**MATHEMATICS PAPER IIA - MAY 2009**

**ALGEBRA & PROBABILITY**

**TIME : 3hrs** **Max. Marks.75**

**Note: This question paper consists of three sections A,B and C.**

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## SECTION A

**Very Short Answer Type Questions.** **10X2 =20**

**Note : Attempt all questions. Each question carries 2 marks.**

1. Prove that the roots of $(x-a)(x-b) = h^2$.

2. If $\alpha, \beta$ and 1 are the roots of $x^3-2x^2-5x+6 = 0$ then find $\alpha$ and $\beta$.

3. If $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \end{bmatrix}$ *and* $B = \begin{bmatrix} -3 & 4 & 0 \\ 4 & -2 & -1 \end{bmatrix}$ then show that (A+B)' = A'+ B'

4. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$ and det A = 45, find x.

5. If ${}^nP_7 = 42\,{}^nP_5$ and ${}^nC_r = 210$, find n.

6. Find the number of ways of selecting 3 girls and 3 boys out of 7 girls and 6 boys.

7. Find the range of the infinite series $\frac{1}{1!}+\frac{1+2}{2!}+\frac{1+2+3}{3!}+....$

8. A problem in calculus is given to two students A and B whose chances of solving it are 1/3 and ¼. Find the probability of the problem being solved, if both of them try independently.

9. Find the range of x for which the binomial expansion $(7+3x)^{-5}$ is valid.

10. If the mean and variance of a binomial variable X are 2.4 and 1.44 respectively then find p and q.

## SECTION B

**Short Answer Type Questions.** **5x4 =20**

**Note: Answer any FIVE questions. Each question carries 4 marks.**

11. If the expression $\frac{x-p}{x^2-3x+2}$ takes all real values for $x \in R$, then find the bounds for p.

12. If $\theta - \phi = \frac{\pi}{2}$, then show that $\begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix}\begin{pmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{pmatrix} = 0$

13. Find the number of numbers that are greater than 4000 which can be formed using 0,2,4,6,8.

14. Prove that $3 \le r \le n$, $n\text{-}3c_r + 3.\ n\text{-}3c_{r-1} + 3.\ n\text{-}3c_{r-2} + n\text{-}3c_{r-3} = nc_r$

15. Resolve $\frac{x^2-3}{(x+2)(x^2+1)}$ into partial fractions.

16. If $y = x + \frac{x^3}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ....$, then show that $x = y - \frac{y^2}{2!} + \frac{y^3}{3!} - \frac{y^4}{4!} + ...$

17. If $E_1, E_2$ are any two events of a random experiment and P is a probability function, then

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

## SECTION C

**Long Answer Type Questions.** **5X7 =35**

**Note: Answer any Five of the following. Each question carries 7 marks.**

18. Solve the equation $x^4 + x^3 - 16x^2 - 4x + 48 = 0$, the product of two of the roots being 6.

19. Show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{bmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{bmatrix} = \left(a^3+b^3+c^3-3abc\right)^2$

20. Solve the following simultaneous linear equations by using 'Cramer's rule.

$3x + 4y + 5z = 18$

$2x - y + 8z = 13$

$5x - 2y + 7z = 20$

21. Find the sum of the series $\frac{3.5}{5.10} + \frac{3.5.7}{5.10.15} + \frac{3.5.7.9}{5.10.15.20} + .....$

22. If the $2^{nd}$, $3^{rd}$ and $4^{th}$ terms in the expansion of $(a + x)^n$ are respectively, 240, 720, 1080 find a, x, n.

23. A, B, C are three horses in a race. The probability of A to win the race is twice that of B, and probability of B is twice that of C. What are the probabilities of A, B and C to win the race.

24. The range of random variable X is {0, 1, 2} given that $P(X = 0) = 3c^3$
$P(X = 1) = 4c - 10c^2$; $P(X = 2) = 5c - 1$
i) Find the value of c ii) $P(X < 1)$, $P(1 < X \leq 2)$ and $P(0 < X \leq 3)$

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