MEASUREMENT, UNITS & DIMENSIONS

Exercise

1. **In Poiseuille's method of determination of coefficient of viscosity which physical quantity requires greater accuracy in measurement?**

Sol: $V = \frac{\pi p r^2}{8\eta l}$ .The accuracy in measurement is directly proportional to power of quantity. Since the power of radius is highest, radius of capillary tube requires greater accuracy in measurement.

2. **In an experiment of simple pendulum the errors in the measurement of length of the pendulum L and time period T are 3 % and 2 % respectively. Find the maximum percentage error in the value of $\frac{L}{T^2}$.**

Sol: Maximum percentage error in $\frac{L}{T^2} = \left(\frac{\Delta L}{L}\times 100\right) + 2\left(\frac{\Delta T}{T}\times 100\right) = 3 + 2\,(2) = 7\ \%$

3. **The measured mass and volume of a body are 2.42 g and 4.7 cm^2 respectively with possible errors 0.01 g, and 0.1 cc. Find the maximum error in density.**

Sol: m = 2.42 gm , V = 4.7 cc

Maximum error is density = $\left(\frac{\Delta m}{m} + \frac{\Delta V}{V}\right)\times 100 = \left(\frac{0.01}{2.42} + \frac{0.1}{4.7}\right)\times 100 = 2\%$.

4. **The heat generated in a circuit is dependent on the resistance, current and time of flow of electrical current. If the errors measured in the above are 1%, 2% ad 1% respectively. Find the maximum error in measuring the heat .(Heat developed = $\frac{I^2 RT}{J}$ calories)**

Sol: $Q = \frac{i^2 RT}{J} = 2\left(\frac{\Delta I}{I}\times 100\right) + \left(\frac{\Delta R}{R}\times 100\right) + \left(\frac{\Delta T}{T}\times 100\right) = 2(2) + 1 + 1 = 6\ \%$

5. **We know density of a cube can be measured by measuring its mass and the length of its side. If the maximum errors in the measurement of mass and length are 3 % and 2% respectively, find the maximum error in the measurement of the density of the cube.**

Sol: Maximum error is density = $\frac{m}{V} = \frac{m}{l^3} = \left(\frac{\Delta m}{m} + 3\frac{\Delta l}{l}\right)\times 100 = 3 + 3\,(2) = 9\ \%$

Given $\frac{\Delta r}{r}\times 100 = 1$

From $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{\Delta V}{V}\times 100 = 3\left(\frac{\Delta r}{r}\times 100\right)$

$\Rightarrow \frac{\Delta V}{V}\times 100 = 3(1) = 3\%$.

6. **An experiment measures quantities a, b,c and then x is calculated as $x = \frac{ab^2}{c^3}$. If the percentage errors in a,b,c are $\pm 1\%, \pm 3\%$ *and* $\pm\ 2\%$ respectively, the percentage error in x can be**

Sol: $x = \frac{ab^2}{c^3}$

$\Rightarrow \frac{\Delta x}{x}\times 100 = \left(\frac{\Delta a}{a}\times 100\right) + 2\left(\frac{\Delta b}{b}\times 100\right) + 3\left(\frac{\Delta c}{c}\times 100\right) = 1 + 2(3) + 3(2) = 13\ \%$.

7. The percentage error in the mass and speed are 2 % and 3 % respectively. How much will be the maximum error in kinetic energy calculated using mass and velocity ?

Sol: K.E = $\frac{1}{2}m\upsilon^2 \Rightarrow \frac{\Delta K.E}{K.E}\times 100 = \left(\frac{\Delta m}{m}\times 100\right) + 2\left(\frac{\Delta \upsilon}{\upsilon}\times 100\right) = 2 + 2(3) = 8\%$

8. A gas bubble from an explosion under water oscillates with a period proportional to $P^a \rho^b E^c$, where P is the static pressure, ρ is the density of water and E is the total energy of the explosion. Find the values of a,b and c.

Sol: Given, $T = P^a \rho^b E^c$

Writing dimensional formulae both sides

$T = \left(ML^{-1}T^{-2}\right)^a \left(ML^{-3}\right)^b \left(ML^2T^{-2}\right)^c$

$T = M^a L^{-a} T^{-2a} M^b L^{-3b} M^c L^{2c} T^{-3c}$

$T = M^{o+b+c} L^{-o-3b+2c} T^{-2a-2c}$

Comparing the powers,

$a + b + c = 0$ .....(1)

$-a - 3b + 2c = 0$ ….(2)

$-2a - 2c = 1$ ……(3)

Solving the above, we get $a = \frac{-5}{6}$; $b = \frac{1}{2}$ and $c = \frac{1}{3}$

9. The period T_0 for a planet of mass m about the sun in a circular orbit of radius R depends on m,R and G where G is the gravitational constant. Find expression for time period by dimensional methods.

Sol :Let $T \propto M^A G^B R^C \Rightarrow T = KM^A G^B R^C$ ……(1)

Writing dimensional formulae both sides

$T = M^A (M^{-1} L^3 T^{-2})^B L^C$

$T = M^{A-B} L^{3B+C} T^{-2B}$

Comparing the powers,

$A - B = 0 \Rightarrow A = B$ ……(1)

$3B + C = 0$ …….(2)

$-2B = 1$ ….(3)

Solving the above, we get $A = \frac{-1}{2}$; $B = \frac{-1}{2}$ and $C = \frac{3}{2}$

$\therefore T = KM^{-1/2} G^{-1/2} R^{3/2}$ *or* $\boxed{T = K\sqrt{\frac{R^3}{GM}}}$

10. The frequency 'n' of a vibrating string depends upon its length l, linear density m (mass per unit length) and the tension T in the string. Derive an expression for the frequency of the string.

Sol: Let $n \propto l^A m^B T^c \Rightarrow n = Kl^A m^B T^C$

Writing dimensional formulae

$T^{-1} = L^A \left(ML^{-1}\right)^B \left(MLT^{-2}\right)^C$

$T^{-1} = M^{B+C}\ L^{A-B+C}\ T^{-2C}$

Comparing the powers,

$B + C = 0$ …… (1)

$A - B + C = 0$ …….. (2)

$-2C = -1$ …… (3)

Solving the above, we get A = -1; $B=\frac{-1}{2}$ and $C=\frac{1}{2}$

$\therefore n=\frac{K}{l}\sqrt{\frac{T}{m}}$

11. Derive an expression for the rate of flow of a liquid through a capillary tube Assume that the rate of flow depends on (i) the pressure gradient $\left(\frac{p}{l}\right)$, (ii) the radius r , and (iii) the coefficient of viscosity η. The value of the constant $K=\frac{\pi}{8}$.

Sol: $\frac{V}{t}\propto\left(\frac{P}{l}\right)^{A} r^{B}\eta^{C} \Rightarrow \frac{V}{t}=K\left(\frac{p}{l}\right)^{A} r^{B}\eta^{C}$ ……..(i)

Writing dimensional formulae both sides

$L^{3}T^{-1}=\left(ML^{-2}T^{-2}\right)^{A}L^{B}\left(ML^{-1}T^{-1}\right)^{C}$

Comparing the powers,

A+ C = 0 ……..(1)

$-2A+B-C=3$…..(2)

$-2A-C=-1$ ……(3)

Solving the above, we get $A=1$; $B=4$ *and* $C=-1$

$\therefore\frac{V}{t}=\frac{\pi pr^{4}}{8\eta l}$.

12. If energy E, velocity v and Time T are chosen as the fundamental quantities, find dimensional formula of surface tension ?

Sol: Surface tension $S\propto E^{A}V^{B}T^{C} \Rightarrow S=KE^{A}V^{B}T^{C}$.....(1)

$MT^{-2} = K\left(ML^{2}T^{-2}\right)^{A}\left(LT^{-1}\right)^{B}T^{C}$

Solving the above, we get $A=1$; B = -2 and C = -2

$\therefore S=K\,EV^{-2}T^{-2}$.

13. If the unit of length is doubled, mass is tripled and time is halved, how the unit of pressure is effected?

Sol: P= $ML^{-1}T^{-2}$ or $P=\frac{M}{LT^{2}}$ _____(1)

New pressure, $P_1=\frac{3M}{2L\left(\frac{T}{2}\right)^{2}}$ = $P_1=6P$

14. If unit of mass is taken as 1 kg, and that of time as 1 minute and acceleration due to gravity is taken as 9.81 ms^{-2}. What is the unit of energy?

Sol: Let $E\ \alpha m^{A}g^{B}t^{C}$

Or $E=Km^{A}g^{B}t^{C}$ _____(1)

Or $ML^{2}T^{-2}=M^{A}\left(LT^{-2}\right)^{B}T^{C}$

Solving the above, we get $A=1$; B = 2 and C = 2

$\therefore E=mg^{2}t^{2}$

Here K = 1

$\therefore$ Given m = 1 g = 9.81 t = 60 sec

$\therefore$ E = $1\times(9.81)^{2}\times(60)^{2}$ or E = 346450 J.