a = \frac{-25}{20} = -1.25 ms^{-2}$

7. The acceleration –displacement graph of a particle moving in a straight line in given as in the figure. The initial velocity of the particle is zero. Find the velocity of the particle when displacement of the particle is s = 12m

Sol:



Area under the curve as $= \frac{v^2 - u^2}{2}$

Here u = 0

$\therefore as = \frac{v^2}{2}$

$$\therefore V = \sqrt{2(area\ under\ \ a-s\ graph)}$$

$$= \sqrt{\left[\frac{1}{2}(2)(2)+6\times 2+\frac{1}{2}(2+4)2+\frac{1}{2}(2)4\right]2} \quad = \sqrt{2\times 24} = 4\sqrt{3}\ ms^{-1}$$

8. Two bodies begin to fall freely from the same height. The second one begins to fall $'\tau'$s after the first. After what time the 1st body begins to falls will the distance between the bodies equal to?

Sol: If the first body falls after a time't', then the second fall after $t-\tau$

$$\therefore H_1 = \frac{gt^2}{2}\ ;\ H_2 = \frac{g(t-\tau)^2}{2}$$

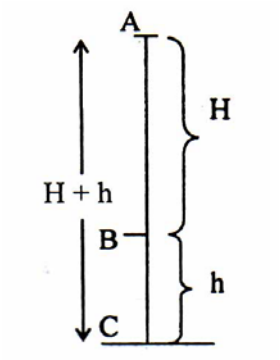
Therefore $l = H_1 - H_2 = gt\tau - \frac{1}{2}g\tau^2$

And $\frac{l}{g\tau} = t - \frac{\tau}{2}$

$$\therefore t = \frac{l}{g\tau} + \frac{\tau}{2}$$

9. One body fall freely from a point A at a height (H +h) while another body is projected upwards with an initial velocity v_0 from point C at the same time as the first body begins to fall. What should be the velocity v_0 of the second body so that the bodies meet at a point B at a height h? What is the maximum height attained by the second body for the given initial velocity? What is H = h?

Sol: If the first travels a distance 'H' downwards, then the second travels a distance H upwards



$$\therefore H = \frac{gt^2}{2} \quad .....(1)$$

And $h = ut - \frac{gt^2}{2}$ .....(2)

From (1) and (2), solving for u we get

$$u = (H+h)\sqrt{\frac{g}{2H}} \quad .....(3)$$

$$H_{\max} = \frac{u^2}{2g}$$

$$H_{\max} = \frac{u^2}{2g} = \frac{(H+h)^2}{4H} \qquad (\text{here } H_{\max} > h).$$

When H = h we have $u = \sqrt{2gh}$ as $H_{\max} = h$.

10. A balloon starts from rest, moves vertically upwards with acceleration $g/8 ms^{-2}$. A stone falls the balloon after 9s from the start. Find the time taken by the stone to reach the ground $(g = 9.8 ms^{-2})$.

Sol: Consider a stone is falling from a it 'h' from balloon

$\therefore s = h, u = 0$, a = g/8, t = 8s in

$$s = ut + \frac{1}{2}at^2$$

$$h = \frac{1}{2}\left(\frac{g}{8}\right)8^2 = 4g$$

The velocity of the balloon at this height can be calculated as below v = u + at

$= 0 + \left(\frac{g}{8}\right)8 = g$ This is the initial velocity of stone

$\therefore$ H = h = 4g, u = v = g in

$$-H = ut - \frac{1}{2}gt^2$$

$$-4g = gt - \frac{1}{2}gt^2$$

$$t^2 - 2t - 8 = 0$$

Solving for t we get t = 4 and -2 s. Ignoring negative value of time, t = 4 s

11. A ball is thrown with the velocity $u = 10 ms^{-1}$ at an angle of $\alpha = 40^0$ to the horizon. Find (i) the height of which the ball will rise to, (ii) the distance x from the point of projection to the point to the point where it reaches to the ground and (iii) the time during which the ball will be in motion (neglect the air resistance)

Sol: **i)** Here $u = 10 m^{-1}$ and $\alpha = 40^0$

$$H_{\max} = \frac{u^2 \sin^2\theta}{2g} = \frac{(10)^2 \sin^2 40^0}{2 \times 9.8} = 2.1m$$

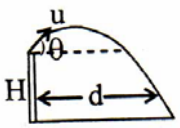
ii) $R = \frac{u^2 \sin 2\theta}{g} = \frac{(10)^2 \sin(2 \times 40^0)}{g} = 10m$

iii) $T = \frac{2u \sin\theta}{g} = \frac{2 \times 10 \times \sin 40^0}{9.8} = 1.3s$.

12. A ball is thrown from the top of a tower of 61m high with a velocity $24.4 ms^{-1}$ at an elevation of 30^0 above the horizon. What is the distance from the foot of the tower to the point where the ball is hits the ground?

Sol: From the figure $-H - ut - \frac{1}{2}gt^2$ and substituting $u = u_y = u \sin\theta = (24.4 \times \sin 30^0) ms^{-1}$; H = 61m, we get

$$-61 = (24.4 \sin 30^0)t - \frac{1}{2}gt^2$$



t = 5s ball hits the ground after '5'sec

$\therefore$ Horizontal acceleration.

$u_x = (24.4)\cos 30^0$. As there is no horizontal acceleration from the figure horizontal distance

$d = (u \cos\theta)t = (24.4 \times \cos 30^0) ms^{-1} \times 5s = 105.65m$

PROBLEMS

1. A car moving on a straight road accelerates from a speed of .1 m/s to a speed of 6.9 m/s in 5.0s. What was its average acceleration?

Sol: Given initial velocity 9u) = .1 m/s
Final velocity (v0 = 6.9 m/s
Time (t) = 5 sec
Acceleration (a) =?

$$a = \frac{v-u}{t}$$

$$= \frac{6.9-4.1}{5}$$

$$= \frac{2.8}{5} = 0.56\, m/s^2$$

2. A bullet moving with speed 150 m/s strikes a tree and penetrates 3.5cm before stopping. Find the magnitude of its acceleration and the time taken to stop.

Sol: Given initial velocity (u) = 150 m/s
Distance travelled (S) = $3.5\times10^{-2}\, m$
Final velocity (v) = 0
A = ? and t = ?

From $v^2 - u^2 = 2aS \Rightarrow a = \frac{v^2-u^2}{2S}$

$$a = \frac{0^2 - 150^2}{2\times0.035}$$

$$= \frac{-150\times150}{2\times0.035} = -32128.5 = 3.124\times10^5\, m/s^2$$

Time (t) $= \frac{v-u}{a}$

$$= \frac{0-150}{a}$$

$$= \frac{0-150}{-3.21\times10^5} = 4.67\times10^{-4}\ \text{sec}$$

3. A motor vehicle travelled the first third of a distance S at a speed of $v_1 = 10\, kmph$, the second third at a speed of $v_2 = 20\, kmph$ and the last third at a speed of $v_3 = 60\, kmph$. Determine the mean speed of the vehicle over the entire distance s.

Sol: Given $v_1 = 10\, kmph$, $v_2 = 20\, kmph$
$v_3 = 60\, kmph$, v = ?

Mean speed $mean = \frac{total\ dis\tan ce}{total\ time}$

$$= \frac{\frac{S}{3}+\frac{S}{3}+\frac{S}{3}}{\frac{S}{3\times10}+\frac{S}{3\times20}+\frac{S}{3\times60}}$$

$$v = \frac{3}{\frac{1}{10}+\frac{1}{20}+\frac{1}{60}}$$

$$\frac{1}{10}+\frac{1}{20}+\frac{1}{60}=\frac{3}{v}$$

$$v=\frac{3}{0.166}=18.07\,kmph$$

4. A motorist drives north for 35.0 minutes at 85 Km/h and then stops for 15 minutes. He next continues north, travelling 130 km in 2.00 hours. a) What is his total displacement? (b) What is his average velocity/

Sol: Given $t_1=35\min$ $v_1=85\,kmph$

$t_2=15\min,$ $S_2=0$

$t_3=2\,hours,$ $S_3=130\,km$

Initial displacement $(S_1)=\frac{85\times35}{60}$

= 49.58 + 0 + 130 = 179.58 km

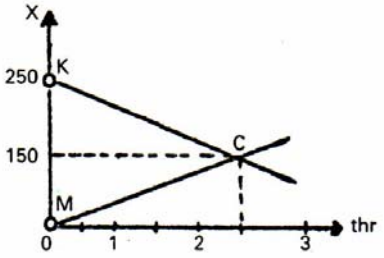
b) Average velocity (v) = $\frac{S_1+S_2+S_3}{t_1+t_2+t_3}$

$$=\frac{49.58+0+130}{\frac{35}{60}+\frac{15}{60}+2}$$

$=\frac{179.58\times60}{170}$ = 63.38 (or) = 63.4 kmph

5. The distance between the towns M and K is 250 km. Two cars set off simultaneously from the towns towards each other. The car from M travels at a speed of v_1=60 km/hr and the one from K at a speed of v_2=40 km/hr. Draw graphs if position versus time for each car and use them to determine the time that will meet and the time that will elapse before they meet. Solve the problem by analytical method?

Sol: A graph is drawn taking distance between the two towns on Y-axis and time is taken on X-axis. The graph is two straight lines which are meet at 'C' as sho0wn in figure.
From figure it is obtained the cars will meet at 150km from M after 2.5 hours.



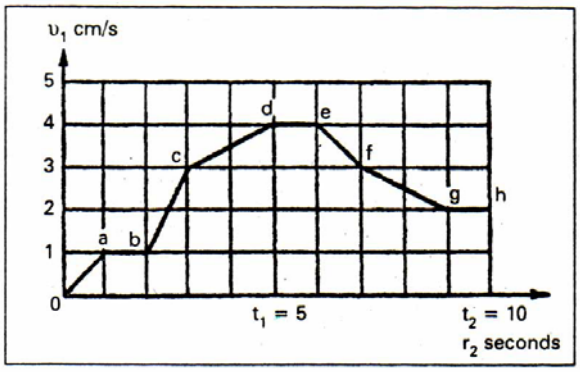
$S_A=60\times t,\ \ S_B=40t$

$S_A+S_B=t(40+60)$

250=100t $\Rightarrow$ t=2.5hrs.

6. Determine the mean velocity and the mean acceleration of a point in 5 and 10 seconds if it moves as shown in the fig.

Sol:



Mean velocity at t=5sec

$$(V_1) = \frac{Total\ dis\tan ce}{Total\ time}$$

$$= \frac{\frac{1}{2}+1+1+1+1+6}{5}$$

=2.1 cm/s

Mean acceleration$= \frac{1+2+0.5}{5}$

$= 0.8 cm/s^2$

At t=10sec

Mean velocity $= \frac{14+10.5}{10}$

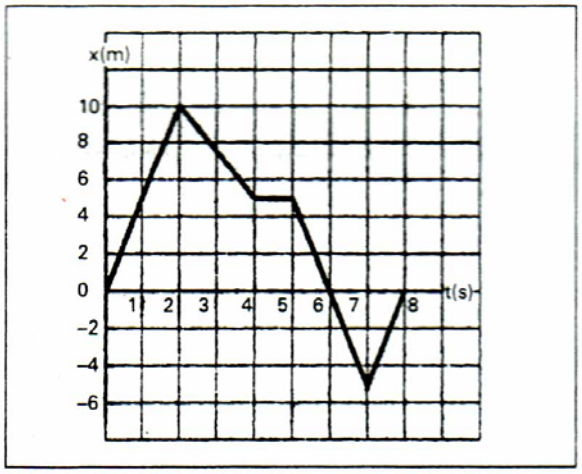
=2.45 cm/s

Mean acceleration$= \frac{14+10.5}{10}$

$= 0.2 cm/s^2$

7. Find the average velocity in the time intervals a)0 to 2 s b)0 to 4s C0 2s to 4s d)4s to 7s e) 0 to 8 s. also find the instantaneous velocity at a)t=1.0s b) t=3.0 s c)t=4.5 s d)t=7.5 s in the graph.

Sol:



i) a) at average interval '0' to 2s

Average velocity $= \frac{Total\ displacement}{Total\ time}$

$v = \frac{1}{2} \times \frac{10 \times 2}{2} = 5\,m/s$

b) '0' to 4s $\Rightarrow v = \frac{4.8}{4} = 1.2\,m/s$

c) 2s to 4s $\Rightarrow v = \frac{-5}{2} = -2.5\,m/s$

d) 4 to 7 s $\Rightarrow v = \frac{-9.9}{3} = -3.3\,m/s$

e) 0 to 8s $\Rightarrow v = 0$

ii) a) t=1.0sec

$\Rightarrow$ Instantaneous velocity $(V_{in}) = \frac{\Delta v}{\Delta t}$

$= \frac{5}{1}$=5 m/s

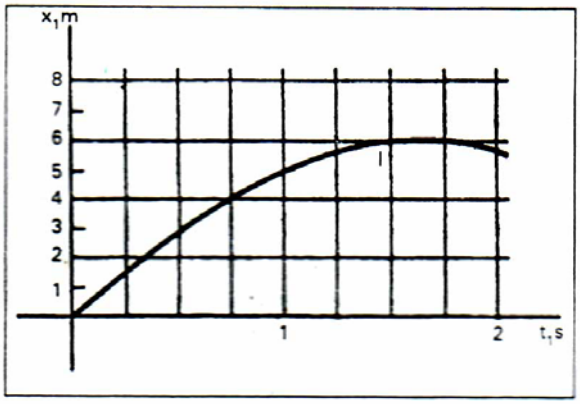
b) t= 3 sec $\Rightarrow V_{in} = \frac{-7.5}{3} = -2.5\,m/s$

c) t=4.5 sec $\Rightarrow V_{in} = 0$ since slope is zero.

d) t=7.5 sec $\Rightarrow V_{in} = \frac{-3.75}{7.5} = -0.5\,m/s$

8. Find the average velocity for the time interval $\Delta t = t_2 - 0.75$ when t_2 is 1.75, 1.5, 1.25 and 1.0 s. what is the instantaneous velocity at t=0.75 s ?

Sol:



$\Delta t_1 = t_2 - 0.75$

$t_2 = 1.75 \Rightarrow \Delta t = 1.75 - 0.75 = 1\text{sec}$

$\Rightarrow average\,velocity = \frac{6-4}{1} = \frac{2}{1}\,m/s$

$t_2 = 1.5 \Rightarrow \Delta t = 0.75$

$\Rightarrow v_a = \frac{6-4}{0.75} = \frac{2}{0.75} = 2.66 = 2.7\,m/s$

$t_2 = 1.25 \Rightarrow \Delta t = 0.50$

$\Rightarrow v_a = \frac{5.6-4}{0.50} = \frac{1.60}{0.5} = 3.2\,m/s$

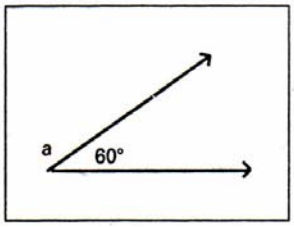
$t_2 = 1.08 \Rightarrow \Delta t = 0.25$

$$\Rightarrow v_a = \frac{5-4}{0.25} = \frac{1}{0.25} = 4\,m/s$$

Instantaneous velocity $(V_{in}) = \frac{5.1-3}{1-0.5} = \frac{2.1}{0.5} = 4.2\,m/s$

9. Velocity of a particle at time t=0 is $2\,ms^{-1}$ **A constant acceleration of** $2\,ms^{-2}$ **acts on the particle for 1 second at an angle of** 60^0 **with its initial velocity. Find the magnitude of velocity and displacement of the particle at the end of t=1 s.**

Sol:



at t=0 $\Rightarrow$ u=2 m/s

T=1 sec

$$v_x = u + a_x t = 2 + \left(a\cos 60^0\right)t$$

$$= 2 + 2\cos 60 \times 1 = 2 + 1 = 3\,m/s$$

$$v_y = u_y + a_y t = 0 + \left(a\sin 60^0\right)x$$

$$= 2.\frac{\sqrt{3}}{2}.1 = \sqrt{3}$$

$$\therefore v = \sqrt{v_x^{\,2} + v_y^{\,2}} = \sqrt{9+3}$$

$$= \sqrt{12} = 2\sqrt{3}\,m/s$$

$$x = ut + \frac{1}{2}a_x t^2 = 2\times 1 + \frac{1}{2}a\cos 60^0 \times 1^2$$

$$= 2 + \frac{1}{2}\times 2\times\frac{1}{2} = 2 + 0.5 = 2.5$$

$$y = ut + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}\left(a\sin 60^0\right)$$

$$= 0 + \frac{1}{2}.2.\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

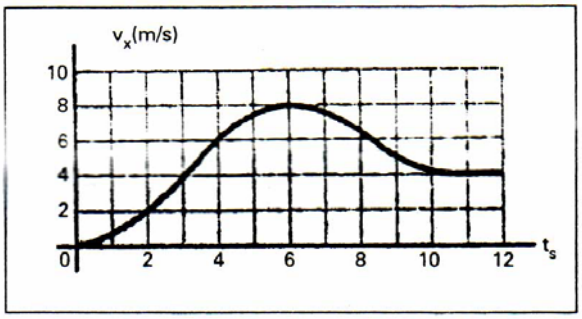
Total distance traveled (s)= $\sqrt{x^2 + y^2}$

$$= \sqrt{(2.5)^2 + \frac{3}{4}}$$

$$= \sqrt{6.25 + \frac{3}{4}} = \sqrt{\frac{28}{4}} = \sqrt{7}\,m$$

Displacement of the particle at the end of 1s= $\sqrt{7}\,m$ =2.645 m

10. Graph of velocity (V) versus time 't' for motion of a motorbike which starts from rest and moves along a straight road is given. A) Find the average acceleration for the time interval $t_0 = 0\,to\,t_1 = 6.0\,s$ **b) estimate the time at which the acceleration has its greatest positive value and the value of the acceleration at that instant. C) When is the acceleration is zero? d) Estimate the maximum negative value of the acceleration and the time at which it occurs.**



Sol: a) Average acceleration $a_{avg} = \frac{v_2 - v_1}{t_2 - t_1}$

$$= \frac{8-0}{6-0} = \frac{8}{6} = \frac{4}{3} = 1.33 m/s^2$$

b) t =3 $\Rightarrow a_{\max} = \frac{6-4}{4-3} = \frac{2}{1} = 2 m/s^2$

c) a =slope of line at t= 6sec.

= Horizental line at t=6 sec=0

And a=0 at t>10 sec

= it is a straight line indicates $\Delta v = 0$

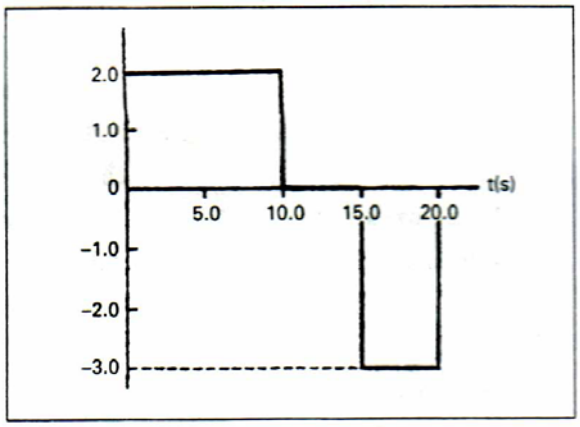
Retardation(or) negative acceleration

=slope of the graph at t=8 sec.

$\frac{6.5-5}{9-8} = \frac{1.5}{1} = 1.5 m/s^2$

$a_{neg} = 1.5 m/s^2$

11. A particle starts from rest and accelerates a shown in the graph (fig.) Determine a) the particles speed at t=10 s and at t=20 s and b) the distance travelled in the first 20 s.



Sol: a) At t= 10 sec $\Rightarrow$ particle speed and t= 20 sec

upto 10 sec

Here initial velocity u=0

From $v_1 = u + at \Rightarrow v_1 = 0 + 2 \times 10$

=20 m/s

In between 15 to 20 sec. $v_2 = v_1 + at$

$= 20 - 3 \times 5$

=20-15=50 m/s

b) Distance travelled in first 20sec is $s = ut + \frac{1}{2}at^2$

$$= 0 \times 10 + \frac{1}{2} \times 2 \times 10^2$$

s_1 =100 m

Distance travelled in 10 to 15 sec

$s_2 = vt = 20\times 5 = 100\,m$

Distance travelled 15-20 sec

$\Rightarrow s_3 = ut + \frac{1}{2}at^2$

$= 20\times 5 + \frac{1}{2}\times(-3)\times 5^2$

$= 100 - 1.5\times 25 = 62.5$

Total distance travelled $= s_1 + s_2 + s_3$

=100+100+62.5

=262.5 m

12. A stone is dropped from the top of a cliff. It is seen to hit the ground below after 4.2s. how height is the cliff ?

Sol: Given initial velocity (u)=0($\therefore$ body is dropped)

Time (t) =4.2sec.

Height of the cliff (h)= ?

$s = ut + \frac{1}{2}at^2$

$s = 0 + \frac{1}{2}\times 9.8\times 4.2^2$

$s = 4.9\times 17.64 = 86.436\,(or)\,86.44\,m$

13. How long does it take a brick to reach the ground if dropped from a height of 65m? What will be its velocity just before it reaches the ground?

Sol: Height (h) = 65 m

Initial velocity (u) = 0

V=?

Velocity before it reach the ground v= $\sqrt{2gh}$

$= \sqrt{2\times 9.8\times 65} = \sqrt{1274}$

=35.69 m/s

Time(t) = $\sqrt{\frac{2h}{g}}$

$= \sqrt{\frac{2\times 65}{9.8}} = \sqrt{\frac{65}{4.9}}$

$= \sqrt{13.26} = 3.64\,\text{sec}$

14. A helicopter is ascending vertically with a speed of $8.0\,ms^{-1}$. At a height of 120 m above the earth, a package is dropped from a window. How much time does it take for the package to reach the ground?

Sol: Initial speed (u)=8 m/s

Height (h) = 120 m

Time taken the package to reach the ground (t) =?

From $S = ut + \frac{1}{2}at^2$

$$-120 = 8t - \frac{1}{2}9.8t^2$$

$$4.9t^2 - 8t - 120 = 0$$

$$t = \frac{8 \pm \sqrt{64 + 4 \times 4.9 \times 120}}{2 \times 4.9}$$

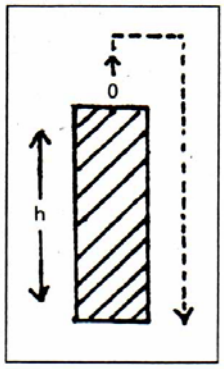
$$t = \frac{8 \pm \sqrt{64 + 2352}}{9.8} = \frac{8 \pm \sqrt{2416}}{9.8}$$

$$t = \frac{8 \pm 49.15}{9.8} = \frac{57.15}{9.8}$$

t= 5.83 sec

15. A ball is thrown straight upward with a speed v from a point h meters above the ground. Show that the time taken for the ball to strike the ground is $\frac{v}{g}\left[1+\sqrt{1+\frac{2hg}{v^2}}\right]$.

Sol: Initial velocity (u) = v
Height(S) = hm
Time (t) = ?



A = -g

From $s = ut + \frac{1}{2}at^2$

$$-h = vt - \frac{1}{2}gt^2$$

$$gt^2 - 2vt - 2h = 0$$

$$t = \frac{2v \pm \sqrt{4v^2 - 4g - 2h}}{2g}$$

$$= \frac{2v \pm \sqrt{4v^2 + 8gh}}{2g}$$

$$= \frac{v \pm \sqrt{v^2 + 2gh}}{g}$$

$$= \frac{v}{g}\left[1+\sqrt{1+\frac{2gh}{v^2}}\right]$$

16. A girl is standing on the top edge of an 18m high building. She tosses a coin up ward with a speed of $7.0\,ms^{-1}$. How long does it take for the coin to hit the ground? How fast is the coin going just before it strikes the ground? $(g = 10ms^{-2})$

Sol: Height (h)=s=18m

$g = 10 ms^{-2}$

Initial velocity (u) = 7 m/s

$-h = ut - \frac{1}{2}at^2$

$-18 = 7t - \frac{1}{2}10t^2$

$5t^2 - 7t - 18 = 0$

$\therefore t = \frac{7 \pm \sqrt{49 - 4.5 - 18}}{2.5}$

$= \frac{7 \pm \sqrt{49 + 360}}{10}$

$= \frac{7 \pm 20.22}{10}$

$t = \frac{27.22}{10} = 2.722 \sec$

Just before the coin reaches the ground; its velocity is given by

$v^2 = u^2 + 2aS$

$= 7^2 + 2 \times 10.18$

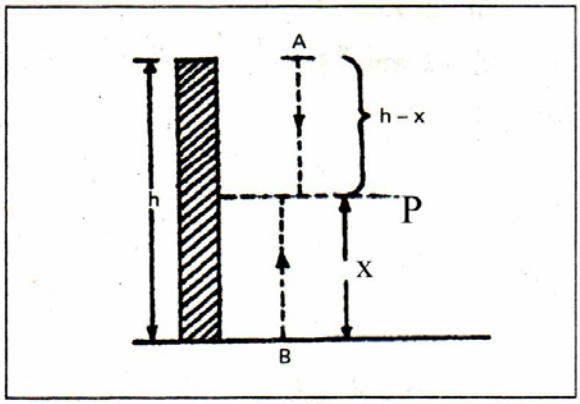
$v^2 = 49 + 360$

$v^2 = 409$

$v = \sqrt{409} = 20.2 m/s$

17. A stone is allowed to fall from the top of a tower 300 m height and at the same time another stone is projected vertically up from the ground with a velocity $100 ms^{-1}$. Find when and where the two stones meet?

Sol:



Given height of the tower(h) = 300 m

Projected body velocity (u) = 100 m/s(vertically)

Let 'P' be the point where the two stones meet.

Time taken to meet the stones (t) $= \frac{h}{u} = \frac{300}{100} = 3 \sec$

If 'x' is the distance above ground where two stones meets, then

$x = 100 \times 3 + \frac{1}{2}.9.8 \times 9 \quad \left[\because \text{from } s = ut + \frac{1}{2}at^2 \right]$

$= 300 - 4.9 \times 9$

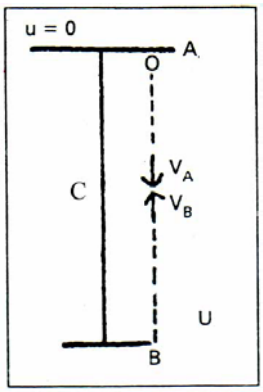
$= 300 - 44.1 = \boxed{255.9 m}$

18. Ball A is dropped from the top of a building at the same instant that a ball B is thrown vertically upward from the ground. When the balls collide, they are moving in opposite directions and the speed of A is twice the speed of B. At what fraction of the height of the building did the collision occur?

Sol: Velocity $(V_A) = 2V_B$ (at collide point)

Let 'h' be the height of the body 'A" from ground

Let two bodies are collide at a height 'x' from the ground



AC = h – x

BC = x

For 'A' body $h - x = \frac{1}{2}gt^2$ .....(1)

For 'B' body $x = ut - \frac{1}{2}gt^2$ .....(2)

Velocity at 'C' $V_A = gt$

For 'B' body $V_B = u - gt$

But $V_A = 2V_B$

$\Rightarrow gt = 2(u - gt)$

$\Rightarrow u - gt = \frac{gt}{2}$

$u = \frac{gt}{2} + gt = \frac{3gt}{2}$

$u = \frac{3gt}{2}$

$\frac{(1)}{(2)} \Rightarrow \frac{h-x}{x} = \frac{\frac{1}{2}gt^2}{4t - \frac{1}{2}gt^2}$

$\frac{h}{x} - 1 = \frac{gt^2}{2ut - gt^2}$

$\frac{h}{x} = \frac{gt^2 + 2ut - gt^2}{2ut - gt^2}$

$\frac{h}{x} = \frac{2ut}{2ut - gt^2} = \frac{2u}{2u - gt}$

Substitute 'u' in above equation

$$\frac{h}{x}=\frac{2.\frac{3gt}{2}}{2.\frac{3gt}{2}-gt}=\frac{3gt}{3gt-gt}=\frac{3gt}{2gt}$$

$$\frac{h}{x}=\frac{3}{2}\Rightarrow\frac{x}{h}=\frac{2}{3}\Rightarrow x=\frac{2h}{3}$$

19. An object falls from a bridge that is 45m above the water. If falls directly into a small row-boat moving with constant velocity that was 12m from the point of impact when the object was released. What was the speed of the boat?

Sol: Given height of the bridge (h) = 45m

Time taken by the stone to drop into boat = Time taken by the boat to travel 12m

$$\therefore t=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2\times45}{10}}=\sqrt{\frac{90}{10}}=3\text{sec}$$

Distance travel by boat = 12m

$\therefore$ Velocity of the boat $\frac{s}{t}=\frac{12}{3}=4m/s$

20. One body falls freely from a point A at a height (H +h) while another body is projected upwards with an initial velocity v_0 from point C at some time as first body begins to fall. How long before or after the first body starts to fall and with what initial velocity should a body be projected upwards from point C to satisfy…….
i) The bodies meet at point B at the height 'h'.
ii) the height 'h' to be the maximum height the projected body reaches?
[If H < h, the second body should be thrown after some delay; when H = h, the bodies should be simultaneously released and thrown; when H < h the second body should be thrown before the first begins to fall].

Sol: i) Time taken by the I body to reach ground $(t_1)=\sqrt{\frac{2h}{g}}$

Time taken by the || body to reach ground $(t_2)=\sqrt{\frac{2H}{g}}$

Difference in their time interval $(t)=t_1-t_2$

$$=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2H}{g}}$$

$$t=\frac{\sqrt{2gh}-\sqrt{2gH}}{g}$$

ii) maximum height = h, initial velocity = u

$$h=\frac{u^2}{2g}\Rightarrow u^2=2gh$$

$$u=\sqrt{2gh}$$

21. Two balls are dropped to the ground from different heights. One ball is dropped 2.0s after the other, but both strike the ground at the same time 5.0s after the 1st is dropped. a) What is the difference in the height s from which they were dropped? (b) From what height was the first ball dropped?

Sol: Given difference in the time between two balls (t) = 2sec

For first ball: Time taken by first ball to reach ground $(t_1)=5\text{sec}$

$S=h_1$ & initial velocity (u) = 0 (since dropped)

$S = h_1 = 0 \times t + \frac{1}{2} g t^2$

$\Rightarrow \frac{1}{2} \times 9.8 \times 5^2$

= 122.5m

a) Difference in heights = 122.5 – 44.1 = 78.4 m

b) height of the first ball = 122.5m

For second ball:

Time taken to reached ground = 5 – 2 = 3 sec

Distance travelled $S_2 = h_2$

$h_2 = \frac{1}{2} g t^2 = \frac{1}{2} \times 9.8 \times 3^2$

= 4.9 x 9 = 44.1 m

22. Drops of water fall at regular intervals from the roof of a building of height H = 16m, the first drop striking the ground at the same moment as the fifth drop detaches from the roof. Find the distance between the successive drops.

Sol: Given height of the house (H) = 16m

Time taken by the 1st drop to reach ground $(t) = \sqrt{\frac{2H}{g}}$

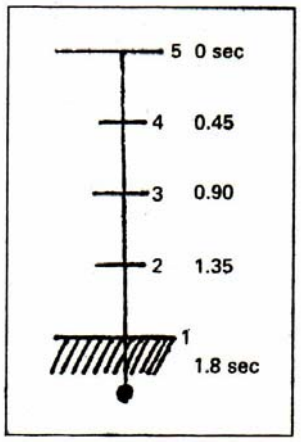
$= \sqrt{\frac{2 \times 16}{9.8}} = \sqrt{3.265} = 1.81 \text{sec}$

Time interval between each drop $= \frac{t}{n-1} = \frac{1.81}{5-1} = \frac{1.81}{4} = 0.45 \text{sec}$

Distance travelled by 2nd drop

$h_2 = \frac{1}{2} \times 9.8 \times 1.35 \times 1.35$ = 8.93 m

Distance between 1st and 2nd drops = 16-8.93 = 7.06m



Distance travelled by 3rd drop $(h_3) = \frac{1}{2} \times 9.8 \times 0.90 \times 0.90$ = 3.97 m

Distance between 2nd and 3rd drop = 8.93 – 3.97 = 4.96 m

Distance travelled by 4th drop $(h_4) = \frac{1}{2} \times 9.8 \times 0.45 \times 0.45$ = 0.992 m

Distance between 3rd and 4th drop = 3.97 – 0.992 = 2.9778 =3m

$\therefore$ distance between 4th and 5th drop = 0.9922 – 0 =0.9922 m

23. A stone is thrown vertically upward with a speed of $10.0\,ms^{-1}$ from the edge of a cliff 65m high. How much later will it reach the bottom of the cliff? What will be its speed just before hitting the bottom?

Sol: Given velocity (u) = 10 m/s
Height of the cliff (h) = 65m
Time (t) = ?

From the equation $-h = ut - \frac{1}{2}gt^2$

$$-65 = 10\times t - \frac{1}{2}\times 9.8.t^2$$
$$-65 = 10 - 4.9t^2$$
$$4.9t^2 - 10t - 65 = 0$$
$$t = \frac{10 \pm \sqrt{100 + 4\times 4.9\times 65}}{9.8}$$
$$= \frac{10 \pm \sqrt{100 + 1274}}{9.8}$$
$$= \frac{10 \pm \sqrt{1374}}{9.8}$$
$$= \frac{10 \pm 37.06}{9.8}$$
$$t = \frac{47.06}{9.8} = 4.8\,\text{sec}$$

Speed of the stone
$$v = \sqrt{2gh}$$
$$= \sqrt{2\times 9.8\times 65} = \sqrt{19.6\times 65} = \sqrt{1274}$$
= 35.69 m/s

24. If an object reaches a maximum vertical height of 23.0m when thrown vertically upward on earth how high would it travel on the moon where the acceleration due to gravity is about one sixth that on the earth? Assume that initial velocity is the same.

Sol: Acceleration due to gravity on earth = g

Acceleration due to gravity on moon = $(g_M) = \frac{g}{6}$

$$\therefore H_E = \frac{u^2}{2g} \Rightarrow height\,(H) = 23m$$
$$\frac{u^2}{2g} = 23m$$
$$H_M = \frac{u^2.6}{2g}$$
$$H_M = 138\,m$$

25. A ball is thrown at an angle 30^0 to the horizontal with an initial velocity 20m/s. find its (i) time of light ii) maximum height reaches (iii) horizontal ranges $(e'g' = 10\,ms^2)$

Sol: Given $\theta = 30^0$; initial velocity (u) = 20m/s
$g = 10\,m/s^2$

i) Time of light $(t) = \frac{2u\sin\theta}{g}$

$$= \frac{2\times 20\times \sin 30}{10}$$

$$2\times 2\times \frac{1}{2} = 2\sec$$

ii) maximum height reaches $(H) = \frac{u^2 \sin^2 \theta}{2g}$

$$= \frac{20\times 20\times \sin^2 30}{2\times 10}$$

$$= 20\times \frac{1}{4} = 5m$$

iii) Horizontal range $(R) = \frac{u^2 \sin 2\theta}{g}$

$$R = \frac{20\times 20\times \sin 60}{10}$$

$$= 40\times \frac{\sqrt{3}}{2} = 20\sqrt{3}\, m$$

26. A shell is fixed from a long –range gun with an initial velocity $u = 1000\, ms^{-1}$ at an angle 30^0 to the horizon. How long will the shell be in the air? At what distance from the gun will it fall to the ground ? The gun and the point where the shell lands are on the same horizontal line.

Sol: Given initial (u) = 1000 m/s

$\theta = 30^0$

Time of height $(T) = \frac{2u \sin \theta}{g}$

$$= \frac{2\times 1000\times \sin 30}{9.8}$$

$$= 2\times 1000\times \frac{1}{2}\times \frac{1}{9.8}$$

$$= \frac{1000}{9.8} = 102.04 \text{ sec}$$

Range $(R) = \frac{u^2 \sin 60}{g}$

$$= \frac{1000\times 1000\times \sqrt{3}}{9.8\times 2}$$

$$= \frac{1732\times 10^6}{19.6}$$

$$= 88.367\times 10^3 = 88367 \text{ m}$$

27. Two bodies are thrown with the same initial velocity at angles α and $(90-\alpha)$ to the horizon. What is the ratio of the maximum heights reached by the bodies.

Sol: Initial velocities of two bodies are equal angles $= \alpha, 90-\alpha$

Rage of the maximum heights $= \frac{H_1}{H_2} + ?$

$$\frac{H_1}{H_2} = \frac{\frac{u^2 \sin\alpha}{2g}}{\frac{u^2 \sin^2(90-\alpha)}{2g}}$$

$$= \frac{\sin^2\alpha}{\sin^2(90-\alpha)} = \frac{\sin^2\alpha}{\cos^2\alpha}$$

$$\frac{H_1}{H_2} = \tan^2\alpha$$

28. A hunter aims the gun at a monkey hanging from a height tree branch some distance away. At the instant the bullet is shot the monkey drops from the branch, hoping to avoid the bullet. Show analytically that the monkey made the wrong move. Ignore the air resistance.

Sol: Time taken by monkey to reach ground $(t_1) = \sqrt{\frac{2h}{g}}$.....(1)

The path of the bullet from the gun is like a horizontal projectile $\Rightarrow$ vertical velocity $(v_y) = 0$

$S = h \Rightarrow$ time taken by bullet to reach ground

$$h = ut + \frac{1}{2}at_2^2$$

$$h = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \sqrt{\frac{2h}{g}} \text{ .....(2)}$$

From (1) and (2) $t_1 = t_2$

Both bullet and monkey reach the ground simultaneously

$\therefore$ bullet hits the monkey.

29. Two balls are projected from the same point in the direction inclined at 60^0 and 30^0 to the horizontal. If they attain the same height, what is the ratio of their velocities of projections? What is the ratio if they have same horizontal range?

Sol: Given $\theta_1 = 60^0$ and $\theta_2 = 30^0$

Height is same $\Rightarrow H_1 = H_2$

Ratio of the velocities of projection = ?

$$\frac{u_1}{u_2} = ?$$

$$H_1 = \frac{u_1^2 \sin^2\theta_1}{2g} \text{ and } H_2 = \frac{u_2^2 \sin^2\theta_2}{2g}$$

$$H_1 = H_2 \Rightarrow \frac{u_1^2 \sin^2\theta_1}{2g} = \frac{u_2^2 \sin^2\theta_2}{2g}$$

$$\frac{u_1^2}{u_2^2} = \frac{\sin^2\theta_2}{\sin^2\theta_1} = \frac{\sin^2 30}{\sin^2 60}$$

$$= \frac{\left(\frac{1}{2}\right)^2}{\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{4} \times \frac{4}{3} = \frac{1}{3}$$

$$\frac{u_1}{u_2} = \frac{1}{\sqrt{3}}$$

If the two bodies have same range i.e., $R_1 = R_2$

$$\frac{u_1^2 \sin 2\theta_1}{g} = \frac{u_2^2 \sin 2\theta_2}{g}$$

$$\Rightarrow \frac{u_1^2}{u_2^2} = \frac{\sin 120}{\sin 60} = \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} = 1$$

30. A stone is thrown horizontal with the velocity u = 15 m/s from a tower of height H = 25m. Find (i) the time during which the stone is in motion. (ii) the distance from the tower base to the point where the stone will touch the ground, (iii) the velocity v with which it will touch the ground and (iv) the angle θ the trajectory of the stone makes with the horizontal at the point stone touches the ground. (air resistance is to be neglected).

Sol: Given height of the town (H) = 25m

Velocity (u 0 = 15 m/s

i) Time of height $(t) = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 25}{9.8}}$

$$= \sqrt{\frac{25}{4.9}} = \sqrt{5.102}$$

= 2.258 sec = 2.26 sec

ii) Horizontal distance (R) = u x t

R = 15 x 2.26 = 33.9 m

iii) Velocity (v) = ?

from $v^2 - u^2 = 2aS$

$v^2 = u^2 + 2aS$

$= 15^2 + 2 \times 9.8 \times 2.26$ = 889.44

V = 29.82 m/s

Angle made with the horizontal

$$\theta = \tan^{-1} \sqrt{\frac{2gh}{u}}$$

$$= \tan^{-1} \sqrt{\frac{2 \times 9.8 \times 2.5}{15}}$$

$$= \tan^{-1} (1.4757)$$

$= 55.87 = 56^0$

31. A body is thrown with the velocity $u = 12.0 ms^{-1}$ at an angle of $\alpha = 45^0$ to the horizon dropped to the ground at the distance s from the point where it was thrown. From what height h should stone be thrown in a horizontal direction with the same initial velocity u for it to fall at the same spot?

Sol: Given initial velocity of the body (u) = 12 m/s

Angle $(\theta) = 45^0$

Distance travelled in horizontal direction(R) = S

$$R(S) = \frac{u^2 \sin 2\theta}{g} = \frac{12^2 \sin 90}{9.8}$$

$$= \frac{144}{9.8} = 14.7\, m$$

$R = 14.7\, m \Rightarrow h = ?$

$$R = u \times \sqrt{\frac{2h}{g}} \Rightarrow R^2 = u^2 . \frac{2h}{g} \Rightarrow h = \frac{R^2 . g}{2u^2}$$

$$h = \frac{14.7 \times 14.7 \times 9.8}{2 \times 12 \times 12}$$

$= \frac{2117.68}{288}$ = 7.35 m

32. A ball is tossed from an upper storey window of a building. The ball is given an initial velocity of $8.00\,ms^{-1}$ at an angle of 20.0^0 below the horizontal. It strikes the ground 3.00 s later. (a) How far horizontally from the base of the building does the ball strike the ground? (b) find the height from which the ball was thrown. (c) How long does it take the ball to reach a point 10.0m below the level of launching?

Sol: Velocity of ball (u) = 8m/s, $\theta = 20^0$

Time (t) = 3sec

a) Horizontal distance = $u\cos\theta \times t$

$= 8\cos 20 \times 3$

$= 8 \times 0.9397 \times 3$

=22.6 m

b) Height (h) $= (u\sin\theta)\times t + \frac{1}{2}gt^2$

$= 8 \times \sin 20 \times 3 + \frac{1}{2} \times 9.8 \times 9$

=8.208+44.1

=52.31 m

c)The ball is thrown from a height of 44.1m

$h_1 = u\sin\theta t_1 + \frac{1}{2}gt_1^2$

$10 = (8\sin 20)t_1 + \frac{1}{2} \times 9.8 \times t_1^2$

$10 = 2.736t_1 + 4.9t_1^2$

$4.9t_1^2 - 2.736t_1 - 10 = 0$

$t_1 = \frac{-2.736 \pm \sqrt{(2.736)^2 \pm 4 \times 4.9 \times 10}}{2 \times 4.9}$

$t_1 = \frac{-2.736 \pm \sqrt{203.8}}{2 \times 4.9}$

$t_1 = \frac{-2.736 + 14.265}{2 \times 4.9}$

$= \frac{11.5288}{9.8} = 1.176\,\text{sec} = 1.18\,\text{sec}$

33. A food packet is dropped from an aeroplane moving at at speed of 360 kmph in a horizontal direction from a height of 500m. find its (i) time of descent and (ii) the distance at which the food packet reaches the ground from the point on the ground which is vertically below the point of dropping (take g = $10m/s^2$ **)**

Sol: Velocity of aeroplane (V) = 360kmph

$= 360 \times \frac{5}{18} = 100\,m/s$

Height (h) = 500m

i) Time of descent $(t) = \sqrt{\frac{2h}{g}}$

$= \sqrt{\frac{2 \times 500}{10}}$ = 10 sec

ii) Horizontal range $(R) = u \times \sqrt{\frac{2h}{g}}$ = 100 x 10 = 1000 m