VECTORS
1. ADDITION-SUBTRACTION

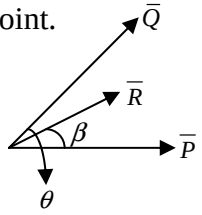
POINTS TO REMEMBER

1. A quantity which has only magnitude but no direction is called a scalar quantity.
   Ex: Mass, distance, Electric flux.
2. A quantity which has both magnitude and direction and which obeys the laws of addition is called vector quantity
   Ex: displacement, velocity
3. A vector is represented geometrically by an arrow making an angle with a reference line. Length of the arrow is proportional to the magnitude, angle specifies the direction.
4. Vectors having same magnitude and which have same direction are called equal vectors. Their corresponding components are equal.
5. A vector which has the same magnitude as that of another and which is opposite in direction is called a negative vector
6. A vector whose magnitude is zero and which has vno specific direction is called a null vector.
7. A vector which has unit magnitude is called a unit vector. $\hat{A} = \frac{\bar{A}}{|\bar{A}|}$ $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along X,Y and Z directions respectively.
8. **Triangle law:** If two vectors are represented in magnitude and direction by the two sides of a triangle taken in order, then the third side taken in the reverse order represents their sum or resultant in magnitude as well as in direction.

   Parallelogram law: If two vectors $\vec{P}$ and $\vec{Q}$ are represented by the two sides of a parallelogram drawn from a point, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.

   If two vectors $\bar{P}$ and $\bar{Q}$ are at an angle θ with each other and if the resultant makes an angle β with the vector $\bar{P}$ then

   $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ and $Tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta}$

   Polygon law : If a number of vectors are represented by the sides of a polygon taken in the same order, the resultant is represented by the closing side of the polygon taken in the reverse order.
9. Vector addition follows commutative, associative and distributive laws.
10. If two vectors $\bar{P}$ and $\bar{Q}$ have equal magnitudes x, then $R = 2x\cos\frac{\theta}{2}$.
11. Subtraction of two vectors is the addition of one vector and the opposite vector of the other $\bar{P} - \bar{Q} = \bar{P} + (-\bar{Q})$
12. Consider a vector $\bar{A}$ represented along $\overrightarrow{OA}$ in two co-ordinate system, which makes an angle θ with positive X-axis. The X and Y components of this vector are given by
    $A_x = |\bar{A}|Cos\theta$ and $A_y = |\bar{A}|Sin\theta$
13. If the vector makes an angle α with X-axis, β With Y axis and γ with Z-axis. Then
    $A = A_x i + A_y j + A_z k$ and $|\bar{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$
    $Cos\alpha^2 + Cos^2\beta + Cos^2\gamma = 1$ (Law of cosines)

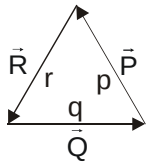
14. **Equilibrium**: It is the state of a body in which there is no acceleration i.e., net force acting on a body is zero.

15. The forces whose lines of action pass through a common point (called the point of concurrence) are called concurrent forces.

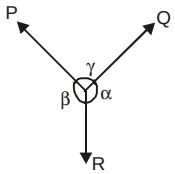
16. Resultant force is the single force which produces the same effect as a given system of forces acting simultaneously.

17. A force which when acting along with a given system of forces produces equilibrium is called the equilibrant.

18. **Triangle law of forces:** If a body is in equilibrium under the action of three coplanar forces, then these forces can be represented in magnitude as well as direction by the three sides of a triangle taken in order. $\frac{\vec{P}}{p}=\frac{\vec{Q}}{q}=\frac{\vec{R}}{r}$ Where p, q, r are sides of a triangle. $\vec{P}$, $\vec{Q}$, $\vec{R}$ are coplanar vectors.



19. **Lami's theorem:** When three coplanar forces $\vec{P}$, $\vec{Q}$ and $\vec{R}$ keep a body in equilibrium, then $\frac{P}{\sin\alpha}=\frac{Q}{\sin\beta}=\frac{R}{\sin\gamma}$.

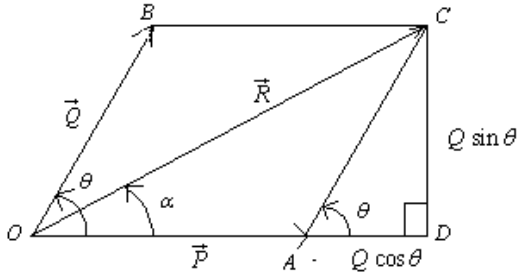


SHORT ANSWER QUESTIONS

1. State parallelogram law of vectors. Derive an expression for the magnitude and direction of the resultant vector.
(2010June, 2010March, 2009March, 2008March, 2006June, 2005- March, 2004March, 2003June, 2003-March, 2002-June)

Ans. **Statement:** If two vectors are represented both in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then their resultant is represented in magnitude and direction by the diagonal passing through the same point.

Expression:



Consider two Vectors $\vec{P}$ and $\vec{Q}$ are drawn from 'O' as a common initial point. The angle between them is θ. A parallelogram OACB is constructed using these vectors as adjacent sides. $\overrightarrow{OA}$ represents the vector $\vec{P}$ and $\overrightarrow{OB}$ represents vector $\vec{Q}$. The diagonal $\overrightarrow{OC}$ drawn from the point 'O' represents the resultant $\vec{R}$ both in magnitude and direction.

i.e., $\vec{R}=\left(\vec{P}+\vec{Q}\right)$

Extend the OA and draw a perpendicular from C on to OA extended..

From the parallelogram OACB,

$\angle BOA=\angle CAD=\theta$; $OA=BC=\left|\vec{P}\right|$

OB=AC=Q and OC=$\left|\vec{R}\right|$

In the right angled triangle ODC,

$OC^2=OD^2+DC^2$

$R^2 = (OA + AD)^2 + DC^2$

$R^2 = OA^2 + AD^2 + 2OA.AD + DC^2 -----(1)$

Now, in the right angle triangle ADC

$AD = AC\cos\theta = Q\cos\theta$ and $DC = AC\sin\theta = Q\sin\theta -----(2)$

Hence $AD^2 + DC^2 = Q^2\cos^2\theta + Q^2\sin^2\theta = Q^2 ----(3)$

Using . (1), (2) and (3), $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

Direction of the resultant:

If the resultant $\vec{R}$ makes an angle α with it

$$\tan\alpha = \frac{DC}{OD} = \frac{DC}{OA + AD}$$

$OA = P, AD = Q\cos\theta, DC = Q\sin\theta$

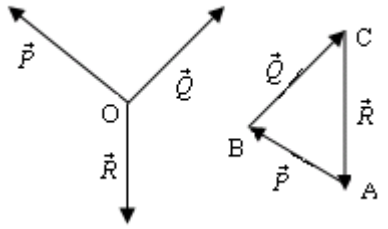
$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} \Rightarrow \alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$$

2. State triangle law of vectors and explain it. (2001-March)

Ans. If two vectors are represented both in magnitude and direction by the two sides of a triangle taken in order, the resultatant is represented in both magnitude and direction by the third side of the triangle taken in the reverse order.

OR

If three vectors (forces) acting simultaneously at a point keep it in equilibrium, then they can be represented in magnitude and direction by three sides of a triangle taken in order.



Let the three vectors (forces) $\vec{P}, \vec{Q}$ and $\vec{R}$ act simultaneously at a point O. The sides AB,BC and CA of triangle ABC represent these vectors in magnitude and direction as shown in above figure. According to the law of vector addition, the resultant of vectors $\vec{P}$ and $\vec{Q}$ represented by $\overrightarrow{AB}$ and $\overrightarrow{BC}$ respectively, is obtained by joining the initial point A of vector $\vec{P}$ to the terminal point C of vector $\vec{Q}$, then the vector $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \vec{P} + \vec{Q}$, This is equal in magnitude but opposite in direction to vector $\vec{R} = \overrightarrow{CA}$

Hence $\vec{P} + \vec{Q} + \vec{R} = \vec{O}$ i.e the three vectors $\vec{P}, \vec{Q}$ *and* $\vec{R}$ keep the point 'O' in equilibrium.

The ratio of the vectors and their corresponding sides are equal.

$\frac{\vec{P}}{p} = \frac{\vec{Q}}{q} = \frac{\vec{R}}{r}$ Where p, q, r are sides of a triangle. $\vec{P}$, $\vec{Q}$, $\vec{R}$ are coplanar vectors.

3. Define unit vector, null vector and position vector. (2008-March)

Ans. **Unit Vector:** A vector whose magnitude equal unit is called a unit vector. A unit vector has no units and dimensions.

If $\vec{A}$ is vector then the unit vector along its direction is given by $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

In Cartesian co-ordinate system, unit vectors along positive X, Y and Z axes are represented by $\hat{i}, \hat{j}$ and $\hat{k}$ respectively and $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$

Null Vector (or) Zero Vectors: $\rightarrow$ A vector whose magnitude is zero and which has no specific direction is called a null vector. e.g. 1) the cross product of two parallel vectors is a null vector. 2) The acceleration of a particle moving at constant (uniform) velocity.

Position Vector: The position of a particle is described by a position vector which is drawn from the origin of a reference frame. The position vector helps to locate the particle in space.

The position of a particle P is indicated as $\overrightarrow{OP}(=\vec{r})$ by a position vector in X, Y and Z co-ordinate system is given by $\overrightarrow{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

its magnitude $|\overrightarrow{OP}| = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

The unit vector along '$\vec{r}$' is and given by $\hat{r} = \dfrac{\vec{r}}{|\vec{r}|} = \dfrac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$

4. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, prove that the angle between $\vec{a}$ and $\vec{b}$ is 90^0.

Ans. Let θ be the angle between $\vec{a}$ and $\vec{b}$

$|\vec{a} + \vec{b}| = \sqrt{a^2 + b^2 + 2ab\cos\theta}$ and $|\vec{a} - \vec{b}| = \sqrt{a^2 + b^2 - 2ab\cos\theta}$

Given that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}| \Rightarrow \sqrt{a^2 + b^2 + 2ab\cos\theta} = \sqrt{a^2 + b^2 - 2ab\cos\theta}$

$\Rightarrow a^2 + b^2 + 2ab\cos\theta = a^2 + b^2 - 2ab\cos\theta \Rightarrow 4ab\cos\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^0$

Angle between the two vectors $\theta = 90^0$

VERY SHORT ANSWER QUESTIONS

1. The vertical component of a vector is equal to its horizontal component. What is the angle made by the vector with x-axis?

Ans. Let θ be the angle made by a vector $\vec{A}$ with x-axis. Then

X-component of $\vec{A} = A_x = A\cos\theta$ and Y-component of $\vec{A} = A_y = A\sin\theta$

But, $A_x = A_y \Rightarrow A\cos\theta = A\sin\theta \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45^0$

2. A vector $\vec{V}$ makes an angle with the horizontal. The vector is rotated through an angle. Does this rotation change the vector $\vec{V}$?

Ans. When a vector is rotated through certain angle its components may change, but the magnitude of the vector does not change.

3. Two forces of magnitudes 3 units and 5 units act at 60^0 with each other. What is the magnitude of their resultant?

Ans. Magnitude of the resultant = $\sqrt{(3)^2 + (5)^2 + 2\times 3\times 5\cos 60^0}$

$$= \sqrt{9+25+2\times3\times5\left(\frac{1}{2}\right)} = \sqrt{49} = 7\,units$$

4. $\vec{A} = \vec{i} - \vec{j}$ **What is the angle between the vector and x-axis?**

Ans. Let 'θ' be angle made by the vector with x-axis $\Rightarrow \cos\theta = \frac{A_x}{|\vec{A}|} = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^0$, in the fourth quadrant.

5. **When two right angled vectors of magnitude 7 units and 24 units combine, what is the magnitude of their resultant?**

Ans. The angle between the two vectors is 90^0

Magnitude of their resultant $= \sqrt{(7)^2 + (24)^2 + 2\times7\times24\cos90^0}$

$$= \sqrt{49+576} = \sqrt{625} = 25\,units$$

6. **If** $\vec{P} = 2\hat{i} + 4\hat{j} + 14\hat{k}$ **and** $\vec{Q} = 4\hat{i} + 2\hat{j} + 10\hat{k}$ **find the magnitude of** $\vec{P} + \vec{Q}$.

Ans. $\vec{P} + \vec{Q} = \left(2\hat{i} + 4\hat{j} + 14\hat{k}\right) + \left(4\hat{i} + 2\hat{j} + 10\hat{k}\right) = 6\hat{i} + 6\hat{j} + 24\hat{k}$

$$\left|\vec{P} + \vec{Q}\right| = \sqrt{36+36+576} = \sqrt{648}\,units$$

7. **A force** $2\hat{i} + \hat{j} - \hat{k}$ **Newton acts on a body which is initially at rest. At the end of 20 seconds the velocity of the body is** $4\hat{i} + 2\hat{j} + 2\hat{k}\,ms^{-1}$. **What is the mass of the body?**

Ans. Force acting on the body, $\vec{F} = m\vec{a} = m\left(\frac{\vec{v} - \vec{u}}{t}\right)$

Initial velocity $\vec{u} = 0$

Final velocity $\vec{v} = 4\hat{i} + 2\hat{j} + 2\hat{k}$

Time t = 20 s

$$\vec{a} = \frac{\vec{v} - \vec{u}}{t} = \frac{4\hat{i} + 2\hat{j} + 2\hat{k}}{20} = \frac{\sqrt{6}}{10}\,m/\sec^2$$

Force $\vec{F} = 2\hat{i} + \hat{j} - \hat{k}\,N$

$$\left|\vec{F}\right| = \sqrt{4+1+1} = \sqrt{6}N$$

F = ma $\Rightarrow \sqrt{6} = m\times\frac{\sqrt{6}}{10} \Rightarrow$ Mass of the body m = 10kg

8. **Can a vector of magnitude zero have non-zero components?**

Ans. No. If the components of a vector are non-zero components, then its magnitude will never be equal to zero.

9. **What are the units of a unit vector?**

Ans. A unit vector can not have units and dimensions.

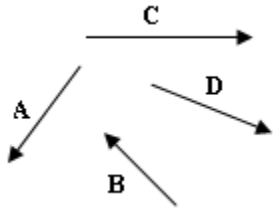
10. **Can two vectors of unequal magnitude add up to give the zero vectors? Can three unequal vectors add up to give zero vectors?**

Ans. 1) No. Two vectors of unequal magnitude cannot add up to give zero vectors.
2) Yes. Three vectors of unequal magnitude can add up to give zero vectors.

11. During practice a player throws a ball high in to the air and then runs in a straight line and catches it. Which had the greater displacement, the player or the ball?

Ans. Both, the ball and the player will have equal horizontal displacements.

12. In the figure there are four forces that are equal in magnitude. Identify any three of them which act on a body simultaneously and keep it with unchanged velocity.



Ans. If the resultant of the three forces acting on the body is equal to zero, then the velocity of the body will remain constant. Hence the three forces A, B and C produce zero resultant.

13. Does vector subtraction obey commutative law?

Ans. No, vector subtraction does not obey commutative law. $\vec{A}-\vec{B}\neq\vec{B}-\vec{A}$

14. $\vec{A}=\vec{i}+\vec{j}$ **What is the angle between the vector and x-axis? (2007-May)**

Ans. Let 'θ' be angle made by the vector with x-axis $\Rightarrow \cos\theta=\frac{A_x}{|\vec{A}|}=\frac{1}{\sqrt{2}}\Rightarrow\theta=45^0$, in the first quadrant.

15. A force $20\hat{i}+10\hat{j}+5\hat{k}$ Newton acts on a body of mass 5Kg. What is the acceleration produced? (2008-May)

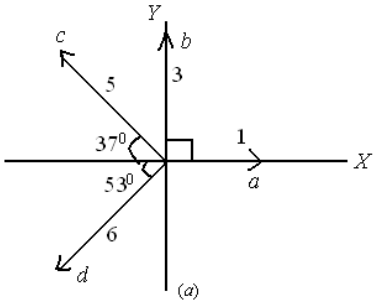
Ans. $a=\frac{|\vec{F}|}{m}=\frac{\sqrt{400+100+25}}{5}=\frac{\sqrt{525}}{5}=\sqrt{21}\ m/s^2$

16. **If the resultant of two unit vectors $\vec{a}$ and $\vec{b}$ is also a unit vector, find the angle between $\vec{a}$ and $\vec{b}$.(March 2011)**

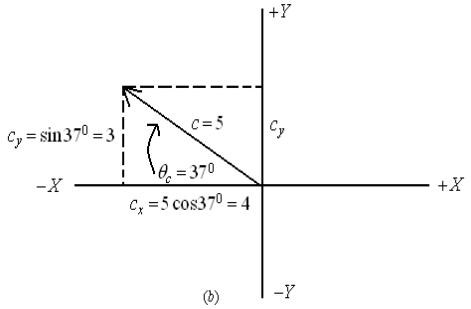
Ans: $R^2=P^2+Q^2+2PQ\,Cos\theta\Rightarrow 1=1+1+2\cos\theta\Rightarrow\theta=120^o$

Exercise 1

1. Find the resultant of the vectors shown in figure by the component method.

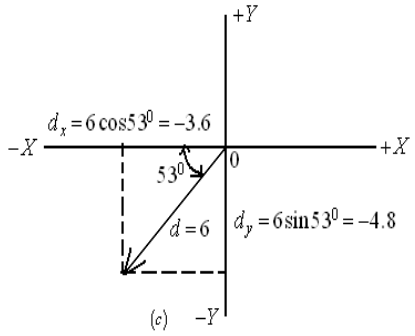


Sol. First we find the components of all the vectors along 'x' and 'y' axes. Then add all the 'x' components of the vectors to find 'x' component 'R_X of the resultant, and add all the 'y' components of the vectors to find 'y' component R_Y of the resultant. Then using Pythagorean theorem we can find the resultant. See fig a,b,c,d.



(a) Vector '$\vec{a}$' is directed along +x-axis. Hence its x component a_X=+1.0. It has no 'y' component i.e., a_Y=0.

Vector '$\vec{b}$' is directed along +y-axis. Its y component b_Y=+3.0 and 'x' component b_X=0.



x component of '$\vec{c}$' is $c_x = c\cos\theta_c = 5\cos 37^0 = 4\, along - ve\, X - direction.$

y component of $\vec{c}$ is $c_y = c\sin\theta_c = 5\sin 37^0 = 3$ *along* $+ve$ *y direction*

Similarly x component of $\vec{d}$ is $d_x = d\cos\theta_d = 6\cos 53^0 = 3.6\, along - ve\, x\, direction$

y component of $\vec{d}$ is $d_y = d\sin\theta_d = 6\sin 53^0 = 4.8\, along - ve\, Y - direction.$ Now,

Vector	x component	y component
$\vec{a}$	$a_X = +1.0$	$a_Y = 0$
$\vec{b}$	$b_x = 0$	$b_y = +3.0$
$\vec{c}$	$c_x = -4.0$	$c_y = +3.0$
$\vec{d}$	$d_x = -3.6$	$d_y = -4.8$
Resultant	$R_x = -6.6$	$R_y = +1.2$

R_x And R_y are perpendicular.

$\therefore R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-6.6)^2 + (1.2)^2} = 6.7$

$Tan\,\alpha = \dfrac{R_y}{R_x} = \dfrac{1.2}{-6.6} = -0.18$; $\alpha = \tan^{-1}(-0.18) = 10.3^0$ With negative X-axis in clock wise direction or 169.7º with positive x-axis in anti clock wise direction

2. Two vectors $\vec{a}$ and $\vec{b}$ have equal magnitudes of 12.7 units. These vectors are making angles 28.2º and 133.2º with the x axis respectively. Their sum is r. Find (a) the x and y components of r (b) the magnitude of r (c) the angle 'r' makes with the +x axis

Sol. The components of the vectors $\vec{a}$ and $\vec{b}$ are

Then $a_x = a\cos 28.2^0 = 12.7\cos 28.2^0 = 11.2$

$a_y = a\sin 28.2^0 = 12.7\sin 28.2^0 = 6.0$

$b_x = b\cos 133.2^0 = 12.7\cos 133.2^0 = -8.7$

$b_y = b\sin 133.2^0 = 12.7\sin 133.2^0 = 9.3$

The x component of the resultant $\vec{r}$ is $r_x = a_x + b_x = 11.2 + (-8.7) = 2.5$

The y component of the resultant $\vec{r}$ is $r_y = a_y + b_y = 6 + 9.3 = 15.3$

Then the magnitude of $\vec{r}$ is $\sqrt{r_x^2 + r_y^2} = \sqrt{(2.5)^2 + (15.3)^2} = 15.5$

The angle $\vec{r}$ makes with +x axis is given by $\tan\alpha = \dfrac{r_y}{r_x} = \dfrac{15.3}{2.5} = 6.12$

$\therefore \alpha = \tan^{-1}(6.12) = 80.7^0$

3. A vector has x component of -25.0 units and y component of 40.0 units. Find the magnitude and direction of the vector.

Sol. Let $\vec{A} = A_x\hat{i} + A_y\hat{j}$.

$A_x = -25.0$ and $A_y = 40.0$.

$\therefore A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25)^2 + (40.0)^2} = 47.16 = 47.2$ Units.

And $\tan\alpha = \dfrac{A_y}{A_x} = \dfrac{40.0}{-25.0} = -1.6$

$\therefore \alpha = \tan^{-1}(-1.6) = -58.0^0$ With negative x-axis. (This is in clock wise direction.)

This is equivalent to $(180^0 - 58.0^0)$ in anticlockwise direction which is considered to be +ve direction in measuring the angles.

Hence magnitude of the vector is 47.2 units and it makes 122^0 with + x-axis.

4. If vectors $\vec{A}$ and $\vec{B}$ are $3\hat{i} - 4\hat{j} + 5\hat{k}$ and $2\hat{i} + 3\hat{j} - 4\hat{k}$ respectively, find the unit vector parallel to $\vec{A} + \vec{B}$.

Sol. $\vec{A} + \vec{B} = (3\hat{i} - 4\hat{j} + 5\hat{k}) + (2\hat{i} + 3\hat{j} - 4\hat{k}) = 5\hat{i} - \hat{j} + \hat{k}$

$\therefore |\vec{A} + \vec{B}| = \sqrt{5^2 + (-1)^2 + (1)^2} = \sqrt{27}$

Then the unit vector of $\vec{A} + \vec{B}$ which is parallel to $\vec{A} + \vec{B}$ is

$$\frac{\vec{A} + \vec{B}}{|\vec{A} + \vec{B}|} = \frac{(5\hat{i} - \hat{j} + \hat{k})}{\sqrt{27}}$$

5. If magnitude of the resultant of two vectors equal in magnitude, is equal to the magnitude of either of the vectors, what is the angle between them?

Sol. If $\vec{P}, \vec{Q}$ are the vectors and their resultant is $\vec{R}$, then $|\vec{P}| = |\vec{Q}| = |\vec{R}|$. If θ is the angle between $\vec{P}$ and $\vec{Q}$, then

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

Or $P = \sqrt{P^2 + P^2 + 2P^2\cos\theta} = \sqrt{2P^2 + 2P^2\cos\theta} = \sqrt{2P^2(1 + \cos\theta)}$

But $1 + \cos\theta = 2\cos^2\dfrac{\theta}{2}$

Hence $P = \sqrt{2P^2.2\cos^2\dfrac{\theta}{2}} = \sqrt{4P^2\cos^2\dfrac{\theta}{2}}$

$P = 2P\cos\dfrac{\theta}{2} \Rightarrow \dfrac{1}{2} = \cos\dfrac{\theta}{2}$.

$\therefore \theta = 120^0$

6. Two vectors $\vec{A}$ and $\vec{B}$ have precisely equal magnitudes. For the magnitude of $\vec{A} + \vec{B}$ to be larger than the magnitude of $\vec{A} - \vec{B}$ by a factor n, what must be the angle between them?

Sol. $|\vec{A} + \vec{B}| = n|\vec{A} - \vec{B}|$ and $|\vec{A}| = |\vec{B}|$

If θ is the angle between the vectors $\vec{A}$ *and* $\vec{B}$

$$A^2 + B^2 + 2AB\cos\theta = n^2\left[A^2 + B^2 - 2AB\cos\theta\right]$$

$$2A^2 + 2A^2\cos\theta = n^2\left[2A^2 - 2A^2\cos\theta\right]$$

$$\left[1+\cos\theta\right] = n^2\left[1-\cos\theta\right]$$

$$2\cos^2\left(\frac{\theta}{2}\right) = n^2 2\sin^2\left(\frac{\theta}{2}\right) \Rightarrow \tan\frac{\theta}{2} = \frac{1}{n} \Rightarrow \theta = 2\tan^{-1}\left(\frac{1}{n}\right)$$

Exercise 2

1. If $\vec{A} = 3\hat{i} - 4\hat{j}$ and $\vec{B} = -\hat{i} - 4\hat{j}$ calculate (a) $\vec{A}+\vec{B}$ (b) $\vec{A}-\vec{B}$ (c) $\left|\vec{A}+\vec{B}\right|$ (d) $\left|\vec{A}-\vec{B}\right|$ and (e) the direction of $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$.

Sol. (a) $\vec{A}+\vec{B} = 3\hat{i} - 4\hat{j} - \hat{i} - 4\hat{j} = 2\hat{i} - 8\hat{j}$

(b) $\vec{A}-\vec{B} = 3\hat{i} - 4\hat{j} - \left(-\hat{i} - 4\hat{j}\right) = 4\hat{i}$

(c) $\left|\vec{A}+\vec{B}\right| = \sqrt{2^2 + (-8)^2} = \sqrt{4+64} = \sqrt{68}$ *units*

(d) $\left|\vec{A}-\vec{B}\right| = \sqrt{4^2} = 4$ *units*

(e) Angle between $\vec{A}+\vec{B}$ and X-axis $\Rightarrow \alpha = Tan^{-1}\left(\frac{-8}{2}\right) = Tan^{-1}(-4) = 76^0$ clock wise

$\vec{A}-\vec{B} = 4\hat{i}$, that is $\vec{A}-\vec{B}$ is along X-axis

2. A bird moves in such a way that it has a displacement of 12m towards east, 5m towards north and 9m vertically upwards. Find the magnitude of its displacement.

A. The resultant of 12 and 5m which are perpendicular to each other is $R = \sqrt{12^2 + 5^2} = 13m$. The displacement 12m and 9m are again perpendicular to each other and hence their resultant is $R^1 = \sqrt{13^2 + 9^2} = \sqrt{250} = 5\sqrt{10}m$

3. A person runs along a circular path of radius 5m. If he completes half of the circle find (a) the magnitude of the displacement vector. (b) How far the person ran? (c) What is the magnitude of the displacement if he runs all the way around the circle?

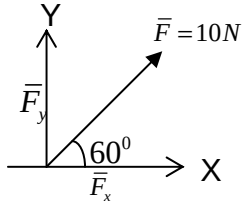
Sol. Let r be the radius of the circle. R = 5m

(a) The magnitude of displacement vector is the shortest distance between initial and final positions. Hence the magnitude of the displacement vector = 2r =10m

(b) The distance traveled by person is equal to the perimeter of the semicircle i.e. $\pi r = 5\pi$ *m*

(c) If the person runs all the way around the circle, the displacement of the person becomes zero.

4. **The force acting on a particle makes an angle of 60^0 with the positive x-axis. If the magnitude of the force is 10N what are its x-and y-components**?



A. $\left|\vec{F}_x\right| F\cos\theta = 10\cos 60^0 = 5\text{N}$

$\left|\vec{F}_y\right| F\sin 60^0 = 10\times\sqrt{3} = 5\sqrt{3}N$

5. **One of the rectangular components of a velocity of $20m^{-1}$ is $10m^{-1}$. Find the other component**.

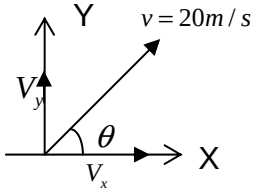
A. $V_x = v\cos\theta = 10$

$20\cos\theta = 10$

$\cos\theta = \frac{1}{2}$

$\theta = 60^0$

$\therefore v_y = v\sin\theta = 10\sin 60 = 5\sqrt{3}m/s$



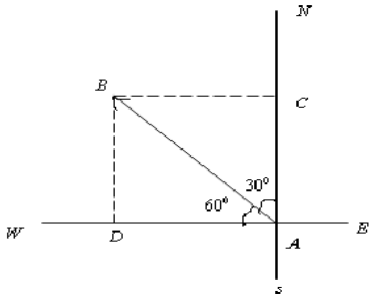
6. To go from town A to town B a plane must fly about 1780 km at an angle of 30º west of north. How far north of A is B? How far west?

Sol. From the figure , $\cos 30^0 = \frac{AC}{AB}$

$\Rightarrow AC = AB\cos 30^0 = 1780\times\frac{\sqrt{3}}{2}km$

$= 890\sqrt{3}\,km = 1541.48km$

That means B is $890\sqrt{3}$ km North of A



$\cos 60^0 = \frac{AD}{AB} \Rightarrow AD = AB\cos 60^0 = 1780\times\frac{1}{2} = 890\,km$

That means B is 890 Km west of A

7. Vector 'A' has an X and Y components of -8.70 cm and 15.0 cm respectively. Vector 'B' has X and Y components of 13.2 cm and -6.60 cm respectively. If A-B+3C=0 what are the components of 'C'?

Sol. $\vec{A} = A_x\hat{i} + A_y\hat{j} = -8.7\hat{i} + 15\hat{j}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} = 13.2\hat{i} - 6.6\hat{j}$

$\vec{C} = C_x\hat{i} + C_y\hat{j}$

Given that $\vec{A} - \vec{B} + 3\vec{C} = 0$

$-3\vec{C} = \left(\vec{A} - \vec{B}\right) = \left(-8.7\hat{i} + 15\hat{j}\right) - \left(1 + 13.2\hat{i} - 6.6\hat{j}\right) = -21.9\hat{i} + 21.6\hat{j}$

$\vec{C} = 7.3\hat{i} - 7.2\hat{j}$

$C_x = 7.3cm$ and $C_y = -7.2cm$

8. Displacement vectors $\vec{A} = \left(3\hat{i} + 3\hat{j}\right)m, \vec{B} = \left(\hat{i} - 4\hat{j}\right)m$ and $\vec{C} = \left(-2\hat{i} + 5\hat{j}\right)m$. Use the components method to determine

(a) The magnitude and direction of the vector $\vec{D} = \vec{A} + \vec{B} + \vec{C}$ and (b) the magnitude and direction of $\vec{E} = -\vec{A} - \vec{B} + \vec{C}$

Sol. (a) $\vec{D} = \vec{A} + \vec{B} + \vec{C} = 3\hat{i} + 3\hat{j} + \hat{i} - 4\hat{j} - 2\hat{i} + 5\hat{j}$

$\vec{D} = D_x\hat{i} + D_y\hat{j} = 2\hat{i} + 4\hat{j}$

Angle between $\vec{D}$ and X-axis is $\alpha \Rightarrow Tan\alpha = \frac{D_y}{D_x} = \frac{4}{2} = 2$

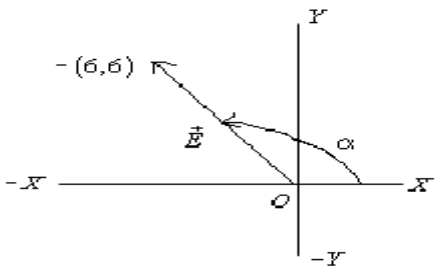
$\Rightarrow \alpha = Tan^{-1}(2) = 63.43^0$

(b) $\vec{E} = E_x\vec{i} + E_y\vec{j} \qquad \vec{E} = -\vec{A} - \vec{B} + \vec{C}$

$= -3\vec{i} - 3\vec{j} - \vec{i} + 4\vec{j} - 2\vec{i} + 5\vec{j} = -6\vec{i} + 6\vec{j}$

Magnitude of

$\vec{E} = |\vec{E}| = \sqrt{E_x^2 + E_y^2} \Rightarrow \sqrt{6^2 + 6^2} = 6\sqrt{2}m$



Let 'α' be the angle between $\vec{E}$ and X-axis

$\Rightarrow Tan\alpha = \dfrac{E_y}{E_x} = \dfrac{6}{-6} = -1$

$\Rightarrow \alpha = 135^0$ Anti clock wise

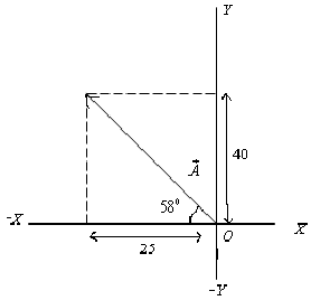
9. A vector has an X component of -25.0 units and a Y component of 40.0 units. Find the Magnitude and direction of this vector

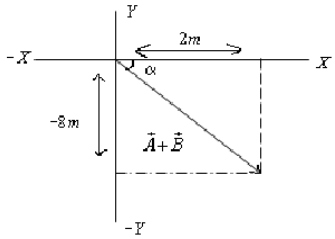
Sol. $\vec{A} = A_x\hat{i} + A_y\hat{j} = -25\hat{i} + 40\hat{j}$

Magnitude of $\vec{A}$

$\Rightarrow |\vec{A}| = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25)^2 + 40^2} = \sqrt{2225} = 47.17\,units$

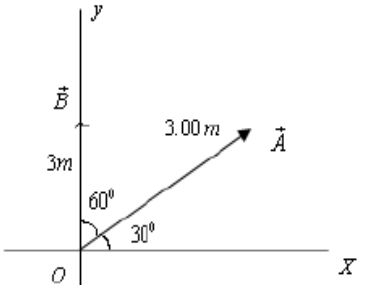
Let 'α' be the angle between $\vec{A}$ and X-axis





$\Rightarrow Tan\alpha = \dfrac{A_y}{A_x} = \dfrac{40}{-2.5} = -1.6 \Rightarrow \alpha = Tan^{-1}(-1.6) = 58^0$ clock wise with negative x-axis.

10. Two displacement vectors A and B are shown in figure below and each vector has a magnitude of 3m. Find graphically (a) $\vec{A}+\vec{B}$ (b) $\vec{A}-\vec{B}$ (c) $\vec{B}-\vec{A}$ (D) $\vec{A}-2\vec{B}$. The direction of all these are to be represented counter clock wise with respect to x-axis.



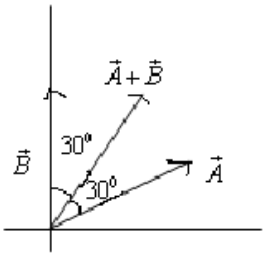
Sol. (a) $|\vec{A}+\vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos 60^0}$

$= \sqrt{3^2 + 3^2 + 2\times 3\times 3\times \dfrac{1}{2}}$

$= \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$ units

Angle between $\vec{A}+\vec{B}$ and $\vec{A} = \alpha$

$\Rightarrow Tan\alpha = \dfrac{B\sin 60^0}{A + B\cos 60^0} = \dfrac{1}{\sqrt{3}}$

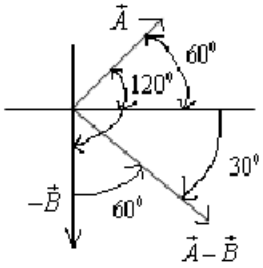


$\Rightarrow \alpha = 30^0$

The angle between $\vec{A}+\vec{B}$ and x-axis is 60º

(b) $\left|\vec{A}-\vec{B}\right| = \sqrt{A^2+B^2-2AB\cos 60^0}$

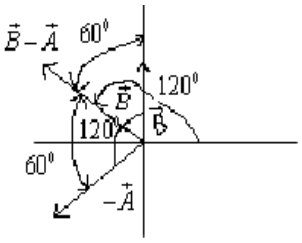
$= \sqrt{3^2+3^2-2\times3\times3\times\frac{1}{2}} = \sqrt{9+9-9} = \sqrt{9} = 3$ *units*



Angle made by $\vec{A}-\vec{B}$ with X-axis is 30^0 clock wise or 330^0 anticlock wise

(c) $\left|\vec{B}-\vec{A}\right| = \sqrt{B^2+A^2-2AB\cos 60^0}$

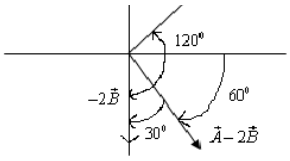
$= \sqrt{3^2+3^2-2\times3\times3\times\frac{1}{2}} = \sqrt{9+9-9} = \sqrt{9} = 3 units$



Angle made by $\vec{B}-\vec{A}$ with X-axis is 120º anticlock wise

(d) $\left|\vec{A}-2\vec{B}\right| = \sqrt{A^2+4B^2-2A(2B)\cos 60^0}$

$\sqrt{3^2+4\times3^2-2\times3\times2\times3\times\frac{1}{2}} = 3\sqrt{3}m$



Angle between $\vec{A}-2\vec{B}$ and x-axis is 60^0 clockwise or 300^0 anticlockwise.

11. **An ant moves on the floor of a room making two displacements. One of magnitude 3m and another of magnitude 4m. Show how the displacements should be combined to get the resultant displacement of magnitude i) 7m, ii) 1m and iii) 5m**

A. P = 3m : Q = 4m

i) R = 7m

$R^2 = P^2+Q^2+2pQ\cos\theta$

$49 = 9+16+2\times3\times4\cos\theta$

$24 = 2\times3\times4\cos\theta$

$\cos\theta = 1 \Rightarrow \theta = 0^0$

ii) R = 1m

$1 = 9+16+2\times3\times4\cos\theta$

$-24 = 2\times3\times4\cos\theta$

$\theta = 180^0$

iii) R = 5m

$R^2 = P^2+Q^2+2PQ\cos\theta$

$25 = 9+16+2\times3\times4\cos\theta$

$-24 = 2\times3\times4\cos\theta$

$\cos\theta = 0 \Rightarrow \theta = 90^0$

12. **In the above problem, If the angle between the two displacement is i)** 60^0 **and ii)** 120^0 **what will be the magnitude of the resultant displacement ?**

A. i) $\theta = 60^0$

$R^2 = P^2+Q^2+2PQ\cos\theta$

$= 9+16+2\times3\times4\cos 60$

$= 9+16+12 = 37$

$R = \sqrt{37}m = 6.082m$

ii) $\theta = 120^0$

$R^2 = 9+16+2\times3\times4\cos120$

$= 9+16-12$

$\sqrt{13} = 6.606m$

13. A car makes a displacement of 100 m towards east and then 200 m towards north. Find the magnitude and direction of the resultant.

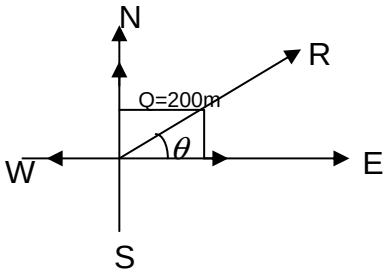
A. $R^2 = P^2 + Q^2$

$= 1w^2 + 2w^2$

$R = 100\sqrt{5}m$

$\therefore R = 223.7m$

From the figure



$Tan\theta = \frac{Q}{P} = \frac{200}{100} = \frac{2}{1}$

$\theta = Tan^{-1}2 = 63^0 26$

North of east

14. If $A = 6.00\hat{i} - 8.00\hat{j}$ units, $B = 8.00\hat{i} + 3.00\hat{j}$ units and C= $26.0\hat{i} + 19.0\hat{j}$ units, determine a and b when $a\vec{A} + b\vec{B} + \vec{C} = 0$

Sol. $a\vec{A} + b\vec{B} + \vec{C} = 0$

$a\left(6\hat{i} - 8\hat{j}\right) + b\left(8\hat{i} + 3\hat{j}\right) + 26\hat{i} + 19\hat{j} = 0$

$\left(6a + 8b\right)\hat{i} + \left(-8a + 3b\right)\hat{j} = -26\hat{i} - 19\hat{j}$

$6a - 8b = -26 ------ (i)$

$-8a + 3b = -19 ------ (ii)$

Solving we get, a = 5 and b = 7

15. What is the condition for the vectors $2\hat{i} + 3\hat{j} - 4\hat{k}$ and $3\hat{i} - a\hat{j} + b\hat{k}$ to be parallel?

Sol. Let $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{B} = 3\hat{i} - a\hat{j} + b\hat{k}$

If $\vec{A}$ and $\vec{B}$ are parallel to each other, then $\vec{A} = C\vec{B}$

C is a non zero constant $A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = C\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$

$$A_x = CB_x; A_y = Cb_y; A_z = CB_z \Rightarrow C = \frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} \Rightarrow \frac{2}{3} = \frac{3}{-a} = \frac{-4}{b}$$

$$a = \frac{-9}{2} = -4.5;\ b = \frac{-12}{2} = -6$$

16. A body starts with a velocity $2\hat{i} - 3\hat{j} + 11\hat{k}$ m/s and moves with an acceleration of $10\hat{i} + 10\hat{j} + 10\hat{k}$ ms^{-2}. What is its velocity after 0.25 s?

Sol. Final velocity $\vec{v} = \vec{u} + a\vec{t}$

Initial velocity $\vec{u} = 2\hat{i} - 3\hat{j} + 11\hat{k}$ ms^{-1}

Acceleration $\vec{a} = 10\hat{i} + 10\hat{j} + 10\hat{k}$ ms^{-2}

Time t=0.25sec

$\vec{v} = \left(2\hat{i} - 3\hat{j} + 11\hat{k}\right) + \left(10\hat{i} + 10\hat{j} + 10\hat{k}\right)0.25$

$2\hat{i} - 3\hat{j} + 11\hat{k} + \frac{5}{2}\hat{i} + \frac{5}{2}\hat{j} + \frac{5}{2}\hat{k}$

$= 4.5\hat{i} - 0.5\hat{j} + 13.5\hat{k}$ m/s

ASSES YOURSELF

1. Identify the vector and scalar quantities among the following
(a) temperature (b) force (c) volume of the milk in a bottle (d) bus fare to Hyderabad from your village/town (e) height of charminar (f) velocity of horse-cart you travel (g) the age of the earth.

Ans. Vectors; Force, velocity of horse-cart
Scalars : temperature, volume of the milk, bus fare, height, the age of the earth.

2. Can we add a vector quantity to a scalar quantity?

Ans. A vector can be added to another vector of same kind but a vector can not be added with a scalar quantity.

3. The magnitudes of two vectors A and B are 8 units and 5 units respectively. What are the maximum and minimum possible values for the magnitude of the resultant vector R=A+B?

Ans. $R_{\max} = A + B = (8+5)$ = 13 units And $R_{\min} = A - B = (8-5)$ = 3 units.

4. If $\vec{A}$ and $\vec{B}$ are two vectors, under what conditions $\left|\vec{A}+\vec{B}\right| = \left|\vec{A}\right| + \left|\vec{B}\right|$, under what conditions the resultant vector equals to zero?

Ans. $\left|\vec{A}+\vec{B}\right| = \left|\vec{A}\right| + \left|\vec{B}\right|$ When these vectors have same direction. $\left|\vec{A}+\vec{B}\right| = 0$ when they are oppositely directed.

5. A displacement vector in x-y plane has magnitude 25 m and direction 30º with x-axis. What are its x and y components?

Ans. $s_x = 25\cos 30^0 = 25\left(\frac{\sqrt{3}}{2}\right) = 25 \times 0.866 = 21.65\,m$

$s_y = 25\sin 30^0 = 25\left(\frac{1}{2}\right) = 12.5\,m$

6. A certain force has magnitude of 200 N, its x- component is -80 N. What is its y component and its direction. Two possible answers exist find them.

Ans. $F^2 = F_x^2 + F_y^2$

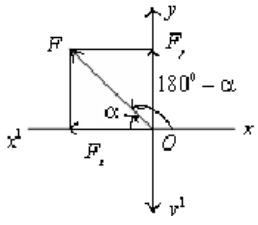
$F_y^2 = F^2 - F_x^2$

$F_y = \sqrt{F^2 - F_x^2} = \sqrt{(200)^2 - (-80)^2}$

$F_y = \sqrt{4000 - 6400}$

$F_y = \sqrt{33600}$

$F_y = \pm 183\,N$ …… (1)



(1) If $F_y = +183N$ *and* $F_x = -80\,N$

$$\tan\alpha = \frac{F_y}{F_x} = \frac{183}{80} = 2.2875 \Rightarrow \alpha = 66^0 23^1$$

$180^0 - \alpha = 180^0 - 6623^1 = 113^0 37^1$ with positive x-axis anticlockwise

(2) If $F_y = -183N$ *and* $F_x = -80\,N$

$$\tan\beta = \frac{F_y}{F_x} = \frac{183}{80} = 2.2875 \Rightarrow \beta = 66^0 23^1$$

$180^0 + \beta = 180^0 + 66^0 23^1 = 246^0 23^1$ with positive x-axis anticlockwise

7. If $|\vec{A}| = |\vec{B}|$, what can you conclude about the components of $\vec{A}$ & $\vec{B}$.

Ans. All their components are equal.

8. If one component of a vector is not zero, can the magnitude of the vector be zero?

Ans. The magnitude of vector A is given by

$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$. Hence if any component is non zero, A can never be zero.

9. Under what circumstances can the component of a vector be equal to the magnitude of the vector?

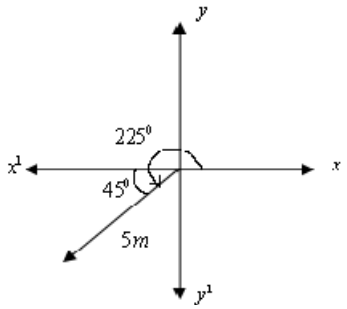
Ans. If vector is directed along positive direction of one of the three axes X,Y or Z, then the magnitude of the vector is the same as the component associated with the direction, other components become zero.

10. Write the following vectors in their unit vector form in cartesian coordinate system in two dimensions (i.e. consider x-y plane only).

a) a velocity vector of 10 ms^{-1} at angle of elevation of 60º

b) a vector $\vec{A}$ of magnitude A= 5 m and angle $\theta = 225^0$

c) a displacement from the origin to the point $x = 14\,m$, $y = -6m$



Ans. (a) $\vec{V} = 10\cos 60^0\,\hat{i} + 10\sin 60^0\,\hat{j}$

$\vec{V} = \left(5\hat{i} + 8.66\,\hat{j}\right) ms^{-1}$

(b)

$s_x = +5\cos 45^0 = \frac{5}{\sqrt{2}}m$ $\qquad$ $s_y = +5\sin 45^0 = \frac{5}{\sqrt{2}}m$

$\vec{s} = s_x\left(-\hat{i}\right) + s_y\left(-\hat{j}\right)$ $\qquad$ $\vec{s} = -\frac{5}{\sqrt{2}}\hat{i} - \frac{5}{\sqrt{2}}\hat{j}$ $\qquad$ $\vec{s} = \left(-3.54\hat{i} - 3.54\,\hat{j}\right)m$

c) $\vec{r} = x\hat{i} + y\hat{j} = \left(14\hat{i} - 6\,\hat{j}\right)m$

11. If the resultant of three vectors is zero, must they all be in the same plane?

Ans. Yes, the vectors must be in the same plane.