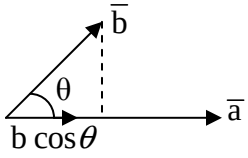
VECTORS
3. PRODUCT OF VECTORS

POINTS TO REMEMBER

1. When a vector is multiplied by a scalar it nature may (or) may not change.
   Ex:(1) If a vector is multiplied by a number like 1,2,3 m its nature does not change.
   (2) It mass is multiplied by acceleration, and then a force vector is obtained which is different from acceleration.

Scalar Product:

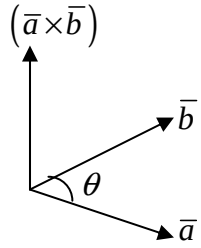
1. 1.Scalar product (or) dot product is defined as the product of the magnitudes of two vectors and the cosine of the angle between them.
   $\bar{a}.\bar{b} = ab\cos\theta$



2. The dot product of two perpendicular vector is zero (i.e.) if $\theta = 90^o$, $\bar{a}.\bar{b} = 0$
3. $i.i = j.j = k.k = 1$ And $i.j + j.k = k.i = 0$
4. If $A = A_x i + A_y j + A_z k$ and $\bar{B} = B_x i + B_y j + B_z k$
5. $\bar{A}.\bar{B} = A_x B_x + A_y B_y + A_y B_z$
6. And $Cos\theta = \dfrac{\bar{A}.\bar{B}}{AB}$
   a. W = $\vec{F}.\vec{S}$ (dot product of force and displacement is work)
   b. P = $\vec{F}.\vec{V}$ (dot product of force and velocity is power)
   c. E_p = $m\vec{g}.\vec{h}$ (dot product of gravitational force and vertical displacement is P.E)
   d. Magnetic flux, ϕ = $\vec{A}.\vec{B}$ (dot product of area vector and magnetic flux density vector)

Cross Product:

1. Cross product (or) vector product of two vectors is a vector which is the product of the magnitudes of the two vectors and the sine of the angle between them.
2. $\bar{a}\times\bar{b} = ab\sin\theta\,.\,\hat{n}$
   Where $\hat{n}$ is a u nit vector along $\bar{a}\times\bar{b}$.



3. $\bar{A}\times\bar{B} \neq \bar{B}\times\bar{A}$ and $\bar{A}\times\bar{B}$ =- $-\bar{B}\times\bar{A}$ (Anti - commutative).
4. The cross product of two parallel vectors is a null vector. (i.e) If $\theta = 0^o$. Then $\bar{A}\times\bar{B} = 0$
5. $i\times i = j\times j = k\times k = 0$
6. $i\times j = k$ $\quad$ $j\times i = -k$
   $j\times k = i$ $\quad$ $k\times j = -i$
   $k\times i = j$ $\quad$ $i\times k = -j$
7. If $\bar{a} = a_x i + a_y j + a_z k$ and $\bar{b} = b_x i + b_y j + b_z k$

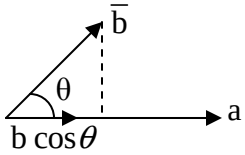
$$\bar{a}\times\bar{b}=\begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

8. Torque is the cross product of radius vector and force vector, $\vec{\tau}=\vec{r}\times\vec{F}$
9. Angular momentum is the cross product of radius vector and linear momentum, $\vec{L}=\vec{r}\times\vec{p}$
10. Linear velocity in circular motion may be defined as the cross product of angular velocity and radius vector. $\vec{V}=\vec{\omega}\times\vec{r}$
11. The area of the triangle formed by $\vec{A}$ and $\vec{B}$ as adjacent sides is $\frac{1}{2}\left|\vec{A}\times\vec{B}\right|$.
12. The area of the parallelogram formed by $\vec{A}$ and $\vec{B}$ as adjacent sides is $\left|\vec{A}\times\vec{B}\right|$.
13. If $\vec{P}$ and $\vec{Q}$ are diagonals of a parallelogram, then area of parallelogram= $\frac{1}{2}\left|(\vec{P}\times\vec{Q})\right|$.

LONG ANSWER QUESTION

1. Define scalar and vector products and explain their properties. Give two examples for each.

A. **Scalar Product or Dot Product**: The dot product of two vectors is the product of the magnitude of one of the vector quantities and the scalar component of the second vector in the direction of first vector .



The dot product of two vectors $\vec{a}$ and $\vec{b}$ is defined as $\vec{a}.\vec{b}=ab\cos\theta$ .Here a is the magnitude of $\vec{a}$, $\vec{b}$ is the magnitude of $\vec{b}$ and θ is the angle between $\vec{a}$ and $\vec{b}$.

$$\vec{a}.\vec{b}=a(b\cos\theta) \quad or \quad b(a\cos\theta)$$

In position vector form $\vec{a}=a_x\hat{i}+a_y\hat{j}+a_z\hat{k}$.

and $\vec{b}=b_x\hat{i}+b_y\hat{j}+b_z\hat{k}$ then

$$\vec{a}.\vec{b}=\left(a_x\hat{i}+a_y\hat{j}+a_z\hat{k}\right)\left(b_x\hat{i}+b_y\hat{j}+b_z\hat{k}\right)$$

$$= a_xb_x+a_yb_y+a_zb_z$$

$\hat{i}.\hat{i}=\hat{j}.\hat{j}=\hat{k}.\hat{k}=1$, as these unit vectors are parallel to each other and cos 0=1

$\hat{i}.\hat{j}=\hat{j}.\hat{k}=\hat{k}.\hat{i}=0$, as these unit vectors are perpendicular to each other and cos 90 = 0.

Properties of Scalar Product

i) Scalar product obeys commutative law.

In scalar product of two vectors, the order of multiplication of the vectors is not important. $\vec{A}.\vec{B}=\vec{B}\vec{A}$

ii) Scalar product obeys distributive law. $\vec{A}.(\vec{B}+\vec{C})=\vec{A}.\vec{B}+\vec{A}.\vec{C}$

iii) If two vectors $\vec{A}$ *and* $\vec{B}$ are in the same direction i.e. If the angle between the two vectors is zero, the scalar product has maximum value. $\vec{A}.\vec{B}=AB$

iv) The scalar product of two perpendicular vectors is zero. $\vec{A}.\vec{B}=0$

v) Scalar product can be negative. If two vectors are anti-parallel (i.e., angle between them is 180^0) then $\vec{A}.\vec{B}=AB\cos 180^0=-AB$

Example of scalar product

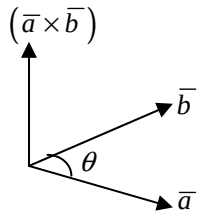
1. The dot product of force and the displacement gives the work done (W) by the force $W=\vec{F}.\vec{S}=F\ S\ \cos\theta$

2. The dot product of force and velocity is power. $P = \vec{F}.\vec{V} = FV\cos\theta$

Vector product or Cross Product:

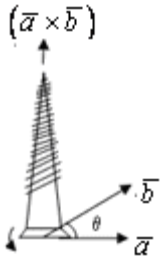
Cross product (or) vector product of two vectors is a vector which is the product of the magnitudes of the two vectors and the sine of the angle between them.

$\bar{a}\times\bar{b}=ab\sin\theta\,.\,\hat{n}$ Where $\hat{n}$ is a u nit vector along $\bar{a}\times\bar{b}$.



The direction of $\bar{a}\times\bar{b}$ can be known from right hand thumb rule (or) Cork screw rule.

Let a right - handed screw whose axis is perpendicular to the plane containing $\bar{a}$ *and* $\bar{b}$ be turned from $\bar{a}$ *to* $\bar{b}$ through an angle θ between them. Then the direction of advancement of the screw gives the direction of the vector $\bar{a}\times\bar{b}$.



If $\bar{a}=a_x i+a_y j+a_z k$ and $\bar{b}=b_x i+b_y j+b_z k$

$$\bar{a}\times\bar{b}=\begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$=(a_y b_z-a_z b_y)\hat{i}-(a_x b_z-a_z b_x)\hat{j}+(a_x b_y-a_y b_x)\hat{k}$$

Properties of Vector Product

i. Cross product of vectors do not obey commutative law.

$\bar{A}\times\bar{B}\neq\bar{B}\times\bar{A}$ and $\bar{A}\times\bar{B}=-\bar{B}\times\bar{A}$ (Anti - commutative).

ii. Cross product obeys distributive law.

$\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$

iii. The cross product of two parallel vectors is a null vector. (i.e) If $\theta=0^o$. Then $\bar{A}\times\bar{B}=0$

iv. The area of the triangle formed by $\vec{A}$ and $\vec{B}$ as adjacent sides is $\frac{1}{2}\left|\vec{A}\times\vec{B}\right|$.

v. The area of the parallelogram formed by $\vec{A}$ and $\vec{B}$ as adjacent sides is $\left|\vec{A}\times\vec{B}\right|$.

vi. $i\times i=j\times j=k\times k=0$

$i\times j=k$ $\qquad$ $j\times i=-k$

$j\times k=i$ $\qquad$ $k\times j=-i$

$k \times i = j$ $\quad\quad$ $i \times k = -j$

Example of Vector Product

1. Angular momentum is the cross product of radius vector and linear momentum, $\vec{L} = \vec{r} \times \vec{p}$
2. Linear velocity in circular motion may be defined as the cross product of angular velocity and radius vector. $\vec{V} = \vec{\omega} \times \vec{r}$

SHORT ANSWER QUESTIONS

1. Define scalar product. Give its properties and two examples. (2009-March, 2008-March, 2001-July)

A. **Scalar Product or Dot Product**: The dot product of two vectors is the product of the magnitude of one of the vector quantities and the scalar component of the second vector in the direction of first vector.

The dot product of two vectors $\vec{a}$ and $\vec{b}$ is defined as $\vec{a}.\vec{b} = ab \cos\theta$ .Here a is the magnitude of $\vec{a}$, $\vec{b}$ is the magnitude of $\vec{b}$ and θ is the angle between $\vec{a}$ and $\vec{b}$.

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In position vector form $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$.

and $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$ then

$\vec{a}.\vec{b} = \left(a_x\hat{i} + a_y\hat{j} + a_z\hat{k}\right)\left(b_x\hat{i} + b_y\hat{j} + b_z\hat{k}\right)$

$= \quad a_xb_x + a_yb_y + a_zb_z$

$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$, as these unit vectors are parallel to each other and cos 0=1

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Properties of Scalar Product

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ii) Scalar product obeys distributive law. $\vec{A}.(\vec{B} + \vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

iii) If two vectors $\vec{A}$ *and* $\vec{B}$ are in the same direction i.e. If the angle between the two vectors is zero, the scalar product has maximum value. $\vec{A}.\vec{B} = AB$

iv) The scalar product of two perpendicular vectors is zero. $\vec{A}.\vec{B} = 0$

v) Scalar product can be negative. If two vectors are anti-parallel (i.e., angle between them is 180^0) then $\vec{A}.\vec{B} = AB\cos 180^0 = -AB$

Examples

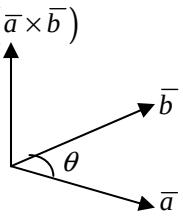
1. The dot product of force and the displacement gives the work done (W) by the force $W = \vec{F}.\vec{S} = F\ S\ \cos\theta$
2. The dot product of force and velocity is power. $P = \vec{F}.\vec{V} = FV\cos\theta$

2. Define vector product. Give its properties and two examples. (March 2011, March 2006, June 2002)

Ans. **Vector product or Cross Product:**

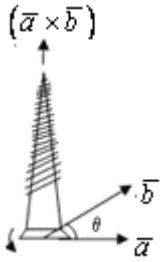
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If $\bar{a} = a_x i + a_y j + a_z k$ and $\bar{b} = b_x i + b_y j + b_z k$

$$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$= (a_y b_z - a_z b_y)\hat{i} - (a_x b_z - a_z b_x)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$$

Properties of Vector Product

i. Cross product of vectors do not obey commutative law.

$\bar{A}\times\bar{B} \neq \bar{B}\times\bar{A}$ and $\bar{A}\times\bar{B} = -\bar{B}\times\bar{A}$ (Anti - commutative).

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Examples

1. Angular momentum is the cross product of radius vector and linear momentum, $\vec{L} = \vec{r}\times\vec{p}$
2. Linear velocity in circular motion may be defined as the cross product of angular velocity and radius vector. $\vec{V} = \vec{\omega}\times\vec{r}$

VERY SHORT ANSWER QUESTIONS

1. The mass of a body is 5 kg and its velocity is $4\hat{i}+2\hat{j}-\hat{k}\,ms^{-1}$. What is the momentum of the body?(May 2009)

Ans. Momentum of the body p = mv

Mass of the body m = 5kg

Velocity of the body $\vec{v}=4\hat{i}+2\hat{j}-\hat{k}$

$\Rightarrow \vec{p}=5\left(4\hat{i}+2\hat{j}-\hat{k}\right)=20\hat{i}+10\hat{j}-5\hat{k}\ kgms^{-1}=\sqrt{525}Kg-m/s$

2. Find the values of $\left|\vec{a}.\vec{b}\right|^2+\left|\vec{a}\times\vec{b}\right|^2$ and $\left|\vec{a}.\vec{b}\right|^2-\left|\vec{a}\times\vec{b}\right|^2$

Ans Let ' θ ' be the angle between the vectors ' $\vec{a}$ ' and ' $\vec{b}$ '

$\left|\vec{a}.\vec{b}\right|^2+\left|\vec{a}\times\vec{b}\right|^2=(ab\cos\theta)^2+(ab\sin\theta)^2=a^2b^2\left(\cos^2\theta+\sin^2\theta\right)=a^2b^2$ and

$\left|\vec{a}.\vec{b}\right|^2-\left|\vec{a}\times\vec{b}\right|^2=(ab\cos\theta)^2-(ab\sin\theta)^2=a^2b^2\left(\cos^2\theta-\sin^2\theta\right)$

3. **A force of $\vec{F}=2\hat{i}+3\hat{j}+4\hat{k}\,N$ acts on a body and displaces it through $\vec{s}=\hat{i}+2\hat{j}+\hat{k}\,m$ what is the work done. (2005-June)**

Ans. $W=\vec{F}.\vec{S}=\left(2\hat{i}+3\hat{j}+4\hat{k}\right).\left(\hat{i}+2\hat{j}+\hat{k}\right)=2+6+4=\ 12J$

4. **If $\vec{F}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{S}=6\hat{i}+2\hat{j}+5\hat{k}$, then find the work done.(2010March)**

Ans. $W=\vec{F}.\vec{S}=\left(3\hat{i}+4\hat{j}+5\hat{k}\right).\left(6\hat{i}+2\hat{j}+5\hat{k}\right)=18+8+25=51$ units.

5. Can $\left|\vec{a}\times\vec{b}\right|=\left|\vec{a}.\vec{b}\right|$ find the angle between $\vec{a}$ and $\vec{b}$? (2001-July; 2006-March)

Ans. The angle between $\vec{a}$ and $\vec{b}$ is 45^0.

$\left|\vec{a}.\vec{b}\right|=\left|\vec{a}\times\vec{b}\right| \Rightarrow ab\cos\theta=ab\sin\theta \Rightarrow \frac{\sin\theta}{\cos\theta}=1\Rightarrow\tan\theta=1\Rightarrow\theta=45^0$

6. If $\vec{F}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{S}=6\hat{i}+2\hat{j}+5\hat{k}$ then find the work done. (March2010)

Ans. $W=\vec{F}.\vec{S}=\left(3\hat{i}+4\hat{j}+5\hat{k}\right).\left(6\hat{i}+2\hat{j}+5\hat{k}\right)$

$=(3)(6)+(4)(2)+(5)(5)\ =\ 51$units.

7. Can a scalar product of two vectors be negative? Explain. (2000-March)

Ans. Scalar product can be negative. If θ is the angle between $\vec{P}$ and $\vec{Q}$ and if $\theta>90^0$ then $\vec{P}.\vec{Q}$ becomes negative. The work done against friction is negative.

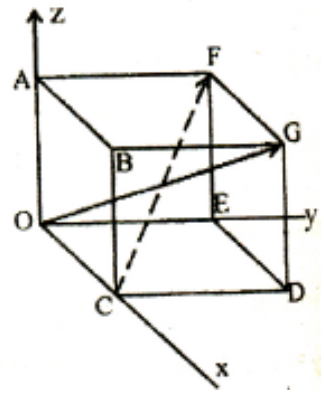
Exercise 1

1. Find the angles between the diagonals of a cube with edges of length a.

Sol. OABCDEFG is a cube of side 'a', CF and OG are the diagonals.

If OC is along +x axis, OE along y-axis and OA along Z-axis.

Then $\overrightarrow{OG} = a\hat{i} + a\hat{j} + a\hat{k}$, $\overrightarrow{CF} = -a\hat{i} + a\hat{j} + a\hat{k}$



The magnitude of $\left|\overrightarrow{OG}\right| = OG = \sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$

and that of $\left|\overrightarrow{CF}\right| = \sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$

Dot product between these vectors

$\overrightarrow{OG}.\overrightarrow{CF} = \left(a\hat{i} + a\hat{j} + a\hat{k}\right).\left(-a\hat{i} + a\hat{j} + a\hat{k}\right)$

$= (OG)(CF)\cos\theta = \left(a\sqrt{3}\right).\left(a\sqrt{3}\right)\cos\theta$

$-a^2 + a^2 + a^2 = 3a^2\cos\theta$

$a^2 = 3a^2\cos\theta$

$\therefore \cos\theta = \frac{1}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right) = 70.5^0$

2. Find the angle between two vectors $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{B} = \hat{i} - \hat{k}$

Sol. $A = \left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{2^2 + 1^2 + (-1)^2} = \sqrt{6}$

$B = \left|\vec{B}\right| = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(1)^2 + (0)^2 + (-1)^2} = \sqrt{2}$

$\vec{A}.\vec{B} = \left(2\hat{i} + \hat{j} - \hat{k}\right).\left(\hat{i} - \hat{k}\right) = 2 + 1 = 3$

$\cos\theta = \frac{\vec{A}.\vec{B}}{AB} \Rightarrow \quad \Rightarrow \cos\theta = \frac{3}{\sqrt{6}\sqrt{2}} = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^0$

3. How much work you do on an object as you lift it to a height 'h' from the ground?

Sol.. Consider a body of mass 'm' .is the mass of the body. Let 'g' be the acceleration due to gravity. In order to lift the object, a force F equal to its weight mg must be applied on it. . Let 'h' be the displacement in the upward direction. The lifting force and displacement are in the same direction.



$\therefore$ Work $= \vec{F}.\vec{h} = Fh\cos 0^0 = mgh$

This work is stored in the object as its potential energy and hence the gravitational potential energy of an object of mass 'm' which is raised to a height 'h' is mgh.

4. If $\vec{F} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{r} = \hat{i} - \hat{j} + 6\hat{k}$ find $\vec{r} \times \vec{F}$.

Sol. $\vec{r}\times\vec{F}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & +6\\ 2 & 3 & -1\end{vmatrix}=\hat{i}(1-18)-\hat{j}(-1-12)+\hat{k}(3+2)=-17\hat{i}+13\hat{j}+5\hat{k}$

5. Find the unit vector perpendicular to $\vec{A}=2\hat{i}+3\hat{j}+\hat{k}$ and $\vec{B}=\hat{i}-\hat{j}+\hat{k}$

Sol. If $\vec{C}=\vec{A}\times\vec{B}$ then, the direction of $\vec{C}$ will be perpendicular to the plane containing $\vec{A}$ and $\vec{B}$. The unit vector perpendicular to $\vec{A}$ and $\vec{B}$ is given by

$\hat{n}=\dfrac{\vec{C}}{|\vec{C}|}=\dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

$\therefore \vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 1\\ 1 & -1 & 1\end{vmatrix}=\hat{i}(3+1)+\hat{j}(1-2)+\hat{k}(-2-3)=4\hat{i}-\hat{j}-5\hat{k}$

$|\vec{A}\times\vec{B}|=\sqrt{(4)^2+(-1)^2+(-5)^2}=\sqrt{42}$

$\therefore\quad \hat{n}=\dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}=\dfrac{4\hat{i}-\hat{j}-5\hat{k}}{\sqrt{42}}$

Exercise 2

1. If $\vec{F}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{S}=6\hat{i}+2\hat{j}+5\hat{k}$ find the work done by the force. (2007-March)

Sol. Work done $W=\vec{F}.\vec{S}$

$=(3\hat{i}+4\hat{j}+5\hat{k}).(6\hat{i}+2\hat{j}+5\hat{k})=3\times6+4\times2+5\times5=18+8+25=51 units$

2. If $\vec{F}=\hat{i}+2\hat{j}+\hat{k}$ and $\vec{V}=4\hat{i}-\hat{j}+7\hat{k}$, find $\vec{F}.\vec{V}$

Sol. $\vec{F}.\vec{V}=(\hat{i}+2\hat{j}+\hat{k}).(4\hat{i}-\hat{j}+7k)=4-2+7=9$ unit

3. Find the angle between $\vec{A}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{B}=-\hat{i}+\hat{j}-2\hat{k}$

Sol. We know that Dot product of the vectors $\vec{A}$ and $\vec{B}$ is $\vec{A}.\vec{B}=|\vec{A}||\vec{B}|\cos\theta$

θ is the angle between $\vec{A}$ and $\vec{B}$

$\cos\theta=\dfrac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}=\dfrac{(\hat{i}+2\hat{j}-\hat{k}).(-\hat{i}+\hat{j}-2\hat{k})}{\sqrt{6}\times\sqrt{6}}$

$=\dfrac{-1+2+2}{6}=\dfrac{3}{6}=\dfrac{1}{2}\Rightarrow\theta=60^0$

4. If $\vec{P}=2\hat{i}+3\hat{j}-4\hat{K}$ and $\vec{Q}=5\hat{i}+2\hat{j}+4\hat{k}$, find the angle between the two vectors.

Sol. Dot product of $\vec{P}$ and $\vec{Q}$ is $P.Q=|\vec{P}||\vec{Q}|\cos\theta$

θ is the angle between $\vec{P}$ and $\vec{Q}$, $\cos\theta = \dfrac{\vec{P}.\vec{Q}}{\left|\vec{P}\right|\left|\vec{Q}\right|} = \dfrac{\left(2\hat{i}+3\hat{j}-4\hat{k}\right).\left(5\hat{i}+2\hat{j}+4\hat{k}\right)}{\sqrt{29}\times\sqrt{45}}$

$\cos\theta = \dfrac{10+6-16}{\sqrt{29}\times\sqrt{45}} = 0$

$\theta = 90^0$

5. A force $\vec{F} = 5\hat{i}-3\hat{j}+2\hat{k}$ moves a particle from $\vec{r}_1 = 2\hat{i}+7\hat{j}+4\hat{k}$ to $\vec{r}_2 = 5\hat{i}+2\hat{j}+8\hat{k}$ calculate the work done.

Sol. Work done $W = \vec{F}.\vec{S}$

Displacement of the particle $\vec{S} = \vec{r}_2 - \vec{r}_1$

$5\hat{i}+2\hat{j}+8\hat{k}-\left(2\hat{i}+7\hat{j}+4\hat{k}\right) = 3\hat{i}-5j+4\hat{k}$

$\Rightarrow W = \vec{F}.\vec{S} = \left(5\hat{i}-3\hat{j}+2\hat{k}\right).\left(3\hat{i}-5\hat{j}+4\hat{k}\right)$ = 15+15+8 = 38 units

6. The magnitude of scalar and vector products of two vectors are $4\sqrt{3}$ and $\sqrt{144}$ respectively. What is the angle between the two vectors?

Sol. Given that $\vec{A}.\vec{B} = 4\sqrt{3} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta$ ---------- (1)

$\left|\vec{A}\times\vec{B}\right| = \sqrt{144} = \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta$ ---------- (2)

$\dfrac{2}{1} \Rightarrow \dfrac{AB\sin\theta}{AB\cos\theta} = \dfrac{\sqrt{144}}{4\sqrt{3}} = \sqrt{\dfrac{144}{48}} = \sqrt{3} \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = 60^0$

7. A force of $2\hat{i}+3\hat{j}+2\hat{k}$ N acts a body for 4 s and produces a displacement of $3\hat{i}+4\hat{j}+5\hat{k}\,m$ Calculate the power? (June2010)

Sol. Power $P = \dfrac{workdone}{time} = \dfrac{\vec{F}.\vec{S}}{t}$

Force $\vec{F} = 2\hat{i}+3\hat{j}+2\hat{k}\,N$

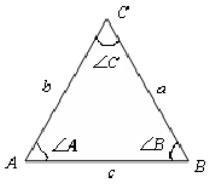
Displacement $\vec{S} = 3\hat{i}+4\hat{j}+5\hat{k}$ m

$\Rightarrow P = \dfrac{\vec{F}.\vec{S}}{t} = \dfrac{\left(2\hat{i}+3\hat{j}+2\hat{k}\right).\left(3\hat{i}+4\hat{j}+5\hat{k}\right)}{4} = \dfrac{6+12+10}{4} = \dfrac{28}{4} = 7\ watt$

8. In a triangle of sides a, b, c and angles A, B, C show that a/sinA=b/sinB=c/sinC

Sol. $\angle A \rightarrow$ Angle between AB and AC; $\angle B \rightarrow$ Angle between AB and BC; $\angle C \rightarrow$ Angle between AC and CB

Area of the triangle



$$\frac{1}{2}|AB\times AC|=\frac{1}{2}|AB\times CB|=\frac{1}{2}|AC\times CB|$$

$$\Rightarrow \frac{1}{2}c\times b\times \sin A=\frac{1}{2}c\times a\times \sin B=\frac{1}{2}b\times a\times \sin C$$

Divide the above equation

by abc $\Rightarrow \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}\Rightarrow \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

9. The adjacent sides of a parallelogram are $\vec{P}=2\hat{i}-3\hat{j}+\hat{k}$ and $\vec{Q}=-2\hat{i}+4\hat{j}-\hat{k}$. What is the area of the parallelogram?

Sol. The area of the parallelogram is given by = $\left|\vec{P}\times\vec{Q}\right|$

$$\vec{P}\times\vec{Q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -3 & 1\\ -2 & 4 & -1\end{vmatrix}=\hat{i}\left|(-3\times-1)-(4\times1)\right|-\hat{j}\left|(2\times-1)-(1\times-2)\right|+\hat{k}\left|(2\times4)-(-2\times-3)\right|$$

$=\hat{i}(+3-4)-\hat{j}(-2+2)+\hat{k}(8-6)=-\hat{i}+2\hat{k}$ and

$\left|\vec{P}\times\vec{Q}\right|=\sqrt{(-1)^2+2^2}=\sqrt{1+4}=\sqrt{5}$ sq.units

7. **At what angle two vectors of magnitudes A + B and A – B act, so that their resultant is $\sqrt{3A^2+B^2}$?**

A. $R^2=P^2+Q^2+2PQ\cos\theta(or)$

$3A^2+B^2=(A+B)^2+(A-B)^2+2(A+B)(A-B)\cos\theta$

(or) $3A^2+B^2=A^2+B^2+2\bar{A}.\bar{B}+A^2-2\bar{A}.\bar{B}+2(A^2-B^2)\cos\theta$

$(A^2-B^2)=2(A^2-B^2)\cos\theta\Rightarrow\cos\theta=\frac{1}{2}$

$\theta=60^0$

ASSES YOURSELF

1. Prove that $\vec{P}.\vec{P}=P_x^2+P_y^2+P_z^2=P^2$

Ans. $\vec{P}.\vec{P}=\left(P_x\hat{i}+P_y\hat{j}+P_z\hat{k}\right).\left(P_x\hat{i}+P_y\hat{j}+P_z\hat{k}\right)=P_x^2+P_y^2+P_z^2=P^2$

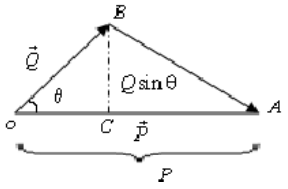
$\therefore \vec{P}.\vec{P}=P^2$

2. In the figure what happens when you lower the object?



Ans. When the object is lowered, the work is said to be done by the force of gravity and P.E of the body decreases

3. Prove that $\frac{1}{2}|P\times Q|$ is equal to the area of a triangle with the vectors as adjacent sides.

Ans. 

Area of the triangle OAB

$$=\frac{1}{2}(OA)(BC)=\frac{1}{2}(P)(Q\sin\theta)=\frac{1}{2}(PQ\sin\theta)=\frac{1}{2}\left|\vec{P}\times\vec{Q}\right|$$