MATHEMATICS PAPER IIB - MARCH 2010

COORDINATE GEOMETRY & CALCULUS

TIME : 3hrs **Max. Marks.75**

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. **10X2 =20**

Noe : Attempt all questions. Each question carries 2 marks.

1. Obtain the parametric equation of each of the following circle $x^2+y^2-6x+4y-12=0$

2. Find the center and radius of the sphere $x^2+y^2+z^2-2x-4y-6z=11$

3. Find the value of k if the lines $2x+3y+4=0 \, and \; x+y+k=0$ are conjugate w.r.t $y^2=8x$

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5. Find the nth derivative of $\log\left(4-x^2\right)$

6. Evaluate $\int \left(\frac{1+x\log x}{x}\right)dx \; on \; (0,\infty)$

7. Evaluate $\int \frac{e^x(1+x)}{\cos^2\left(xe^x\right)}dx \; on \; I \subset R \setminus \left\{x \in R : \cos\left(xe^x\right)=0\right\}$

8. Evaluate $\int_0^{\frac{\pi}{2}} \cos^5 x \, \sin^4 x \, dx$

9. Find the area of the region enclosed by the given curves $x=4-y^2, x=0$

10. Find the order and degree of $\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{5/3}=0$

SECTION B

SHORT ANSWER TYPE QUESTIONS. **5X4 =20**

Note : Answer any FIVE questions. Each question carries 4 marks.

11. Find the equation of the circle whose center lies on X-axis and passing throgh the points (-2,3),(4,5).

12. Show that the equations of the commmon tangents to the circle $x^2+y^2=2a^2$ and the parabola $y^2=8ax$ are $y=(x+2a)$.

13. Find the eccentricity , foci and the equations of directrices of the following ellipse $4x^2 + y^2 - 8x + 2y + 1 = 0$

14. Show that the polar equation of a conic in the standard form is $\frac{l}{r} = 1 + e\cos\theta$
('l' is semilatus rectum, e is eccentricity)

15. Evaluate $\int \frac{dx}{5 + 4\cos x}$

16. Solve the differential equation $\left(x^2 + y^2\right)dy = 2xydx$

17. Solve the differential equation $\left(1 + x^2\right)\frac{dy}{dx} + y = e^{Tan^{-1}x}$

SECTION C

LONG ANSWER TYPE QUESTIONS. 5X7 =35

Note: Answer any Five of the following. Each question carries 7 marks.

18. Show that the circles $x^2+y^2-6x-2y+1=0$; $x^2+y^2+2x-8y+13=0$ touch each other. Find the point of contact and the equation of common tangent at the point of contact.

19. Find the limiting points of the coaxial system determined by the circles

$x^2+y^2+10x-4y-1=0, x^2+y^2+5x+y+4=0.$

20. If the polar of P with respect to the Parabola $y^2=4ax$ touches the circle $x^2+y^2=4a^2$, then show that P lies on the curve $x^2-y^2 = 4a^2$

21. If $y = \cos\left(m\log x\right), x > 0,$ then show that $x^2y_2 + xy_1 + m^2y = 0$ and hence deduce that $x^2y_{n+2} + \left(2n+1\right)xy_{n+1} + \left(m^2 + n^2\right)y_n = 0$

22. Evaluate $\int \frac{x+1}{\left(x^2 + 3x + 12\right)}dx$

23. Evaluate $\int_0^1 \frac{\log\left(1+x\right)}{1+x^2}dx$

24. Find the approximate value of π from $\int_0^1 \frac{1}{1+x^2}dx$ by using Simpson's rule by dividing [0, 1] into 4 equal parts.

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