Code No: A109210201 **R09** **Set No. 2**

**II B.Tech I Semester Examinations,December 2011**
**MATHEMATICS-III**
**Common to ICE, E.COMP.E, ETM, EIE, ECE, EEE**

**Time: 3 hours** **Max Marks: 75**

**Answer any FIVE Questions**
**All Questions carry equal marks**

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1. (a) S.T. $x^3 = \frac{2}{5}P_3(x) + \frac{3}{5}P_1(x)$.
   (b) Express f(x) = 2x+10x$^3$ interms of legendre polynomials. [15]

2. (a) If G = (V, E) be an undirected graph with 'e' edges. Then prove that the sum of the degrees of all the vertices of the graph is twice the number of vertices.
   (b) Write the adjacency Matrix to represent the graph (Figure 1). [8+7]

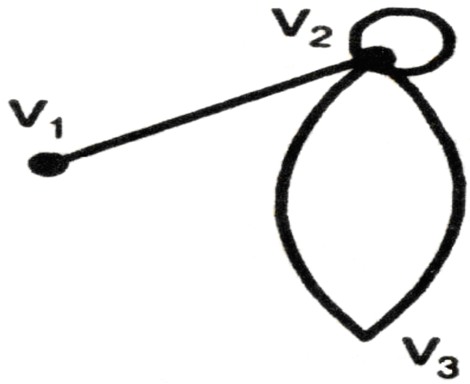


Figure 1:

3. (a) S.T. $\int_0^{\pi/2} \sqrt{\cos\theta} d\theta = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$
   (b) Evaluate $4\int_0^{\alpha} \frac{x^2}{1+x^4} dx$ Using $\beta - \Gamma$ functions [15]

4. (a) Evaluate $\int\limits_0^{\infty} \frac{Sin\, x}{x^2+4x+5}\, dx$
   (b) Evaluate by Residue theorem $\int\limits_c \frac{z-1}{(z+1)^2(z-2)}\, dz$ where C: $|z - i| = 2$ [7+8]

5. (a) Show that the transformation $w = \cos\ \ z$ maps the half of the z-plane to the right of the imaginary axis into the entire w-plane.
   (b) Show that the transformation maps the family of line parallel to y-axis to ellipse. [8+7]

6. (a) Evaluate $\int\limits_c \frac{3z^2+7z+1}{z+1} dz$ where C: $|z + i| = 1$

(b) Evaluate $\int\limits_c \frac{z^2-z+1}{z-1}dz$ where C: $|z| = 1/2$ taken in anticlockwise sense [15]

7. For the function f(z) = $\frac{2z^3+1}{z^2+z}$ find

(a) Find the Taylsor's series expansion of about z =3.

(b) Explain f(z)= Cos z in Taylor's series about z = [15]

8. P.T. The function f(z) defined by f(z) = $\begin{cases} \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2} & ; \ (z \neq 0) \\ 0 & ; \ (z = 0) \end{cases}$ is continuous and the C-R equations are satisfied at the origin, yet $f^1(0)$ does not exists. [15]

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Code No: A109210201 **R09** **Set No. 4**

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**Time: 3 hours** **Max Marks: 75**

**Answer any FIVE Questions**
**All Questions carry equal marks**

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1. (a) Express the following interms of legendre polynomials $4x^3-2x^2-3x+8$.

   (b) Evaluate $\int_{-1}^{1} x^2U_3(x)dx$. [15]

2. (a) Find whether the function u = $\log|z|^2$ is harmonic. If so, find the analytic function whose real part is u.

   (b) Separate the real and imaginary parts of $i^{log(1+i)}$. [15]

3. (a) Evaluate $\int_0^1 \frac{x^2}{\sqrt{1-x^5}}dx$ interms of Beta function.

   (b) S.T. $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left\{\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right\}$ [15]

4. (a) Evaluate $\int_C \frac{z-3}{z^2+2z+5}$ where C is

   i. $|z| = 1$
   ii. $|z+1-i| = 2$
   iii. $|z+1+i| = 2$ [15]

   (b) Evaluate $\int_C \frac{5z^2-3z+2}{(z-1)^3}dz$ where c is any simple closed curve enclosing z = 1 [15]

5. (a) Evaluate $\int_0^{\infty} \frac{1}{1+x^6}\,dx$

   (b) Evaluate by Residue theorem $\int_c \frac{4z^2-4z+1}{(z-2)(z^2+4)}\,dz$ where C: $|z| = 1$. [8+7]

6. (a) Draw graph having the given property or explain why no such graph exists.

   i. Graph with 4 vertices of degree 1, 1,3and 3
   ii. Graphs with five vertices of degree 0,1,2,2 and 3
   iii. Graphs with six vertices each of degree 3
   iv. Simple Graphs with 4 vertices of degree 1, 1, 3, and 3

   (b) Prove that number of edges in a bipartite graph with n vertices is at most ( $n^2/4$)

   [8+7]

7. Find the bilinear transformation which maps $z_1 = 1, z_2 = i, z_3 = -1$ in to the points $w_1 = 2, w_2 = i, w_3 = -2$ respectively. Find the fixed and critical points of this transformation. [15]

8. (a) Represent the function f (z) = $\frac{1}{z(z+2)^3(z+1)^2}$ in Laurent series with in $\frac{5}{4} \leq |z| \leq \frac{7}{4}$

   (b) Define for a complex function (z)

   i. Isolated Singularity
   ii. Removable Singularity
   iii. Essential singularity [15]

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Code No: A109210201 **R09** **Set No. 1**

**II B.Tech I Semester Examinations,December 2011**
**MATHEMATICS-III**
**Common to ICE, E.COMP.E, ETM, EIE, ECE, EEE**

**Time: 3 hours** **Max Marks: 75**

**Answer any FIVE Questions**
**All Questions carry equal marks**

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1. (a) P.T. $\int_0^\alpha e^{-y^{1/m}} dy = m\Gamma(m)$
   (b) Express $J_2(x)$ interms of $J_0(x)$ & $J_1(x)$. [15]
2. (a) P.T. $(1-x^2)P_n^1(x) = (n+1)\{xP_n(x) - P_{n+1}(x)\}$
   (b) Express the following interms of legendre polynomials $1+x-x^2$. [15]
3. (a) If f = u + iv is analytic in a domain D and uv is constant in D, then prove that f(z) is constant.
   (b) Find the general and primial values of $\log(1+i\sqrt{3})$. [15]
4. Expand $\frac{7z-2}{(z+1)z(z-2)}$ about the point z = -1 in the region $1 < |z+1| < 3$ as Laurent's series. [15]
5. (a) Evaluate $\int\limits_c \frac{z^3+z^2+2z-1}{(z-1)^3} dz$ where C: $|z| = 3$
   (b) Evaluate $\int\limits_c \frac{z^4}{(z+1)(z-i)^2} dz$ where 'C' is the ellipse $9x^2+4y^2=36$ [15]
6. (a) Draw the undirected graph represented by the adjacency matrix A given below.
$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$
   (b) Show that the following graph (Figure 2) is not planar. [7+8]
7. (a) By the method of contour integration prove that $\int\limits_0^\infty \frac{Cos\, mx}{a^2+x^2} dx = \frac{\pi}{2a} e^{-ma}$
   (b) Evaluate by Residue theorem $\int\limits_c \frac{z^2}{(z-1)^2(z+2)} dz$ where $|z| = 3$ [8+7]
8. Find the bilinear transformation which maps $1+i, -i, 2-i$ of the z-plane in to the points 0,1, irespectively of the w-plane. Find the fixed and critical points of this transformation. [15]

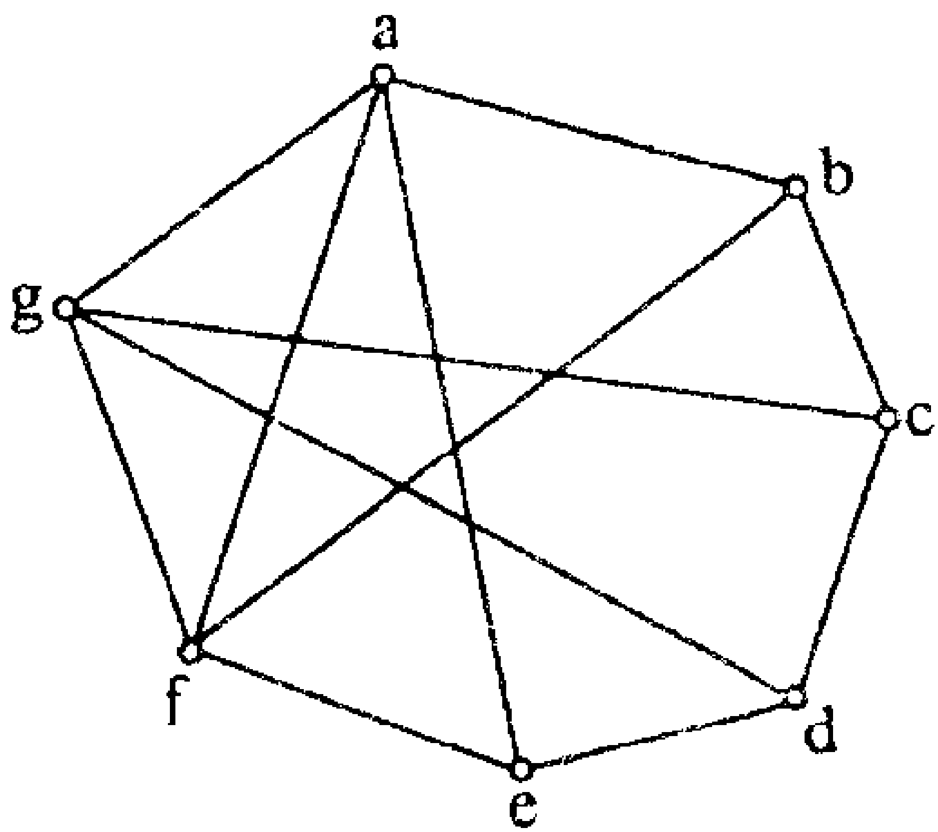


Figure – 2

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Code No: A109210201 **R09** **Set No. 3**

**II B.Tech I Semester Examinations,December 2011**
**MATHEMATICS-III**
**Common to ICE, E.COMP.E, ETM, EIE, ECE, EEE**

**Time: 3 hours** **Max Marks: 75**

**Answer any FIVE Questions**
**All Questions carry equal marks**
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1. (a) P.T. $P_n^1(-1) = (-1)^{n-1}\frac{n(n+1)}{2}$
   (b) Write $T_2(x) + T_1(x) + T_0(x)$ as a polynomial. [7+8]

2. (a) S.T. $\int_0^1 (1 - n\sqrt{x})^m \, dx = \frac{m!n!}{(m+n)!}$ where m&n are the integers.
   (b) S.T. $\frac{\beta(p,q+1)}{q} = \frac{\beta(p+1,q)}{p} = \frac{\beta(p,q)}{p+q}$ where p>0, q>0 [15]

3. (a) Evaluate $\int\limits_{(0,0)}^{(1,1)} ((x - y^2)dx + 2xydy)$ along $x_0$ curve y = x
   (b) Evaluate $\int\limits_{z=0}^{z=1+i} \{x^2 + 2xy + i(y^2 - x)\} \, dz$ along $y = x^2$ [15]

4. State and prove Taylor's Theorem of complex function f (z). [15]

5. (a) Find the in- degree and out- degree of each vertex of the following graph (Figure 3).
   (b) Show that every complete graph is regular. [8+7]

6. (a) If $W = \phi + i\Psi$ represents the complex potential for an electric field & $\Psi = x^2 - y^2 + \frac{x}{x^2+y^2}$, determine the function $\phi$.

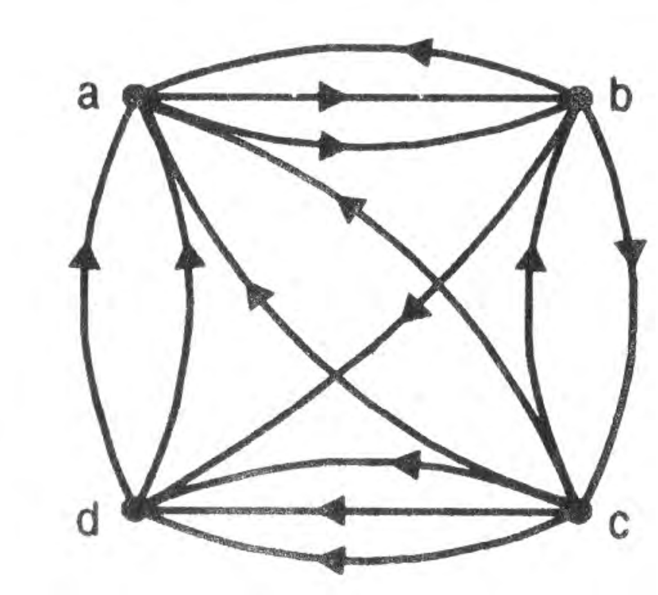


Figure 3:

(b) If f(z) = u+iv is an analytic function of z & u-y = $e^x$ (cosy-siny) find f(z) interms of z. [15]

7. (a) Evaluate $\int_{-\infty}^{\infty} \frac{x^{2}}{(1+x^2)(x^2+4)} dx$

(b) Evaluate by Residue theorem $\int_{c} \frac{Sinz}{z^6} dz$ where C: $|z| = 2$ [7+8]

8. Find the bilinear transformation which maps $z_1 = 1 - 2i, z_2 = 2 + i, z_3 = 2 + 3i$ in to the points $w_1 = 2 + 2i, w_2 = 1 + 3i, w_3 = 4$ respectively. Find the fixed and critical points. [15]

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