QUADRATIC EQUATIONS AND EXPRESSIONS

1. A polynomial of the form $f(x)=ax^2+bx+c \ \ (a\neq 0)$ is called a quadratic expression

2. Any equation of the form $ax^2+bx+c=0 \ (a\neq 0)$ is called a quadratic equation

3. The roots of the quadratic equation $ax^2+bx+c=0$ are $\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

4. The sum of the roots of the equation $ax^2+bx+c=0$ is $-\dfrac{b}{a}$

5. The product of the roots of the equation $ax^2+bx+c=0$ is $\dfrac{c}{a}$

6. The quantity b^2-4ac is called the discriminate of the equation $ax^2+bc+c=0$ this is denoted by Δ

7. i) If $\Delta>0$ and is a perfect square then the roots of the equation are real, distinct and rational

ii) If $\Delta>0$ and is not a perfect square then the roots of the equation are real, distinct and irrational if $a+\sqrt{b}$ is one root then $a-\sqrt{b}$ is the other root

iii) If $\Delta=0$ the roots are equal and each root is equal to $-\dfrac{b}{2a}$. The expression can be written as a perfect square.

Theorem : Let a, b, $c\in R$ and $a\neq 0$. If the equation $ax^2+bx+c=0$ has imaginary roots then ax^2+bx+c and a have same sign

Proof Since the equation has imaginary roots $b^2-4ac<0$

$\therefore 4ac-b^2>0$

$$ax^2+bx+c=a\left\{x^2+\frac{b}{a}x+\frac{c}{a}\right\}$$

$$=a\left\{(x)^2+2(x)\frac{b}{2a}+\frac{b^2}{4a^2}-\frac{b^2}{4a^2}+\frac{c}{a}\right\}$$

$$=a\left\{\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a^2}\right\}$$

$$\frac{ax^2+bx+c}{a}=\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a^2}$$

Here $\left(x+\frac{b}{2a}\right)^2>0$ and $\frac{4ac-b^2}{4a^2}>0$

$$\therefore \frac{ax^2+bx+c}{a}>0$$

$\therefore ax^2+bx+c$ and a have same sign

Theorem 2 : If a, b, $c\in R$ **and** $a\neq 0$ **such that the equation** $ax^2+bx+c=0$ **has real roots** α **and** β **with** $\alpha<\beta$ **then**

i) for $\alpha<x<\beta$ ax^2+bx+c **and a have opposite signs**

ii)for $x<\alpha$ **or** $x<\beta$ ax^2+bx+c **and a have same sign**

Proof: Since α,β are the roots of $ax^2+bx+c=0$ we have

$$ax^2+bx+c=a(x-\alpha)(x-\beta)$$

$$\frac{ax^2+bx+c}{a}=(x-\alpha)(x-\beta)$$

i) When $\alpha<x<\beta$

$\alpha<x$ and $x<\beta$

$x-\alpha>0$ and $x-\beta<0$

$$\therefore (x-\alpha)(x-\beta)<0$$

$$\therefore \frac{ax^2+bx+c}{a}<0$$

$\therefore ax^2+bx+c$ and a have opposite sign

ii) when $x<\alpha$

Here $x<\alpha$ and $\alpha<\beta$

$\therefore x<\beta$

$x-\alpha<0$ and $x-\beta<0$

$(x-\alpha)(x-\beta)>0$

$\therefore \dfrac{ax^2+bx+c}{a}>0$

$\therefore ax^2+bx+c$ and a have same sign

When $x>\beta$

$x>\beta$ and $\alpha>\beta$

$x>\beta$ and $x>\alpha$

$x-\alpha>0$ and $x-\beta>0$

$(x-\alpha)(x-\beta)>0$

$\dfrac{ax^2+bx+c}{a}>0$

$\therefore ax^2+bx+c$ and a have same sign.

Theorem 3: **Suppose a, b,** $c\in R$ $a\neq 0$ and $f(x)=ax^2+bx+c$

i) If a > 0 then f (x) has absolute minimum at $x=-\dfrac{b}{2a}$ and the minimum value is $\dfrac{4ac-b^2}{4a}$

ii) If a < 0 then f(x) has absolute maximum at $x=-\dfrac{b}{2a}$

and the maximum value is $\dfrac{4ac-b^2}{4a}$

Proof : $f(x)=ax^2+bx+c$

$f'(x)=2ax+b$

For extreme values $f'(x)=0$

$\therefore 2ax+b=0\Rightarrow x=-\dfrac{b}{2a}$

$f''(x)=2a$

If a > 0 then $f''\left(-\frac{b}{2a}\right)>0$ f(x) has minimum value at $x=-\frac{b}{2a}$

Minimum value $= f\left(-\frac{b}{2a}\right)=a\left\{\frac{b^2}{4a^2}\right\}+b\left(\frac{-b}{2a}\right)+c$

$=\frac{b^2}{4a}-\frac{b^2}{2a}+c=\frac{4ac-b^2}{4a}$

If a <0 then $f''\left(-\frac{b}{2a}\right)<0$ hence f(x) has maximum value at $x=-\frac{b}{2a}$

$\therefore$ maximum value $= f\left(-\frac{b}{2a}\right)=a\left(-\frac{b}{2a}\right)^2+b\left(-\frac{b}{2a}\right)+c$

$=\frac{b^2}{4a}-\frac{b^2}{2a}+c=\frac{4ac-b^2}{4a}$

The quadratic equation with α,β as roots is $x^2-(\alpha+\beta)x+\alpha\beta=0$

I. QUADRATIC EXPRESSION AND EQUATION OF ONE VARIABLE

Very Short Answer Questions :

1. Find the roots of the following equations

i) $x^2-7x+12=0\Rightarrow(x-3)(x-4)=0\Rightarrow x=3$ *or* $x=4$

ii) $-x^2+x+2=0\Rightarrow x^2-x-2=0\Rightarrow(x-2)(x+1)=0\Rightarrow x=2$ *or* $x=-1$

iii) $2x^2+3x+2=0\Rightarrow x=\frac{-b\pm\sqrt{b^2+4ac}}{2a}$

here a = 2 b = 3 c = 2

$\therefore x=\frac{-3\pm\sqrt{9-16}}{2(2)}=\frac{-3\pm i\sqrt{7}}{4}$

iv) $\sqrt{3}x^2+10x-8\sqrt{3}=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=-\frac{-10\pm\sqrt{100-4(\sqrt{3})(-8\sqrt{3})}}{2\times\sqrt{3}}$

$$x=\frac{-10\pm14}{2\sqrt{3}}\Rightarrow x=\frac{2}{\sqrt{3}},-\frac{2}{\sqrt{3}}$$

v) $6\sqrt{5}x^2-9x-3\sqrt{5}=0$

$a=6\sqrt{5}\qquad b=-9\qquad c=-3\sqrt{5}$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{9\pm\sqrt{81+360}}{12\sqrt{5}}$$

$$x=\frac{9\pm21}{12\sqrt{5}}\Rightarrow x=\frac{30}{12\sqrt{5}}\ x=-\frac{12}{12\sqrt{5}}$$

$$\therefore x=\frac{\sqrt{5}}{2}\ ,\ -\frac{1}{\sqrt{5}}$$

2. Form quadratic equation whose roots are

i) 2, 5 **ii)** $\frac{m}{n},-\frac{n}{m}\ (m\neq0,\ n\neq0)$ **iii)** $\frac{p-q}{p+q},-\frac{p+q}{p-q}$

iv) $7\pm2\sqrt{5}$ **v)** $-3\pm5i$

Sol: **i)** $\alpha=2$ **,** $\beta=5$

Quadratic equation $x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-7x+10=0$

ii) $\alpha=\frac{m}{n}$**,** $\beta=-\frac{n}{m}$

$$\alpha+\beta=\frac{m^2-n^2}{mn},\ \alpha\beta=-1$$

Equation is $x^2-\left(\frac{m^2-n^2}{mn}\right)x-1=0$

$mnx^2-(m^2-n^2)x-mn=0$

iii) $\alpha = \frac{p-q}{p+q}$, $\beta = \frac{(p+q)}{p-q}$

$$\alpha+\beta = \frac{(p-q)^2-(p+q)^2}{(p+q)(p-q)} = \frac{-4pq}{p^2-q^2}$$

$$\alpha\beta = -1$$

Equation is $x^2 + \frac{4pqx}{p^2-q^2} - 1 = 0$

$$(p^2-q^2)x + 4pqx - (p^2-q^2) = 0$$

iv. $\alpha = 7+2\sqrt{5}$ **;** $\beta = 7-2\sqrt{5}$

$\alpha+\beta = 14$ $\quad\alpha\beta = 49-20 = 29$

Equation is $x^2 - 14x + 29 = 0$

v. $\alpha = -3+5i$ **;** $\beta = -3-5i$

$\alpha+\beta = -6$ $\quad\alpha\beta = (-3)^2-(5i)^2 = 9-25i^2 = 34$

Equation is $x^2 - (\alpha+\beta)x + \alpha\beta = 0$

s $x^2 + 6x + 34 = 0$

3. Find the nature of the roots of the following equations without finding the roots

i) $2x^2 - 8x + 3 = 0$ Here a = 2, b = -8, c = 3

$\Delta = b^2 - 4ac = (-8)^2 - 4(2)(3) = 64-24 = 40 > 0$

40 is not perfect square

$\therefore$ Roots are real distinct and irrational

ii) $9x^2 - 30x + 25 = 0$

a = 9, b = 30, c = 25

$\Delta = b^2 - 4ac = (30)^2 - 4(9)(25) = 0$

$\therefore$ Roots are real and equal

iii) $x^2-12x+32=0$

a = 2, b = -12, c = 32

$\Delta=b^2-4ac=144-128=16=4^2>0$

$\Delta>0$ and is a perfect square

$\therefore$ Roots are real distinct and rational

4. **If** α,β are the roots of the equation $ax^2+bx+c=0$ find the values of the following

i) $\frac{1}{\alpha}+\frac{1}{\beta}$ ii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ iii) $\alpha^4\beta^7+\alpha^7\beta^4$ iv) $\left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right)^2$ v) $\frac{\alpha^2+\beta^2}{\alpha^{-2}+\beta^{-2}}$

Sol: Since α,β are roots of $ax^2+bx+c=0$

$\alpha+\beta=-\frac{b}{a}$; $\alpha\beta=\frac{c}{a}$

i) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{\left(-\frac{b}{a}\right)}{\left(\frac{c}{a}\right)}=-\frac{b}{c}$

ii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}=\frac{b^2-2ac}{\frac{a^2}{\frac{c^2}{a^2}}}=\frac{b^2-2ac}{c^2}$

iii) $\alpha^4\beta^7+\alpha^7\beta^4=\alpha^4\beta^4\{\alpha^3+\beta^3\}$

$=\alpha^4\beta^4\{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)\}$

$=\left(\frac{c}{a}\right)^4\left\{-\frac{b^3}{a^3}+\frac{3bc}{a^2}\right\}=\frac{(3abc-b^3)c^4}{a^7}$

iv) $\frac{\alpha^2+\beta^2}{\alpha^{-2}+\beta^{-2}}=\frac{\alpha^2+\beta^2}{\frac{1}{\alpha^2}+\frac{1}{\beta^2}}=\frac{(\alpha^2+\beta^2)\alpha^2\beta^2}{(\alpha^2+\beta^2)}=(\alpha\beta)^2=\frac{c^2}{a^2}$

5. **If the equation** $x^2-15-m(2x-8)=0$ **has equal roots, find the values of m.**

Sol: Given equation is $x^2-2mx+(8m-15)=0$

This has equal roots

$\therefore b^2-4ac=0$

$4m^2-4(8m-15)=0\Rightarrow m^2-8m+15=0$, m = 3 or 5

6. For what values of m will the equation $(m+1)x^2+2(m+3)x+(m+8)=0$ has equal roots

Sol: Given equation is $(m+1)x^2+2(m+3)x+(m+8)=0$

This has equal roots

$\therefore b^2-4ac=0$

$4(m+3)^2-4(m+8)(m+1)=0$

$m^2+6m+9-m^2-9m-8=0$

$-3m+1=0\Rightarrow m=\frac{1}{3}$

7. If α and β are the roots of the equation $x^2+px+q=0$ form a quadratic equation whose roots are $(\alpha-\beta)^2$ and $(\alpha+\beta)^2$

Sol: Given that α,β are the roots of the equation $x^2+px+q=0$

$\therefore \alpha+\beta=-p$, $\alpha\beta=q$

Let $P=(\alpha-\beta)^2$, $Q=(\alpha+\beta)^2$ be the roots of new equation

$\therefore P=(\alpha+\beta)^2-4\alpha\beta=p^2-4q$, $Q=p^2$

$P+Q=2p^2-4q$; $PQ=P^2(P^2-4q)$

$\therefore$ Equation is $x^2-(2p^2-4q)x-p^2(p^2-4q)=0$

8. Prove that the roots of $(x-a)(x-b)=h^2$ are always real

Sol: $(x-a)(x-b)=h^2\Rightarrow x^2-(a+b)x+ab-h^2=0$

Discriminate = b^2-4ac

$=(a+b)^2-4(1)\{ab-h^2\}$

$=(a+b)^2-4ab+4h^2=(a-b)^2+(2h)^2\geq 0$

$\because (a-b)^2 \geq 0$ and $(2h)^2 \geq 0$

$\therefore \Delta \geq 0$ Hence roots are always real

9. Find the condition that are roots of the quadratic equation $ax^2+bx+c=0$ shall be n times the other, where n is a positive integer

Sol: Let $\alpha, n\alpha$ be the roots of $ax^2+bx+c=0$

$$\therefore \alpha+n\alpha=-\frac{b}{a} \Rightarrow \alpha=-\frac{b}{a(n+1)}$$

And $(\alpha)(n\alpha)=\frac{c}{a} \Rightarrow n\alpha^2=\frac{c}{a}$

$$\therefore n\left\{\frac{b^2}{a(n+1)^2}\right\}=\frac{c}{a} \Rightarrow nb^2=(n+1)^2 ac$$

10. Find two consecutive positive even integer the sum of whose squares is 340

Sol: Let x, x + 2 be two consecutive even integers given that $x^2+(x+2)^2=340$

$2x^2+4x+4-340=0 \Rightarrow 2x^2+4x-336=0$

$x^2+2x-168=0 \Rightarrow (x+14)(x-12)=0$

$\therefore x=-14$, x = 12

$\therefore$ we want even positive integer

$\therefore$ even integers are 12, 14

SHORT ANSWER QUESTIONS:

1. If x_1, x_2 are the roots of the quadratic equation $ax^2+bx+c=0$ and $c \neq 0$ find the value of $(ax_1+b)^{-2}+(ax_2+b)^{-2}$ in terms of a, b, c

Sol: Since x_1, x_2 are the roots of the equation $ax^2+bx+c=0$ $x_1+x_2=-\frac{b}{a}$, $x_1x_2=\frac{c}{a}$

$$ax_1^2+bx_1+c=0 \Rightarrow ax_1+b=-\frac{c}{x_1} \; ; \; ax_2^2+bx_2+c=0 \Rightarrow ax_2+b=-\frac{c}{x_2}$$

$$(ax_1+b)^{-2}+(ax_2+b)^{-2}=\left(-\frac{c}{x_1}\right)^{-2}=\frac{x_1^2+x_2^2}{c^2}$$

$$\therefore \frac{(x_1+x_2)^2-2x_1x_2}{c^2}=\frac{b^2-2ac}{a^2c^2}$$

2. **If α,β are the roots of the equation $ax^2+bx+c=0$ find a quadratic equation whose roots are $\alpha^2+\beta^2$ and $\alpha^{-2}+\beta^{-2}$**

Sol: Since α,β are the roots of the equation $ax^2+bx+c=0$, $\alpha+\beta=-\frac{b}{a}$, $\alpha\beta=\frac{c}{a}$

Let $P=\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\frac{b^2-2ac}{c^2}$

$$Q=\alpha^{-2}+\beta^{-2}=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}=\frac{b^2-2ac}{c^2}$$

$\therefore$ Equation is $x^2-(P+Q)x+PQ=0$

$$\therefore x^2-\frac{(b^2-2ac)}{a^2c^2}(a^2+c^2)x+\frac{(b^2-2ac)^2}{a^2c^2}=0$$

$$\therefore a^2c^2x^2-(b^2-2ac)(a^2+c^2)x+(b^2-2ac)^2=0$$

3. **Solve the equation $2x^4+x^3-11x^2+x+2=0$**

Sol: Given equation is $2x^4+x^3-11x^2+x+2=0$ dividing both sides with x^2

$$\therefore 2x^2+x-11+\frac{1}{x}+\frac{2}{x^2}=0$$

$$2\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)-11=0$$

Let $x+\frac{1}{x}=a \Rightarrow \left(x+\frac{1}{x}\right)^2=a^2$

$$\therefore x^2+\frac{1}{x^2}=a^2-2$$

$$\therefore 2\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)-11=0$$

$$\Rightarrow 2(a^2-2)+a-11=0$$

$$2a^2+a-15=0$$

$2a^2+6a-5a-15=0$

a = - 3 or a = 5/2

ii) Suppose $a=\frac{5}{2}$

$x+\frac{1}{x}=\frac{5}{2}\Rightarrow 2x^2-5x+2=0$

x = 2 or $x=\frac{1}{2}$

4. **Solve** $3^{1+x}+3^{1-x}=10$

Sol: $3^{1+x}+3^{1-x}=10\Rightarrow(3^1)(3^x)+\frac{3^1}{3^x}=10$

Let $3^x=a\Rightarrow 3a+\frac{3}{a}=10$

$3a^2-10a+3=0\Rightarrow 3a^2-9a-a+3=0$

$(3a-1)(a-3)=0\Rightarrow a=\frac{1}{3}\ ;a=3$

i) $a=\frac{1}{3}$

$3^x=a\Rightarrow 3^x=3^{-1}\Rightarrow x=-1$

ii) a = 3

$3^x=a\Rightarrow 3^x=3^1\Rightarrow x=1$

5. $4^{x-1}-3\ 2^{x-1}+2=0\ \ (2^{x-1})^2-3.2^{x-1}+2=0$ let $2^{x-1}=a\Rightarrow a^2-3a+2=0$ (a – 1)(a – 2)= 0
a = 1, a = 2

Sol: i) If a = 1

$a=2^{x-1}$

$\therefore 2^{x-1}=1\Rightarrow 2^{x-1}=2^0$

$x-1=0\Rightarrow x=1$

ii) if a = 2

$a = 2^{x-1}$

$2 = 2^{x-1} \Rightarrow x - 1 = 1$ Hence x = 2

6. $\sqrt{\frac{x}{x-3}} + \sqrt{\frac{x-3}{x}} = \frac{5}{2}$

Let $\sqrt{\frac{x}{x-3}} = a$

$\therefore a + \frac{1}{a} = \frac{5}{2}$

$2a^2 - 5a + 2 = 0$

$2a^2 - 4a - a + 2 = 0$

$(2a-1)(a-2) = 0$

$a = \frac{1}{2}$, a = 2

i) Suppose $a = \frac{1}{2}$

$\sqrt{\frac{x}{x-3}} = \frac{1}{2}$

$\frac{x}{x-3} = \frac{1}{4}$ {By squaring on both sides}

4x = x – 3

$3x = -3 \Rightarrow x = -1$

ii) Suppose a = 2

$\sqrt{\frac{x}{x-3}} = 2$

$\frac{x}{x-3} = 4 \Rightarrow x = 4x - 12$

$3x = 12 \Rightarrow x = 4$

7. $\sqrt{\frac{3x}{x+1}}+\sqrt{\frac{x+1}{3x}}=2$

Let $\sqrt{\frac{3x}{x+1}}=a$

$a+\frac{1}{a}=2$

$a^2-2a+1=0$

$(a-1)^2\ 0\Rightarrow a=1$

$\sqrt{\frac{3x}{x+1}}=1\Rightarrow\frac{3x}{x+1}=1$

$3x=x+1\Rightarrow 2x1$

$x=\frac{1}{2}$

8. $2\left(x+\frac{1}{x}\right)^2-7\left(x+\frac{1}{x}\right)+5=0$

Let $x+\frac{1}{x}=a$

$2a^2-7a+5=0$

$2a^2-2a-5a+5=0$

$2a(a-1)-5(a-1)=0$

(2a – 5) (a – 1) = 0

ii) Suppose $a=\frac{5}{2}$

$x+\frac{1}{x}=\frac{5}{2}\Rightarrow 2x^2+2=5x$

$2x^2-5x+2=0$

$2x^2-4x-x+2=0$

$2x(x-2)-(x-2)=0$

$(2x-1)(x-2)=0 \Rightarrow x=2 or \frac{1}{2}$

9. $\left(x^2+\frac{1}{x^2}\right)-5\left(x+\frac{1}{x}\right)+6=0$ let $x+\frac{1}{x}=a$

$x^2+\frac{1}{x^2}=a^2-2\ a^2-2-5a+6=0 \Rightarrow a^2-5a+4=0$

$(a-1)(a-4)=0 \Rightarrow a=1\ ; a=4$

Case i) a = 1

$x+\frac{1}{x}=1 \Rightarrow x^2-x+1=0$

$x=\frac{1\pm\sqrt{1-4}}{2}=\frac{1\pm i\sqrt{3}}{2}$

Case (ii) Suppose a = 4

$\therefore x+\frac{1}{x}=4 \Rightarrow x^2-4x+1=0$

$x=\frac{4\pm\sqrt{16-4}}{2}=2\pm\sqrt{3}$

10. **(x – 1) (x – 3) (x – 5) (x – 7) = 9**

Sol: (x – 1) (x – 3) (x – 5) (x – 7) = 9

(x -1) (x – 7)(x – 3) (x – 5) = 9

$\{x^2-8x+7\}\{x^2-8x+15\}=9$

Let $x^2-8x=a$

$\therefore (a+7)(a+15)=9$

$a^2+22a+96=0$

(a + 16) (a + 6) = 0

a = - 16; a = -6

Suppose a = - 16

$x^2-8x=-16\Rightarrow x^2-8x+16=0$

$x=\frac{8\pm\sqrt{64-24}}{2}$

$x=\frac{8\pm2\sqrt{10}}{2}=4\pm\sqrt{10}$

11. $\sqrt{3x+1}-\sqrt{x-1}=2$ Squaring on both sides

$\therefore 3x+1+x-1+2\sqrt{(3x+1)(x-1)}=4$

$2\sqrt{(3x+1)(x-1)}-4(1-x)$

Squaring on both sides

$3x^2-2x-1=4(1+x^2-2x)$

$x^2-6x+5=0$

(x – 1) (x – 5) = 0

X = 1; x = 5

For x = 1

$\sqrt{3x+1}-\sqrt{x-1}$

$=\sqrt{3x+1}-\sqrt{1-1}=2$

For x = 5

$\sqrt{3x+1}-\sqrt{x-1}$

$=\sqrt{3\times5+1}-\sqrt{5-1}=4-2=2$

$\therefore x=1,\ 5$

12. $\sqrt{2x+1}+\sqrt{3x+2}=\sqrt{5x+3}$

Sol: Squaring on both sides

$2x+1+3x+2+2\sqrt{3x+2}\ \sqrt{2x+1}=5x+3$

$2\sqrt{(3x+2)(2x+1)}=0$

(3x + 2) (2x +1) = 0

$x=-\frac{2}{3}$; $x=-\frac{1}{2}$

If $x=-\frac{2}{3}$

L.H.S. of the equation

$=\sqrt{2x-\frac{2}{3}+1}+\sqrt{3x-\frac{2}{3}+2}$

$=\sqrt{-\frac{1}{3}}+0\neq R.H.S$

If $x=-\frac{1}{2}$

L.H.S of the equation

$\sqrt{2x-\frac{1}{2}+1}+\sqrt{-\frac{3}{2}+2}=0+\sqrt{\frac{1}{2}}$

R.H.S of the equation $=\sqrt{3x+3}$

$=\sqrt{-\frac{5}{2}+3}=\sqrt{\frac{1}{2}}$

$\therefore x=-\frac{1}{2}$ Is only the solution

13. Find a quadratic equation for which the sum of the roots is 7 and the sum of the squares of the roots is 25.

Sol: Let α,β be the roots of the equation

Given that $\alpha+\beta=7$

$\alpha^2+\beta^2=25$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$25=9-2\alpha\beta\Rightarrow 2\alpha\beta=24$

$\alpha\beta=12$ $\therefore$ Equation is $x^2-(\alpha+\beta)x+\alpha\beta=0$, $\therefore x^2-7x+12=0$