VECTORS
2. RELATIVE VELOCITY

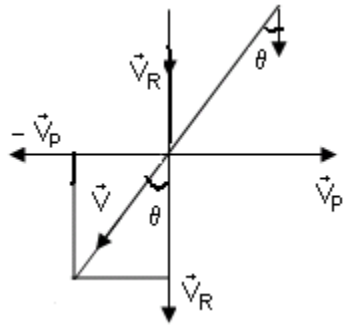
POINTS TO REMEMBER

1. **Relative velocity:** When the distance between two bodies is altering either in magnitude or direction or both, then each is said to have a relative velocity with respect to the other. Relative velocity is vector difference of velocities.
   a. The relative velocity of body 'A' w.r.t. 'B' is given by $\vec{V}_R = \vec{V}_A - \vec{V}_B$
   b. The relative velocity of body 'B' w.r.t. 'A' is given by $\vec{V}_R = \vec{V}_B - \vec{V}_A$
   c. $\vec{V}_A - \vec{V}_B$ and $\vec{V}_B - \vec{V}_A$ are equal in magnitude but opposite in direction
   d. $\left|\vec{V}_R\right| = \left|\vec{V}_A - \vec{V}_B\right| = \sqrt{V_A^2 + V_B^2 - 2.V_A V_B.\cos\theta}$
   e. For two bodies moving in the same direction, relative velocity is equal to the difference of velocities. ($\theta = 0°.\cos 0 = 1$)
      $\left|\vec{V}_R\right| = V_A - V_B$
   f. For two bodies moving in opposite direction, relative velocity is equal to the sum of their velocities. ($\theta = 180°; \cos 180 = -1$)
      $\therefore \left|\vec{V}_R\right| = V_A + V_B$
   g. If they move at right angle to each other, then the relative velocity = $\sqrt{v_1^2 + v_2^2}$.

Rain-Umbrella

1. Rain is falling vertically downwards with a velocity $\vec{V}_R$ and a person is traveling with a velocity $\vec{V}_P$. Then the relative velocity of rain with respect to the person is $\vec{V} = \vec{V}_R - \vec{V}_P$.

   Relative velocity = $|\vec{V}| = \sqrt{V_R{}^2 + V_P{}^2}$.



2. The direction of relative velocity (or) the angle with the vertical at which an umbrella is to be held is given by

$$\text{Tan}\theta = \frac{Velocity\,of\ \ the\ person}{Velocity\,of\ \ the\,rain}.$$

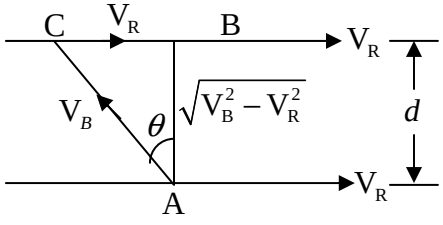
Motion of the boat in a river:

If $\vec{V}_B$ and $\vec{V}_R$ are the velocities of a boat and river flow respectively**,** then to cross the river

1. Shortest Path:

$$Sin\theta = \frac{V_R}{V_B} \qquad \text{And} \qquad \theta = Sin^{-1}\left(\frac{V_R}{V_B}\right)$$

and $90 + Sin^{-1}\left(\frac{V_R}{V_B}\right)$ with stream.

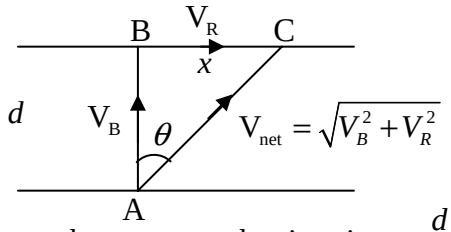


Resultant velocity = $\sqrt{V_B^2 - V_R^2}$

Time taken to cross the river $t = \dfrac{d}{\sqrt{V_B^2 - V_R^2}}$

2. Shortest time:

Resultant velocity = $\sqrt{V_B^2 + V_R^2}$



Time taken to cross the river is $t = \dfrac{d}{V_B}$

Also $Tan\theta = \dfrac{V_R}{V_B} = \dfrac{x}{d} \Rightarrow Drift\ (x) = \dfrac{V_R}{V_B} d$

SHORT ANSWER QUESTIONS

1. What is relative motion? Explain it?

Ans. **Relative motion**: The motion of a body with respect to another body is called the relative motion. And the corresponding velocity is called the relative velocity.

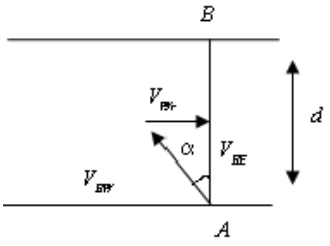
Explanation: Consider two persons A and B moving with velocities $\vec{V}_A$ *and* $\vec{V}_B$ in two different directions making an angle θ with each other

1. The relative velocity of body 'A' w.r.t. 'B' is given by $\vec{V}_R = \vec{V}_A - \vec{V}_B$
2. The relative velocity of body 'B' w.r.t. 'A' is given by $\vec{V}_R = \vec{V}_B - \vec{V}_A$
3. $\vec{V}_A - \vec{V}_B$ and $\vec{V}_B - \vec{V}_A$ are equal in magnitude but opposite in direction
4. $\left|\vec{V}_R\right| = \left|\vec{V}_A - \vec{V}_B\right| = \sqrt{V_A^2 + V_B^2 - 2.V_A V_B.\cos\theta}$
5. For two bodies moving in the same direction, relative velocity is equal to the difference of velocities. (θ = 0°.cos 0 = 1) $\left|\vec{V}_R\right| = V_A - V_B$
6. For two bodies moving in opposite direction, relative velocity is equal to the sum of their velocities. (θ =180°;cos180 = –1)
7. $\therefore \left|\vec{V}_R\right| = V_A + V_B$
8. If they move at right angle to each other, then the relative velocity = $\sqrt{v_1^2 + v_2^2}$.
9. If $\left|\vec{V}_A\right| = \left|\vec{V}_B\right| = V$, *then* $V_{Rel} = 2V \sin\dfrac{\theta}{2}$

2. Show that a boat must move at an angle of 90^0 with respect to river water in order to cross the river in a minimum time?

Ans. Let d be the (AB) width of the river.

Suppose that a boat moves with a velocity V_{BW} such that it makes an angle α with the line AB upstream as shown in figure. V_{BW} is the velocity of boat with respect to water.



V_{WE} is the velocity of river water with respect to the earth.

V_{BE} is the velocity of boat with respect to the earth.

From the above figure, $\sin\alpha = V_{WE}/V_{BW}; \alpha = \sin^{-1}(V_{WE}/V_{BW})$

and $V_{BE} = \sqrt{V_{BW}^2 - V_{WE}^2}$

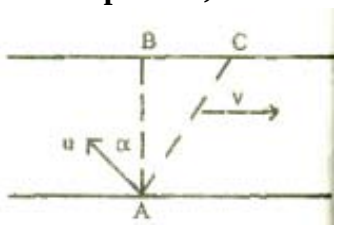
The time taken by the boat to cross the river in the shortest possible path is

$$t = AB/V_{BE} = \frac{d}{\sqrt{V^2{}_{BW} - V^2{}_{WE}}} = \frac{d}{V_{BW}\cos\alpha}$$

If the boat is directed along the line perpendicular to the river current (AB), then $\alpha = 0^0$ and the boat takes minimum time to cross the river and $t_{min} = \frac{d}{V_{BW}\cos 0^0} = \frac{d}{V_{BW}}$

Exercise 1

1. A man in a boat attempts to cross a river from Point A. If he rows the boat perpendicular to the banks, he reaches point C at a distance s = 120 m down stream from point B, in 10 minutes. If the man rows his boat at an angle α to the straight line AB he reaches the point B in 12.5 minutes. Find a) the width of the river (b) velocity of the boat relative to the water (c) velocity of the current and (d) the angle α. Assume the velocity of the boat relative to the water to be constant and of the same magnitude in both cases.



Sol. Let the velocity of the boat with respect to water be u, the velocity of the water current with respect to the banks is v and the width of the river l.

When boat is rowed perpendicular to the banks , the resultant velocity of the boat with respect to the banks is along AC, the boat moves along the river with velocity v. Suppose the boat moves a distance BC=S along the river in time t_1.

Then $BC = s = vt_1 = 120\,m$ ---------(i)

Exactly in this time t_1 boat moves across the river and travels a distance l, (width of the river), with velocity u.

Then $l = ut_1$ ------ (ii)

Now we consider the case when the boat moving with velocity 'u' making an angle α with the line AB.

The relative velocity of the boat with respect to the bank along the river is $v - u\sin\alpha = 0$, or $v = u\sin\alpha$ ----- (iii)

The relative velocity of the boat with respect to the bank and perpendicular to it is .

If the time taken to reach the point B in this case is t_2, then $(u\cos\alpha)t_2 = l$ ----(iv)

From (i), (ii),(iii) & (iv)

$t_1 = s/v; t_2 = l/u\cos\alpha$

$\sin\alpha = s/l; \cos\alpha = t_1/t_2$

we know that $\sin^2\alpha + \cos^2\alpha = 1$. From this identity

$$(s/l)^2 + (t_1/t_2)^2 = 1 \Rightarrow \frac{s^2}{l^2} = 1 - \frac{t_1^2}{t_2^2} = \frac{t_2^2 - t_1^2}{t_2^2} \Rightarrow l^2 = \frac{s^2 t_2^2}{\sqrt{t_2^2 - t_1^2}}$$

$l = st_2 / \sqrt{t_2^2 - t_1^2}$ And substituting the values we get $l = 200\,m$

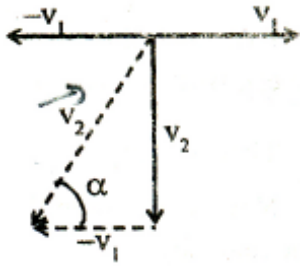
Using (i) and (ii) we get

$u = 20\,m/\min; v = 12\,m/\min$

And $\alpha = \sin^{-1}(v/u) = \sin^{-1}(12/20) = \sin^{-1}(0.6) = 36.86^0\,(or\ 36^0\,50^1)$

2. A pipe is fixed to a moving cart such that the axis of the pipe makes an angle α with the floor of the cart. The cart moves uniformly along a horizontal path with velocity $v_1 = 2ms^{-1}$. If rain drops which are falling with a velocity $v_2 = 6ms^{-1}$ vertically down reach the bottom of the pipe without touching the walls of the pipe, find the value of α. Assume that the air resistance is negligible and velocity 'v' of the drops is constant.

Sol. The velocity of the rain drops with respect to the pipe (or cart) should be along the axis of the pipe. Let v_1 is the velocity of the cart and v_2 be the velocity of the rain drops with respect to the earth. The velocity of the rain drops with respect to the cart is



$v_{21} = v_2 - v_1$

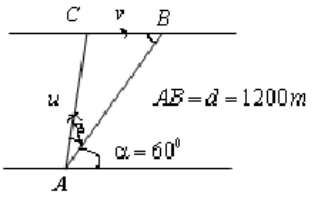
From the figure, $Tan\ \alpha = v_2 / v_1 = 6/2 = 3$

$\alpha = \tan^{-1}(3) = 71^0$ or $\alpha = \tan^{-1}(3) = 71^0 35^1$

Exercise 2

1. A ferry boat travels between two points A and B on the banks of a river (fig) always following the line AB. The distance S between points A and B in 1,200 m. The velocity of the river current $v = 1.9\,ms^{-1}$ is constant over the entire width of the river. The line AB makes an angle $\alpha = 60^0$ with the direction of the current. With what velocity 'u' and at what angle β to the line AB should the ferry boat move to cover the distance AB and back in a time t=5 minutes? The angle β remains the same during the passage from A to B and from B to A.

Sol.



Velocity of water current = V=1.9 m/sec

Velocity of Ferry boat = u=?

Time taken by the Ferry boat to cover the distance AB=t_1 and

$t_1 = \dfrac{d}{u\cos\beta}$

$u\cos\beta$ is the component of velocity of boat (u) along AB

time taken by the ferry boat to cover the distance BA = t_2

and $t_2 = \dfrac{d}{u\cos\beta}$ (since angle β remains same during the passage of boat from A to B and B to A)

The total time taken by the boat to cover the distance AB and back $t = t_1 + t_2 = 5$ minutes

$\Rightarrow t = 300\sec = \dfrac{2d}{u\cos\beta} = \dfrac{2400}{u\cos\beta}$

$u\cos\beta = 8 - - - - - - (1)$

From ΔACB, we can write $\frac{V}{\sin\beta}=\frac{u}{\sin\alpha}$

$u\sin\beta = V\sin\alpha = 1.9\times\sin 60^0$

$=\frac{1.9\times\sqrt{3}}{2}$ ---------- (2)

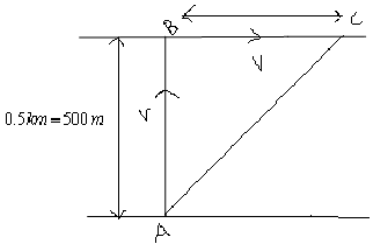
$\frac{(2)}{(1)}\Rightarrow Tan\beta=\frac{1.9\times\sqrt{3}}{2\times 8}=\frac{1.9\times 1.732}{16}=0.2056$

$\Rightarrow\beta=\tan^{-1}(0.2056)=11^0 37^1$

$u\cos\beta=8$

Velocity of the Ferry boat $u=\frac{8}{\cos\beta}=\frac{8}{\cos(11^0 37^1)}=\frac{8}{0.9797}=8.16 m/s$

2. A boat moves perpendicular to the bank with a velocity of 7.2 km/h. The current carries it 150 m downstream, find (1) the velocity of the current, (2) the time required to cross the river. The width of the river is 0.5 km.



Sol. V_b=velocity of the boat and side AB represents V_b

V_c=velocity of water current and side BC represents V_c

V_b=7.2 kmph = 2 m/sec

1) Time taken by the boat to cross the river $t=\frac{d}{v_b}=\frac{500}{2}=250\sec=4.16\min$

2) Drift experienced by the boat= $BC=V_c\times t=150m\Rightarrow V_c=\frac{BC}{t}=\frac{150}{250}=0.6m/\sec$

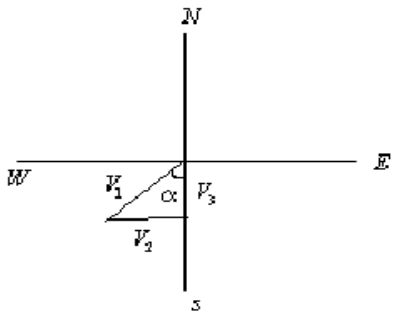
3. An aeroplane is flying with the velocity of $V_1=800\,kmph$ relative to the air. A wind with velocity of $V_2=15ms^{-1}$ is blowing from West to East. What is the velocity of the aeroplane with respect to the earth? And what should be the angle with the meridian to fly the aeroplane (1) south ward (2) North ward (3) west ward (4) East ward

Sol. (a) $V_1=800\,kmph=800\times\frac{5}{18}m/\sec$ =222.22 m/se

$V_2=15m/\sec$

The velocity of the aeroplane moving South wards =V_3

$\alpha=\sin^{-1}\left(\frac{V_2}{V_1}\right)=\sin^{-1}(0.0675)=3^0 52^1$ West of south



$V_3=V_1\cos\alpha=V_1\cos(3^0 52^1)=222.22\times 0.9979=221.7\,m/s$

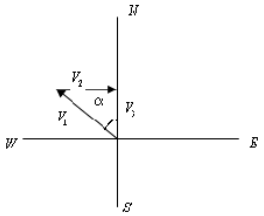
(b) The velocity of the aeroplane moving south wards $=V_3=V_1\cos\alpha$

$\sin\alpha = \left(\frac{V_2}{V_1}\right) = 3^0 52^1$ West of North

$\Rightarrow V_3 = V_1 \cos\left(3^0 52^1\right) = 221.7\, m/\sec$

(c) Velocity of the aeroplane moving towards west =
$V_2 - V_1 = 222.22 - 15 = 207.22\, m/s$

$\therefore$ Aeroplane moves west wards making an angle 90º with the meridian



(d) Velocity of the aero plane moving towards East
$= V_2 + V_1 = 237.22\ m/s$

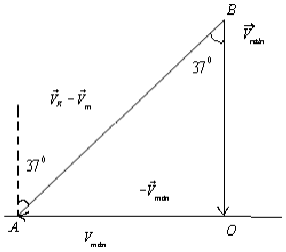
Aero plane moves eastwards making an angle 90^o with the meridian

4. A man is traveling at 10.8 kmph in a topless car on a rainy day. He holds an umberella at an angle of 37^0 with the vertical so that he does not wet. If rain drops falls vertically downwards, what is the rain velocity?

Sol. Let 'OA" be representing the velocity of man
'OB' be representing the velocity of rain
From ΔOAB

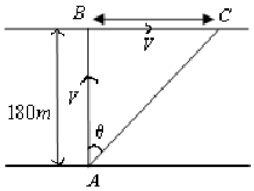
$Tan37^0 = \frac{OA}{OB} = \frac{V_{man}}{V_{Rain}} \Rightarrow \frac{3}{4} = \frac{V_{man}}{V_{rain}}$

$\Rightarrow$ Velocity of rain $= V_{Rain} = \frac{4}{3} \times V_{man} = \frac{4}{3}\left(10.8 \times \frac{5}{18}\right) = 4\, ms^{-1}$



5. A swimmer is capable of swimming 1.65 ms^{-1} in still water (a) If she swims directly across a 180 m wide river whose current is 0.85 m/s, how far downstream (from a point opposite her starting point) will she reach ? (b) how long will it take her to reach the other side ?

Sol.



$V_s =$ velocity of the swimmer = 1.65 m/s and side AB represents V_s

$V_c =$ velocity of water current = 0.85 m/s and side BC represents V_c

$BC \rightarrow$ The drift experienced by swimmer

1) Time taken to reach the other side

$t = \frac{d}{V_s} = \frac{180}{1.65} = \frac{1200}{11}\sec = 1.82\min$

2) Drift experienced by the swimmer $BC = V_c \times t = = 0.85 \times \frac{1200}{11} = 92.72\,\text{m}$

ASSES YOURSELF

1. Two boats traveling at the same speed set off across a river at the same time. One heads straight across and is pulled downstream some what by the current. The other one heads upstream at an angle so as to arrive at a point opposite the starting point. Which boat reaches the opposite side first?

Ans. The boat which heads straight across reaches the opposite side first because it takes minimum time to cross i.e., $t = W / V_b$ where W= width of the river and V_b = boat velocity