Oscillations

- A motion that repeats itself at regular intervals of time is called **periodic motion.**
- If thebody is given a small displacement from theposition, a force comes into play which tries tobring the body back to the equilibrium point,giving rise to oscillations or vibrations.
- The smallest interval of time afterwhich the motion is repeated is called itsperiod.
- The reciprocal of T gives the number ofrepetitions that occur per unit time. Thisquantity is called the frequency of the periodicmotion. It is represented by the symbol v . Therelation between v and T is

  $v = 1/T$
- Anyperiodic function can be expressed as asuperposition of sine and cosine functionsof different time periods with suitablecoefficients.

- $x(t) = A \cos(\omega t + \phi)$

  The quantity A is called the amplitude of the motion. It is a positive constant which representsthe magnitude of the maximum displacement ofthe particle in either direction. The cosine functionvaries between the limits ±1, so thedisplacement x(t) varies between the limits ± A.

  The time varying quantity, $(\omega t + \phi)$ is called the **phase** of the motion. It describes the state of motion at a given time.

  The constant ϕ is called the **phase constant** (or **phase angle**). The value of ϕ depends on the displacement and velocity of the particle at t = 0.

  The constant ω, called the angular frequency of the motion, is related to the period T.

  The angular frequency is,

$\omega = 2\pi / T$

The SI unit of angular frequency is radians per second.

- Simple harmonic motionis the projection of uniform circular motionon a diameter of the circle in which thelatter motion takes place.
- Simple harmonic motion is the motionexecuted by a particle subject to a force, which is proportional to the displacement of theparticle and is directed towards the meanposition.

  $F(t) = ma = -m\omega^2 x(t)$

  $F(t) = -k\, x(t)$ where $k = m\omega^2$

- A particle executing simple harmonic motion has, at any time, kinetic energyK = ½ mv2 and potential energy U = ½ kx2. If no friction is present the mechanicalenergy of the system, E = K + U always remains constant even though K and U changewith time.

- A particle of mass m oscillating under the influence of a Hooke's law restoring forcegiven by $F = – k x$ exhibits simple harmonic motion with

  $\omega = \sqrt{(k/m)}$

  $T = 2\pi \sqrt{(m/k)}$

  Such a system is also called a linear oscillator.

- Simple Pendulum

  The restoring torque τ is given by,

  $\tau = –L (Fg \sin\theta)$

  where the negative sign indicates that the torque acts to reduce θ, and L is the length of the moment arm of the force $Fg \sin \theta$ about the pivot point. For rotational motion we have,

  $\tau = I \alpha$

  where I is the pendulum's rotational inertia about the pivot point and α is its angular acceleration about that point.

  $\omega = \sqrt{(mgL/I)}$

Period of oscillation = $2\pi \sqrt{(L/g)}$

- The mechanical energy in a real oscillating system decreases during oscillations because external forces, such as drag, inhibit the oscillations and transfer mechanical energy to thermal energy. The real oscillator and its motion are then said to be damped. If the damping force is given by Fd = –bv, where v is the velocity of the oscillator and b is a damping constant, then the displacement of the oscillator is given by,

  $x(t) = A e^{-bt/2m} \cos(\omega' t + \phi)$

  where ω', the angular frequency of the damped oscillator, is given by

  $\sqrt{(k/m)(b^2/4m^2)}$

  If the damping constant is small then $\omega' \approx \omega$, where ω is the angular frequency of the un-damped oscillator. The mechanical energy E of the damped oscillator is given by

$E(t) = 1/2\, kA^2 e^{bt/m}$

- If an external force with angular frequency ωd acts on an oscillating system with natural angular frequency ω, the system oscillates with angular frequency ωd.

  The amplitude of oscillations is the greatest when

  ωd= ω

  a condition called resonance

Sample Examples

- A body oscillates with SHMaccording to the equation (in SI units),$x = 5 \cos [2\pi t + \pi/4]$.At t = 1.5 s, calculate the (a) displacement,(b) speed and (c) acceleration of the body.

  Solution

  The angular frequency ω of the body= $2\pi\ s^{-1}$ and its time period T = 1 s.

  At t = 1.5 s

  (a) displacement = (5.0 m) cos $[(2\pi\ s{-}1) \times 1.5\ s + \pi/4]$

  = (5.0 m) cos $[(3\pi + \pi/4)] = -5.0 \times 0.707$ m= –3.535 m

  (b) the speed of the body= – (5.0 m)$(2\pi\ s{-}1)$ sin $[(2\pi\ s{-}1) \times 1.5\ s + \pi/4]$

  = – (5.0 m)$(2\pi\ s{-}1)$ sin $[(3\pi + \pi/4)] = 10\pi \times 0.707\ m\ s^{-1}$

  = 22 m s^{-1}

  (b) the acceleration of thebody= $-(2\pi\ s^{-1})^2 \times$ displacement

  = – $(2\pi\ s^{-1})^2 \times (-3.535\ m) = 140\ m\ s^{-2}$

- Which of the followingfunctions of time represent (a) simpleharmonic motion and (b) periodic but notsimple harmonic? Give the period for eachcase.

  (1) $\sin \omega t - \cos \omega t$

  (2) $\sin^2 \omega t$

  Solution

  (a) $\sin \omega t - \cos \omega t$

  $= \sin \omega t - \sin (\pi/2 - \omega t)$

  $= 2 \cos (\pi/4) \sin (\omega t - \pi/4)$

  $= \sqrt{2} \sin (\omega t - \pi/4)$

  This function represents a simple harmonicmotion having a period $T = 2\pi/\omega$ and aphase angle $(-\pi/4)$ or $(7\pi/4)$

  (b) $\sin^2 \omega t$

  $= ½ - ½ \cos 2 \omega t$

  The function is periodic having a period$T = \pi/\omega$. It also represents a harmonicmotion with the point of equilibrium occurring at ½ instead of zero.