MATHEMATICS PAPER IIB - MAY 2009

COORDINATE GEOMETRY & CALCULUS

TIME: 3hrs   **Max. Marks.75**

Note: This question paper consists of three sections A,B and C.

SECTION A

Very Short Answer Type Questions. $10 \times 2 = 20$

Note : Attempt all questions. Each question carries 2 marks.

1. If the canter of the circle $x^2 + y^2 + ax + by - 12 = 0$ is (2,3) find the values of a, b and the radius of the circle.
2. Find the equation of the sphere that passes through the point (4, 3, -1) and having its centre (3,8,1).
3. Find the coordinates of the points on the parabola $y^2 = 2x$ whose focal distance is 5/2.
4. Find the equations of the tangents to the hyperbola $3x^2 - 4y^2 = 12$ which is parallel to the line
   y=x-7
5. Find the nth derivative of $f(x) = \log(8x^3 + 36x^2 + 54x + 27)$
6. Evaluate $\int \sec^2 x \cos ec^2 x \, dx$

7. Evaluate $\int \dfrac{e^x (1+x)}{(2+x)^2} dx$ b

8. Evaluate $\int\limits_{-\pi/2}^{\pi/2} \sin^2 x \cos^4 x \, dx$

9. Find the area of the enclosed by the curve $f(x) = \sin x$ in the interval $[0, 2\pi]$
10. Form the differential equation corresponding to $y = cx - 2c^2$, where c is a parameter.

SECTION B

Short Answer Type Questions.   **5X4 = 20**

Note: Answer any FIVE questions. Each question carries 4 marks.

11. Show that x+y+1=0 touches the circle $x^2 + y^2 - 3x + 7y + 14 = 0$ and find the point of contact.

12. Prove that the poles of tangents to the parabola $y^2 = 4ax$ w.r.t the parabola $y^2 = 4bx$ lie on

    Parabola.

13. One focus of hyperbola located at (1, -3) and corresponding directrix in the line y =2. Find the equation of hyperbola if its eccentricity is 3/2.

14. If PSQ is chord passing through the focus S of a conic and '*l*' is semi lotus rectum, show that

$$\frac{1}{SP}+\frac{1}{SQ}=\frac{2}{l}$$

15. Evaluate $\int\frac{1}{(1-x)(4+x^2)}dx$

16. Solve $(x^2-y^2)\,dx - xy\,dy = 0$

17. Solve $(1+y^2)dx=(\tan^{-1}y-x)dy$

SECTION C

Long Answer Type Questions.   **5X7 =35**

Note: Answer any Five of the following. Each question carries 7 marks.

18. Find the equation of the circle whose centre lies on X-axis and passing through the points

    (-2,3),(4,5).

19. In the limiting points of the coaxial system determined by the circles $x^2+y^2+2x-6y=0$ and $2x^2+2y^2-10y+5=0$.

20. Find eccentricity, coordinates of foci and equations of directories of the ellipse

    $9x^2+16y^2-36x+32y-92 = 0$

21. If $y=\dfrac{\sinh^{-1}x}{\sqrt{1+x^2}}$, then show that $(1+x^2)y_2+3xy_1+y=0$ and hence deduce that

    $(1+x^2)y_{n+2}+(2n+3)xy_{n+1}+(n+1)^2\,y_n=0$

22. Obtain the reduction formula for $I_n=\int\sin^n xdx$, n being a positive integer, $n\geq2$ and deduce the value of

    $\int\sin^4 x\,dx$

23. Show that $\displaystyle\int_0^{\pi/2}\frac{x}{\sin x+\cos x}dx=\frac{\pi}{2\sqrt2}\log\left(\sqrt2+1\right)$

24. Divinding [0,6] into 6 equal parts evaluate $\int_0^6 x^3dx$ approximately by using Trapezoidal rule and Simpson's rule.