

MATHEMATICS

SECTION – I

(SINGLE CORRECT CHOICE TYPE)

This section contains 10 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct

41. Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + 1$ is real. Then a cannot take the value

- a) -1 b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{3}{4}$

Ans. D

$$\text{Let } Z = x + iy \quad (y \neq 0)$$

$$a = z^2 + z + 1 \text{ becomes}$$

$$a = (x + iy)^2 + (x + iy) + 1$$

$$(x^2 - y^2 + x + 1) + i(2xy + y) = a + i0$$

Comparing real & imaginary parts

$$x^2 - y^2 + x + 1 = a \text{ —————(1)}$$

$$2xy + y = 0 \text{ —————(2)}$$

$$\text{From (2) } x = \frac{-1}{2} \text{ as } y \neq 0$$

Put value of x in (1)

$$\left(-\frac{1}{2}\right)^2 - y^2 - \frac{1}{2} + 1 = a$$

$$\frac{1}{4} + \frac{1}{2} - y^2 = a$$

$$y^2 = \frac{3}{4} - a > 0$$

$$a < \frac{3}{4}$$

42. If $\lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$, then

a) $a = 1, b = 4$

b) $a = 1, b = -4$

c) $a = 2, b = -3$

d) $a = 2, b = 3$

Ans. B

$$\lim_{x \rightarrow \infty} \frac{x^2 + x + 1}{x + 1} - ax - b = 4$$

$$\lim_{x \rightarrow \infty} \frac{x^2(1-a) + x(1-a-b) + 1-b}{x+1} = 4$$

For existence of limit coefficient of $x^2 = 0$

$$1 - a = 0$$

$$a = 1$$

$$\lim_{x \rightarrow \infty} \frac{x(1-a-b) + 1-b}{1+x} = 4$$

$$1 - a - b = 4$$

$$b = -4$$

43. Let $P = [a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \leq i, j \leq 3$. If the determinant of P is 2, then the determinant of the matrix Q is

a) 2^{10}

b) 2^{11}

c) 2^{12}

d) 2^{13}

Ans. D

$$\text{as } b_{ij} = 2^{i+j} a_{ij}$$

$$b_{11} = 4a_{11}, b_{12} = 8a_{12}, b_{13} = 16a_{13}$$

$$b_{21} = 8a_{21}, b_{22} = 16a_{22}, b_{23} = 32a_{23}$$

$$b_{31} = 16a_{31}, b_{32} = 32a_{32}, b_{33} = 64a_{33}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = 2$$

$$\begin{aligned}
 B &= \begin{bmatrix} 4a_{11} & 8a_{12} & 16a_{13} \\ 8a_{21} & 16a_{22} & 32a_{23} \\ 16a_{31} & 32a_{32} & 64a_{33} \end{bmatrix} \\
 &= 4 \times 8 \times 16 \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 4a_{31} & 4a_{32} & 4a_{33} \end{bmatrix} \\
 &= 4 \times 8 \times 16 \times 2 \times 4 \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \\
 &= 4 \times 8 \times 16 \times 2 \times 4 \times 2 \\
 B &= 2^{13}
 \end{aligned}$$

44. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinates axes. Another ellipse E_2 passing through the point $(0, 4)$ circumscribes the rectangle R. The eccentricity of the ellipse E_2 is

- a) $\frac{\sqrt{2}}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{1}{2}$ d) $\frac{3}{4}$

Ans. C

Given Ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$

As ellipse is bounded by $x = \pm a$ & $y = \pm b$

Sides of rectangle are

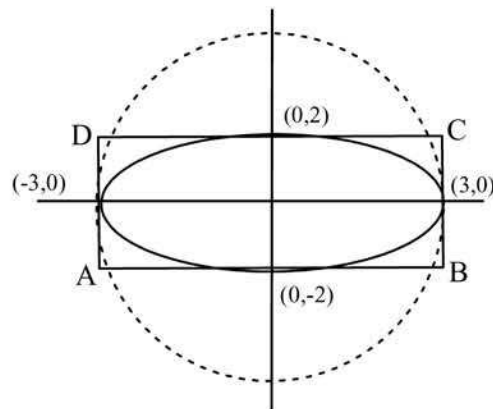
$$x = \pm 3 \text{ \& } y = \pm 2$$

$$C(3, 2), D(-3, 2)$$

$$A(-3, -2), B(3, -2)$$

As new ellipse

Circumscribe the rectangle & passes through $(0, 4)$.



Let new ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Passes through $(0, 4) \Rightarrow \frac{16}{b^2} = 1 \Rightarrow b^2 = 16$

Passes through $(3, 2) \Rightarrow \frac{9}{a^2} + \frac{4}{16} = 1$

$$a^2 = 12$$

So, Vertical ellipse $\frac{x^2}{12} + \frac{y^2}{16} = 1$

$$a^2 = b^2(1 - e^2)$$

$$12 = 16(1 - e^2)$$

$$e^2 = 1 - \frac{12}{16} = \frac{1}{4}$$

$$e = \frac{1}{2}$$

45. The function $f : [0, 3] \rightarrow [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$ is

a) one-one and onto

b) onto but not one-one

c) one-one but not onto

d) neither one-one nor onto

Ans. B

$$f : [0, 3] \rightarrow [1, 29]$$

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

$$f'(x) = 6x^2 - 30x + 36$$

$$= 6(x - 2)(x - 3)$$

$$f''(x) = 6(2x - 5)$$

$\therefore x = 2$ is pt of maxima.

$x = 3$ is pt of minima.

$$\& f(2) = 29 > 0$$

$$f(3) > 0$$

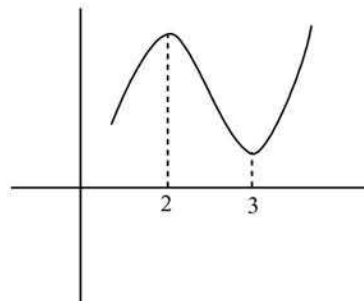
so curve is

$\therefore f''$ is not one one

$$\text{Min value} = f(0) = 1$$

$$\text{Max value} = f(2) = 29$$

$\therefore$ so f'' is Onto.



46. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is

a) $20(x^2 + y^2) - 36x + 45y = 0$

b) $20(x^2 + y^2) + 36x - 45y = 0$

c) $36(x^2 + y^2) - 20x + 45y = 0$

d) $36(x^2 + y^2) + 20x - 45y = 0$

Ans. A

Let $P(h, k) \& Q(x_1, y_1)$

As P lies of $4x - 5y = 20$

$$\therefore P\left(h, \frac{4h - 20}{5}\right)$$

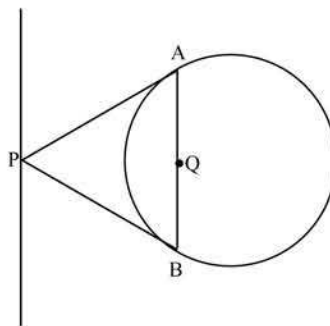
Equation of chord of contact

$$AB \Rightarrow S_1 = 0$$

$$xh + y\left(\frac{4h - 20}{5}\right) = 9$$

$$5xh + y(4h - 20) = 45 \text{ -----(1)}$$

Equation of chord AB whose mid point is $Q(x_1, y_1)$



$$xx_1 + yy_1 = x_1^2 + y_1^2 \text{-----}(2)$$

Compare (1) & (2)

$$\frac{5h}{x_1} = \frac{4h-20}{y_1} = \frac{45}{x_1^2 + y_1^2}$$

$$h = \frac{9x_1}{x_1^2 + y_1^2} \quad \& \quad 4h - 20 = \frac{45y_1}{x_1^2 + y_1^2}$$

On eliminating h

$$\frac{49x_1}{x_1^2 + y_1^2} - 20 = \frac{45y_1}{x_1^2 + y_1^2}$$

$$20(x_1^2 + y_1^2) = 36x_1 - 45y_1$$

$\therefore$ Locus is

$$20(x^2 + y^2) - 36x + 45y = 0$$

47. The point P is the intersection of the straight line joining the points Q (2, 3, 5) and R (1, -1, 4) with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point T (2, 1, 4) to QR, then the length of the line segment PS is

- a) $\frac{1}{\sqrt{2}}$ b) $\sqrt{2}$ c) 2 d) $2\sqrt{2}$

Ans. A

$$\text{Equation of } QR \rightarrow \frac{x-1}{1} = \frac{y+1}{4} = \frac{z-4}{1} = \lambda$$

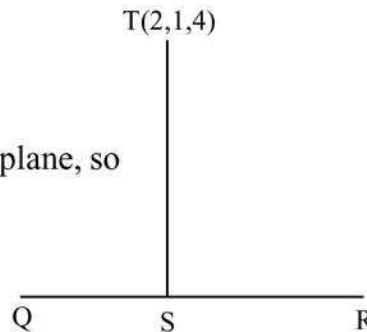
Dr's of QR (1, 4, 1)

Co-ordinates of $P(1+\lambda, -1+4\lambda, 4+\lambda)$

As P is point of Intersection of QR & given plane, so

$$5(1+\lambda) - 4(-1+4\lambda) - (4+\lambda) = 1$$

$$\lambda = \frac{1}{3}$$



$$\therefore P\left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right)$$

$$\text{Let } S(1+\mu, -1+4\mu, 4+\mu)$$

$$\text{Dir's of } TS(\mu-1, 4\mu-2, \mu)$$

$$TS \perp QR \text{ So,}$$

$$1(\mu-1) + 4(4\mu-2) + \mu = 0$$

$$\mu = \frac{1}{2}$$

$$\therefore S\left(\frac{3}{2}, 1, \frac{9}{2}\right)$$

$$\begin{aligned} \therefore PS &= \sqrt{\left(\frac{4}{3} - \frac{3}{2}\right)^2 + \left(1 - \frac{1}{3}\right)^2 + \left(\frac{13}{3} - \frac{9}{2}\right)^2} \\ &= \frac{1}{\sqrt{2}} \end{aligned}$$

48. Let $f(x) = \begin{cases} x^2 \left| \cos \frac{\pi}{x} \right|, & x \neq 0 \\ 0, & x = 0 \end{cases}, x \in \mathbb{R}$, then f is

- a) differentiable both at $x = 0$ and at $x = 2$
- b) differentiable at $x = 0$ but not differentiable at $x = 2$
- c) not differentiable at $x = 0$ but differentiable at $x = 2$
- d) differentiable neither at $x = 0$ nor at $x = 2$

Ans. B

$$f(x) = \begin{cases} x^2 \left| \cos \frac{\pi}{x} \right|; & x \neq 0 \\ 0; & x = 0 \end{cases}$$

At $x=0$

$$RHD \Rightarrow \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{h^2 \cos \frac{\pi}{h} - 0}{h}$$

$$= \lim_{h \rightarrow 0} \left(h \cos \frac{\pi}{h} \right) = 0$$

$$LHD \Rightarrow \lim_{h \rightarrow 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h \rightarrow 0} \frac{h^2 \left[\cos \left(-\frac{\pi}{h} \right) \right] - 0}{-h}$$

$$= \lim_{h \rightarrow 0} \left(-h \cos \left(\frac{\pi}{h} \right) \right) = 0$$

$\therefore$ Differentiable at $x=0$

At $x=2$

$$RHD \Rightarrow \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{(2+h)^2 \left[\cos \frac{\pi}{2+h} \right] - 4 \left[\cos \frac{\pi}{2} \right]}{h} = \infty$$

$$LHD \Rightarrow \lim_{h \rightarrow 0} \frac{f(2-h) - f(2)}{-h} = \lim_{h \rightarrow 0} \frac{(2-h)^2 \left[\cos \frac{\pi}{2-h} \right] - 4 \left[\cos \frac{\pi}{2} \right]}{-h} = -\infty$$

$\therefore$ Not Differentiable at $x=2$

49. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

a) 75 b) 150 c) 210 d) 243

Ans. B

5 Balls to 3 persons, so that all persons get atleast one ball.

Case-I

Total ways

$$= ({}^3P_3 \times {}^3C_2 \times {}^3C_1) \times 3$$

2	2	1
2	1	2
1	2	2

$$= 10 \times 3 \times 1 \times 3 = 90$$

Case-II

Total ways

$$= ({}^3C_0 \times {}^3C_1 \times {}^3C_2 \times {}^3C_3) \times 3$$

$$= 10 \times 2 \times 1 \times 3 = 60$$

$$\therefore \text{Total ways} = 90 + 60 = 150$$

3	1	1	
1	3	1	
1	1	3	

50. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{11/2}} dx$ equals (for some arbitrary constant K)

a) $\frac{1}{(\sec x + \tan x)^{11/2}} \left[\frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right] + K$

b) $\frac{1}{(\sec x + \tan x)^{11/2}} \left[\frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right] + K$

c) $\frac{1}{(\sec x + \tan x)^{11/2}} \left[\frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right] + K$

d) $\frac{1}{(\sec x + \tan x)^{11/2}} \left[\frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right] + K$

Ans. C

$$\int \frac{\sec x}{(\sec x + \tan x)^{9/2}} dx$$

$$\text{Let } \sec x + \tan x = t \quad \text{--- (1)}$$

$$\text{So, } (\sec x \tan x + \sec^2 x) dx = dt$$

$$\sec x dx = \frac{dt}{t} \quad \text{--- (2)}$$

$$\sec x - \tan x = \frac{1}{t} \text{-----(3)}$$

From (1) & (3)

$$\sec x = \frac{1}{2} \left(t + \frac{1}{t} \right)$$

So, Integral becomes

$$\int \frac{\sec x \cdot \sec x}{(\sec x + \tan x)^{9/2}} dx$$

$$\int \frac{\frac{1}{2} \left(t + \frac{1}{t} \right) dt / t}{t^{9/2}}$$

$$\frac{1}{2} \int \frac{t^2 + 1}{t^{13/2}} dt$$

$$= \frac{1}{2} \left[\int t^{-9/2} dt + \int t^{-13/2} dt \right]$$

$$= \frac{1}{2} \left[\frac{t^{-7/2}}{-7/2} + \frac{t^{-11/2}}{-11/2} \right] + K$$

$$= \frac{t^{-7/2}}{-7} + \frac{t^{-11/2}}{-11} + K$$

$$= -\frac{1}{7(\sec x + \tan x)^{7/2}} - \frac{1}{11(\sec x + \tan x)^{11/2}}$$

$$= -\frac{1}{(\sec x + \tan x)^{11/2}} \left(\frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right) + K$$

SECTION – II

(MULTIPLE CORRECT CHOICE TYPE)

This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONE OR MORE is/ are correct

51. A ship is fitted with three engines E_1 , E_2 and E_3 . The engines function independently of each other with respective probabilities $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1 , X_2 and X_3 denote respectively the events that the engines E_1 , E_2 and E_3 are functioning. Which of the following is (are) true ?

- a) $P[X_1^c | X] = \frac{3}{16}$ b) $P[\text{Exactly two engines of the ship are functioning} | X] = \frac{7}{8}$
 c) $P[X | X_2] = \frac{5}{16}$ d) $P[X | X_1] = \frac{7}{16}$

Ans. B,D

$$\text{A) } \frac{P(X_1^c \cap X)}{P(X)} = \frac{\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}}{\frac{1}{4}} = \frac{1}{8}$$

$$P(X) = \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right) \times 2 + \left(\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}\right) \times 2 = \frac{1}{4}$$

$$\text{B) } P[\text{Exactly two engines of the ship are functioning} | X] = \frac{\left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right) \times 2 + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}}{\frac{1}{4}} = \frac{7}{8}$$

$$\text{C) } P(X | X_2) = \frac{P[X_2 \cap X]}{P(X_2)} = \frac{\left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}\right) \times 2}{\frac{1}{4}} = \frac{5}{8}$$

$$\text{D) } P(X | X_1) = \frac{P[X \cap X_1]}{P(X_1)} = \frac{\left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right) \times 2 + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}}{\frac{1}{2}} = \frac{7}{16}$$

52. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, parallel to the straight line $2x - y = 1$. The points of contact of the tangents on the hyperbola are

a) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ b) $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ c) $(3\sqrt{3}, -2\sqrt{2})$ d) $(-3\sqrt{3}, 2\sqrt{2})$

Ans. A, B

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

Tangent to the hyperbola is of the form

$$y = 2x + \lambda$$

$$\lambda^2 = 9(4) - 4 = 36 - 4 = 32$$

$$\lambda = \pm 4\sqrt{2}$$

$$\text{Tangent is: } 2x - y \pm 4\sqrt{2} = 0$$

$$\text{Point of contact is } \left(-\frac{a^2l}{n}, \frac{b^2m}{n}\right) = \left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

53. Let S be the area of the region enclosed by $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 1$. Then

a) $S \geq \frac{1}{e}$ b) $S \geq 1 - \frac{1}{e}$ c) $S \leq \frac{1}{4} \left(1 + \frac{1}{\sqrt{e}}\right)$ d) $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}} \left(1 - \frac{1}{\sqrt{2}}\right)$

Ans. A, B, D

$$I = \int_0^1 e^{-x^2} dx \geq \int_0^1 e^{-x} dx = 1 - \frac{1}{e}$$

$$I \geq 1 - \frac{1}{e}$$

$$e^{-1} \leq e^{-x^2} \leq 1$$

$$\int_0^1 e^{-1} dx \leq \int_0^1 e^{-x^2} dx \leq \int_0^1 dx$$

$$\therefore \int_0^1 e^{-x^2} dx \geq \frac{1}{e}$$

54. If $y(x)$ satisfies the differential equation $y' - y \tan x = 2x \sec x$ and $y(0) = 0$, then

a) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$

b) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$

c) $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$

d) $y\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$

Ans. A,D

$$\frac{dy}{dx} - y \tan x = 2x \sec x$$

$$IF = e^{-\int \tan x} = \cos x$$

$$\text{General solution is } y(\cos x) = \int (2x \sec x) \cos x \, dx$$

$$= x^2 + c$$

$$y = x^2 \sec x + c \sec x$$

$$\longrightarrow (0,0)$$

$$0 = 0 + c \Rightarrow c = 0$$

$$y = x^2 \sec x$$

$$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16}(\sqrt{2})$$

$$= \frac{\pi^2}{8\sqrt{2}}$$

$$y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9} \times 2$$

$$\frac{dy}{dx} = x^2 \sec x \tan x + 2x \sec x$$

$$y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}(2)\sqrt{3} + 2\left(\frac{\pi}{3}\right)2$$

$$= \frac{2\pi^2}{3\sqrt{3}} + \frac{4\pi}{3}$$

55. Let $\theta, \phi \in [0, 2\pi]$ be such that

$$2 \cos \theta (1 - \sin \phi) = \sin^2 \theta \left(\tan \frac{\theta}{2} + \cot \frac{\theta}{2} \right) \cos \phi - 1,$$

$$\tan(2\pi - \theta) > 0 \text{ and } -1 < \sin \theta < -\frac{\sqrt{3}}{2}.$$

Then ϕ cannot satisfy

a) $0 < \phi < \frac{\pi}{2}$

b) $\frac{\pi}{2} < \phi < \frac{4\pi}{3}$

c) $\frac{4\pi}{3} < \phi < \frac{3\pi}{2}$

d) $\frac{3\pi}{2} < \phi < 2\pi$

Ans. A, C, D

$$\tan(2\pi - \theta) > 0, -1 < \sin \theta < -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3} \right) \Rightarrow 0 < \cos \theta < \frac{1}{2} \text{ ————— (1)}$$

$$2 \cos \theta (1 - \sin \phi) = \sin^2 \theta \left(\tan \theta / 2 + \cot \theta / 2 \right) \cos \phi - 1$$

$$\Rightarrow \sin(\theta + \phi) = \cos \theta + \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} < \sin(\theta + \phi) < 1 \quad (\text{from (1)})$$

$$\frac{13\pi}{6} < \theta + \phi < \frac{17\pi}{6}$$

$$\Rightarrow \frac{2\pi}{3} < \phi < \frac{7\pi}{6} \quad (\text{from (1)})$$

cannot satisfied by A, C, D

SECTION -III

(INTEGERANSWERTYPE)

This section contains 5 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened.

56. Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x = 1$ and a local minimum at $x = 3$. If $p(1) = 6$ and $p(3) = 2$, then $p'(0)$ is

Ans. 9

$$P(x) = ax^3 + bx^2 + cx + d$$

$$P(1) = 6 \Rightarrow a + b + c + d = 6$$

$$P(3) = 2 \Rightarrow 27a + 9b + 3c + d = 2$$

$$P'(1) = 0 \Rightarrow 3a + 2b + c = 0$$

$$P'(3) = 0 \Rightarrow 27a + 6b + c = 0$$

$$a = 1, b = -6, c = 9$$

$$P'(x) = 3ax^2 + 2bx + c$$

$$P'(0) = c = 9$$

57. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$, then $|2\vec{a} + 5\vec{b} + 5\vec{c}|$ is

Ans. 3

sol. $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$$

$$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \frac{-3}{2}$$

$$3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2|\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}| = 0$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = 0$$

$$\Rightarrow \bar{a} + \bar{b} + \bar{c} = \bar{0} \quad \text{--- (1)}$$

Taking dot product with (1) by $\bar{a}, \bar{b}$ & $\bar{c}$

$$\text{then } \bar{a} \cdot \bar{b} = \frac{-1}{2}, \bar{b} \cdot \bar{c} = \frac{-1}{2}, \bar{c} \cdot \bar{a} = \frac{-1}{2}$$

$$\therefore |2\bar{a} + 5\bar{b} + 5\bar{c}| = 3$$

58. The value of $6 + \log_{\frac{3}{2}} \left(\frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \dots \right)$ is

Ans. 4

sol. $x = \frac{1}{3\sqrt{2}} \sqrt{4 - x}$

$$3\sqrt{2} x = \sqrt{4 - x}$$

squaring on both sides

$$18x^2 = 4 - x$$

$$18x^2 + x - 4 = 0$$

$$18x^2 + 9x - 8x - 4 = 0$$

$$9x(2x+1) - 4(2x+1) = 0.$$

$$(2x+1)(9x-4) = 0$$

$$x = -\frac{1}{2}, \frac{4}{9}$$

$$\Rightarrow x = \frac{4}{9} \quad (\because x > 0)$$

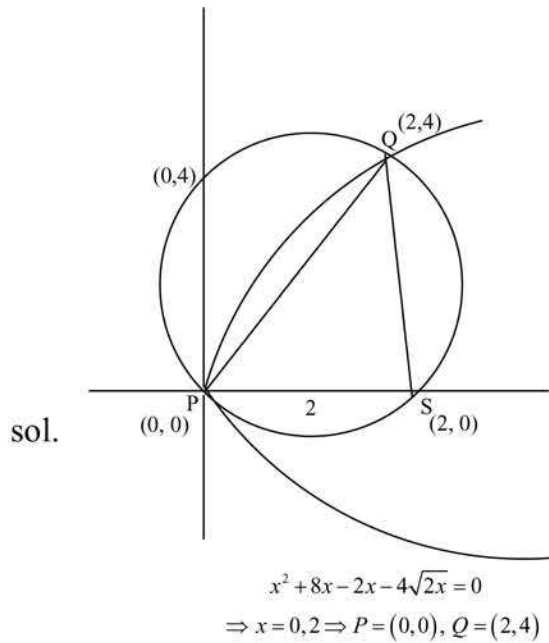
$$\therefore \text{Required answer is } 6 + \log_{3/2} \left(\frac{3}{2} \right)^{-2}$$

$$= 6 - 2$$

$$= 4$$

59. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is

Ans. 4



area of ΔSPQ

$$= \frac{1}{2} \times 2 \times 4$$

$$= 4$$

60. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is

Ans. 5

sol. $f(x) = -x + x^2 - 1, x \leq -1$

$$= -x + 1 - x^2, -1 < x < 0$$

$$= x + 1 - x^2, 0 \leq x < 1$$

$$= x + x^2 - 1, x \geq 1$$

$$f'(x) = 2x - 1, x < -1$$

$$= -2x - 1, -1 < x < 0$$

$$= 1 - 2x, 0 \leq x < 1$$

$$= 1 + 2x, x \geq 1$$

differentiable at $x = -\frac{1}{2}, \frac{1}{2}$,

not differentiable at $-1, 0, 1$

The number of points = 5

