## Test Paper Code : GP

**Time : 3 Hours          Max. Marks : 300**

### INSTRUCTIONS

1. The question-cum-answer booklet has **60** pages and has **66** questions. Please ensure that the copy of the question-cum-answer booklet you have received contains all the questions.

2. Write your **Registration Number, Name and the name of the Test Centre** in the appropriate space provided on the right side.

3. Write the answers to the objective questions against each Question No. in the **Answer Table for Objective Questions**, provided on Page No. **13**. Do not write anything else on this page.

4. Each objective question has **4 choices** for its answer : (A), (B), (C) and (D). Only **ONE** of them is the correct answer. There will be **negative marking** for wrong answers to objective questions. The following marking scheme for objective questions shall be used :

   (a) For each correct answer, you will be awarded 3 **(Three)** marks.

   (b) For each wrong answer, you will be awarded –1 **(Negative one)** mark.

   (c) Multiple answers to a question will be treated as a wrong answer.

   (d) For each un-attempted question, you will be awarded **0 (Zero)** mark.

   (e) Negative marks for objective part(s) will be carried over to total marks.

5. Answer the subjective question only in the space provided after each question.

6. Do not write more than one answer for the same question. In case you attempt a subjective question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing last only will be evaluated.

7. All answers must be written in blue/black/ blue-black ink only. Sketch pen, pencil or ink of any other colour should not be used.

8. All rough work should be done in the space provided and scored out finally.

9. No supplementary sheets will be provided to the candidates.

10. **Clip board, log tables, slide rule, calculator, cellular phone or electronic gadgets in any form are NOT allowed.**

11. The question-cum-answer booklet must be returned in its entirety to the Invigilator before leaving the examination hall. Do not remove any page from this booklet.

**READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY**

| REGISTRATION NUMBER |
|---|

Name :

Test Centre :

**Do not write your Registration Number or Name anywhere else in this question-cum-answer booklet.**

I have read all the instructions and shall abide by them.

.......................................    ....................................

Signature of the Candidate

I have verified the information filled by the Candidate above.

.......................................    ....................................

Signature of the Invigilator

## IMPORTANT NOTE FOR CANDIDATES

- **Geology Section** : **Q. Nos. 1–15 (Objective Questions) and Q. Nos. 46–52 (Subjective Questions)**
- **Physics Section** : **Q. Nos. 16–30 (Objective Questions) and Q. Nos. 53–59 (Subjective Questions)**
- **Mathematics Section** : **Q. Nos. 31–45 (Objective Questions) and Q. Nos. 60–66 (Subjective Questions)**
- Select any _TWO_ Sections.
- Attempt objective and subjective questions of the selected _TWO_ Sections.
- Questions 1-45 (objective questions) carry _three_ marks each and questions 46-66 (subjective questions) carry _fifteen_ marks each.
- Write the answers to the objective questions in the _Answer Table for Objective Questions_ provided on page 13 only.

## GEOLOGY SECTION

Q.1   A crescent-shaped dune with horns pointing down wind is

(A)   transverse dune

(B)   parabolic dune

(C)   barchan dune

(D)   star dune

Q.2   Match the stratigraphic Group in **Column-1** with its respective Supergroup in **Column-2**.

| Column-1 | Column-2 |
|---|---|
| P. Panchet Group | 1. Aravalli Supergroup |
| Q. Champaner Group | 2. Dharwar Supergroup |
| R. Hindoli Group | 3. Gondwana Supergroup |
| S. Bababudan Group | 4. Bhilwara Supergroup |

(A)   P-3, Q-1, R-4, S-2

(B)   P-1, Q-4, R-3, S-2

(C)   P-2, Q-4, R-1, S-3

(D)   P-2, Q-1, R-4, S-3

Q.3
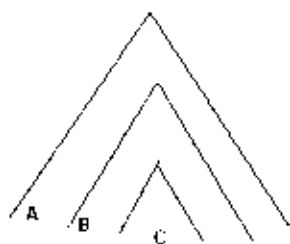


If bed "C" is the youngest and "A" is the oldest, then the above diagram represents

(A)  anticline                              (B)  antiform
(C)  synformal anticline                    (D)  antiformal syncline

Q.4  Lines joining points of equal dip on the outer and inner arc of a fold are called

(A)  isogyres        (B)  isogons        (C)  isochrones        (D)  contours

Q.5  Match the terms in **Group-1** with those in **Group-2**.

| **Group-1** | **Group-2** |
| --- | --- |
| P. Liquefaction | 1. Glacier |
| Q. Cirques | 2. Terrace |
| R. Drainage | 3. Dendritic |
| S. Flood Plain | 4. Earthflow |
|  | 5. Karst topography |

(A)  P-4, Q-1, R-3, S-2                    (B)  P-1, Q-2, R-5, S-3
(C)  P-5, Q-1, R-3, S-4                    (D)  P-4, Q-5, R-3, S-2

Q.6  If $\lambda_1, \lambda_2, \lambda_3$ (where $\lambda_1 > \lambda_2 > \lambda_3$) are the three principal axes of strain ellipsoid, then plane strain is represented by

(A)  $\lambda_1 > \lambda_2 = 1 > \lambda_3$            (B)  $\lambda_1 = 1 > \lambda_2 > \lambda_3$
(C)  $\lambda_1 > \lambda_2 = \lambda_3$               (D)  $\lambda_1 = \lambda_2 > \lambda_3$

Q.7  Earth's main magnetic field is generated due to

(A)  magnetic materials present in the crust  (B)  inter planetary interactions
(C)  electric current present in the outer core  (D)  polar wandering

Q.8  Consider the reaction given below related to contact metamorphism of calcareous sediments. Choose the correct mineral to complete the reaction

$$\text{calcite} + \text{quartz} = \underline{\hspace{2cm}} + CO_2$$
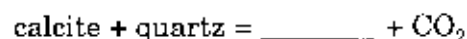
(A)  anorthite        (B)  diopside        (C)  ankerite        (D)  wollastonite

Q.9 The $(SiO_4)^{4-}$ tetrahedra in minerals of garnet group are all

(A) linked unidirectionally defining a chain

(B) unlinked and isolated with cations between them

(C) linked to form two-dimensional layers

(D) polymerized and occur in three dimensions

Q.10 Match the type of deposits in **Group-1** with their geographical location in **Group-2**.

| **Group-1** | **Group-2** |
|---|---|
| P. Baryte | 1. Tongpal |
| Q. Magnesite | 2. Byrapur |
| R. Chromite | 3. Mangampeta |
| S. Tin | 4. Chalk Hills |
|  | 5. Pulivendla |

(A) P-3, Q-1, R-4, S-2          (B) P-4, Q-3, R-5, S-1

(C) P-3, Q-4, R-2, S-1          (D) P-2, Q-1, R-5, S-3

Q.11 Sediments with unstratified clays consisting angular, subangular and rounded blocks of rock, mostly polished and striated are known as

(A) varve          (B) polymictic conglomerate

(C) claystone          (D) till

Q.12 An ultramafic rock with olivine, orthopyroxene and clinopyroxene as major minerals is classified as

(A) dunite          (B) peridotite          (C) kimberlite          (D) pyroxenite

Q.13 Which one of the following ore minerals gives off visible light when it is struck by ultraviolet rays?

(A) wolframite          (B) wurtzite          (C) scheelite          (D) selenite

Q.14 The twin plane related to "Manebach law" in orthoclase is

(A) 001          (B) 110          (C) 0$\bar{1}$0          (D) 201

Q.15 Gravity ($g$) variation from Equator to Pole on the earth's surface is the combined effect of several factors. Which one of the following statements is **NOT** true?

(A) Gravity decreases at Equator due to larger centrifugal acceleration at the Equator

(B) Gravity at Equator increases due to excess mass of Equatorial bulge

(C) Gravity reduces at Equator due to longer Equatorial radius

(D) Gravity reduces at Pole due to shorter Polar radius

## PHYSICS SECTION

Q.16 A point charge $q$ is placed at $(0, 0, d)$ from a thin, infinite conducting sheet kept in the $xy$-plane $(z = 0)$. If the sheet is grounded, the electric field at a point $(0, 0, -r)$ is

(A) dependent on $q$ and $d$ only  (B) zero

(C) dependent on $q$, $d$ and $r$  (D) dependent on $q$ and $r$ only

Q.17 An infinitely long straight conductor and a rectangular frame made of a conducting wire are kept in such a way that the two long sides of the frame are parallel to the straight conductor. The distances between the conductor and the two long sides of the frame are $l_1$ and $l_2$ $(l_2 > l_1)$. If the length of the frame is $h$, the mutual inductance between the conductor and the frame is

(A) $\dfrac{\mu_0 h}{2\pi} \ln\left(\dfrac{l_2}{l_1}\right)$  (B) $\dfrac{\mu_0}{2\pi h} \ln\left(\dfrac{l_2}{l_1}\right)$

(C) $\dfrac{\mu_0 h}{2\pi}\left(\dfrac{l_2}{l_1}\right)$  (D) $\dfrac{\mu_0}{2\pi h}\left(\dfrac{l_2}{l_1}\right)$

Q.18 Continuity equation for an incompressible fluid of density $\rho$ and having a velocity field $\vec{v}(x, y, z, t)$ and pressure $p(x, y, z, t)$ is

(A) $\vec{\nabla} \cdot \vec{v} = 0$  (B) $\vec{\nabla} \times \vec{v} = 0$

(C) $\dfrac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\, \vec{v} = -\dfrac{1}{\rho}\vec{\nabla} p$  (D) $\dfrac{1}{2}\rho v^2 + p = \text{constant}$

Q.19 The X-ray diffraction pattern of a material contains only reflections with all-even or all-odd Miller indices. Then the crystal structure will be

(A) simple cubic  (B) hexagonal

(C) face centred cubic  (D) body centred cubic

Q.20 An ideal gas is the one for which

(A) $C_V$ is a function of volume only

(B) $C_V$ is a function of volume and temperature

(C) $\left(C_p / C_V\right)$ is not a function of temperature

(D) $C_p$ and $C_V$ depend on temperature only

Q.21 One mole of an ideal gas is kept in one half of an adiabatic container and the other half is empty. The two halves of the container are separated by an adiabatic wall. The gas is now allowed to expand freely into the empty half by removing the wall. Then,

(A) there is an increase in the entropy of the gas
(B) there is an increase in the temperature of the gas
(C) there is a decrease in the entropy of the gas
(D) there is a decrease in the temperature of the gas

Q.22 Two X-ray tubes operate with the accelerating voltages are $V$ and $2V$. The work function of the material of the cathodes in them are $\phi$ and $2\phi$, respectively. Then the ratio of the minimum wavelengths of the continuous X-rays produced by them is

(A) $1:4$      (B) $1:1$      (C) $4:1$      (D) $2:1$

Q.23 The ratio of binding energy of an alpha particle to that of a deuteron ($^2_1$H) is
$(m_p c^2 = 938.3 \text{ MeV}, m_n c^2 = 939.6 \text{ MeV}, m_\alpha c^2 = 3727.2 \text{ MeV}, m_d c^2 = 1875.7 \text{ MeV})$

(A) 11      (B) 12      (C) 13      (D) 14

Q.24 The de Broglie wavelength of a particle having a mass $900 \text{ MeV}/c^2$ and kinetic energy $2$ MeV is (Planck's constant, $h = 4.2 \times 10^{-21}$ MeV sec.)

(A) $3.8 \times 10^{-12}$ m   (B) $6.2 \times 10^{-13}$ m    (C) $2.1 \times 10^{-14}$ m   (D) $4.0 \times 10^{-15}$ m

Q.25 In a certain medium, the wave number $k$ and the frequency $\omega$ are related by the dispersion relation $\omega^2 = c^2 k^2 (1 + ak^2)$, where $c$ and $a$ are constants. If $v_g$ is the group velocity and $v_p$ is the phase velocity, then

(A) $v_g = v_p$

(B) $\dfrac{v_g}{v_p} = \dfrac{(1 + ak^2)}{(1 + 2ak^2)}$

(C) $v_g \cdot v_p = c^2$

(D) $v_g \cdot v_p = c^2(1 + 2ak^2)$

Q.26 Consider the following forced oscillator equation:
$\dfrac{d^2x}{dt^2} + 2\beta \dfrac{dx}{dt} + 16x = 9\cos(5t)$, where the damping coefficient $\beta$ is a constant. As $\beta \to 0$, the general solution of $x(t)$ is given by $x(t) = \cos(5t + \varphi) + A\cos(4t + \psi)$. The phase, $\varphi$, tends to

(A) $\pi$      (B) $-\pi$      (C) $0$      (D) $\pi/2$

Q.27  Superposition of two sinusoidal waves $x(t) = A\sin(\omega_1 t)$ and $y(t) = B\sin(\omega_2 t)$ produces the following Lissajous figure. The ratio $\dfrac{\omega_1}{\omega_2}$ is
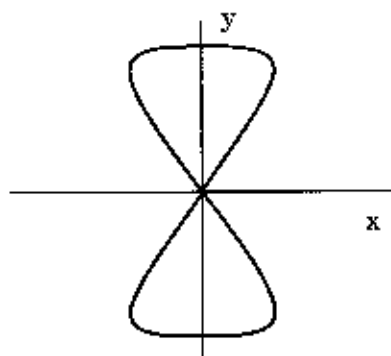


(A)  1/2          (B)  1          (C)  2          (D)  4

Q.28  The equivalent capacitance between the points A and B in the following circuit is
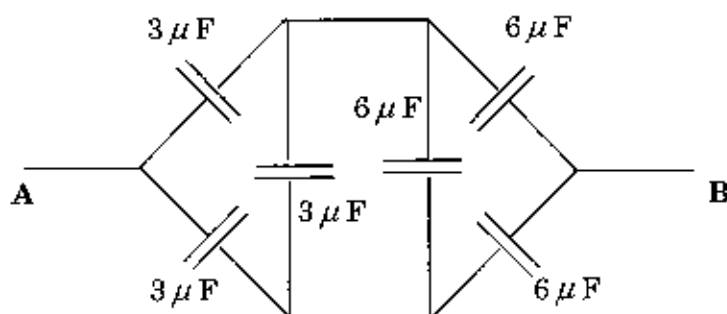


(A)  $3\,\mu F$          (B)  $4\,\mu F$          (C)  $6\,\mu F$          (D)  $9\,\mu F$

Q.29  Which of the following **CANNOT** be a solution of the one dimensional wave equation:
$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u(x,t)}{\partial t^2}$$

(A)  $A\cos k(x + ct)$                    (B)  $A\exp[ik(x - ct)]$

(C)  $A\exp[-\alpha(x - ct)^2]$              (D)  $\dfrac{A}{(x^2 - c^2 t^2)}$

Q.30  For a completely inelastic collision of two particles in one dimension, which of the following statements is **NOT** correct?

(A)  The kinetic energy is conserved
(B)  The linear momentum is conserved
(C)  The particles move with a common velocity after the collision
(D)  The total energy is conserved

## MATHEMATICS SECTION

Q.31 The critical or stationary points of $f(x,y) = (x^2 + y^2)e^{5x}$ are

(A) (0, 0), (2, 0)

(B) (1, 1), (−2/5, 0)

(C) (0, 0), (−2/5, 0)

(D) (0, 0), (2/5, 0)

Q.32 The series $\sum_{n=1}^{\infty} \dfrac{n}{2n+1}$

(A) converges to 0

(B) converges to 1/2

(C) converges to 1

(D) diverges

Q.33 Let $A_n = \left[\dfrac{n}{n+1}, \dfrac{n+1}{n+2}\right]$, $n = 1, 2, 3 \ldots$, be closed subsets of the real line **R**. Then $\bigcup_{n=1}^{\infty} A_n$ is the set

(A) (1/2, 1)

(B) [1/2, 1)

(C) (1/2, 1]

(D) [1/2, 1]

Q.34 For random samples of size $n$ drawn from a normal population with unit variance, the variance of $\sum_{i=1}^{n} (X_i - \overline{X})^2$ is

(A) $n-1$

(B) $n$

(C) $n^2$

(D) $2(n-1)$

Q.35 The value of the integral $\oint_C (x + y)dx + x^2 dy$, where $C$ is the triangle with vertices (0, 0), (2, 0) and (2, 4) in the anticlockwise direction is

(A) 5/3

(B) 10/3

(C) 20/3

(D) 40/3

Q.36 Which one of the following functions **CANNOT** be characteristic function of a random variable?

(A) $\sin t$

(B) $\cos t$

(C) $e^{it}$

(D) $e^{-it}$

Q.37 For a constant vector $\vec{a}$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, consider the following statements:

I.   $curl(\vec{a} \times \vec{r}) = 2\,grad(\vec{a} \cdot \vec{r})$      II.   $div\,[(\vec{a} \cdot \vec{r})\,\vec{r}] = 2(\vec{a} \cdot \vec{r})$

then,

(A)   both statements are true          (B)   only statement I is true

(C)   only statement II is true          (D)   both statements are false


Q.38 An iterative sequence of Newton-Raphson method to find the value of the reciprocal of a positive integer $N$ is given by

(A)   $x_{n+1} = x_n - Nx_n^2$          (B)   $2x_{n+1} = x_n - Nx_n^2$

(C)   $x_{n-1} = 2x_n - Nx_n^2$          (D)   $x_{n+1} = -x_n + 2Nx_n^2$


Q.39 For the backward difference operator $\nabla$, the operator $\nabla(1-\nabla)^{-1/2}$ equals

(A)   $\delta$          (B)   $\nabla$          (C)   E          (D)   $\Delta$


Q.40 Let $X$ be a binomial variable such that $3P(X = 3) = 2P(X = 2)$. Then the mean of $X$ is

(A)   1/3          (B)   1/2          (C)   1          (D)   2


Q.41 The set of all limit points of the set $\{\, x : x \in \mathbf{R} \text{ and } x < 1 \text{ or } x > 2 \,\}$ is

(A)   $\{1, 2\}$          (B)   $(-\infty, 1) \cup (2, \infty)$

(C)   $(-\infty, 1] \cup [2, \infty)$          (D)   $[1, 2]$


Q.42 The solution of the initial value problem $\dfrac{dy}{dx} = \dfrac{\sin x}{y+2}$, $y(0) = 0$, is

(A)   $y(y+2) = 4(1 - \cos x)$          (B)   $y^2 = 2(1 - \cos x)$

(C)   $y(y+4) + \cos x = 0$          (D)   $y(y+4) = 2(1 - \cos x)$

Q.43 If $e^{2x}$ and $xe^{2x}$ are particular solutions of a second order homogeneous differential equation with constant coefficients, then the equation is

(A) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 0$

(B) $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 0$

(C) $\dfrac{d^2y}{dx^2} - 4y = 0$

(D) $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = 0$

Q.44 The rank of the matrix $\begin{bmatrix} 1 & 2 & 4 & 2 \\ 1 & 1 & 3 & 2 \\ 2 & 4 & 8 & 4 \end{bmatrix}$ is

(A) 1       (B) 2       (C) 3       (D) 4

Q.45 The value of $\displaystyle\int_{0}^{2+i} \bar{z}\, dz$ along the straight line $x = 2y$ is

(A) $\dfrac{5}{2}$       (B) $\dfrac{3+4i}{2}$       (C) $1 - \dfrac{i}{2}$       (D) 5

Space for rough work

Space for rough work

**Space for rough work**

Space for rough work

## Answer Table for Objective Questions

Write the Code of your chosen answer only in the 'Answer' Column against each Question No. Do not write anything else on this page.

| Question No. | Answer | Do not write in this column | Question No. | Answer | Do not write in this column |
|---|---|---|---|---|---|
| 01 | | | 31 | | |
| 02 | | | 32 | | |
| 03 | | | 33 | | |
| 04 | | | 34 | | |
| 05 | | | 35 | | |
| 06 | | | 36 | | |
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**FOR EVALUATION ONLY**

| Correct | | Marks | (+) |
|---|---|---|---|
| Incorrect | | Marks | (−) |
| Total Marks in Q. Nos. 1-15 | | ( ) | |

| Correct | | Marks | (+) |
|---|---|---|---|
| Incorrect | | Marks | (−) |
| Total Marks In Q. Nos. 16-30 | | ( ) | |

| Correct | | Marks | (+) |
|---|---|---|---|
| Incorrect | | Marks | (−) |
| Total Marks In Q. Nos. 31-45 | | ( ) | |

## GEOLOGY SECTION

Q.46    Draw and explain

(a)    how the increase in temperature, confining pressure and pore fluid pressure would
influence the ductility and strength of materials?                                               (9)

(b)    the difference between (i) Hinge line and Fold axis (ii) Plane cylindrical fold and
non-plane non-cylindrical fold.                                                                  (6)

Q.47   (a)   Draw and describe different types of Plate margins.     (9)

      (b)   Explain the Wilson Cycle.     (6)

Q.48 (a) Arrange the following in chronological order (Oldest to Youngest) :

    (i) Chotanagpur Gneiss, Ladakh Granitoid, Peninsular Gneiss

    (ii) Rajmahal volcanics, Dalma volcanics, Panjal volcanics

    (iii) Dhok Pathan Formation, Jhamarkotra Formation, Nagthat Formation    (9)

  (b) Give the chronostratigraphic divisions of "Siwalik Group".    (6)

A

Q.49    (a)    (i)    What is the mathematical relationship among retardation, birefringence and thickness of an anisotropic mineral in thin section?

(ii)    Draw a cross-section (parallel to optic axis) of ray velocity surface for uniaxial negative minerals.      (6)

(b)    Why anisotropic minerals exhibit interference colours under Crossed Polars?      (9)

Q.50 (a) Explain the formation of laterite-bauxite from basaltic rocks. (6)

(b) (i) What is the main host rock associated with Pb-Zn mineralization in Zawar?

(ii) Give names of two Pb-Zn mines in Zawar belt.

(iii) What is the mode of occurrence of the ore body in Zawar? (9)

Q.51 (a) Consider a binary system involving forsterite-silica as end members related to magmatic crystallization.

     (i) What is peritectic point?

     (ii) Indicate the formation of a binary compound at peritectic point with the chemical reaction. (6)

  (b) (i) Explain the formation of "graphic texture".

     (ii) Give the characteristic mineral assemblage for basic igneous rocks metamorphosed to amphibolite and eclogite facies.

     (iii) Give an example each of silica, phosphate and sulfate minerals commonly found in chemical sediments. (9)

Q.52    (a)    Explain the demerits of Kant and Laplace Nebular hypothesis in explaining the
               origin of the solar system.                                                (9)

        (b)    Describe Airy and Prat hypothesis of isostacy with suitable diagrams.      (6)

### PHYSICS SECTION

Q.53 A conducting sphere of radius $R_1$ is coated over the entire surface with a linear dielectric material of dielectric constant (relative permittivity) $\kappa$ up to a radius of $R_2$ $(R_2 > R_1)$. A positive charge $Q$ is given to the conductor.

  (a) Find the bound (polarization) charges on the dielectric layer at $r = R_1$ and $r = R_2$. (9)

  (b) The electric fields in the region $r < R_1$ and $r > R_2$. (6)

Q.54  The longest wavelength in the Balmer series of hydrogen atom is given as 656 nm.

(a)  What is the longest wavelength of the Lyman series in this case?  (9)

(b)  Positronium can be considered as a system of an electron and a positron in which both these particles orbit each other. What will be the longest wavelength in the Lyman series for positronium?  (6)

Q.55 A Carnot engine working between hot and cold reservoirs of absolute temperatures $T_1$ and $T_2$ respectively, has an efficiency of 50%.

(a) Find out the ratio of the heat that is absorbed from the hot reservoir to that released to the cold reservoir. (9)

(b) If the absolute temperature of the hot reservoir is made $2T_1$ while keeping the temperature of the cold reservoir unchanged, what will be the new efficiency? (6)

Q.56 A solid cylinder of mass $m$, radius $a$ and height also $a$ and a solid sphere of the same mass and the same radius are connected to the two ends of a thin rod of mass $m/5$ and length $4a$. The line joining the centres of mass of the cylinder and the sphere coincides with the axis of the rod. The vertical cross section of the assembly is shown in the figure below. The whole assembly is suspended vertically by a massless wire passing through its centre of mass. The torsional constant of the wire is $76\, ma^2/\sec^2$. Calculate the frequency of the torsional oscillation about the suspension in the horizontal plane. (15)
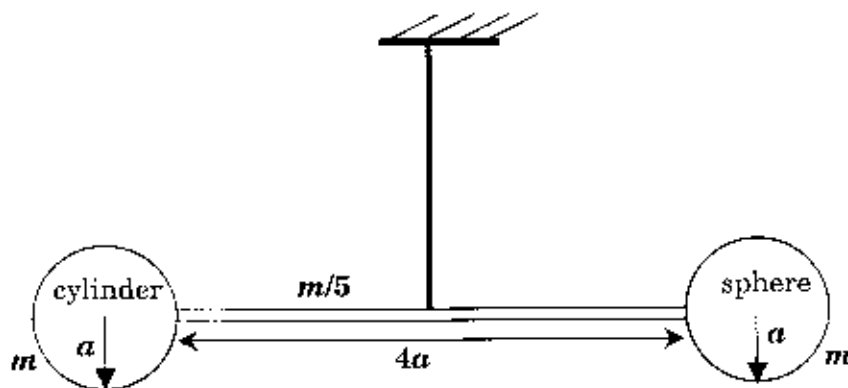
Q.57  A ray of light passing through the origin **O** along the x-axis reaches the point **A** on the screen **S**, placed perpendicular to the x-axis. A glass plate of thickness $t$ and refractive index $\mu$ is introduced in the path, making an angle $\theta$ with the x-axis, as shown in the figure. Calculate the extra optical path that the ray has to travel to reach the point **A'** on the screen. (15)
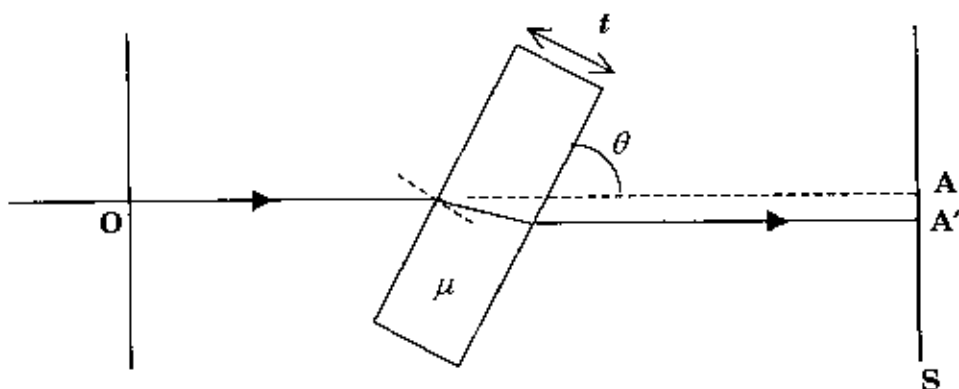
Q.58 A constant magnetic field $\vec{B} = B_0\hat{j}$ and a constant electric field $\vec{E} = E_0\hat{j}$ exist in a region. A sinusoidally varying electric field $\vec{E'} = E_1\sin(\omega t)\hat{k}$ is also applied in the same region. A particle with positive charge $q$ and mass $m$ is ejected from the origin with the velocity $v_0\hat{i}$ at time $t = 0$.

(a) Find the equations of motion for the three velocity components of the particle. (9)

(b) What is the $y$ coordinate of the particle at time $t$? (6)

Q.59  Suppose a comet of mass $\sqrt{20} \times 10^{13}$ kg is moving in a parabolic orbit around the Sun and has a possibility of colliding at its perihelion with Venus. Assume that Venus is moving in a circular orbit around the Sun and takes 225 days to complete a revolution around the Sun. Calculate the angular momentum of the comet about the Sun. Neglect the interaction between the comet and Venus.

(Mass of the Sun $= 2 \times 10^{30}$ kg. Assume gravitational constant $G = \left(\dfrac{20}{3}\right) \times 10^{-11}$ in SI units and $\pi^2 = 10$.) (15)

## MATHEMATICS SECTION

Q.60 (a) For the sequence $\{x_n\}$, where $x_1$ is a positive real number strictly less than 3 and $x_{n+1} = \dfrac{12(1 + x_n)}{13 + x_n}$, $n = 1, 2, 3 \ldots$,

    (i) prove that $x_n < 3$, for all $n = 1, 2, 3 \ldots$

    (ii) show that $\{x_n\}$ is strictly monotone increasing and

    (iii) check whether $\{x_n\}$ is convergent.         (9)

(b) For the function $f(x,y) = \begin{cases} \dfrac{3x^3 - 5y^3}{x^2 + y^2} & \text{, if } x^2 + y^2 \neq 0 \\ 0 & \text{, if } x^2 + y^2 = 0 \end{cases}$ , find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at $(0, 0)$. (6)

Q.61 (a) Find the radius of convergence and the exact interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty}\frac{x^{n}}{\sqrt{n+1}}$ . (9)

(b) For the function $f(z)=|z|^{2}$ of complex variable $z$,

  (i)   find the derivative of $f(z)$ at $z=0$ and

  (ii)   check the analyticity of $f(z)$ at $z=0$ . (6)

Q.62   (a)   Show that the following equation is exact and then solve it:

$$\left\{(x+1)e^x - e^y\right\}dx = xe^y dy .$$

                    (6)

     (b)   Find the general solution of $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 10y = 25x^2 + 3$.

                    (9)

Q.63 (a) Suppose that the probability of a successful strike when drilling for oil is 0.05. The conditional probability of rock condition R when there is oil below the rock is 0.8 and it is 0.5 in the absence of oil. If the condition of the rock at a site is observed to be R, what is the chance of striking oil at this site? (6)

(b) If $X$ and $Y$ have the joint probability density function,

$$f(x, y) = 2(x + y - 2xy), \ \ 0 \le x \le 1, \ \ 0 \le y \le 1,$$

then find the correlation coefficient between $X$ and $Y$. (9)

Q.64 (a) Evaluate the integral $\int_{0}^{1/2}\dfrac{dx}{1+2x}$ up to four decimal places using Simpson's one-third rule with four equal subintervals. (6)

(b) Five measurements of the heat producing capacity (in millions of calories per ton) of coal from a mine are, 11, 9, 12, 10 and 8. Assuming that the population follows a normal distribution, find 99% confidence interval for the mean of the population. $(t_{0.005,4}=4.6041,\ t_{0.005,5}=4.0322)$ (9)

Q.65 (a) If $\vec{v} = x^2 z \hat{i} + y \hat{j} - xz^2 \hat{k}$ and $S$ is the surface of the closed cylinder $x^2 + y^2 = 16$, $z = 0$ and $z = 4$, evaluate the integral $\iint_S \vec{v} \cdot \hat{n} \, dA$. (6)

(b) Find a function $\phi(x, y, z)$ such that
$$d\phi(x, y, z) = yz \, dx + (z + xz + z^2) dy + (y + xy + 2yz) dz$$
and $\phi(0, 1, -1) = 0$. (9)

Q.66  For the matrix $A = \begin{vmatrix} -1 & 0 & 2 \\ 0 & 2 & 3 \\ 0 & 1 & 0 \end{vmatrix}$, find

    (i)   **all** eigenvalues of $A$,

    (ii)  **all possible** eigenvectors corresponding to every eigenvalue of $A$ and

    (iii)  the dimension of the eigenspace corresponding to each eigenvalue of $A$.    (15)

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