Moving Charges and Magnetism

- The total force on a charge q moving with velocity v in the presence of magnetic and electric fields B and E, respectively is called the Lorentz force. It is given by the expression:

  $F = q (v \times B + E)$

  The magnetic force $q (v \times B)$ is normal to v and work done by it is zero.
- A straight conductor of length l and carrying a steady current I experiences a force F in a uniform external magnetic field B,

  $F = I\, l \times B$

  where $|l| = l$ and the direction of l is given by the direction of the current.
- In a uniform magnetic field B, a charge q executes a circular orbit in a plane normal to B. Its frequency of uniform circular motion is called the cyclotron frequency and is given by:

$V_c = qB/2\pi m$

This frequency is independent of the particle's speed and radius. This fact is exploited in a machine, the cyclotron, which is used to accelerate charged particles.

- The Biot-Savart law asserts that the magnetic field dB due to an element dl carrying a steady current I at a point P at a distance r from the current element is:

  $dB = (\mu 0/4\pi)*I*[(dl \times r)/r^3]$

  To obtain the total field at P, we must integrate this vector expression over the entire length of the conductor.
- The magnitude of the magnetic field due to a circular coil of radius R carrying a current I at an axial distance x from the centre is

  $B = (\mu 0 I R^2)/(2(x^2 + R^2)^{3/2}$

  At the center this reduces to

  $B = \mu 0 I/2R$

- Ampere's Circuital Law: Let an open surface S be bounded by a loop Then the Ampere's law states that 0

  $C _\int B_dl = \mu I$

where I refers to the current passing through S. The sign of I is determined from the right-hand rule. We have discussed a simplified form of this law. If B is directed along the tangent to every point on the perimeter L of a closed curve and is constant in magnitude along perimeter then,

$BL = \mu_0 I_e$

where $I_e$is the net current enclosed by the closed circuit.

- The magnitude of the magnetic field at a distance R from a long, straight wire carrying a current I is given by:

  $B = \mu_0 I/2\pi R$

  The field lines are circles concentric with the wire.

- The magnitude of the field B inside a long solenoid carrying a current I is

  $B = \mu_0 nI$

  where n is the number of turns per unit length.

  For a toroid one obtains,

  $B = \mu_0 NI/2\pi r$

  where N is the total number of turns and r is the average radius.

- Parallel currents attract and anti-parallel currents repel.
- A planar loop carrying a current I, having N closely wound turns, and an area A possesses a magnetic moment m where,

  $m = N I A$

  and the direction of m is given by the right-hand thumb rule : curl the palm of your right hand along the loop with the fingers pointing in the direction of the current. The thumb sticking out gives the direction of m (and A)

  When this loop is placed in a uniform magnetic field B, the force F on it is: $F = 0$

  And the torque on it is,

  $\tau = m \times B$

  In a moving coil galvanometer, this torque is balanced by a counter torque due to a spring, yielding

  $k\phi = NI AB$

  where ϕ is the equilibrium deflection and k the torsion constant ofthe spring.

- An electron moving around the central nucleus has a magnetic moment μ_l given by:

  $\mu_l = el/2m$

  where l is the magnitude of the angular momentum of the circulating electron about the central nucleus. The smallest value of μl is called the Bohr magneton μ_B and it is $\mu_B = 9.27\times10^{-24}$ J/T

- A moving coil galvanometer can be converted into a ammeter by introducing a shunt resistance rs, of small value in parallel. It can be converted into a voltmeter by introducing a resistance of a large value in series.

Sample Examples

- The horizontal component of the earth's magnetic fieldat a certain place is 3.0×10^{-5} T and the direction of the field is fromthe geographic south to the geographic north. A very long straightconductor is carrying a steady current of 1A. What is the force perunit length on it when it is placed on a horizontal table and thedirection of the current is (a) east to west; (b) south to north?

  Solution

  $F = I\, l \times B$

  $F = IlB \sin\theta$

  The force per unit length is$f = F/l = I\, B \sin\theta$

  (a) When the current is flowing from east to west,$\theta = 90°$

Hence,f = I B

$= 1 \times 3 \times 10^{-5} = 3 \times 10^{-5}$ N m^{-1}

This is larger than the value 2×10^{-7} Nm^{-1} quoted in the definitionof the ampere. Hence it is important to eliminate the effect of theearth's magnetic field and other stray fields while standardisingthe ampere.

The direction of the force is downwards. This direction may beobtained by the directional property of cross product of vectors.

(b) When the current is flowing from south to north,

$\theta = 0^{o}$

$f = 0$

Hence there is no force on the conductor.

- A solenoid of length 0.5 m has a radius of 1 cm and ismade up of 500 turns. It carries a current of 5 A. What is the magnitude of the magnetic field inside the solenoid?

  Solution

  The number of turns per unit length is,

  n = 500/0.5 = 1000 turns/m

  The length l = 0.5 m and radius r = 0.01 m. Thus, l/a = 50 i.e., l >> a.

  Hence, we can use the long solenoid formula,

  $B = \mu_0 n\, l$

  $= 4\pi \times 10^{-7} \times 10^{3} \times 5$

  $= 6.28 \times 10^{-3}$ T