MEASUREMENTS, UNITS AND DIMENSIONS

Units and dimensions

1. A unit is the standard reference of the physical quantity which is used for comparison of the same physical quantity.
2. **Physical quantity:** A quantity that can be measured is called physical quantity. Physical quantities are classified as
   a) Fundamental physical quantities and
   b) Derived physical quantities
3. **Fundamental physical quantities**: The physical quantities which can exist independently are called fundamental quantities
   E.g.: length, mass, time, temperature, etc.
4. The units of fundamental quantities are called fundamental units
   E.g.: meter, kilogram, second, Kelvin etc
5. **Derived physical quantities** The physical quantities which can be derived from fundamental quantities are called derived quantities
   Ex: Speed, acceleration, force, work etc.
6. The units of the derived physical quantities are called derived units
   E.g.: ms^{-1}, ms^{-2}, Newton, joule etc.
7. **Systems of units :** -To measure the fundamental quantities length, mass and time, there are three systems of units

System	Units		
	Length	Mass	Time
F.P.S	Foot	Pound	Second
C.G.S	Centimetre	Gram	Second
M.K.S	Metre	Kilogram	Second

8. But these three quantities are not sufficient to derive all the quantities.
9. In this system there are seven fundamental quantities and two supplementary quantities are there.
10. Fundamental quantities and their units are given in table

s.no	Name of physical quantity	Unit	Symbol
1	Length	Meter	M
2	Mass	Kilogram	Kg
3	Time	Second	S
4	Electric current	Ampere	A
5	Thermodynamics temperature	Kelvin	K

6	Quantity of matter	Mole	Mol
7	Luminous intensity	Candela	Cd
8	Plane angle	Radian	Rad
9	Solid	Angle	Sr

11. **Definitions of radian & Steadman**
    I. **Radian (rad): -** The angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle is known as radian.

    $2\pi radian = 360^0$ And $1rad = 57^0 17' 45"$

    II. **Steradian (Sr) : -** The solid angle subtended at the centre of a sphere by its surface whose area is equal to the square of the radius of the sphere
12. **Rules for writing units in SI**
    i. Full names of units even if they are named after scientists should not be written with initial capital letter.

    E.g.: -Unit of force is mentioned as Newton but not as Newton

    iii) The symbols for a unit named after a scientist should have a capital letter Eg: 'N' for Newton, 'W' for watt etc.

    iv) The symbols of the other units are written as small letters .E.g.: 'm' for meter, 's' for second etc.

    v) Full stop or punctuation marks should not be used after symbol of units e.g.:10 kg but not 10 kg.

    vi) Symbols for units should not be take plural form. E.g.: 50 meters is 50m but not 50 ms
13. **Metre** : A metre is equal to 1650763.73 times the wavelength of the light emitted in vacuum due to electronic transition from $2p^{10}$ state to $5d^5$ state in Krypton–86. But in 1983, 17^{th} General Assembly of weights and measures, adopted a new definition for the metre in terms of velocity of light. According to this definition, metre is defined as the distance traveled by light in vacuum during a time interval of 1/299, 792, 458 of a second.
14. **Kilogram**: The mass of a cylinder of platinum–iridium alloy kept in the International Bureau of weights and measures preserved at Serves near Paris is called one kilogram.
15. **Second**: The duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of cesium–133 atom is called one second.
16. **Ampere** : The current which when flowing in each of two parallel conductors of infinite length and negligible cross–section and placed one metre apart in vacuum, causes each conductor to experience a force of $2x10^{-7}$ Newton per metre of length is known as one ampere.
17. **Kelvin**: The fraction of 1/273.16 of the thermodynamic temperature of the triple point of water is called Kelvin.
18. **Candela**: The luminous intensity in the perpendicular direction of a surface of a black body of area 1/600000 m^2 at the temperature of solidifying platinum under a pressure of 101325 Nm^{-2} is known as one candela.
19. **Mole**: The amount of a substance of a system which contains as many elementary entities as there are atoms in $12x10^3$ kg of carbon–12 is known as one mole.
20. **Radian:** The angle made by an arc of the circle equivalent to its radius at the centre is known as radian. 1 radian = $57^o 17^l 45^{ll}$.

21. **Steradian** : The angle subtended at the centre by one square metre area of the surface of a sphere of radius one metre is known as steradian.
22. **Dimensions :** The powers to which the fundamental quantities are to be raised to represent a physical quantity are called dimensions of that physical quantity

23. **Dimensional formula :** The expression showing the powers to which the fundamental quantities must be raised to represent a physical quantity is called dimensional formula

24. **Dimensional constants**: Plank's constant, Universal gas constant, Universal gravitational constant, Boltzmann's constant, Stefan's constant etc are called dimensional constants.
25. **Dimensionless quantities**: Angle, strain relative density, coefficient of friction etc are called dimensionless quantities.
26. **Principle of Homogeneity of dimensions**: The Physical quantities having the same dimensions may be either added or subtracted or equated.
27. **Applications of dimensional method of analysis**
28. Dimensional formulae are used along with principle of homogeneity of dimensions
    1) to convert one system of units in to another
    2) to check the correctness of an equation.
    3) to derive relation between different physical quantities.
29. **Limitations of dimensional method of analysis**
    1) The value of dimensionless constant (or) proportionality constants cannot be found by dimensional method. They have to be determined experimentally
    2) The equation containing trigonometrically, exponential and logarithmic functions can not be analyzed.
    3) If the physical quantity is a sum or a difference of two or more quantities, we can not use dimensional methods to derive an equation for the physical quantity. It is just possible to verify the relation dimensionally
    4) It is difficult to apply dimensional methods, if the physical quantity depends upon more than three fundamental quantities the length, mass and time.

Dimensional formula for some physical quantities

1. Speed = $\frac{Dis\tan ce}{Time} = \frac{L}{T} = LT^{-1}\left(ms^{-1}\right)$
2. Velocity = $\frac{Displacemnt}{Time} = \frac{L}{T} = LT^{-1}\left(m/s\right)$
3. Acceleration = $\frac{Velocity}{Time} = \frac{LT^{-1}}{T} = LT^{-2}\left(m/s^2\right)$
4. Force = ma = $MLT^{-1}\left(N\right)$
5. Universal gravitational constant $G = \frac{Fd^2}{m_1 m_2} = M^{-1}L^3T^{-2}\left(Nm^2/kg^2\right)$
6. Work or Energy = Force × Displacement $= MLT^{-2} \times L = ML^2T^{-2}\left(J\right)$
7. Momentum = Mass × Velocity $= M \times LT^{-1} = MLT^{-1}\left(kgm/s\right)$
8. Pressure $= \frac{force}{area} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}\left(pa\right)$
9. Density $= \frac{Mass}{Volume} = \frac{M}{L^3} = ML^{-3}\left(kg/m^3\right)$

10. Impulse = $F \times t = MLT^{-2} \times T = MLT^{-1}\left(Ns\right)$
11. Frequency n $= M^0L^0T^{-1}\left(Hz\right)$
12. Power = $\frac{w}{t} = \frac{ML^2T^{-2}}{T} = ML^2T^{-2}\left(watt\right)$
13. Kinetic energy = $\frac{1}{2}mv^2 = M \times \left(LT^{-1}\right)^2 = ML^2T^{-2}\left(J\right).$
14. Angular velocity = $\frac{Angular\ displacement}{Time} = \frac{1}{T} = T^{-1}\left(rad/s\right)$
15. Angular acceleration= $\frac{Angular\ velocity}{Time} = \frac{T^{-1}}{T} = T^{-2} = M^0L^0T^{-2}\left(rad/s^2\right)$
16. Linear density $P = \frac{Mass}{Length} = \frac{M}{L} = ML^{-1}\left(kg/m\right)$
17. Stress = $\frac{Restoring\ force}{Area} = \frac{F}{A} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}\left(N/m^2\right)$
18. Strain = $\frac{Change\ in\ \text{dim}ension}{Original\ \text{dim}ension} = M^0L^0T^0\left(no\ units\right)$
19. Elastic modulus $E = \frac{Stress}{Strain} = ML^{-1}T^{-2}\left(N/m^2\right)$
20. Velocity gradient = $\frac{\text{Change in velocity}}{\text{Distance}} = \frac{\text{LT}^{-1}}{L} = T^{-1}\left(S^{-1}\right)$
21. Moment of Inertia = I = $mr^2 = ML^2\left(kgm^2\right)$
22. Surface tension $= \frac{F}{L} = \frac{MLT^{-2}}{L} = MT^{-2}\left(N/m\right)$
23. Planck's constant = $h = \frac{E}{v} = \frac{ML^2T^{-2}}{T^{-1}} = ML^2T^{-1}\left(Js\right)$
24. Angular momentum = mvr $= MLT^{-1} \times L = ML^2T^{-1}\left(kgm^2/s\right)$
25. Torque (Moment) τ = Force × arm $= MLT^{-2} \times L = ML^2T^{-2}\left(Nm\right)$
26. Universal gas constant $R = \frac{PV}{nt} = \frac{ML^{-1}T^{-2}L^3}{K} = ML^2T^{-2}K^{-1}\left(J\ mol^{-1}k^{-1}\right)$
27. Specific heat = $\frac{Energy}{Mass \times Temp} = \frac{ML^2T^{-2}}{M \times K} = L^{-1}T^{-2}K\left(J/kgk\right)\left(\therefore S = \frac{Q}{mst}\right)$
28. Latent Heat = $\frac{Q}{m} = \frac{ML^2T^{-2}}{M} = L^2T^{-2}\left(Jkg^{-1}\right)$
29. Boltzmann constant K = $\frac{Energy}{Temperature} = \frac{ML^2T^{-2}}{K} = ML^2T^{-2}.K^{-1}\left(JK^{-1}\right)$

SHORT ANSWER QUESTIONS

1. What are fundamental quantities? State their units in SI.

Ans. **Fundamental physical quantities**

The physical quantities which can exist independently are called fundamental quantities

Eg: length, mass, time, temperature, etc.
The units of fundamental quantities are called fundamental units
Eg: metre, kilogram, second, kelvin etc

s.no	Name of phy quantity	Unit	Symbol
1	Length	Metre	M
2	Mass	Kilogram	Kg
3	Time	Second	S
4	Electric current	Ampere	A
5	Thermodynamics temperature	Kelvin	K
6	Quantity of matter	Mole	Mol
7	Luminous intensity	Candela	Cd
8	Plane angle	Radian	Rad
9	Solid	Angle	Sr

2. Explain the uses of dimensional methods with examples. (June2006, March2005)

Ans. 1.To verify the correctness of an equation connecting physical quantities

Ex. $S = ut + \frac{1}{2}at^2$

If every term of the equation is having same dimensional formula. Then dimensional the equation is correct. In the given equation

$S = ut + \frac{1}{2}at^2$

S = L

$ut = LT^{-1} \times T = L$

$\frac{1}{2}at^2 = LT^{-2} \times T^2 = L$

Since every term of the equation is having same dimensional formulae dimensionally the equation is correct

2.To convert one system of units into another system of units

Ex. $1J = 10^7 erg$

Erg and Joule are units of work in CGS and MKS systems respectively.

The dimensional formulae of work is ML^2T^{-2}

Let M_1, M_2, are the masses; L_1, L_2 are lengths; T_1, T_2 are times in CGS and MKS system respectively

Let $n_1u_1 = n_2u_2 \quad \Rightarrow n_1\left(M_1L_1^2T_1^{-2}\right) = n_2\left(M_2L_2^2T_2^{-2}\right)$

$$\frac{n_1}{n_2} = \left(\frac{M_2}{M_1}\right)\left(\frac{L_2}{L_1}\right)^2\left(\frac{T_2}{T_1}\right)^{-2}$$

$$\frac{n_1}{n_2} = \left(\frac{1kg}{1g}\right)\left(\frac{1m}{1cm}\right)^2\left(\frac{1s}{1s}\right)^{-2} = \left(\frac{10^3 g}{g}\right)\left(\frac{10^2 cm}{cm}\right)(1)^{-2} \quad = 10^3 \times 10^2 = 10^7$$

$\therefore 1J = 10^7 erg$

3.To obtain the equations connecting physical quantities

Ex. $V = \sqrt{\frac{E}{d}}$

Velocity of sound depends upon elastic modulus and density

$v \propto E^a d^b$

$\Rightarrow v = KE^a d^b$

$LT^{-1} = \left(ML^{-1}T^{-2}\right)^a \left(ML^{-3}\right)^6$

$L^1T^{-1} = M^{a+b} \times L^{-a-3b} \times T^{-2a}$

Comparing the similar terms, we get $a = \frac{1}{2}$ and $b = \frac{-1}{2}$

$\therefore V = K\sqrt{\frac{E}{d}}$

Experimentally k = 1, $V = K\sqrt{\frac{E}{D}}$

3. Is there any limit for the number of fundamental quantities? What are supplementary fundamental quantities? Define their units.

Ans. Yes, there are seven number of fundamental quantities are enough to express all physical quantities Supplementary fundamental quantities are

1) Plane angle 2) Solid angle

Radian: - It is the unit of plane angle. It is defined as the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle

1 rad = $57^0 17^1 45^{11}$

Steradian: - It is the unit of solid angle. It is defined as the solid angle subtended at the centre of sphere by its surface whose area is equal to the square of the radius.

4. What are limitations of dimensional methods?

Ans **Limitations of dimensional method of analysis**

1) The value of dimensionless constant (or) proportionality constants cannot be found by dimensional method. They have to be determine experimentally

2) The equation containing trigonometrically, exponential and logarithmic functions can not be analyzed.

3) If the physical quantity is a sum or a difference of two or more quantities, we can not use dimensional methods to derive an equation for the physical quantity. It is just possible to verify the relation dimensionally

4) It is difficult to apply dimensional methods, if the physical quantity depends upon more than three fundamental quantities the length, mass and time.

5. **Using the dimensional methods verify the correctness of the following relations.**

i) $a = \frac{V^2}{r}$ **ii)** $F = mr\omega^2$ **iii)** $S.T. = \frac{rhdg}{2}$ **. In the above formulae 'r' stands for radius 'm' for mass, 'ω' for angular velocity ,'v' for linear velocity , 'h' for height, 'd' for density and S.T for surface tension.**

Ans. i) $a = \frac{V^2}{r}$

Dimensions for $a = L^1T^{-2}$

Dimensions for $V^2 = \left(LT^{-1}\right)^2 = L^2T^{-2}$

Dimensions for r = L

$\therefore \frac{V^2}{r} = \frac{L^2T^{-2}}{L} = LT^{-2}$

The dimensions of a and $\frac{V^2}{r}$ are the same and hence the equation is correct.

ii) $F = mr\omega^2$

Dimensions for $F = MLT^{-2}$.

Dimensions for radius r = L

Dimensions (angular velocity) $^2 = \omega^2 = \left(T^{-1}\right)^2 = T^{-2}$

Dimensions of mass m = M

The dimensions of F and $mr\omega^2$ are the same and hence the equation is dimensionally correct.

iii) S.T = $\frac{rhdg}{2}$

Dimensional formula for S.T = MT^{-2}

Dimensional formula for radius r = L

Dimensional formula for height h = L

Dimensional formula for density $d = ML^{-3}$.

Dimensional formula acceleration due to gravity $g = LT^{-2}$.

Dimensional formula $\frac{rhdg}{2} = LLML^{-3}LT^{-2} = MT^{-2}$

The dimensional formulae for S.T. and $\frac{rhdg}{2}$ are the same and hence the equation is dimensionally correct.

6. **The velocity of an object varies with time as $v = At^2 + Bt + C$. If units of v and t are expressed in SI, find the units of constants A, B and C. (May '07)**

Ans. Velocity $v = At^2 + Bt + C$

Dimensional formula of velocity $v = LT^{-1}$

From the principle of homogeneity, in a physical equation both sides contain same dimensional formulae.

$\therefore AT^2 = LT^{-1} \Rightarrow A = \frac{LT^{-1}}{T^2} = LT^{-3}$; $Bt = LT^{-1} \Rightarrow B = \frac{LT^{-1}}{T} = LT^{-2}$; $C = LT^{-1}$

Unit of A is metre /sec^3. ; Unit of B is metre / sec^2. ; Unif of C is metre / sec.

VERY SHORT ANSWER QUESTIONS

1. What are fundamental and derived quantities? Given examples? (June2006)

Ans. A physical quantity which is independent of any other quantity is a fundamental quantity
Eg : length, mass, time, temperature etc.
The physical quantities which can be derived from other fundamental physical quantities are called derived physical quantities
Eg : Area, volume, pressure, density etc.

2. What is meant by "dimensions" of a physical quantity? What is meant by dimensional formula?

Ans. **Dimension:** The dimensions of physical quantities are the powers to which the fundamental quantities are raised to obtain the unit of derived quantity

Eg : The dimensional formula of energy is $\left[ML^2T^{-2}\right]$

Here the dimensions of mass is +1
The dimensions of length is +2
The dimensions of time is -2

Dimensional formula: The expression showing the powers to which the fundamental quantities are raised to obtain the unit of derived physical quantity is called the dimensional formula

Eg: Dimensional formula of force is $\left[ML^2T^{-2}\right]$

3. Which physical quantity has negative dimensions of mass?

Ans. The physical quantities with negative dimensions in mass are universal gravitational constant, capacitance etc.

Dimensional formula of G $=\left[M^{-1}L^3T^{-2}\right]$

4. State two constants which have dimensions. (June2005)

Ans. Two constants having dimensions are universal gas constant $\left[M^{-1}L^3T^{-2}\right]$ and Planck's constant $\left[ML^2T^{-1}\right]$

5. What is the physical quantity represented by $\sqrt{gR}$?

Ans. $\sqrt{gR} = \sqrt{gR} = \sqrt{M^0L^1T^{-2} \times M^0L^1T^0} = L^1T^{-1} = Velocity$

6. What is the physical quantity represented by PV

Ans. PV $= \left[ML^{-1}T^{-2}\right] \left[M^0L^3T^0\right] = \left[ML^2T^{-2}\right]$

This is the dimensional formula for work and energy

7. If density of wood is 0.8 gm/cc. find its value in SI?

Ans.

$$1g/cm^3 = \frac{10^{-3}kg}{10^{-6}m^3} = 1000kg/m^3$$

$$0.8g/cm^3 = 0.8 \times 1000kg/m^3 \quad = 800kg/m^3$$

8. If units of length and force are increased by four times, how much increase will be there in unit of pressure? (March2006)

A. Pressure = $\frac{Force}{Area}$ *or* $P = \frac{F}{L^2}$ And $P_1 = \frac{4F}{16L^2} = \frac{F}{4L^2}$

$\therefore P_1 = \frac{P}{4}$ Hence pressure decreases by 4 times.

9. If units of mass, length and time are doubled, what happens to unit of energy? (March2009, March2008, May 2007)

A. $E = \frac{ML^2}{T^2}$

Now, $E_1 = \frac{2M \times 4L^2}{4T^2}$ or $E_1 = 2E$ Energy gets doubled.

10. If force F, length L and time T are fundamental quantities, find the dimensional formula of mass?

A. Force = Mass ×Acceleration $\Rightarrow Mass = \frac{Force}{Acceleration} = \frac{F}{LT^{-2}} = F^1L^{-1}T^2$

Dimensional formula for mass = $FL^{-1}T^2$.

11. Prove that energy per unit volume is pressure. (March 2008)

A. $\frac{Energy}{Volume} = \text{Pr}essure$

Substituting the dimensional formulae on both sides

$\frac{ML^2T^2}{L^3} = ML^{-1}T^{-2} \Rightarrow ML^{-1}T^{-2} = ML^{-1}T^{-2}$.Thus proved.

12. The air pressure in a tyre is $10Nm^{-2}$. Express it in dynes/sq.cm.

A. 1N = 10^5 dyne and 1m = 10^2 cm

$\therefore 1m^{-2} = \left(10^2 cm\right)^{-2} = 10^{-4} cm^{-2}$

$\therefore 10Nm^{-2} = 10 \times 10^5 dyne \times 10^{-4} cm^{-2} = 10^6 \times 10^{-4} dynecm^{-2} = 10^2 dyne/sq.cm$

13. Which physical quantity has same dimensional formula as surface tension?

A. MT^{-2}, force constant or spring constant has the same dimensional formula as surface tension.

14. Find out the quantity $\sqrt{g\lambda}$ using dimensional formula where g is acceleration due to gravity and λ is wavelength of water waves.

A. $\sqrt{g\lambda} = \sqrt{LT^{-2}L} = \sqrt{L^2T^{-2}} = LT^{-1} = \text{Velocity}$

15. Which physical quantities have units Jm^{-1} and Jm^{-2}?

A. i) $Jm^{-1} = \frac{ML^2T^{-2}}{L} = MLT^{-2} = \text{Force}$

ii) $Jm^{-2} = \frac{ML^2T^{-2}}{L} = MT^{-2} = \text{SurfaceTension}$

16. State two physical quantities having units as Pa.

A. (i) pressure and (ii) Modulus of Elasticity.

17. If energy E and volume V are the fundamental quantities, which physical quantity has dimensional formula EV^{-1} ?

A. $EV^{-1} = \frac{\text{Energy}}{\text{Volume}} = \frac{ML^2T^{-1}}{L^3} = ML^{-1}T^2 = \text{Pressure}$

18. Surface tension for water is 70 dynes/cm. Express it in SI.

A. $70\,\text{dyne/cm} = 70 \times \frac{10^{-5}\,\text{N}}{10^{-2}\,\text{m}} = 70 \times 10^{-3}\,\text{N/m}$

19. What is the convenience in using SI ?

A. As SI units at length and mass make use of decimal system, conversions within the system are simpler and more convenient. It is internationally accepted and hence a usage of unit facilitates free exchange of scientific and industrial work.

20. Write the dimensional formula for pressure and energy. (March-2011)

A. $\text{Pressure} = \left[ML^{-1}T^2\right]$ and $\text{Energy} = \left[ML^2T^{-2}\right]$

Exercise 1

1. Prove that Planck's constant and angular momentum have same dimensions. (June 2010, March-2010, March-2008)

A. E = hν where E = energy, v = frequency and h = Planck's constant.

$\therefore h = \frac{E}{v} = \frac{ML^2T^{-2}}{M^\circ L^\circ T^{-1}} = \left[ML^2T^{-1}\right]$

$\therefore$ Angular momentum (L) = Iω

Where I is moment of inertia, ω is angular velocity.

$\therefore L = \left[ML^2T^0\right]\left[M^\circ L^\circ T^{-1}\right] = \left[ML^2T^{-1}\right]$

$\therefore$ Angular momentum, Planck's constant has same dimensions.

2. If E,M,L and G denote energy, mass, angular momentum and Universal Gravitational constant respectively, prove that $\frac{EL^2}{M^5G^2}$ is a dimensionless quantity.

Sol. Taking dimensional formulae of E,M,L and G

Energy (E) = $\left[ML^2T^{-2}\right]$

Mass (M) = $\left[ML^\circ T^\circ\right]$

Angular momentum (L) = $\left[ML^2T^{-1}\right]$

Universal Gravitational constant $(G) = \left[M^{-1}T^3T^{-2}\right]$

$\frac{EL^2}{M^5G^2} = \frac{\left[ML^2T^{-2}\right]\left[ML^2T^{-1}\right]^2}{\left[ML^\circ T^\circ\right]^5\left[M^{-1}L^3T^{-2}\right]^2} = \left[M^\circ L^\circ T^\circ\right]$ = dimensionless

3. If the equation of state of a gas is expressed as $\left(P+\frac{a}{V^2}\right)(V-b)=RT$ where P is the pressure, V is the volume and T the absolute temperature and a,b,R are constants, then find the dimensions of 'a' and 'b'.

A. From principle of homogeneity,

$\frac{a}{V^2}$ = pressure $\Rightarrow \frac{a}{\left[M^{\circ}L^{3}T^{0}\right]^2}=\left[M^{1}L^{-1}T^{-2}\right]$

$\therefore a=\left[M^{1}L^{5}T^{-2}\right]$

Similarly , b = volume

$\therefore b=\left[M^{\circ}L^{3}T^{\circ}\right]$

4. The dimensional formula of the product of two physical quantities P and Q is $\left[ML^{2}T^{-2}\right]$. The dimensional formula of $\frac{P}{Q}$ is $\left[ML^{\circ}T^{-2}\right]$. Then which physical quantities are presented by P and Q ? (2005-March)

A. $PQ=\left[ML^{2}T^{-2}\right]$ And $\frac{P}{Q}=\left[ML^{\circ}T^{-2}\right]$

Now $(PQ)\left(\frac{P}{Q}\right)=\left[ML^{2}T^{-2}\right]\left[ML^{\circ}T^{-2}\right]$

$\therefore P=\left[ML^{1}T^{-2}\right]$ = Force

Also $\frac{P}{Q}=\left[M^{1}L^{\circ}T^{-2}\right]$

$\left[\frac{ML^{1}T^{-2}}{Q}\right]=\left[ML^{\circ}T^{-2}\right]$

$Q=[L]$ = Displacement

$\therefore$ P and Q are respectively force and displacement.

5. If pressure P, velocity v and time T are taken as the fundamental (base) quantities, find the dimensional formula of force.

A. Let force $=[F]=\left[P^{a}v^{b}T^{c}\right]$

Force $[F]=\left[M^{1}L^{1}T^{-2}\right]$, pressure $[P]=\left[M^{1}L^{-1}T^{-2}\right]$, velocity $[v]=\left[M^{0}L^{1}T^{-1}\right]$

Time $[T]=\left[M^{0}L^{0}T^{-1}\right]$

$\therefore\left[MLT^{-2}\right]=\left[ML^{-1}T^{-2}\right]^{a}\left[M^{\circ}L^{-1}T^{-1}\right]^{b}\left[M^{\circ}L^{\circ}T^{-1}\right]^{c}$

$\left[MLT^{-2}\right]=\left[M^{a}L^{-a+b}T^{-2a-b+c}\right]$

$\therefore$ Comparing power of M, L, T on both sides, solve for a,b and c

a = 1 ; b = 2 ;c = 2

$\therefore$ Force $(F)=\left[PV^{2}T^{2}\right]$

Exercise 2

1. **A gas bubble from an explosion under water oscillates with a period proportional to $P^a \rho^b E^c$, where P is the static pressure, ρ is the density of water and E is the total energy of the explosion. Find the values of a, b and c.**

Sol: Given, $T = P^a \rho^b E^c$

Writing dimensional formulae both sides

$T = \left(ML^{-1}T^{-2}\right)^a \left(ML^{-3}\right)^b \left(ML^2T^{-2}\right)^c$

$T = M^a L^{-a} T^{-2a} M^b L^{-3b} M^c L^{2c} T^{-3c}$

$T = M^{o+b+c} L^{-o-3b+2c} T^{-2a-2c}$

Comparing the powers,

$a + b + c = 0$ .....(1)

$-a - 3b + 2c = 0$ ….(2)

$-2a - 2c = 1$ ……(3)

Solving the above, we get $a = \frac{-5}{6}$; $b = \frac{1}{2}$ and $c = \frac{1}{3}$

2. The period T_0 for a planet of mass m about the sun in a circular orbit of radius R depends on m,R and G where G is the gravitational constant. Find expression for time period by dimensional methods.

Sol :Let $T \propto M^A G^B R^C \Rightarrow T = KM^A G^B R^C$ ……(1)

Writing dimensional formulae both sides

$T = M^A (M^{-1} L^3 T^{-2})^B L^C$

$T = M^{A-B} L^{3B+C} T^{-2B}$

Comparing the powers,

$A - B = 0 \Rightarrow A = B$ ……(1)

$3B + C = 0$ …….(2)

$-2B = 1$ ….(3)

Solving the above, we get $A = \frac{-1}{2}$; $B = \frac{-1}{2}$ and $C = \frac{3}{2}$

$\therefore T = KM^{-1/2} G^{-1/2} R^{3/2}$ *or* $\boxed{T = K\sqrt{\frac{R^3}{GM}}}$

3. The frequency 'n' of a vibrating string depends upon its length l, linear density m (mass per unit length) and the tension T in the string. Derive an expression for the frequency of the string.

Sol: Let $n \propto l^A m^B T^c \Rightarrow n = Kl^A m^B T^C$

Writing dimensional formulae

$T^{-1} = L^A \left(ML^{-1}\right)^B \left(MLT^{-2}\right)^C$

$T^{-1} = M^{B+C} L^{A-B+C} T^{-2C}$

Comparing the powers,

$B + C = 0$ …… (1)

$A - B + C = 0$ …….. (2)

$-2C=-1$ …… (3)

Solving the above, we get A = -1; $B=\frac{-1}{2}$ and $C=\frac{1}{2}$

$\therefore n=\frac{K}{l}\sqrt{\frac{T}{m}}$

4. Derive an expression for the rate of flow of a liquid through a capillary tube Assume that the rate of flow depends on (i) the pressure gradient $\left(\frac{p}{l}\right)$, (ii) the radius r , and (iii) the coefficient of viscosity η. The value of the constant $K=\frac{\pi}{8}$

Sol: $\frac{V}{t}\propto\left(\frac{P}{l}\right)^A r^B\eta^C \Rightarrow \frac{V}{t}=K\left(\frac{p}{l}\right)^A r^B\eta^C$ ……..(i)

Writing dimensional formulae both sides

$L^3T^{-1}=\left(ML^{-2}T^{-2}\right)^A L^B\left(ML^{-1}T^{-1}\right)^C$

Comparing the powers,

A+ C = 0 ……..(1)

$-2A+B-C=3$…..(2)

$-2A-C=-1$ ……(3)

Solving the above, we get $A=1$; $B=4$ *and* $C=-1$

$\therefore\frac{V}{t}=\frac{\pi pr^4}{8\eta l}$.

5. If energy E, velocity v and Time T are chosen as the fundamental quantities, find dimensional formula of surface tension?

Sol: Surface tension $S\propto E^A V^B T^C \Rightarrow S=KE^A V^B T^C$.....(1)

$MT^{-2}=K\left(ML^2T^{-2}\right)^A\left(LT^{-1}\right)^B T^C$

Solving the above, we get $A=1$; B = -2 and C = -2

$\therefore S=K\,EV^{-2}T^{-2}$.

6. If the unit of length is doubled, mass is tripled and time is halved, how the unit of pressure is effected?

Sol: P= $ML^{-1}T^{-2}$ or $P=\frac{M}{LT^2}$ _____(1)

New pressure, $P_1=\frac{3M}{2L\left(\frac{T}{2}\right)^2}$ = $P_1=6P$

7. If unit of mass is taken as 1 kg, and that of time as 1 minute and acceleration due to gravity is taken as 9.81 ms^{-2}. What is the unit of energy?

Sol: Let $E\ \alpha\, m^A g^B t^C$

Or $E=Km^A g^B t^C$ _____(1)

Or $ML^2T^{-2} = M^A\left(LT^{-2}\right)^B T^C$

Solving the above, we get $A = 1$; B = 2 and C = 2

$\therefore E = mg^2t^2$

Here K = 1

$\therefore$ Given m = 1 g = 9.81 t = 60 sec

$\therefore$ E = $1\times(9.81)^2\times(60)^2$ or E = 346450 J.

ASSESS YOURSELF

1. As far as mechanics is concerned all systems of units are having the same dimensions for a given physical quantity. But for thermodynamics, electricity, magnetism and electromagnetic this is not the case, why?

Ans: This is because in SI we have got dimensions for temperature and current also in addition to length, mass and time.

2. Two physical quantities are having the same dimensions. Does it mean that they are the same? Give an example supporting your answers.

Ans: No, Eg: Torque and work.

3. Two physical quantities are having the same dimensions. Should their units be necessarily the same? Give an example supporting your answer.

Ans: No. Eg: Torque having a unit of Nm and
Work having a unit of J but both is having same dimensions.

4. Why is the dimension of one fundamental (base) quantity interms of any other fundamental (base) quantity is always zero.

Ans: Otherwise, it will not be a fundamental quantity as per the definition.

5. A dimensionally wrong equation in which principle of homogeneity of dimensions violated must be wrong. Can you say that, a dimensionally correct equation should always be right?

Ans: Need not be

6. State about the correctness or otherwise of the following two statements
i) A physical quantity can have dimensions but no units
ii) A physical quantity can have units but no dimensions

Ans: (i) Not correct,
(ii) Correct, Eg: Angular displacement having a unit radium but no dimensions.

7. Can we derive the Cinematic relations $v = u + at, S = ut + \frac{1}{2}at^2$ *and* $v^2 = u^2 + 2as$ **.**
Explain the reason for your answer.

Ans: No. On the hand side there is only one physical quantity, but on the right hand side there is more than one quantity.

8. Coulomb's law is $F = k\frac{q_1 q_2}{r^2}$ **. In C.G.**

S system, Unit of change is so defined that the force in air or vacuum will be 1 dyne when two identical charges of unit magnitude are kept at a distance of . This makes K=1 in C.G.S system.

But is SI the charges 1C is defined as the amount of charge, flowing when 1A current passes for 1S.

That is 1C=1A1S. The Unit of current, ampere is defined through electromagnetic. Consequently the value of F in air or vacuum comes out to be N. When two identical changes of 1C magnitude are placed in air or vacuum at 1m apart. What are the consequences of these results?

Ans: The constant proportionality K has got the units and dimensions.