MATHEMATICS PAPER IA- MAY 2010.
ALGEBRA, VECTOR ALGEBRA AND TRIGONOMETRY

TIME: 3hrs **Max. Marks.75**

Note: This question paper consists of three sections A, B and C.

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. 10X2 =20

Note: Attempt all questions. Each question carries 2 marks.

1. If A = { -2,-1,0,1,2 } and f: A →B is a surjection defined by $f(x) = x^2+x+1$, then find B.
2. find the domain of the function $f(x) = \sqrt{4x - x^2}$.
3. If the vectors are -3**i** +4**j** ++ l**k** **µi** + 8**j** +6**k, are collinear vectors then find** l , µ.
4. Find the vector equation of the line passing through the point (2,3,1) and parallel to the vector (4, – 2, 3)
5. If **(r – 2i + j – 4k).(r + 2i – 2j + 2k)** = 0 is the equation of the sphere, then find its centre.
6. Find the maximum and minimum values of the following functions over R.
   $f(x) = 5\sin x + 12\cos x - 13$
7. Find the period of the function $f(x) = \cos\left(\frac{4x+9}{5}\right)$
8. If $\sinh x = 3$, show that $x = \log_e\left(3+\sqrt{10}\right)$
9. If a = 6, b=5, c=9 , then find angle A.
10. Find the square root of 3+4i.

SECTION B

SHORT ANSWER TYPE QUESTIONS 5X4 =20

Note: Answer any FIVE questions. Each question carries 4 marks.

11. **a, b, c** are non-coplanar vectors. Prove that the following four points are coplanar –a + 4b – 3c,3a + 2b – 5c,–3a + 8b – 5c ,–3a + 2b + c.
12. Find a unit vector perpendicular to the plane determined by the points P(1, 2,3), Q(2, -1,1) and R(1,2,-4).
13. Prove that $\sin 18^o = \frac{\sqrt{5}-1}{4}$
14. If θ_1, θ_2 are solutions of the equation $a\cos 2\theta + b\sin 2\theta = c$, $\tan\theta_1 \neq \tan\theta_2$ *and* $a + c \neq 0$ then find the values of (i) $\tan\theta_1 + \tan\theta_2$ (ii) $\tan\theta_1.\tan\theta_2$

15. Prove that $Tan^{-1}\frac{1}{2}+Tan^{-1}\frac{1}{5}+Tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

16. Show that $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}=\frac{a^2+b^2+c^2}{2abc}$

17. Show that $16\sin^5\theta=\sin 5\theta-5\sin 3\theta+10\sin\theta$

SECTION C

LONG ANSWER TYPE QUESTIONS **5X7 =35**

Note: Answer any Five of the following. Each question carries 7 marks.

18. If $f: A\to B,\ g: B\to C$ be bijections. Then show that $gof: A\to C$ is a bijection.

19. Using mathematical induction, Show that $3.5^{2n+1} + 2^{3n+1}$ is divisible by 17 for all n € N.

20 If the angle between any two diagonals of a cube is θ , then by vector method , prove that cosθ = 1/3

21. If A + B + C = 180^0, then prove that $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}-\cos^2\frac{C}{2}=2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$.

22. If a = 13, b = 14, c = 15, show that $R=\frac{65}{8}, r=4, r_1=\frac{21}{2}$, r_2 = 12 and r_3 = 14.

23. A pillar is leaning towards east and a and b are the angles of elevation of the top of the pillar from two points due west of the pillar at distance a and b respectively. Show that the angle between the pillar and the horizontal is $\text{Tan}^{-1}\left(\frac{b-a}{b\cot\alpha-a\cot\beta}\right)$.

24. Show that one of $\left(\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right)^{\frac{8}{3}}=-1$