S·C·R·A-2010

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

T.B.C. : Q–RBQ2–K–LZ

Test Booklet Series

Serial No. 135161

TEST BOOKLET

A

MATHEMATICS

Paper—III

Time Allowed : Two Hours

Maximum Marks : 200

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET ***DOES NOT*** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. ENCODE CLEARLY THE TEST BOOKLET SERIES **A, B, C** OR **D** AS THE CASE MAY BE IN THE APPROPRIATE PLACE IN THE ANSWER SHEET.
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. ***DO NOT*** write ***anything else*** on the Test Booklet.
4. This Test Booklet contains **100** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ***ONLY ONE*** response for each item.
5. You have to mark all your responses ***ONLY*** on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. All items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator ***only the Answer Sheet***. You are permitted to take away with you the Test Booklet.
9. Sheet/s for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**
    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third (0·33)** of the marks assigned to that question will be deducted as penalty.
    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.
    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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1. What is $\lim_{x \to 0} \frac{\int_0^{x^2} \cos^2 t \, dt}{x \sin x}$ equal to ?

(a) 3/2
(b) 1
(c) −1
(d) None of the above

2. If P_m stands for P(m, m), then what is the value of

$1 + P_1 + 2 \cdot P_2 + 3 \cdot P_3 + \ldots + n \cdot P_n$?

(a) n !
(b) (n + 1) !
(c) (n − 1) !
(d) (n − 2) !

3. What is the value of

$\tan [\cos^{-1}(4/5) + \tan^{-1}(2/3)]$?

(a) 6/17
(b) 7/18
(c) 16/7
(d) 17/6

4. In how many ways can the letters of the word 'MACHINE' be arranged so that the vowels may occupy only odd positions ?

(a) 5040
(b) 576
(c) 288
(d) 275

5. For the function $f(x) = \int_0^x \frac{\sin t}{t} dt$, which one of the following is correct ?

(a) Maximum occurs at $x = n\pi$, where n is even
(b) Minimum occurs at $x = n\pi$, where n is odd
(c) Maximum occurs at $x = n\pi$, where n is odd
(d) None of the above

6. If $x^2 + x + 1 = 0$, then what is the value of

$$\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + \ldots \ldots \left(x^{27} + \frac{1}{x^{27}}\right)^2 ?$$

(a) 27
(b) 42
(c) 45
(d) 54

7. An equilateral triangle has each side equal to 'a'. If the coordinates of its vertices are (x_1, y_1), (x_2, y_2), (x_3, y_3), then what is the square of the determinant $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ equal to ?

(a) $3a^4$
(b) $3a^4/4$
(c) $3a^4/2$
(d) $2a^4$

8. If $x \in R$, then what is the least value of the expression $\frac{x^2 - 6x + 5}{x^2 + 2x + 1}$?

(a) −1
(b) −1/2
(c) −1/3
(d) −1/4

9. If α is an n^{th} root of unity, then what is $1 + 2\alpha + 3\alpha^2 + \ldots + n\alpha^{n-1}$ equal to ?

(a) $n/(1 - \alpha)$
(b) $-n/(1 - \alpha)$
(c) $n/(1 - \alpha)^2$
(d) $-n/(1 - \alpha)^2$

(Contd.)

10. What is the ratio of the two integrals

$$\int_0^{\pi} f(\sin x)\,dx : \int_0^{\frac{\pi}{2}} f(\sin x)\,dx ?$$

(a) 1/2

(b) 2

(c) $\pi/2$

(d) $2/\pi$

11. If the roots of the cubic equation $(z + \alpha\beta)^3 = \alpha^3$, $\alpha \neq 0$ represent the vertices of a triangle, then what is the length of one of the sides of the triangle ?

(a) $\sqrt{3}\,|\alpha|$

(b) $\sqrt{3}\,|\beta|$

(c) $|\alpha| / \sqrt{3}$

(d) $|\alpha\beta| / \sqrt{3}$

12. If A and B are events such that $P(A \cup B) = 0{\cdot}5$, $P(\text{not } B) = 0{\cdot}8$ and $P(A \mid B) = 0{\cdot}4$, then what is P(A) equal to ?

(a) 0·38

(b) 0·32

(c) 0·28

(d) 0·22

13. If $I = \int_1^2 \frac{e^x(1+\sin x)}{x^2}\,dx$, then which one of the following is correct ?

(a) $I = 3/22$

(b) $I = 1/2$

(c) $0{\cdot}5 < I < 18$

(d) $0 < I < (3/22)$

14. What is the left derivative of the function $f(x) = \max\left(|x|, \sqrt{|x|}, x^3\right)$ at $x = -1$?

(a) 1

(b) 3

(c) $-1/2$

(d) -1

15. What is the degree of the differential equation obtained from $y = cx^2 + c^{-1}$ where c is an arbitrary constant ?

(a) 4

(b) 3

(c) 2

(d) 1

16. If the middle term is the greatest term in the expansion of $(2 + 3x)^{10}$, then which one of the following is correct ?

(a) $5/8 < x < 1$

(b) $5/9 < x < 4/5$

(c) $5/8 < x < 5/6$

(d) $5/9 < x < 4/3$

17. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$, then what is f (100) equal to ?

(a) 0

(b) 1

(c) 100

(d) -100

18. If $m \in \mathbb{Z}$ and the equation

$$mx^2 + (2m-1)x + (m-2) = 0$$

has rational roots, then m is of the form :

(a) $n(n+2)$

(b) $n(n+1)$

(c) $n(n-2)$

(d) $n(n+3)$

where $n \in \mathbb{Z}$

(Contd.)

19. Consider the following statements :

1. There exists a natural number n such that $1^3 + 2^3 + 3^3 + ... + n^3 = 776$.
2. For any real number $r \neq 0$, r^{-1}, r^{-2}, r^{-3} is in GP and if $r \neq 1$, then r^{-1}, r^{-2}, r^{-3} is not in HP.

Which of the statements given above is/are correct ?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

20. Consider the following statements :

1. There exist real numbers x, y such that $0 < x < 1$, $0 < y < 1$, $x \neq y$ and $0 < (\sin^{-1} x - \sin^{-1} y) < (\pi/8)$.
2. $\tan(\tan^{-1} y) = y$ is true for only non-negative real numbers.

Which of the statements given above is/are correct ?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

21. Let $A = \{2m : m \in \mathbb{N}, m \text{ is odd}\}$ and $B = \{4t + 1 : t \in \mathbb{N}\}$.

Consider the following statements :

Statement I : For any $n \geq 4$, the number of diagonals of a polygon of n sides is $n(n-3)/2$.

Statement II : For any $n \geq 4$, the number of diagonals of a polygon of n sides is even if $n \notin A \cup B$.

Which one of the following is correct ?

(a) Both statement-I and statement-II are individually true and statement-II is the correct explanation of statement-I.
(b) Both statement-I and statement-II are individually true but statement-II is not the correct explanation of statement-I.
(c) Statement-I is true, but statement-II is false.
(d) Statement-I is false but statement-II is true.

22. Let $x = \sqrt{2} - 1$. Consider the trigonometric equation $\sqrt{x}\cos\theta + \sqrt{x+2}\sin\theta = 2^{3/4}$.

Which one of the following is correct ?

(a) The above trigonometric equation has no solution.
(b) The solution to the above trigonometric equation is $\tan^{-1}(x + 2) + k\pi$ where k ranges through integers.
(c) The solution to the above trigonometric equation is $\tan^{-1}(x + 2) + m\pi$ where m ranges through odd integers.
(d) None of the above

23. Consider the following statements :

Statement I : If θ_1, θ_2, θ_3 are such that $0 < \theta_i < \pi/2$ for $i = 1, 2, 3$ and if $\tan\theta_1 + \tan\theta_2 + \tan\theta_3 = \tan\theta_1 \tan\theta_2 \tan\theta_3$, then $\theta_1 + \theta_2 > \pi/2$, $\theta_2 + \theta_3 > \pi/2$ and $\theta_1 + \theta_3 > \pi/2$.

Statement II : If x, y, z are positive real numbers with $x + y + z = xyz$, then $\frac{x+y}{1-xy}$ is positive.

Which one of the following is correct ?

(a) Both statement-I and statement-II are individually true and statement-II is the correct explanation of statement-I.
(b) Both statement-I and statement-II are individually true but statement-II is not the correct explanation of statement-I.
(c) Statement-I is true, but statement-II is false.
(d) Statement-I is false but statement-II is true.

(Contd.)

24. The sum of the squares of three distinct real numbers which are in GP is s^2. If their sum is αs, then which one of the following is correct ?

(a) $\alpha^2 \in (1/3, 1)$

(b) $\alpha^2 \in (1, 3)$

(c) $\alpha^2 \geq 3$

(d) None of the above

25. If $\alpha, \beta, \gamma \in \left(0, \frac{\pi}{2}\right)$, then the value of $\frac{\sin(\alpha+\beta+\gamma)}{\sin\alpha+\sin\beta+\sin\gamma}$ is :

(a) less than 1

(b) greater than 1

(c) 1

(d) greater than or equal to 1

26. What is the number of ways in which sum of upper faces of four distinct dice can be six ?

(a) 10

(b) 7

(c) 6

(d) 4

27. What is $\int_1^2 \frac{dx}{\sqrt{x+1}\sqrt{x-1}}$?

(a) $\ell n(2+\sqrt{3})$

(b) $\ell n(1+\sqrt{3})$

(c) $\ell n(3+\sqrt{3})$

(d) None of the above

28. If $\frac{1}{\sqrt{a}}\int_1^a \left(\frac{3}{2}\sqrt{x}+1-\frac{1}{\sqrt{x}}\right)dx < 4$, then what value can 'a' take ?

(a) 0

(b) 4

(c) 5

(d) None of the above

29. What is the ratio in which the area bounded by the curves $y^2 = 12x$ and $x^2 = 12y$ is divided by the line $x = y$?

(a) 1 : 2

(b) 15 : 16

(c) 15 : 49

(d) None of the above

30. If f(x), g(x), h(x) are three polynomials of degree 2, what is the degree of the polynomial $\varphi(x)$ where $\varphi(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$?

(a) 3

(b) 2

(c) 1

(d) 0

31. If A and B are two events such that $P(A \cup B) = \frac{1}{2}$ and $P(\overline{A}) = \frac{2}{3}$, then what is $P(\overline{A} \cap B)$?

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) $\frac{1}{2}$

(d) $\frac{1}{6}$

(Contd.)

32. AB is a vertical pole. The end A is on the level ground. C is the middle point of AB. P is a point on the level ground. The portion BC subtends an angle β at P. If AP = n(AB), then what is tan β equal to ?

(a) $\frac{n}{2n^2+1}$

(b) $\frac{n}{n^2+1}$

(c) $\frac{n}{n^2-1}$

(d) None of the above

33. Let P and Q be points on an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Let c_1 and c_2 respectively be the y-coordinate of the points of intersection of the y-axis with the tangents at P and Q. If (x_1, y_1) and (x_2, y_2) are the coordinates of the points P and Q, then which one of the following is correct ?

(a) $c_1y_1 - c_2y_2 = 0$

(b) $c_1y_2 - c_2y_1 = 0$

(c) $c_1y_1 + c_2y_2 = 1$

(d) $c_1y_2 - c_2y_1 = 1$

34. What is the sum of all numbers between 4000 and 4250 formed out of the digits 0, 2, 3 and 4 (no digit is repeated in the formation) ?

(a) 16488

(b) 8433

(c) 8405

(d) None of the above

35. Which one of the following formulas for variance (V) is most likely to involve least rounding off error ?

(a) $V=\frac{1}{n}\sum(X_i-\bar{X})^2$

(b) $V=\frac{1}{n}\sum X_i^2-(\bar{X})^2$

(c) $V=\frac{n\sum X_i^2-(\sum X_i)^2}{n^2}$

(d) All of the above would yield equal rounding off error

36. The probability that a regularly scheduled flight departs on time is 0·80, the probability that it arrives on time is 0·70, and the probability that it departs and arrives on time is 0·60. What is the probability that a plane arrives on time given that it departed on time ?

(a) 0·75

(b) 0·90

(c) 0·42

(d) 0·56

37. Consider two points A(a, b) and B(c, d) where c > a > 0 and d > b > 0, then abscissa of a point on x-axis such that sum of its distances from A and B is minimum, is :

(a) (bc – ad) / (b – d)

(b) (ac + bd) / (b + d)

(c) (ac – bd) / (b – a)

(d) (ad + bc) / (b + d)

38. If f(x) = | sin x – cos x |, then which one of the following is correct ?

(a) $f'(\pi/4)$ does not exist.

(b) $f'(\pi/4)$ exists and is positive.

(c) $f'(\pi/4)$ exists and is negative.

(d) $f'(\pi/4)$ exists and is equal to zero.

(Contd.)

39. A wall measures 40 m by 30 m contains a window of size 15 m by 10 m. The wall is hit by four stones thrown up by a mower. Assuming that each stone hits the wall in a random position independently of other stones, what is the probability that every throw hits the window ?

(a) 1/8

(b) 1/4096

(c) 2401/4096

(d) 1/512

40. What is the probability that a point inside a circle is nearer to the circumference than to the centre ?

(a) 1/4

(b) 1/2

(c) 3/4

(d) 2/3

41. What is the number of integral values of k for which the equation $7 \cos x + 5 \sin x = 2k + 1$ has a solution ?

(a) 4

(b) 8

(c) 10

(d) 12

42. A line makes the same angle θ with each of x-axis and z-axis. If it makes an angle β with the y-axis such that $\sin^2 \beta = 3 \sin^2 \theta$, what is $\cos^2 \theta$ equal to ?

(a) 2/3

(b) 1/5

(c) 3/5

(d) 2/5

43. If the function

$$f(x) = ax + bx^2 \quad (0 \le x < 1)$$
$$= c + 3x \quad (1 < x \le 2)$$

is such that $f(1) = 1$, $f(2) = 4$ and $f(x)$ is differentiable at $x = 1$, what is the value of b ?

(a) –1

(b) –2

(c) 2

(d) 1

44. Triangles are formed by joining vertices of an octagon. Any one of these triangles is selected at random. What is the probability that the selected triangle has no side common with the octagon ?

(a) 1/7

(b) 2/7

(c) 3/7

(d) 5/7

45. AB and CD are the diameters of a circle of radius 1 unit with centre O. PB and PD are tangents at B and D which meet at the point P. If BC = 1 unit, then what is the area of the triangle PBD ?

(a) $3\sqrt{3}$ square units

(b) $3\sqrt{3}/2$ square units

(c) $3\sqrt{3}/4$ square units

(d) $\sqrt{3}/2$ square units

(Contd.)

46. What is the area of a triangle whose vertices are the cube roots of unity ?

(a) $\pi\sqrt{3}$ square units

(b) $3\sqrt{3}/2$ square units

(c) $3\sqrt{3}/4$ square units

(d) $2\pi\sqrt{3}/3$ square units

47. What is the value of the error when $\frac{2}{11}$ is approximated by 0·18 ?

(a) $\frac{1}{55}$

(b) $\frac{1}{550}$

(c) $\frac{1}{110}$

(d) $\frac{1}{1100}$

48. What is $\int_2^6 [f(x)]dx$ equal to where $f(x)=\frac{[(x-1)^2]}{2[x]+1}$ and $[\cdot]$ denotes the greatest integer value function ?

(a) 0

(b) 1

(c) 2

(d) 3

49. If z_1, z_2 and z_3 are complex numbers which lie on a straight line L and if $z_4 = a\,z_1 + b\,z_2 + 4\,z_3$ lies on L, then what are the values of a, b respectively ?

(a) 2, –5

(b) 3, –7

(c) 2, 4

(d) None of the above

50. Consider the equation $L = a^{\sin x} + a^{\sqrt{3}\cos x}$ ($a > 0$, x is real). For any given $a > 0$, the value of L is :

(a) greater than 2

(b) greater than or equal to 2/a

(c) less than 2/a

(d) less than 2

51. What is the set of points (x, y) on the curve that lie on the co-ordinate axes ?

(a) {(–2, 0), (2, 0)}

(b) {(0, 0), (0, 2)}

(c) {(2, 0), (0, 4)}

(d) {(0, 0), (4, 0)}

52. What is the degree of the differential equation corresponding to the family of curves $y = a(x + a)^2$ where 'a' is an arbitrary constant ?

(a) 3

(b) 2

(c) 1

(d) Not defined

(Contd.)

53. If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is $|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|$ equal to ?

(a) 2

(b) $\sqrt{2}$

(c) $2\sqrt{2}$

(d) 4

54. What is the amplitude of $\sin\frac{\pi}{5}+i\left(1-\cos\frac{\pi}{5}\right)$?

(a) $\frac{\pi}{5}$

(b) $\frac{\pi}{10}$

(c) $\frac{\pi}{15}$

(d) $\frac{2\pi}{5}$

55. There are four bus routes between A and B; and three bus routes between B and C. A man can travel round-trip in number of ways from A to C via B. If he does not want to use a bus route more than once, in how many ways can he make round-trip ?

(a) 14

(b) 19

(c) 72

(d) 144

56. What is the number of solutions of the equation $z^2 + |z|^2 = 0$, where $z \neq 0$?

(a) 1

(b) 2

(c) 3

(d) Infinitely many

57. Three squares of a chess board are selected at random. What is the probability of getting two squares of one colour and other of a different colour ?

(a) 3/8

(b) 3/32

(c) 8/21

(d) 16/21

58. Let $f(t)=\begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$, then what is $\lim_{t\to 0}\frac{f(t)}{t^2}$ equal to ?

(a) 0

(b) –1

(c) 2

(d) 3

59. Assume that in a family, each child is equally likely to be a boy or girl. A family with three children is chosen at random. What is the probability that the eldest child is a girl given that the family has at least one girl ?

(a) 1/3

(b) 2/3

(c) 1/2

(d) 4/7

60. If A is a square matrix, then what is $\text{adj}(A^T) - (\text{adj } A)^T$ equal to ?

(a) $2|A|$

(b) $2|A|I$

(c) Null matrix

(d) Unit matrix

(Contd.)

61. If f and g are continuous functions in [0, 1] satisfying $f(x) = f(a - x)$ and $g(x) + g(a - x) = a$ then what is $\int_0^a f(x)\,g(x)\,dx$ equal to ?

(a) $\int_0^a f(x)dx$

(b) $a\int_0^a f(x)dx$

(c) $\frac{a}{2}\int_0^a f(x)dx$

(d) None of the above

62. What is $\lim_{x \to 2} \frac{\sin(e^{x-2} - 1)}{\ell n(x-1)}$ equal to ?

(a) 0

(b) 1

(c) –1

(d) –2

63. What is the number of solutions of the equation $\sin A + \cos A = 1{\cdot}4$ which lie between 0° and 360° ?

(a) 1

(b) 2

(c) 4

(d) None of the above

64. $M(x_1, x_2, \ldots x_n)$ defines a measure of central tendency based on n values $x_1, x_2, \ldots x_n$.

Consider the following measures of central tendency :

1. Arithmetic mean
2. Median
3. Geometric mean

Which of the above measures satisfy/ satisfies the property

$$\frac{M(x_1, x_2, \ldots\ldots x_n)}{M(y_1, y_2, \ldots\ldots y_n)} = M\left(\frac{x_1}{y_1}, \frac{x_2}{y_2}, \ldots \frac{x_n}{y_n}\right)?$$

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) 3 only

(d) 1 and 3

65. Two points P(a, 0) and Q(–a, 0) are given. What is the locus of variable point R such that $\angle RPQ - \angle RQP = \frac{\pi}{4}$?

(a) $x^2 + y^2 - 2xy - a^2 = 0$

(b) $x^2 + y^2 + 2xy - a^2 = 0$

(c) $x^2 - y^2 - 2xy - a^2 = 0$

(d) $x^2 - y^2 + 2xy - a^2 = 0$

66. What is the degree of the differential equation

$$x = 1 + xy\left(\frac{dy}{dx}\right) + \frac{x^2y^2}{2}\left(\frac{dy}{dx}\right)^2 + \frac{x^3y^3}{6}\left(\frac{dy}{dx}\right)^3 + \ldots ?$$

(a) 1

(b) 2

(c) 3

(d) can not be determined

(Contd.)

67. What is the number of ordered pairs (x, y) satisfying $|y| = \cos x$ and $y = \sin^{-1}(\sin x)$ where $|x| \leq 3\pi$?

(a) 2

(b) 4

(c) 6

(d) 8

For the next *02 (two)* items that follow :

A line $y = mx$ bisects the area enclosed by the lines $x = 0$, $y = 0$, $x = 1$ and the curve $y = 3 + 2x - x^2$.

68. What is the value of m ?

(a) 11/12

(b) 11/6

(c) 22/3

(d) 11/3

69. What is the area of the triangle enclosed by $y = mx$, $y = 0$ and $x = 1$?

(a) 11/3 square units

(b) 11/6 square units

(c) 11/12 square units

(d) None of the above

70. AB is the chord of the parabola $y^2 = 4ax$, $a > 0$ with vertex at A. BC is drawn perpendicular to AB meeting the axis of the parabola at C. What is the projection of BC on the axis of the parabola ?

(a) a

(b) 2a

(c) 4a

(d) 8a

For the next *02 (two)* items that follow :

Two vertices of an equilateral triangle are (–1, 0) and (1, 0) and the third vertex lies above the x-axis.

71. What is the radius of circum-circle ?

(a) $\frac{2}{\sqrt{3}}$ unit

(b) $\frac{1}{\sqrt{3}}$ unit

(c) $\frac{1}{2}$ unit

(d) 1 unit

72. What is the radius of the circle inscribed in the triangle ?

(a) $\frac{2}{\sqrt{3}}$ unit

(b) $\frac{1}{\sqrt{3}}$ unit

(c) $\frac{1}{2}$ unit

(d) None of the above

For the next *02 (two)* items that follow :

The adjacent sides of a parallelopiped are given by $\hat{i} + m\hat{j} + \hat{k}$, $\hat{j} + m\hat{k}$, $m\hat{i} + \hat{k}$.

73. What is the value of 'm' so that the volume of the parallelopiped becomes minimum ?

(a) $\frac{1}{\sqrt{3}}$

(b) $\frac{1}{3}$

(c) $\sqrt{3}$

(d) 3

(Contd.)

74. What is the minimum volume of the parallelopiped ?

(a) $\frac{3\sqrt{3}-2}{3\sqrt{3}}$ cubic unit

(b) $\frac{3\sqrt{3}-1}{3\sqrt{3}}$ cubic unit

(c) $\frac{3\sqrt{3}-2}{\sqrt{3}}$ cubic unit

(d) None of the above

75. What is the area bounded by the circle $x^2 + y^2 = r^2$; $r = 1, 2$ and the rays given by $2x^2 - 3xy - 2y^2 = 0$, $y > 0$?

(a) $\frac{3\pi}{2}$ square unit

(b) $\frac{3\pi}{4}$ square unit

(c) $\frac{3\pi}{8}$ square unit

(d) 3π square unit

76. An ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ slides between two perpendicular straight lines. What is the locus of its centre ?

(a) Circle

(b) Parabola

(c) Ellipse

(d) Hyperbola

77. If $\frac{d}{dx}\left(\frac{1+x^2+x^4}{1+x+x^2}\right) = ax + b$, then what is the value of a ?

(a) 1

(b) 2

(c) –1

(d) –2

78. What is the value of $\int_0^{10} (x - [x])\,dx$ where $[\cdot]$ denotes the greatest integer function ?

(a) 0

(b) 1

(c) 5

(d) 10

79. What is the number of solutions of the equation $81^{\sin^2 x} + 81^{\cos^2 x} = 30$; $x \in (0, 2\pi)$?

(a) One

(b) Two

(c) Four

(d) Eight

80. Which one of the following is correct in respect of the function

$f(x) = 5\,\ell n\, x - 24x + 32x^2$; $x > 0$?

(a) It has neither a point of maximum nor a point of minimum

(b) It has a point of maximum, but no point of minimum.

(c) It has a point of minimum, but no point of maximum.

(d) It has a point of maximum and a point of minimum.

(Contd.)

81. Let y be an irrational number such that y^2 is a positive integer. Then $(y + 1)^5$ equals

(a) $(y - 1)^5 + 2m$ for some positive integer m such that the greatest common divisor of m, y^2 is greater than 1.

(b) $(y - 1)^5 + 2p$ for some prime number $p = 8k + 3$ $(k \in \mathbb{Z})$ if $y = \sqrt{2}$.

(c) $(y - 1)^5 + 2t$ for some even integer t if y^2 is even.

(d) $(y - 1)^5 + 2r$ for some even positive integer r if y^2 is odd.

82. Let $x, y \in [0, 1]$ be such that $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$. Which one of the following is ***not*** correct ?

(a) If $x = 0{\cdot}1$, then $y = 0{\cdot}3\sqrt{11}$

(b) If $x = 0{\cdot}2$, then $y = 0{\cdot}4\sqrt{6}$

(c) If $x = 0{\cdot}6$, then $y = 0{\cdot}84$

(d) If $x = \frac{5}{13}$, then $y = \frac{12}{13}$

For the next *02 (two)* items that follow :

Consider the permutations of the letters of the word 'NEUBAKRI'. Let x be the number of words such formed in which the consonants occupy only the odd places. Let y be the number of words such formed which begin with 'NEU'. Let z be the number of words such that they end with 4 vowels. Let t be the total number of permutations of the letters of the word 'NEUBAKRI'.

83. Consider the following :

1. $t = 336\,y$
2. $t = 35\,x$

Which of the above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

84. Consider the following :

1. $x = z$
2. $x - y = 446$

Which of the above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

85. TF is a tower (T denotes the top and F denotes the bottom) of height $30(3 + \sqrt{3})$ m. AB is a building (A denotes the top and B denotes the bottom) of height 60 m. It is given that the angle of elevation of T as seen from B is 60° and the angle of elevation of T as seen from A is α. If $\angle ATB = \theta$, then what is the value of $\sin\theta$?

(a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(b) $\frac{\sqrt{3}-1}{3\sqrt{2}}$

(c) $\frac{1}{\sqrt{3}}$

(d) None of the above

86. If $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{a} + \hat{b} + \hat{c} = \vec{0}$, then what is the value of $\hat{a} \bullet \hat{b} + \hat{b} \bullet \hat{c} + \hat{c} \bullet \hat{a}$?

(a) –3/2

(b) –1

(c) 0

(d) 3

(Contd.)

87. What is the length of latus rectum of the ellipse $5x^2 + 9y^2 = 45$?

(a) 5/3

(b) $\frac{10}{3}$

(c) $\frac{2\sqrt{5}}{3}$

(d) $\frac{\sqrt{5}}{3}$

88. What is the value of m for which the vectors $m\hat{i} + \hat{j} - 2\hat{k}$, $\hat{i} + \hat{j} + 3\hat{k}$, $8\hat{i} + 5\hat{j}$ are coplanar?

(a) –2

(b) 5

(c) 2

(d) –5

89. If for any vector $\vec{a}$,

$$\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = m\vec{a},$$

then what is the value of m ?

(a) 1

(b) 2

(c) 3

(d) 0

90. Consider the following statements :

1. On the set of integers $\mathbb{Z}$, the operation '$\oplus$' defined by $p \oplus q = pq + 2$ is commutative for $p, q \in \mathbb{Z}$.
2. The set of all positive real numbers is a group with respect to multiplication.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

91. Consider the following statements :

1. The set {1, 2, 3, 4, 5, 6, 7} is a group with respect to O_8, where O_8 is defined as taking remainder when two numbers are multiplied and then divided by 8.
2. A group is not necessarily an abelian group.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

For the next *03 (three)* items that follow :

The tangent and normal at the point $P(at^2, 2at)$ to the parabola $y^2 = 4ax$ meet the axis of the parabola at T and G respectively.

92. What is the equation of tangent at P ?

(a) $y = xt + at$

(b) $y = xt^{-1} + at$

(c) $y = ax + t$

(d) None of the above

93. What is the equation of normal at P ?

(a) $y = -tx + 2at + at^3$

(b) $y = -tx - 2at + at^3$

(c) $y = tx + 2at + at^3$

(d) None of the above

(Contd.)

94. What is the angle at which the tangent at P to the parabola is inclined to the tangent at P to the circle through T, P, G ?

(a) $\tan^{-1}(t^2)$

(b) $\cot^{-1}(t^2)$

(c) $\tan^{-1}(t)$

(d) $\cot^{-1}(t)$

95. If the value of third order determinant be 11, then what is the value of the square of determinant formed by its cofactors ?

(a) 11

(b) 121

(c) 1331

(d) 14641

96. What is the solution of the differential equation $(x + 2y^3)\frac{dy}{dx} = y$?

(a) $x = y^3 + cy$

(b) $x^2 = y^3 + cy^2$

(c) $x^2 = y^3 + cy$

(d) None of the above

97. The frequency distribution of a variate X is as follows :

X	11	13	17	21	25
Frequency	13	25	11	17	21

What is the median of the above distribution ?

(a) 13

(b) 17

(c) 21

(d) 25

For the next *03 (three)* items that follow :

A line L passing through the points (1, 0, –1) and (–1, 1, 0) and another line M passes through the points (3, 1, –1) and (4, 5, –2).

98. What is the vector perpendicular to both the vectors ?

(a) $5\hat{i} + \hat{j} + 9\hat{k}$

(b) $5\hat{i} - \hat{j} + 9\hat{k}$

(c) $5\hat{i} + \hat{j} - 9\hat{k}$

(d) None of the above

99. What is the shortest distance between the lines ?

(a) 1 unit

(b) $\frac{10}{\sqrt{107}}$

(c) $\frac{11}{\sqrt{107}}$

(d) None of the above

100. What are the direction ratios of the line of shortest distance ?

(a) $\langle -5, 1, 9 \rangle$

(b) $\langle 5, -1, 9 \rangle$

(c) $\langle 5, 1, -9 \rangle$

(d) None of the above

SPACE FOR ROUGH WORK