Model Paper

B.Sc (III-Year) Examination

(New, *w.e.f* 2010-2011)

Mathematics, Paper-III

(Linear Algebra, Multiple Integrals and Vector Calculus)

Time: 3 Hours **Max.Marks:100**

Part-I (Marks: 6x6=36)

Answer any **six** questions

1. Prove that the four vectors $(1,0,0),(0,1,0)(0,0,1)$ and $(1,1,1)$ in $V_3(C)$ form a linearly dependent set, but, any three of them are linearly independent.
2. If $U(F)$ and $V(F)$ are two vectors spaces and T is a linear transformation from U to V, then prove that the range of T is a sub space of V.
3. Find all characteristic values and characteristic vectors of the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

4. If α, β are vectors in an inner product space V, prove $\|\alpha+\beta\| \le \|\alpha\| + \|\beta\|$.
5. Evaluate $\int_c (3xy\,dx - y^2 dy)$; where c is the curve in the $xy-$plane $y = 2x^2$ from $(0,0)$ to $(1,2)$.
6. Evaluate $\iint xy(x^2+y^2)\,dxdy$ over $[(0,a;0,b)]$.
7. Let $\varphi = x^2y^3z^6$. In what direction from the point $P(1,1,1)$, is the directional derivative of φ a maximum? What is the magnitude of this maximum?
8. By using the Greenôs theorem, show that the area bounded by a simple closed curve c is given by $\frac{1}{2}\oint_c (xdy - ydx)$.

Part-II(Marks:4x16=64)

Answer four questions, choosing **one** question from each section.

Section-A

9 (a) Prove that the necessary and sufficient condition for a non-empty subset W of a vector space $V(F)$ to be a subspace of $V(F)$ is that $a,b \in F$ and $\overline{\alpha},\overline{\beta} \in W \Rightarrow a\overline{\alpha} + b\overline{\beta} \in W$.

(b) Prove that there exists a basis for each finite dimensional vector space.

10. State and prove Rank-Nullity theorem for linear transformations.

Section-B

11. State and prove the Cayley-Hamilton theorem.

12. (a) In an inner product space $V(F)$, prove that $|(\alpha,\beta)| \le \|\alpha\|\,\|\beta\|$.

(b) Apply the Gram-Schmidt process to the vectors $\beta_1 = (1,0,1)$, $\beta_2 = (1,0,-1)$, $\beta_3 = (0,3,4)$ to obtain an orthonormal basis $V_3(R)$ with the standard inner product.

Section-C

13. (a) Assuming that f, φ and ψ are continuous and possesses a continuous derivative φ', then show that $\int_c f(x,y)\,dx = \int_\alpha^\beta f[\varphi(t),\psi(t)]\,\varphi'(t)dt$; where $c : x = \phi(t), y = \psi(t); t \in [\alpha,\beta]$ and φ,ψ defined on $[\alpha,\beta]$.

(b) Prove that the necessary and sufficient condition for the integrability of a bounded function f over a rectangle R is that to every positive number ε, there corresponds a positive number δ, such that for every division D of R whose norm $\le \delta$ the oscillatory sum $S(D) - s(D) < \varepsilon$.

14. (a) If f is continuous in a domain E which is bounded by the curves $y = \phi(x), y = \psi(x);\quad x = a, x = b;\quad$ where ϕ,ψ are continuous and $\psi(x) \le \varphi(x); \forall x \in [a,b]$ then prove $\iint_E f(x,y)dxdy = \int_a^b \left\{ \int_{\psi(x)}^{\phi(x)} f(x,y)dy \right\} dx.$

(b) In the integral $\int_0^{2a}\int_{\frac{x^2}{4a}}^{3a-x}\varphi(x,y)\,dx\,dy$, change the order of integration.

Section-D

15. (a) Define (i) Divergence of a vector (ii) Curl of a vector. If $\vec{A} = x^2yi - 2xzj + 2yzk$ then find $curl\ curl\ \vec{A}$.

(b) Evaluate $\iint_s \vec{A}.\hat{n}\,ds$ for $\vec{A} = \left(x + y^2\right)i - 2xj - 2yzk$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

16. State and prove the divergence theorem of Gauss.