171. $I = \int_0^5 \frac{x^2}{x^2 + (5-x)^2}\, dx$ ... (i)

$I = \int_0^5 \frac{(5-x)^2}{(5-x)^2 + (x)^2}\, dx$ ... (ii) [f(x) = f(a–x)]

$2I = \int_0^5 dx = [x]_0^5$

$2I = 5$

$I = 5/2$

Ans. (d) None of these

172. $\underset{x \to 0}{Lt} \frac{(2^x - 1)}{\sqrt{1+x} - 1} \cdot \frac{\sqrt{1+x} + 1}{\sqrt{1+x} + 1}$

$\underset{x \to 0}{lt} \frac{(2^x - 1)}{x} \cdot \underset{x \to 0}{lt} \left(\sqrt{1+x} + 1\right)$

App lt.

$\Rightarrow \log 2 \times 2 \quad \Rightarrow 2 \log 2$

Ans. (a) 2log 2

173. $\underset{x \to 0}{Lt} \frac{|x-1|}{x-1}$

RHL ... $\underset{x \to 0+}{Lt} \frac{(x-1)}{(x-1)} = 1$

LHL $\underset{x \to 0-}{Lt} \frac{-(x-1)}{(x-1)} = -1$

LHL $\neq$ RHL $\therefore$ f(x) does not exist

(c) Does not exist

174. $f(x) = \frac{x - |x|}{x}$

RHL $\underset{x \to 0+}{Lt} \frac{x - x}{x} = 0$

LHL $\underset{x \to 0-}{Lt} \frac{x - (-x)}{x} = \frac{2x}{x} = 2$

f (0) = 2

$\Rightarrow$ RHL $\neq$ LHL $\neq$ f(0) $\therefore$ f(x) is not continuous at x = 0

Ans. (b) No.

175. $y = \log_3 (\log_3 x)$

$$\frac{dy}{dx} = \frac{d}{dt}\log_3 t . \frac{d}{dx}(\log_3 x)$$

$$= \frac{1}{\log_3 x . \log_3} . \frac{1}{x . \log 3} \Rightarrow \frac{1}{x . \frac{\log x}{\log 3} . (\log 3)^2}$$

$$= \frac{1}{x . \log 3 . \log x}$$

Ans. (a) $= \dfrac{1}{x . \log 3 . \log x}$

176. C I if calculated annually.

$$CI_1 = P\left[\left(1+\frac{20}{100}\right)^2 - 1\right] = \frac{11P}{25}$$

CI if calculated semi annually.

$$CI_2 = P\left[\left(1+\frac{10}{100}\right)^4 - 1\right] = \frac{4641}{10000}P$$

$$\therefore \quad \frac{4641P}{10000} - \frac{11P}{25} = 482 \Rightarrow \frac{241P}{10000} = 482$$

$\therefore$ P = 20,000

Ans. (a) Rs. 20,000

177. Value of annuity (A) = $P\left[\dfrac{(1+i)^n - 1}{i}\right]$

$$= 3000\left[\frac{(1+0.09)^3 - 1}{(0.09)}\right]$$

= 9833.33

Ans. (c) Rs. 9833.33

178. Present Value of Annuity $= \dfrac{10000}{(1+0.05)^{10}}$

$=$ Rs. 7724

Ans. (a) Rs. 7724

179. $A = \dfrac{P}{i}\left[(1+i)^n - 1\right] \Rightarrow \dfrac{45000}{0.06}\left[(1+0.06)^{10} - 1\right]$

$750000 \left[(1+0.06)^{10} - 1\right]$ {Solve by taking log}

A = 517500

$\therefore$ Surplus = 517500 – 5,00,000

= 17,500

Ans. (c) Rs. 17,500

180. Present value of P (rest) of the annuity.

$P = A\left[\dfrac{1-(1+i)^{-n}}{i}\right] \Rightarrow 2000\left[\dfrac{1-(1+0.1)^{-5}}{(0.10)}\right]$

$P = 20000 \left[1-(1.1)^{-5}\right]$

P = 7294 which is less than the Purchase Price.

$\therefore$ leasing is preferable.

Ans. (a) leasing is preferable.

181. The sum of deviations of the given values from their Arithmetic Mean is 0.

Ans. (a) Arithmetic Mean

182. The sum of squares of the deviations of the given values from their Arithmetic Mean is minimum.

Ans. (a) Arithmetic Mean

183. Which is greatly affected by the extreme values – Arithmatic mean

Ans. (a) Arithmetic Mean

184. Which is not amenable to further algebric treatment – Mode and Median

Ans. (d) Both (b) and (c)

185. $\dfrac{a+b}{2} = 15 \Rightarrow a + b = 30$ … (i)

b – a = 4 – (ii) [By eq (i) & (ii)]

$2b = 34 \Rightarrow b = 17$

$\therefore$ a = 13 lower limit = 13

Ans. (c) 13

186. Ans. (b) Refer Properties

187. Ans. (a) Refer Properties

188. Ans. (c) Refer Properties

189. Given, consumer price index in, (say), period I = 120 and consumer price index in (say), period II = 215. The wages of the worker in period I and II are given to be Rs. 1,680 and Rs. 3000 respectively. The real wages of the worker in the current period II with respect to the period I as base, are given by:

$$\text{Rs. } \frac{120}{215} \times 3000 = \text{Rs. } 1674.42$$

Since this wage (Rs. 1674.42) is less than the wages of the worker in the period I (Viz. Rs. 1680) the workers is not better off but worse off R. 5.58 as compared to the period I.

Ans. (a)

190. Purchasing power (P.P) of a rupee in 1994 with respect to the base period 1980 is given by

$$\text{P.P. of a rupee} = \frac{100}{\text{Consumer Price Index for 1994 w.r.t. base 1980}}$$

$$= \text{Rs. } \frac{100}{250} = \text{Re. } 0.40$$

Ans. (a)

191. Let B_1, B_2 and B_3 be the events of drawing a boy from the 1st, 2nd and 3rd group respectively and G_1, G_2 and G_3 be the events of drawing a girl from the 1st, 2nd, and 3rd group respectively then $P(B_1) = 1/4$, $P(B_2) = 2/4$, $P(B_3) = 3/4$ and $P(G_1) = 3/4$, $P(G_2) = 2/4$, $P(G_3) = 1/4$.

The required event of getting 1 girl and 2 boys in a random selection of 3 children can materialize in the following mutually exclusive cases.

(i) Girl from the first group and boys from the 2nd and 3rd group i.e. the event $G_1 \cap B_2 \cap B_3$

(ii) Girl from the 2nd group and boys from 1st and 3rd groups, i.e. the event $B_1 \cap G_2 \cap B_3$ happens.

(iii) Girl from the 3rd group and boys from the 1st and 2nd groups. i.e., the event $B_1 \cap B_2 \cap G_3$ happens.

Hence by the addition theorem of probability, required probability ρ is given by:

P = P(i) + P(ii) + P(iii)

$$= P(G_1 \cap B_2 \cap B_3) + P(B_1 \cap G_2 \cap B_3) + P(B_1 \cap B_2 \cap G_3)$$

$$= P(G_1)\,P(B_2)\,P(B_3) + P(B_1)\,P(G_2)\,P(B_3) + P(B_1)\,P(B_2)\,P(G_3)$$

$$= \frac{3}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{1}{4}$$

$$= \frac{18+6+2}{64}=\frac{26}{64}=\frac{13}{32}$$

192. Let P(A) = x

$\therefore$ $P(B) = \frac{3}{2}\ P(A) = \frac{3}{2}x$

and $P(C) = \frac{1}{2}\ P(B) = \frac{1}{2}\left(\frac{3}{2}x\right) = \frac{3}{4}x$

The events A, B and C are exhaustive

$\therefore$ P (A or B or C) = 1

$\Rightarrow$ P(A) + P(B) + P(C) = 1 ($\because$ A, B, C are mutually exclusive)

$$x+\frac{3}{2}x+\frac{3}{4}x=1$$

$$x\left[\frac{4+6+3}{4}\right]=1$$

$$\therefore \quad x = \frac{4}{13}$$

$$\therefore \quad P(A) = \frac{4}{13}$$

Ans. (b)

193. There are 3+4+2+1 = 10 members in all and a Committee of 4 out of them can be formed in $10C_4$ ways. Hence exhaustive number of Cases is:

$$10C_4 = \frac{10\times9\times8\times7}{4!}=210$$

The probability 'p' that the Committee Consists of the doctor and at least one economist is given by

p = P [One doctor, One economist, 2 others]

+ P [One doctor, Two economist, 1 others]

+ P[One doctor, Three economist]

$$p = \frac{1C_1 \times 3C_1 \times 6C_2}{10C_4} + \frac{1C_1 \times 3C_2 \times 6C_1}{10C_4} + \frac{1C_1 \times 3C_3}{10C_4}$$

$$= \frac{1}{210}\left[\left(1 \times 3 \times \frac{6 \times 5}{2}\right) + (1 \times 3 \times 6) + (1 \times 1)\right]$$

$$= \frac{1}{210}[45 + 18 + 1]$$

$$= \frac{64}{210} = \frac{32}{105} = 0.3048$$

Ans. (a)

194. Let A = event that the company executive travel by plane.

$\therefore$ P(A) = 2/3

Let B = event that the Company executive travel by train.

$\therefore$ P(B) = 1/5

Now the events A and B are mutually exclusive, because he cannot travel by plane and train at the same time.

$\therefore$ The prob. of his traveling by plane or train

Ans. (a)

= P (A or B)

= P(A) + P(B)

= 2/3 + 1/5

$$= \frac{13}{15}$$

Ans. (b)

195. Let A and B denote the events that the contractor will get a 'plumbing' Contract and 'Electric' Contract respectively. Then we are given:

P(A) = 2/3, P($\bar{B}$) = 5/9

$\therefore$ P(B) = 1 − P($\bar{B}$) = 4/9

and P $(A \cup B)$ = Probability that Contractor gets at least one contract.

= 4/5

$\Rightarrow$ P(A) + P(B) − P $(A \cap B)$ = 4/5

2/3 + 4/9 − P $(A \cap B)$ = 4/5

$\Rightarrow$ P $(A \cap B)$ = 2/3 + 4/9 − 4/5

$$= \frac{30+20-36}{45}$$

$$= \frac{14}{45}$$

Hence, the probability that the Contractor will get both the contracts in 14/45

Ans. (a)

196. Standard deviation = σ

$P(x > 60) = 0.05$

$\Rightarrow 1 - P(x \leq 60) = 0.05$

$\therefore \quad P(x \leq 60) = 0.95$

$$\therefore \quad P\left[\frac{x-60}{\sigma} \leq \frac{60-50}{\sigma}\right] = 0.95$$

$$\therefore \quad P\left(\leq \frac{10}{\sigma}\right) = 0.95 \Rightarrow \varphi\left(\frac{10}{\sigma}\right) = \varphi(1.64)$$

$$\Rightarrow \quad \sigma = \left(\frac{10}{1.64}\right) = 6.7 \Rightarrow \text{S.D.} = 6.7$$

Ans. (a) 6.7

197. $P\left(Z \geq \frac{x-10}{20}\right) = 0.10$

$$\therefore \quad \frac{100-x}{20} = 1.28$$

$100 - x = 25.6$

$x = 74.40$

Ans. (c) 74.40

198. $P\left(x \leq \frac{x-100}{20}\right) = 0.10$

$$\therefore \quad \frac{x-100}{20} = 1.28$$

$x = 25.6 + 100 = 125.6$

Ans. (b) 125.6

200. $P = P(x>70)$

$= \quad 1 - P(x \leq 70)$

$$= \quad 1 - P\left[\frac{x-65}{25} \leq \frac{70-65}{5}\right]$$

$= \quad 1 - P\ (z \leq 1)$

$= \quad 1\ 0 - .041$

P = 0.06

Ans. (c) 0.06

Model Test Paper – BOS/CPT – 4

151. $P = \dfrac{100\,A}{100+RT} \Rightarrow \dfrac{100 \times 21315}{100 + 0.045 \times \frac{4}{12}}$

P = 21000

Ans. (a) Rs. 21000

152. $I_1 = 500 \times \dfrac{8}{100} \times 1 = 40$

$I_2 = 1000 \times \dfrac{8}{100} \times \dfrac{3}{4} = 60$

$I_3 = 1000 \times \dfrac{8}{100} \times \dfrac{1}{2} = 40$

∴ Total Amount = 500 + 1000 + 1000+40 + 60 + 60

= 2640

Ans. Rs. 2640

153. $2000 = 1200\left(1 + \dfrac{5}{4 \times 100}\right)^{4n}$

$5 = 3\left(\dfrac{81}{80}\right)^{4n}$ After taking log both side and solved.

log 5 = log 3 + 4n [log 81– log 80]

n = 10 years 3 months

Ans. (a) 10 years 3 months

154. $26500 = 20000\left(1+\frac{r}{100}\right)^4$

After taking log and solving it

r = 7.5%

Ans. (c) 7.5%

155. $CI = 7000\left[\left(1+\frac{7}{100}\right)\left(1+\frac{8}{100}\right)\left(1+\frac{85}{100}\right)-1\right]$

CI = 1776

157. $I_1 = P \times \frac{3}{100} \times 2 = \frac{6P}{100}$

$I_2 = P \times \frac{8}{100} \times 3 = \frac{24P}{100}$

$I_3 = P \times \frac{10}{100} \times 1 = \frac{10P}{100}$

Total interest = 1520

$\therefore \frac{40P}{100} = 1520 \Rightarrow P = \frac{100 \times 1520}{40}$

P = 3800

Ans. Rs. 3800 (a)

158. $A = 7500\left[(1+i)^n\right]$ [I = 0.01 n = 2]

$= 7500\ [1+0.01]^2 \Rightarrow 7500 \times (1.01)^2$

A = 7650.75

Ans. (a) Rs. 7650.75

159. $512.50 = P\left[(1+0.05)^2 - 1\right]$

$512.50 = P \times 0.1025$

$\therefore P = 5000$

Ans. (b) Rs. 5000

160. $1331 = 1000\left[1+\frac{r}{100}\right]^3$

$\left(\frac{11}{10}\right)^3 = \left(1+\frac{r}{100}\right)^3 \Rightarrow 1.1 = 1+\frac{r}{100}$

$\therefore$ 0.1 = r / 100 $\Rightarrow$ r = 10%

Ans. (a) 10%

161. Range = L − S

L … S = 20 $\rightarrow$ (i)

If each item is increased by 15

Range = (L + 15) − (S + 15)

Range = L − S = 20 [from eg (i)]

Ans. (a) 20

162. Range = L − S

$\therefore$ L − S 20

If each item is divided by − 2

$$\text{Range} = \frac{L}{-2} - \frac{S}{-2} = \frac{-1}{2}(L-S)$$

$$\text{Range} = \frac{-1}{2} \times 20 = -10 \text{ [(−) sign ignored]}$$

Range = 10

(because it is difference between largest and smallest data)

Ans. (b) 10

164. In grouped frequency distribution, if the class interval is unequal then quartile deviation is more appropriate.

Ans. (a) Q.D.

165. $$SD = \sqrt{\frac{\sum d^2}{n}} \Rightarrow (4)^2 = \frac{\sum d^2}{10} \Rightarrow \sum d^2 = 160$$

If each item divided by − 2

$$\text{Corrected } \sum (d')^2 = \frac{160}{(-2)^2} = 40$$

$$\therefore \text{ Corrected S.D.} = \sqrt{\frac{\sum (d')^2}{n}} = \sqrt{\frac{40}{10}} = 2$$

S.D. = 2

Ans. (a) 2

166. $\bar{x} = \frac{(5+10+15 .... +125}{25}$

$$\frac{\frac{25}{2}\left[2\times5+(25-1)5\right]}{25} = \frac{130}{2} = 65$$

Average = 65

Ans. (a) 65

167. a, b, c, d, e are five add integers.

$$\text{average} = \frac{a+b+c+d+e}{5} = \frac{a+(a+2)+(a+4)+(a+6)+(a+8)}{5}$$

$\Rightarrow (a + 4)$

Ans. (d) a+4

168. $Av = \frac{180+258+x}{3}$

$$230 = \frac{438+x}{3} \Rightarrow 690-438 = x$$

$\therefore$ x = 252 he should score 252 runs.

Ans. (d) None of these

169. $100 = \frac{(x+2)60 + x.120 + (x-2)180}{(x+2)+x+x-2}$

$300x = 360x - 240$

$\therefore$ $60x = 240$

$x = 4$

Ans. (a) 4

170. $16 = \frac{\sum x}{25} \Rightarrow \sum x = 400$

$$15 = \frac{\sum x^1}{24} \Rightarrow \sum x^1 = 360 \quad \therefore \text{Age of Teacher} = 400 - 360 = 40$$

Ans. 40 Years

171. Ans. (b) Refer Properties

172. Ans. (b) Refer Properties

173. Given two regression lines are

$3x+2y=26 \rightarrow (1)$

$6x+y=31 \rightarrow (2)$

Since the two lines of regression intersect at the point $(\bar{x},\bar{y})$, replacing $\bar{x}$ and $\bar{y}$ by and respectively in the given regression equation, we get.

(1) $\Rightarrow$ $2\bar{y} = 26 - 3\bar{x}$

$\bar{y} = 13 - \frac{3}{2}\bar{x} \rightarrow (3)$

(2) $\Rightarrow$ $6\bar{x} + 13 - \frac{3}{2}\bar{x} = 31$

$\frac{12\bar{x}+26-3\bar{x}}{2} = 31$

$9\bar{x}+26=62$

$9\bar{x}=62-26$

$= \quad 36$

$\bar{x} = \quad 4$

$\therefore$ (3) $\Rightarrow$ $\bar{y} = 13 - \frac{3}{2}$ (4)

$= \quad 13 \quad - \quad 6 = 7$

$\therefore$ $\bar{x} = 4,\ \bar{y} = 7$

Ans. (a)

174. Let us assume that 3x + 24 = 26 $\rightarrow$ (1) represent the regression line of y on x and

6x + y = 31 $\rightarrow$(2) represent the regression line of x on y.

(1) $\Rightarrow$ $2y = 26 - 3x$

$y = 13 - \frac{3}{2} - x$

$\therefore$ $byx = -\frac{3}{2}$

(2) $\Rightarrow$ $6x = 31 - y$

$x = \frac{31}{6} - \frac{1}{6}y$

$\therefore$ $bxy = -\frac{1}{6}$

$$\therefore \quad r^2 = byx \times bxy = \left(-\frac{3}{2}\right)\left(-\frac{1}{6}\right) = \frac{1}{4}$$

$$r = \sqrt{\frac{1}{4}} = \pm\frac{1}{2} = \pm 0.5$$

Ans. (b)

We take the sign of n as negative since both the regression coefficients are negative).

175. Ans. (a) Refer Properties

176. Let E_1, E_2, E_3 denote the events that the probability is solved by X,Y and Z respectively.

Then we have

$P(E_1) = 1/3 \Rightarrow P(\overline{E}_1) = 1 - P(E_1) = 2/3$ –

$P(E_2) = 1/4 \Rightarrow P(\overline{E}_2) = 1 - P(E_2) = 3/4$

$P(E_3) = 1/5 \Rightarrow P(\overline{E}_3) = 1 - P(E_3) = 4/5$

Problem will be solved if at least one of the three is able to solve it. Hence, the required probability that the problem will be solved is given by $P(E_1 \cup E_2 \cup E_3)$

$= \quad 1 - P(\overline{E}_1 \cap \overline{E}_2 \cap \overline{E}_3)$

$= \quad 1 - \left[P\left(\overline{E}_1\right).\, P\left(\overline{E}_2\right).P\left(\overline{E}_3\right)\right]$

$= \quad 1 - 2/3 \times 3/4 \times 4/5$ [Since E_1, E_2, E_3 are independent]

$= \quad 1 - 2/5 = 3/5$

Ans. (c)

177. Given $P(A) = \frac{1}{2}$

$P(B) = \frac{1}{3}$

$P(A \cap B) = \frac{1}{4}$

$$\therefore \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{1}{4} \times \frac{3}{1} = \frac{3}{4}$$

Ans. (a)

ANSWERS

178. Given $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$

$\therefore P(\overline{A} \cap B) = P(B) - P(A \cap B)$

$= \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$

Ans. (c)

179. Given $P(A)\ \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{4}$

$P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$

$= 1 - \{P(A) + P(B) - P(A \cap B)\}$

$= 1 - P(A) - P(B) + P(A \cap B)$

$= 1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4}$

$= \frac{12 - 6 - 4 + 3}{12} = \frac{5}{12}$

Ans. (a)

180. $P(A) = ½$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{4}$

$= P(\overline{A} \cup \overline{B}) = P(\overline{A \cap B})$

$= 1 - P(A \cap B)$

$= 1 - ¼$

$= \frac{4-1}{4} = \frac{3}{4}$

Ans. (b)

181. Given $x_1 = 1, x_2 = 2, x_3 = 3$

$P(x_1) = \frac{1}{2}, P(x_2) = \frac{1}{3}, P(x_3) = 1/6$

$\therefore\ E(x) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$

$= \left(1 \times \frac{1}{2}\right) + \left(2 \times \frac{1}{3}\right) + \left(3 \times \frac{1}{6}\right) = \frac{1}{2} + \frac{2}{3} + \frac{1}{2}$

$$= \frac{3+4+3}{6} = \frac{10}{6} = \frac{5}{3} = 1.666....$$

$= 1.67$

Ans. (c)

182. Given $x_1 = 1, x_2 = 2, x_3 = 3$

$$P(x_1) = \frac{1}{2}, P(x_2) = \frac{1}{3}, P(x_3) = \frac{1}{6}$$

$\therefore \quad V(x) = E(x^2) \ldots [E(x)]^2$

$E(x) = x_1 P(x_1) + x_2 P(x_2) + x_3 P(x_3)$

$$= \left(1 \times \frac{1}{2}\right) + \left(2 \times \frac{1}{3}\right) + \left(3 \times \frac{1}{6}\right)$$

$$E(x) = \frac{1}{2} + \frac{2}{3} + \frac{1}{2}$$

$$= \frac{3+4+3}{6} = \frac{10}{6} = \frac{5}{3}$$

$$E(x^2) = x_1^2 P(x_1) + x_2^2 P(x_2) + x_3^2 P(x_3)$$

$$= \left(1 \times \frac{1}{2}\right) + \left(4 \times \frac{1}{3}\right) + \left(9 \times \frac{1}{6}\right)$$

$$= \frac{1}{2} + \frac{4}{3} + \frac{3}{2}$$

$$= \frac{3+8+9}{6} = \frac{20}{6} = \frac{10}{3}$$

$$\therefore \quad V(x) = \frac{10}{3} - \left(\frac{5}{3}\right)^2$$

$$= \frac{10}{3} - \frac{25}{9}$$

$$V(x) = \frac{30-25}{9} = \frac{5}{9} = .5556$$

Ans. (a)

183. Let x denote the number of defective lamps.

X can assume the values 0, 1, 2, 3

p(X=0) p:probably having 0 bad orange out of 4 bad orange and 3 good orange out of 8 good orange.

$$P(x=0) = \frac{4C_0 \times 8C_3}{12C_3} = \frac{56}{55}$$

$$P(x=1)= \frac{4C_1 \times 8C_2}{12C_3} = \frac{28}{55}$$

$$P(x=2)= \frac{4C_2 \times 8C_2}{12C_3} = \frac{12}{55}$$

P(x=3) = Probability having 3 bad orange out of 4 bad orange and 0 good orange out of 8 good orange.

$$= \frac{4C_3 \times 8C_0}{12C_3} = \frac{1}{55}$$

Probability that at least one orange out of three oranges is good = 1 − P (x=3)

= 1 − 1/55

$$= \frac{55-1}{55} = \frac{54}{55}$$

Ans. (a)

184. Given P(A) = 0.5, P (AB) <_ 0.3

By Addition thereon,

P(A or B) = P(A) + P(B) − P (AB)

∴ P(A) + P(B)−P(AB) < 1 [∴ P(A or B) ≤ 1]

∴ P(B) ≤ 1 − P(A) + P (AB)

≤ 1 − 0.5 + 0.3

P(B) ≤ 0.8

Ans. (a)

185. Let the given events be A, B and P(A) = 2/3 P(B)

Let P(B) = x

∴ P(A) = 2/3 x

The events A and B are exhaustive

∴ P(A or B) = 1

P(A) + P(B) = 1

⇒ 2/3 x + x = 1

5/3 x = 1

x = 3/5

∴ P(B) = 3/5

P(A) = 2/3 x 3/5 = 2/5

P(B) = 3/5 ⇒ odds in favour of B are

3 : 5 − 3 = 3: 2

Ans. (b)

186. Given person variates with parameter = 1

i.e. $\lambda = 1$

By the poison distribution

$$p(x) = \frac{e^{-\lambda}.\lambda^x}{x!}, x > 0$$

∴ The required probability

$$P(3 < \times < 5) = P(x = 4)$$

$$= \frac{e^{-\lambda}.\lambda^4}{4!}$$

$$= \frac{e^{-1}(1)^4}{4!}$$

$$= \frac{0.36783 \times 1}{24}$$

$$P(3 < x < 5) = 0.015326$$

Ans. (a)

187. Given p = 2% = 2/100 = .02

n = 200

∴ $\lambda = np = 200 \times .02 = 4$

The probability of at least 5 defective means.

$$P(x \geq 5) = 1 - P(x < 5)$$

$$= 1 - \{P(x=0) + P(x=1) + P(x=2) + p(x=3) + P(x=4)\}$$

$$= 1 - \left\{\frac{e^{-4}(4)^0}{0!} + \frac{e^{-4}(4)^1}{1!} + \frac{e^{-4}(4)^2}{2!} + \frac{e^{-4}(4)^3}{3!} + \frac{e^{-4}(4)^4}{4!}\right\}$$

$$= \quad 1-e^{-4}\left\{1+4+\frac{4^2}{2!}+\frac{4^3}{3!}+\frac{4^4}{4!}\right\}$$

$$= \quad 1-e^{-4}\left\{1+4+\frac{16}{2}+\frac{64}{6}+\frac{256}{24}\right\}$$

$$= \quad 1-(0.183)\ \{5+8+10.6667+10.667\}$$

$$= \quad 1-(0.83)\ (34.3334)$$

$$= \quad 1\ \ 0.6283$$

P $(x \geq 5) = 0.3717$

Ans. (a)

188. After a man is dealt 4 spade cards from an ordinary pack of 52 cards, there are 52 − 4 = 48 cards left in the pack, out of which 9 are spade cards and 39 are no spade cards. Now, 3 more cards can be dealt to the same man out of the 48 cards in $^{48}C_3$ ways, which determines the exhaustive number of ways.

If none of these 3 additional cards is a spade cards, then the 3 additional cards must be drawn out of the 39 non-spade cards, which can be done in $39C_3$ ways.

The probability that none of the three additional cards dealt to the man is a spade card =

$$\frac{39C_3}{48C_3}$$

Hence, the required probability, 'P' that at least one of the additional cards is a spade cards is given by:

$$p = \quad 1-\frac{39C_3}{48C_3}$$

$$= \quad 1-\frac{39\times38\times37}{3!}\times\frac{3!}{48\times47\times46}$$

$$= \quad 1-\frac{13\times19\times37}{16\times47\times23}$$

$$= \quad 1-\frac{9136}{17296}$$

$$= \quad 10.5282$$

p = 0.4718

Ans. (c)

190. Given P (x = 1) = P (x = 2)

Given x is a poison variable.

$$\therefore \frac{e^{-\lambda}(\lambda)^{(1)}}{1!} = \frac{e^{-\lambda}(\lambda)^{(2)}}{2!}$$

$$\lambda = \frac{\lambda^2}{2}$$

$\lambda = 2$ = variance

Ans. (b)

191. $P = \frac{65}{500}$ $Q = 1 - P = 1 - \frac{65}{500}$

$Q = \frac{435}{500}$

n = 500

SE of Proportion of defectives = $\sqrt{\frac{PQ}{n}}$

$= \sqrt{\frac{65}{500} \times \frac{435}{500} \times \frac{1}{500}}$

SE = 0.015

Ans. (a) 0.015

192. Standard Error of Mean (SE) = $\frac{\sigma}{\sqrt{n}} = \frac{0.75}{\sqrt{100}}$

SE = 0.075

95% Confidence Limit for population mean are given by : x ± 1.96 SE

= 5.6 ± 1.96 × 0.75

= 5.6 ± 0.147

The Confidence level are 5.453 and 5.747

Ans. (a) 5.453 and 5.747

193. Standard Error (SE) = $\frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{128}} = 0.353$

96% confidence limit for population mean are

$\Rightarrow \bar{x} + 2.05 \times SE$

= 28 ± 2.05 × 0.353 ⇒ 28 ± 0.72

The confidence level are 27.272 and 28.728

Ans. (b) 27.272 and 28.728

ANSWERS

194. $P = \frac{65}{500}$, $Q = 1 - P = 1 - \frac{65}{500} \Rightarrow Q = \frac{435}{500}$

SE of proportion of defectives = $\sqrt{\frac{PQ}{n}}$

$= \sqrt{\frac{65}{500} \times \frac{435}{500} \times \frac{1}{500}}$

SE = 0.015

Confidence limits for the population are

$= P \pm 3 \times SE$

$= \frac{65}{500} \pm 3 \times 0.015 \Rightarrow 0.13 \pm 0.045$

Levels are 0.085 and 0.175

or Levels are 8.5% and 17.5%

Ans. (a) 8.5% and 17.5

196. Variance = 4

$\sigma = \sqrt{4} = \pm 2$

Statement is true

Ans. (a) True

197. n = 10 P = 0.3

$\therefore Q = (1 - P) = 1 - 0.3 = 0.7$

$\therefore \sigma = \sqrt{npq}$

$\therefore$ Variance = npq = 10×0.3×0.7

Variance = 2.1

Ans. (a) 2.1

198. When the cost of living increases, the standard of living improves.

Ans. (b) false

199. The 95% confidence limit for the sample mean $(\bar{x})$ is $\bar{x} \pm 1.96 \left(\frac{\sigma}{\sqrt{n}}\right)$ which is not given

Ans. (b) False

200. Mean and variance never be equal

$\therefore$ Statement is false

Ans. (b) False

Model Test Paper – BOS/CPT – 5

151. Let the fraction be x/y. Then according to the given condition of the problem,

$$\frac{3x}{y-3} = \frac{18}{11}$$

33x = 18y – 54

33x – 18y+54 = 0

11x – 6y+18 = 0 .... (i)

and $\frac{x+8}{2y} = \frac{2}{5}$

$\Rightarrow$ 5x + 40 = 4y

5x – 4y+40 = 0 ....(ii)

(i) × 2 $\Rightarrow$ 22x – 12y+36 = 0 ....(iii)

(ii) × 3 $\Rightarrow$ 15x–12y+120 = 0 ....(iv)

(iii) – (iv),we get

7x – 84 = 0

7x= 84

x = 84/7 = 12

(i) $\Rightarrow$ (11) (12) – 6y + 18 = 0

132 – 6y + 18 = 0

6y = 150

y =150/6 = 25

Hence, the required fraction is 12/25

$\therefore$ Ans. (c)

152. Let the two numbers are x and y

Given x + y = 150 % of y

$= \frac{150}{100} \times y$

$x + y = 1.5y$

$x = 0.5\ y$

$x = \frac{1}{2}\ y$

Ans. (a)

153. Let three consecutive even numbers are x, x + 2, x + 4.

Given condition is $x + (x+2) + (x+4) = 60 \times \frac{3}{4} - 15$

$3x + 6 = 30$

$3x = 24$

$x = \frac{24}{3} = 8$

$\therefore$ The middle number = x + 2 = 8 + 2 = 10

Ans. (b)

154. Suppose my present age is x years and my sons present age is y years

Five years ago

my age = (x – 5) years

my son's age = (y – 5) years

According to the first condition of the problem,

$x - 5 = 3(y - 5)$

$x - 5 = 3y - 15$

$\Rightarrow x \quad 3y = 15 + 5$

$\Rightarrow x - 3y = -10$ ....(i)

Ten years later

my age = (x+10) years

my son's age = (y+10) years

According to the second condition of the problem,

$x+10 = 2(y+10)$

$x+10 = 2y+20$

$x-2y = 20-10$

$x - 2y = 10$ ....(i i)

(i) − (ii) ⇒ $y = 20$

(i) ⇒ $x - 60 = -10$

$x = 60 - 10 = 50$

Hence, my presence age = 50 years

and my son•s present age = 20 years

∴ Ans.(a)

155. The compound ratio of 4:3, 9:13, 26:5 and 2:15 is

$$= \frac{4 \times 9 \times 26 \times 2}{3 \times 13 \times 5 \times 15}$$

$$= \frac{16}{25}$$

Ans. (b)

156. We know nPr = $\frac{n!}{(n-4)!}$

$$\therefore \quad 56_{P_{r+6}} = \frac{56!}{\{56-(r+6)\}!}$$

$$= \frac{56!}{(50-r)!}$$

$$54P_{r+3} = \frac{54!}{\{54-(r+3)\}!}$$

$$= \frac{54!}{(51-r)!}$$

Thus, $\frac{56\,P_{r+3}}{54\,P_{r+3}} = \frac{56!}{(50-r)!} \times \frac{(51-r)!}{54!}$

$$= \frac{56 \times 55 \times 54!}{(50-r)!} \times \frac{(51-r)(50-r)!}{54!}$$

$$= \frac{56 \times 55 \times (51-r)}{1}$$

But we are given the ratio as 30800 : 1

$$\therefore \frac{56 \times 55 \times (51-r)}{1} = \frac{30800}{1}$$

(or) $(51 - r)! = \frac{30800}{56 \times 55} = 10 \therefore r = 41$

Ans. (b)

157. He can arrange his schedule in

8P6 = 8 × 7 × 6 × 5 × 4 × 3

= 20160 ways.

Ans. (b)

158. The two Indians can stand together in $^2P_2 = 2! = 2$ ways.

So is the case with the two Americans and the two Russians.

Now these 3 groups of 2 each can stand in a row in $^3P_3 = 3 \times 2 = 6$ ways. Hence by the generalized fundamental principle, the total num ber of ways in which they can stand for a photograph under given conditions is

6 × 2× 2× 2 = 48

Ans. (c)

159. This is the number of combination of 52 cards taken five at a time.

Now applying the formula.

$52C_5 = \frac{52!}{5!(52-5)!}$

$= \frac{52!}{5!47!}$

$= \frac{52 \times 51 \times 50 \times 49 \times 48 \times 47!}{5 \times 4 \times 3 \times 2 \times 1 \times 47!}$

= 2598960

Ans. (a)

160. Let the unit's digit of the number be x and the ten's digit by y. Then

$x + y = 9 \rightarrow (1)$

and the number = 10y + x

Reversing the order of digits of the given number,

Unit's digits becomes y

and ten's digits becomes x

$\therefore$ Now number = 10x + y

According to the given condition of the problem,

$(10x + y) – (x+10y) = 27$

$10x + y – x – 10y = 27$

$9x – 9y = 27$

$x – y = 3 \rightarrow (2)$

Adding (1) and (2), 23 get

$2 \times = 12$

$x = \frac{12}{2} = 6$

$(1) \Rightarrow 6 + y = 9$

$y = 9 – 6 = 3$

$\therefore$ The given number is 36

Ans. (b)

161. $\underset{x\to 0}{\text{Lt}} \frac{9^x - 3^x}{4^x - 2^x} \Rightarrow \underset{x\to 0}{\text{Lt}} \left[\frac{\frac{(9^x-1)-(3^x-1)}{x}}{\frac{(4^x-1)-(2^x-1)}{x}} \right]$

$\underset{x\to 0}{\text{Lt}} \left[\frac{\left(\frac{9^x-1}{x}\right)-\left(\frac{3^x-1}{x}\right)}{\left(\frac{4^x-1}{x}\right)-\left(\frac{2^x-1}{x}\right)} \right] \Rightarrow \frac{\log 9 - \log 3}{\log 4 - \log 2}$

$\Rightarrow \frac{2\log 3 - \log 3}{2\log 2 - \log 2} = \frac{\log 3}{\log 2}$

Ans. (a) $\frac{\log 3}{\log 2}$

162. $\underset{x\to 0}{\text{Lt}} \frac{(5^x-1)^2}{\log(1+x)} = \frac{5^{2x} - 2.5^x + 1}{\log(1+x)}$

$\Rightarrow \underset{x\to 0}{\text{Lt}} \frac{(25^x - 2.5^x + 1)}{\log(1+x)} \Rightarrow \underset{x\to 0}{\text{Lt}} \frac{\left(\frac{25^x-1}{x}\right) - 2\left(\frac{5^x-1}{x}\right)}{\frac{\log(1+x)}{x}}$

App Lt

$\Rightarrow \frac{\log 25 - 2.\log 5}{1} = \frac{\log 25 - \log 25}{1} = 0$

Ans. (d) None of these

163. $\underset{x\to 1}{\text{Lt}}\ f(x) \Rightarrow \underset{x\to 1}{\text{Lt}}\ (x+1)$

App Lt

$\Rightarrow 1 + 1 = 2$

Ans. (a) 2

164. LHL $\underset{x\to 2}{\text{Lt}}\ (x - 1)$

App Lt

$\Rightarrow 2 - 1 = 1 \quad \therefore$ LHL = 1

RHL $\underset{x\to 2}{\text{Lt}}\ (2x - 3)$

App. Lt

LHL 2.2 – 3 = 1

f(2) = 2.2 – 3 = 1 $\quad\quad \therefore$ LHL = RHL = f(2)

$\therefore$ f(x) Continuous at x = 2

Ans. (a) Continuous x = 2

165. $f(x) = \frac{3x^2 + 2x + 7}{x^2 - 3x + 2} = \frac{3x^2 + 2x + 7}{(x-2)(x-1)}$

To be continuous $(x - 2) \neq 0$ & $(x - 1) \neq 0$

$\therefore x \neq 2$ & $x \neq 1$

$\therefore$ Points of discontinuity = 1, 2

Ans. (a) 1, 2

166. Let z = log x

$dz = \frac{1}{x} dx$

dx = x dz

$\therefore I = \int \frac{1}{x \log x} dx = \int \frac{1}{x.z} xdz$

$= \int \frac{1}{z} dz$

= $\log z$

= $\log(\log x) + c$

ans. (b)

167. Let $I = \int \log_{10}^{x} \, dx$

= $\int \log_e^x \cdot \log_{10}^e \, dx$

= $\log_{10}^e \int \log x.1 \, dx$

= $\log_{10}^e \left[\log x.x - \int \frac{1}{x} \times dx\right]$

$I = \log_{10}^e \left[x \log x - x\right] + c$

168. Let $I = \int \frac{4e^{x} + 6e^{-x}}{9e^{x} - 4e^{-x}} dx$

$\therefore \quad I = \int \frac{4e^{2x} + 6}{9e^{2x} - 4} dx$

Let $t = e^{2x}$

$\therefore dt = 2 \; e^{2x} \; dx$

= $2t \; dx$

$\therefore I = \int \frac{4t+6}{9t-4} \frac{dt}{2t} = \int \frac{2t+3}{t(9t-4)} dt$

= $\int \left[\frac{0+3}{t(0-4)} + \frac{2(4/9)+3}{4/9(9t-4)}\right] dt$

= $\int \left[-\frac{3}{4t} + \frac{35}{4(9t-4)}\right] dt$

= $-\frac{3}{4}\log t + \frac{35}{4} \cdot \frac{\log(9t-4)}{9} + c$

= $-\frac{3}{4}\log e^{2x} + \frac{35}{36}\log\left(9e^{2x} - 4\right) + c$

Ans. (a)

169. See formula from the text book

Ans. (c)

170. Put $\sqrt{x^2 - 6x + 100} = t$

$\therefore \quad x^2 - 6^x + 100 = t^2$

(2x – 6) dx = 2t dt

(x – 3) dx = t dt

$\int (x-3)\sqrt{x^2 - 6x + 100}\ dx = \int \sqrt{x^2 - 6x + 100}\ (x-3)\,dx$

$= \quad \int t.tdt$

$= \quad \int t^2\ dt$

$= \quad \int \frac{t^3}{3} + c$

$= \quad \frac{1}{3}(x^2 - 6^x + 100)^{3/2} + c$

Ans. (c)

171. No. of ways in which one or more friends may invited

$= \ 6_{C_1} + 6_{C_2} + 6_{C_3} + 6_{C_4} + 6_{C_5} + 6_{C_6}$

$= 2^6 - 1$ = 63 ways.

Ans. (a) 63 ways.

172. No. of ways of failure of candidate.

$= \quad 4_{C_1} + 4_{C_2} + 4_{C_3} + 4_{C_4}$

$= \quad 2^4 - 1$ = 15 ways.

Ans. (c) 15

173. A voter can vote in the following ways

$254 = n_{C_1} + n_{C_2} + n_{C_3} + n_{C_4} + n_{C_5} + n_{C_6} + n_{C_7} .... n_{C_{n-1}}$

$\therefore$ $254 = 2^n - (n_{C_n} + 1) = 2^n - (1+1)$

$\therefore$ $256 = 2^n$

$\therefore$ $2^8 = 2^n \Rightarrow \therefore n = 8$ Total candidates = 8

Ans. (a) 8

174. No. of words of 3 consonants and 2 vowels among 17 consonants and 5 vowels are

$= 17_{C_3} \times 5_{C_2} \times 5!$

= 816000

Ans. (b) 81,6000

176. The present value of annual profit

V = A.P. (ni)

= 34000 × 3.7079

V = 128886 which is less than initial cost of machine. Machine must not be purchased

Ans. (a) Machine should not be purchased.

177. $40 = 2000 \times \frac{r}{100} \times 4$

$\therefore$ r = -0.5%

Ans. (b) 0.5%

ANSWERS

178. $I_1 = 2000 \times \frac{4}{100} \times 1 = 80$

$I_2 = 3000 \times \frac{14}{100} \times 1 = 420$

Total Interest = 500

$\therefore$ rate of interest = $\frac{500 \times 100}{5000 \times 1} = 10\%$

r = 10%

Ans. (a) r = 10%

179. $I_1 - I_2 = 30 \Rightarrow 1200 \times \frac{R}{100} \times 3 - 1000\ \frac{R}{100} \times 3 = 30$

$\Rightarrow 36R - 30R = 30$

$6R = 30$

$\therefore R = 5\%$

Ans. (c) 5%

180. $40 = 2000 \times \frac{2}{100} \times n$

n = 1 yr.

Ans. (a) = 1 yr.

181. $\frac{a+b}{2} = 20 \Rightarrow a + b = 40 \quad \rightarrow (1)$

$SD = 5 \rightarrow \frac{a-b}{2} = 5$

$\therefore$ a … b = 10 $\rightarrow$ (2)

$\Rightarrow 2a = 50 \Rightarrow a = 25$

$\therefore b = 15$

Ans. (a) 25, 15

182. Mean $= \frac{\sum x}{n}$

$4.4 = \frac{1+2+6+a+b}{5}$

$\therefore$ $a + b = 13 \rightarrow (1)$

$\sigma^2 = \frac{\sum x^2}{N} - \bar{x}^2$

$8.24 = \frac{\sum x^2}{5} - (4.4)^2$

$\sum x^2 = 138$

$1+4+9+a^2+b^2 = 138$

$\therefore$ $a^2 + b^2 = 138 \Rightarrow a^2 + (13 - a)^2 = 138$

$\Rightarrow$ $a^2 - 13a - 36 = 0$ $a = 9, 4$

$\therefore$ Nos $\rightarrow$ 9, 4

183. For individual series, the rank of the median is = $\left(\frac{N+1}{2}\right)^{th}$ term

Ans. (b) $\left(\frac{N+1}{2}\right)^{th}$ term

184. Rank of the median of the series 2, 3, 4, 5, 6, 7

$= \left(\frac{N+1}{2}\right)^{th}$ term $= \left(\frac{6+1}{2}\right)^{th}$ term

= 3.5 th term

Ans. (a) 3.5

185. Regression Eq. $2x + 3y - 10 = 0$

If $y = 50$

$\therefore 2x = +10 - 3 \times 50 = -140$

$x = -70$

None of these

Ans. (d) None of these.

ANSWERS

186. Ans. (c) Refer Properties

187. Ans. (c) Refer Properties

188. Given r (x, y) = 0.4 $\rightarrow$ (1)

We know that r (a X, cY) = $\frac{a \times c}{|a| \times |c|}.r(x,y) \rightarrow (2)$

Using (2) in (1), we get

r (2x, – y) = r (2x, – 1y)

$= \frac{2 \times (-1)}{|2| \times |-1|}.r(x,y)$

$= \frac{-2 \times 0.4}{2 \times 1}$

r (2x, – y) = – 0.4

Ans. (b)

189. Computation of Correlation Coefficient

	x	y	xy	x^2	y^2
	69	70	4830	4761	4900
	85	87	7395	7225	7569
Total	154	157	12225	11986	12469

$\bar{x} = \frac{154}{2} = 77, \quad \bar{y} = \frac{157}{2} = 78.5$

Cov (x,y) = $\frac{\sum x_i y_i}{n} - \bar{x}\,\bar{y} = \frac{12225}{2} - (77)\ (78.5)$

= 68

Sx = $\sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2} = \sqrt{\frac{11986}{2} - (77)^2}$

= 8

Sy = $\sqrt{\frac{\sum y_i^2}{n} - \bar{y}^2} = \sqrt{\frac{12469}{2} - (78.5)^2} = 8.5$

$\therefore$ n = $\frac{\text{Cov}\ (x,y)}{\text{Sx}\ \ \text{Sy}} = \frac{68}{8 \times 8.5} = \frac{68}{68} = 1$

Ans. (a)

190. Computation of correlation Co – efficient.

	x	y	xy	x^2	y^2
	102	50	5100	10404	2500
	109	48	5232	11881	2304
Total	211	98	10332	22285	4804

$$\bar{x} = \frac{211}{2} = 105.5,\ \bar{y} = \frac{98}{2} = 49$$

$$\text{Cov (x, y)} = \frac{\sum xi\ yi}{n} - \bar{x}\ \bar{y} = \frac{10332}{2} - (105.5)(49)$$

$= \quad 5166 - 5169.5 = -3.5$

$$Sx = \sqrt{\frac{\sum xi^2}{n} - \bar{x}^2} = \sqrt{\frac{22285}{2} - (105.5)^2} = 3.5$$

$$Sy = Sx = \sqrt{\frac{\sum yi^2}{n} - \bar{y}^2} = \sqrt{\frac{4804}{2} - (49)^2} = 1$$

$$\therefore \quad n = \frac{\text{Cov } (x, y)}{Sx\ Sy} = \frac{-3.5}{(3.5)\times(1)} = -1$$

Ans. (b)

191. Ans. (a) … Refer Properties

192. Ans. (a) … Refer Properties

193. Ans. (b) … Refer Properties

194. Given X ~ N (μ, σ2), where $\mu = 2$ and $\sigma^2 = 9$

$\sigma = 3$

We want x so that

$P(2 \leq \times \leq x) = 0.4115 \ \rightarrow (1)$

When X = 2, Z = $\frac{x-\mu}{\sigma} = \frac{2-2}{3} = 0$

When x = x, Z = $\frac{x-2}{3}$ = Z1 (Say) $\rightarrow (2)$

From (1), we get $P(0 \leq Z \leq Z_1) = 0.4115$

$\Rightarrow \quad Z_1 = 1.35$ (from Normal Table)

Substituting in (2), we get

$$\frac{x-2}{3} = 1.35$$

$\therefore$ x = 2+3 (1.35)

x = 6.05

Ans. (b)

195. Mean = First moment about origin = 35 (given) $\rightarrow$ (1)

Second moment about 35 = 10 (given)

$\Rightarrow$ Second moment about mean = 10

$\mu 2 = 0 \rightarrow (2)$

Since the given distribution is normal,

$\beta 1 = 0$ and $\beta 2 = 3$

$$\therefore \beta 1 = \frac{\mu 3^2}{\mu 2^3} = 0 \Rightarrow \mu 3 = 0$$

$$\beta 2 = \frac{\mu 4}{\mu 2^2} = 3 \Rightarrow \mu 4 = 3\ \mu_2^2 = 3 \times 10^2 = 300$$

$\therefore$ $\mu 1$=0 (always), $\mu 2 = 10$, $\mu 3 = 0$, $\mu 4 = 300$

Ans. (c)

196. The most commonly used confidence limit is $\rightarrow$ 95%

Ans. (c) 95%

197. Sample mean is statistic

Ans. (b) Statistic

198. Deliberate sampling is – Non \random sampling

Ans. (b) Non random sampling

199. Stratified random sampling issued for Non – Homogeneous population.

Ans. (b) Non – homogeneous

200. Random Sampling is also called lottery sampling

Ans. True

Model Test Paper – BOS/CPT – 6

151. Let given number is x

Then the condition $\frac{1}{5}\left(\frac{1}{3}\left(\frac{1}{2}x\right)\right) = 15$

$$\frac{x}{30} = 15$$

$$x = 450$$

152. Let the number be = x.

Then given the condition = $\frac{3}{4}\left(\frac{1}{5}x\right) = 60$

$$\frac{3x}{20} = 60$$

$$x = \frac{60 \times 20}{3} = 400$$

Ans. (b)

153. Let the number be x.

Given $\frac{4}{5}\left[\frac{3}{8}(x)\right] = 24$

$$\frac{3}{10}x = 24$$

$$x = 24 \times \frac{10}{3} = 80$$

$\therefore$ 250% of x = 250% of 80 = $\frac{250}{100} \times 80 = 200$

Ans. (d)

154. Let the number be x

Given condition $x + x^2 = 182$

$$x^2 + x - 182 = 0$$

$$(x+14)(x-13) = 0$$

$$x = -14,\ x = 13$$

$\therefore$ x = 13 (negative reflected)

Ans. (a)

ANSWERS

155. Let the unit's digit of the number be x and ten's digit by y

Then $x + y = 12 \rightarrow (1)$

and the number = 10y + x

Reversing the order of digits of the given number,

Unit's digit becomes y

and ten's digits becomes x

$\therefore$ New number = 10 x + y

According to the given condition of the problem (10x + y) – (x + 10y) = 18

$9x - 9y = -18$

$x - y = -2 \rightarrow (2)$

Adding (1) and (2) $\Rightarrow 2x = 10$

$x = 5$

$\therefore \Rightarrow y = 7$

$\therefore$ The number is 75

Ans. (a)

156. Let the number of coins is x

Given $10x + \frac{14}{2}x + \frac{18}{4}x = 430$

$$\frac{40x + 28x + 18x}{4} = 430$$

$$\frac{86x}{4} = 430$$

$$x = \frac{430 \times 4}{86} = 20$$

$\therefore$ The one Rupee coins = 10x = 10×20 = 200

The 50 paise coins = 14x = 14×20 = 280

The 25 paise coins = 18x = 18×20 = 360

Ans. (a)

157. First Vessels Contain Milk Ratio 5

First Vessels Contain Water Ratio 2

Second Vessels Contain Milk Ratio 6

Second Vessels Contain Water Ratio 1

Both the Vessels Milk = 5 + 6 = 11

Both the Vessels Water = 2 + 1 = 3

$\therefore$ The new Ratio = 11:3

Ans. (b)

158. Let the two numbers are x and y

Given $8x = 5y \rightarrow (1)$

and $x + 27 = y \rightarrow (2)$

$(2) \Rightarrow x = y - 27$

$(1) \Rightarrow 8(y - 27) = 5y$

$8y - 216 = 5y$

$3y = 216$

$$y = \frac{216}{3} = 72$$

$\therefore$ $x = 72 - 27 = 45$

$\therefore$ Sum of two number = $x + y = 72 + 45 = 117$

Ans. (c)

159. Let their monthly incomes be Rs. 9x and Rs. 7x respectively.

Let their monthly expenditures be Rs. 4y and Rs. 3y respectively.

According ot the given condition of the problem,

$9x - 4y = 200 \rightarrow (1)$

$7x - 3y = 200 \rightarrow (2)$

Multiply (1) by 3, we get

$27x - 12y = 600$

Multiply (2) by 4, we get

$28x - 12y = 800$

Subtracting (3) from (4), we get

$x = 200$

Hence their monthly income are Rs. $(9\times200=1800)$ and Rs. $(7 \times 200 = 1400)$.

Ans. (a)

ANSWERS

160. Let x be the distributed amount of A, B and C

Given

∴ $5x + 11x + 3x = 950$

$19x = 950$

$x = \frac{950}{19} = 50$

∴ The amount of A = 5x = 5 × 50 = 250

The amount of B = 11x = 11 × 50 = 550

∴ The difference of A and B = 300

Ans. (a)

161. $\underset{x\to 1}{\text{Lt}} \frac{e^{-x} - e^{-1}}{x-1} \Rightarrow \underset{x\to 1}{\text{Lt}} \frac{e^{1-x} - 1}{e(x-1)}$

let 1 + h $\to$ x where h $\to$ 0

∴ $\underset{x\to 0}{\text{Lt}} \frac{e^{h} - 1}{e(-h)} = \frac{-1}{e}$

Ans. (b) −1/e

162. $\underset{x\to 0}{\text{Lt}} \frac{(1+x)^n - 1}{x}$

$$= \underset{x\to 0}{\text{Lt}} \frac{(1 + nx + \frac{n(n-1)x^2}{2!} + ...) - 1}{x}$$

$$= \underset{x\to 0}{\text{Lt}} \frac{x\left[n + \frac{n(n-1)^x}{2!} + \frac{n(n-1)(n-2)x^2}{3!}......\right]}{x}$$

App Lt

n + 0 = n

Ans. (c) n

163. $\underset{x\to 0}{\text{Lt}} \frac{(x+2)^{5/3} - (a+2)^{5/3}}{x-a}$

$\underset{x\to 0}{\text{Lt}} \frac{(x+2)^{5/3} - (a+2)^{5/3}}{[(x+2)-(a+2)]}$

App Lt

$$\Rightarrow \frac{5}{3}.(a+2)^{5/3-1} = \frac{5}{3}(a+2)^{2/3}$$

Ans. (a) $\frac{5}{3}(a+2)^{2/3}$

165. $\underset{x\to 0}{Lt}\ \frac{2^x - 3^x}{x} \Rightarrow \underset{x\to 0}{Lt}\ \frac{(2^x-1)-(3^x-1)}{x}$

$$\underset{x\to 0}{Lt}\ \frac{2^x-1}{x} - \underset{x\to 0}{Lt}\ \frac{3^x-1}{x} \qquad \left[\underset{x\to 0}{Lt}\ \frac{a^x-1}{x} = \log e^a\right]$$

$\Rightarrow \log 2 - \log 3$

$\Rightarrow \log\left(\frac{2}{3}\right)$

Ans. (b)

166. $f'(x) = 3x^2 + 2$

$$\int f'(x)\,dx = \int (3x^2+2)\,dx$$

$$f(x) + c = \frac{3x^3}{3} + 2x + c$$

When f (0) = 0 $\Rightarrow$ c = 0

$\therefore$ f(x) = $x^3 + 2x$

$\therefore$ f(2) = $2^3 + 2\,(2)$

= 8 + 4 = 12

Ans. (c)

167. Let $I = \int \frac{x+3}{x^2+6x+4}$

Put $x^2+6x+4 = t$

$\therefore (2x+6)dx = dt$

$(x+3)dx = \frac{dt}{2}$

$$\therefore \int \frac{x+3}{x^2+6x+4}\,dx = \int \frac{dt}{2t}$$

$$= \frac{1}{2}\int \frac{1}{t}\,dt$$

$\frac{1}{2}\log(t)$

$$\therefore \int \frac{x+3}{x^2+6x+4}dx = \frac{1}{2}\log(x^2+6x+4) + c$$

168. $\int e^x \frac{x-1}{(x+1)^3}\,dx = \int \frac{x+1-2}{(x+1)^3} e^x dx$

$$= \int e^x \left[\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}\right] dx$$

$$= \int e^x \{f(x) + f'(x)\} dx$$

$= e^x f(x)$ where $f(x) = \frac{1}{(x+1)^2}$

$$\int e^x \frac{(x-1)}{(x+1)^3} dx = \frac{e^x}{(x+1)^2} + c$$

Ans.(a)

169. $\int (3x+5)^4\,dx = \frac{(3x+5)^{4+1}}{(4+1)(3)} + c$

$$= \frac{(3x+5)^5}{15} + c$$

Ans. (b)

170. $\int \sqrt{7x+5}\,dx$

$$= \int (7x+5)^{\frac{1}{2}}\,dx$$

$$= \frac{(7x+5)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)(7)} + c$$

$$= \frac{(7x+5)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)(7)} + c$$

$$\frac{2(7x+5)^{\frac{3}{2}}}{21}+c$$

171. Voter has option

(i) Two candidates from gentlemen = $3_{c_2} = 3$

(ii) Two candidates from ladies = $3_{c_2} = 3$

(iii) One from each ladies & gentlemen = $3_{c_1} \times 3_{C_1} = 9$

Total options = 3 + 3 + 9 = 15

Ans. (c) 15

172. Total hand shakes in the party = 40_{c_2} = 780

Ans. (a) 780

173. Total triangle formed by m sides = m_{C_3}

$$\Rightarrow \frac{m(m-1)(m-2)(m-3)!}{(m-3)!\,3!}$$

$$\Rightarrow \frac{m(m-1)(m-2)}{6}$$

Ans. (a) $\frac{m(m-1)(m-2)}{6}$

174. Cricket team of 11 among 14 players out of which one wicketkeeper

= $12_{C_{10}} \times 2_{C_1} = 66 \times 2$

$\Rightarrow$ 132

Ans. (b) 132

175. No. of ways in which a particular child goes to circus = $7_{C_2} \times 1 = 21$

Ans. (c) 21

176. $a^x = b^y = c^z = k$ (let)

$\therefore \log a^k = x, \ \log_b{}^k = y, \ \log_c{}^k = z$

$\therefore \log_k a = 1/x, \ \log_k b = 1/y, \ \log_k c = 1/z$

x, y, z in GP

$\therefore y^2 = xz$

$(\log_b{}^k)^2 = (\log_a k).(\log_c k)$

$\therefore \quad \frac{\log_b k}{\log_a k} = \frac{\log_c k}{\log_b k} \Rightarrow \frac{\log_k a}{\log_k k} = \frac{\log_k b}{\log_k c}$

$\therefore$ $\log_a$, $\log_b$ and $\log_c$ in GP

Ans. (b) G.P.

177. $\frac{1}{1024} = 8.\left(\frac{1}{2}\right)^{n-1} \Rightarrow \left(\frac{1}{2}\right)^{13} = \left(\frac{1}{2}\right)^{n-1}$

$\therefore$ n = 14

6th team from end = (14 – 6+1) from beginning = 9th term

$T_9 = 8.\left(\frac{1}{2}\right)^{9-1} = 8.\left(\frac{1}{2}\right)^{8} = \frac{1}{32}$

Ans. (c) 1/32

178. Product of 2nd term from start & last 2nd term from end = (ar) × a(r)$^{n...2}$= $a^2 r^{(n...1)}$

Product of first & last term = $a \times ar^{n-1} = a^2 r^{n-1}$

Hence proved the statement. It is true statement

Ans. (a) True

179. a, b, c in GP $\therefore$ b^2 = ac

a, x, b in AP $\Rightarrow x = \frac{a+b}{2}$

b, y, c in AP $\Rightarrow y = \frac{b+c}{2}$

$\therefore \quad \frac{a}{x} + \frac{c}{y} = \frac{2a}{a+b} + \frac{2c}{b+c} \Rightarrow \frac{2[ab+ac+ac+bc]}{ab+b^2+ac+bc}$

$\Rightarrow \frac{2[ab+ac+ac+bc]}{ab+ac+ac+bc} \quad \{b^2 = ac\}$

= 2

Ans. (c) 2

180. $\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} \Rightarrow \frac{\frac{a+b}{2} + \frac{b+c}{2}}{\frac{(a+b)(b+c)}{4}}$

$\Rightarrow \frac{2(a+2b+c)}{(ab+b^2+ac+bc)} = \frac{2(a+2b+c)}{ab+b^2+b^2+bc}$

$$= \frac{2(a+2b+c)}{b(a+2b+c)} = \frac{2}{b}$$

Ans. (b) $\frac{2}{b}$

181. The number of times a particular item occurs in a given data is called its frequency.

Ans. (b) Frequency

182. Lower class (s) = 10.6

Width = 2.5 (Class interval)

Upper Class (L) of Lightest Class

= S = 10 × C.I.

= 10.6 + 10 x 2.5

= 35.6

Ans. (a) 35.6

183. $\frac{\text{Lower Class} + \text{Upper Class}}{2}$ = Mid Value

$\frac{L + U.Class}{2} = m$

∴ Upper class = (2m – L)

Ans. (c) (2m – L)

184. Mean $= \frac{1.x + 2.2x + 3.3x + ...n.nx}{n(n+1).x/2}$

$$= \frac{(1^2 + 2^2 + 3^2 + ...n^2)x}{n(n+1).x/2}$$

$$= \frac{(1^2 + 2^3 + 3^2 + ...n^2)x}{n(n+1).x/2}$$

$$= \frac{n(n+1)(2n+1)x \times 2}{6n(n+1) - x} = \frac{(2n+1)}{3}$$

Mean $= \frac{(2n+1)}{3}$

Ans. (c) $\frac{2n+1}{3}$

ANSWERS

185. $\bar{x} = 10, n = 4$

$\Sigma x = n.\ \bar{x} = 4 \times 10 = 40$

Corrected $\Sigma x = (40 + 4a)$

Corrected $\bar{x} = \frac{40 + 4a}{4}$

$13 = \frac{40 + 4a}{4}$

$4a = 12$

$a = 3$

Ans. (c) 3

186. From the given data, we observe that

20 + 5 = 25

21 + 4 = 25

and 22 + 3 = 25

Thus, x and y are connected by the linear relation: x + y = 25 $\rightarrow$ (1)

$\Rightarrow$ There is perfect correlation between x and y

$\Rightarrow$ $r = \pm 1 \rightarrow (2)$

From (1) / We get y = 25 – x

$\therefore$ As x increases, y decreases (by the same amount)

$\Rightarrow$ x and y are negatively correlated $\rightarrow$ (3)

From (2) and (3), we conclude that

$r = r(x, y) = -1$

Ans. (c)

187. Ans. (b) Refer Properties

188. Ans. (b) Refer Properties

189. Ans. (b) Refer Properties

190. Ans. (b) Refer Properties

191. Lt X … B (n=6, p). When X denotes the number of successes. Then, by binomial probability law, the probability of r successes is givenby

$p(r) = P(x=r) = 6C_r P^r q^{6-r} \rightarrow (1)$

r = 0, 1, 2, ... 6

Put r = 3 and 4 in (1)

(1) $\Rightarrow$ p(3) = $6C_3 p^3 q^3 = 20p^3 q^3$ = 0.2457 (given)

p(4) = $6C_4\, p^4\, q^2 = 15\, p^4 q^2$ = 0.0819 (given)

$$\frac{p(4)}{p(3)} = \frac{15p^4q^2}{20p^3q^3} = \frac{0.0819}{0.2457} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{3}{4}.\frac{p}{q} = \frac{1}{3}$$

$\therefore$ 9p = 4q = 4 (1 – p)

$\therefore$ 13p = 4

p = 4/13

$\therefore$ q = 1 – p = 1 – 4/13 = 9/13

192. Ans. (a) – Refer Properties

193. Ans. (b) – Refer Properties

194. Ans. (a) – Refer Properties

195. Ans. (b) – Refer Properties

196. Which measure of dispersion has some desirable mathematical properties $\rightarrow$ Standard Deviation.

Ans. (a) Standard Deviation.

197. $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$$\Rightarrow x\sqrt{1+y} = -\, y\sqrt{1+x}$$

Eg. Both side

$$x^2(1+y) = y^2(1+x)$$

$$(x^2 - y^2) = y^2x - x^2y$$

$$(x+y)\,(x-y) = -xy(x-y)$$

$\therefore$ x + y +xy = 0 $\Rightarrow$ $y = \dfrac{-x}{1+x}$

$$\therefore \quad \frac{dy}{dx} = \frac{(1+x)(-1) - (-x)(1)}{(1+x)^2} = \frac{-1-x+x}{(1+x)^2}$$

$$\therefore \quad \frac{dy}{dx} = \frac{-1}{(1+x)^2} \Rightarrow (1+x^2)\frac{dy}{dx} = -1$$

Ans. (c) (–1)

198. $y = x\sqrt{x^2+1} + \log\left(x+\sqrt{x^2+1}\right)$

$$\frac{dy}{dx} = x.\frac{1}{2\sqrt{x^2+1}}.2x + \sqrt{x^2+1}.1 + \frac{1}{x+\sqrt{x^2+1}}\left(1+\frac{1}{2\sqrt{x^2+1}}.2x\right)$$

$$= \frac{x^2}{\sqrt{x^2+1}} + \sqrt{x^2+1} + \frac{\sqrt{x^2+1}+x}{\left(x+\sqrt{x^2+1}\right)}\frac{1}{\sqrt{x^2+1}}$$

$$= \frac{x^2+x^2+1+1}{\sqrt{x^2+1}} = \frac{2(x^2+1)}{\sqrt{x^2+1}} = 2\sqrt{1+x^2}$$

Ans. (c) $2\sqrt{1+x^2}$

199. $y = ae^{mx} + be^{-mx}$

$$\therefore \quad \frac{dy}{dx} = ame^{mx} - bme^{-mx}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} = \left(ame^{mx} - bme^{-mx}\right)$$

$$= am^2e^{mx} + bm^2e^{-mx}$$

$$= m^2\left(ae^{mx} + be^{-mx}\right)$$

$$\frac{d^2y}{dx^2} = m^2y$$

Ans. (c) m^2y

200. $12_{C_5} + 2.12_{C_4} + 12_{C_3} = 14_{C_x}$

$$\left(12_{C_5} + 12_{C_4}\right) + \left(12_{C_4} + 12_{C_3}\right) = 14_{C_x}$$

$$13_{C_5} + 13_{C_4} = 14_{C_x}$$

$$\left[{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\right]$$

$$14_{C_5} = 14_{C_x}$$

x = 5 but value $14_{C_9} = 14_{C_5}$

$\therefore\ 14_{C_x} = 14_{C_9}$

$\therefore$ x = 9

x = 5 or 9

Ans. (c) 5 or 9.

Model Test Paper – BOS/CPT – 7

151. Let the number be x. Then according to the given condition of the problem,

$$\frac{x}{3}=\frac{x+1}{4}+1$$

$$\Rightarrow \frac{x}{3}-\frac{x+1}{4}=1$$

$$\Rightarrow \frac{4x-3(x+1)}{12}=1$$

$$\Rightarrow \frac{x-3}{12}=1$$

$\Rightarrow$ x – 3 = 12

$\Rightarrow$ x = 3 + 12 = 15

Hence the required number is 15

152. Let the fraction = x

And Correct answer = y

$$\therefore \quad \text{Given } \frac{16}{17}x = y \rightarrow (1)$$

$$\text{and } \frac{x}{\frac{16}{17}} = y + \frac{33}{340}$$

$$\text{i.e. } \frac{17}{16}x = y+\frac{33}{340} \rightarrow (2)$$

$$(1) \quad \Rightarrow x = \frac{17}{16}y$$

Substitute x the value of x in equation (2)

$$\frac{17}{16}\times\frac{17}{16}y = y+\frac{33}{340}$$

$$\frac{289}{256}y = y + \frac{33}{340}$$

$$\frac{289}{256}y - y = \frac{33}{340}$$

$$\left(\frac{289-256}{256}\right)y = \frac{33}{340}$$

$$\frac{33}{256}y = \frac{33}{340}$$

$$y = \frac{33}{340} \times \frac{256}{33}$$

$$y = \frac{64}{85}$$

Ans. (a)

153. Let the number be x

Given $\frac{5}{7}\left[\frac{4}{15}(x)\right] = 8 + \frac{2}{5}\left[\frac{4}{9}(x)\right]$

$$\frac{4}{21}x = 8 + \frac{8}{45}x$$

$$\frac{4}{21}x - \frac{8}{45}x = 8$$

$$\left(\frac{180-168}{945}\right)x = 8$$

$$\frac{12}{945}x = 8$$

$$x = 8 \times \frac{945}{12}$$

x = 708.75

Ans. (d)

154. Let two numbers are x and y

Given condition $x + y = 14 \rightarrow (1)$

$y - x = 10 \rightarrow (2)$

Adding (1) and (2) $\Rightarrow 2x = 24$

$x = 12$

$(1) \Rightarrow 12 + y = 14$

$y = 2$

$\therefore$ Product of two numbers = $x \times y = 12 \times 2 = 24$

Ans. (a)

155. Let two numbers are x and y.

Given $x - y = 11 \rightarrow (1)$

And $\frac{x+y}{5} = 9$

i.e. $x + y = 45 \rightarrow (2)$

Adding (1) and (2) $\Rightarrow 2x = 56$

$X = 28$

(1) $\Rightarrow 28 - y = 11$

$-y = 11 - 28$

$= -17$

$y = 17$

$\therefore$ The two numbers are 28, 17.

Ans. (d)

156. Sub duplicate Ratio of 16:49 = $\sqrt{16} : \sqrt{49}$

$=$ 4 : 7

Ans. (a)

157. Duplicate Ratio of 4 : 5 = $4^2 : 5^2$

$=$ 16 : 25

Ans. (a)

158. Triplicate Ratio of 3 : 5 = $3^3 : 5^3$

$=$ 27 : 125

Ans. (a)

159. The sub – triplicate Ratio of 8 : 125 = $\sqrt[3]{8} : \sqrt[3]{125}$

$=$ 2 : 5

Ans. (b)

160. 4^{th} Proportion of 6, 8 and 15 is $\frac{6}{8} = \frac{15}{x}$

$6x = 15 \times 8$

$$x = \frac{15 \times 8}{6}$$

$$= 20$$

Ans. (c)

161. Let the two numbers be x and y. According to the First condition of the problem,

$$\frac{x}{y} = \frac{4}{1}$$

$\Rightarrow$ x = 4y $\rightarrow$ (1)

According to the second condition of the problem,

$$\frac{x+5}{y+5} = \frac{3}{1}$$

x + 5 = 3 (y + 5)

x + 5 = 3y + 15

x – 3y = 15 – 5 = 10 $\rightarrow$ (2)

Put x = 4y from (1) in (2), we get

4y – 3y = 10

y = 10

(1) $\Rightarrow$ x = 4 (10) = 40

Hence the required numbers are 40 and 10.

Ans. (b)

162. Let A having money = 3x

B having money = 4x

C having money = 5x

Given 3x = 300

x = 100

$\therefore$ C = 5x = 5 × 100 = 500

Ans. (c)

163. Let the two numbers be x and y. According to the first condition of the problem.

$$\frac{x}{y} = \frac{5}{6}$$

6x = 5y

6x – 5y = 0 $\rightarrow$ (1)

According to the second condition of the problem

$$\frac{x-5}{y-5}=\frac{4}{5}$$

5(x – 5) = 4 (y – 5)

5x – 25 = 4y – 20

5x – 4y = 5 $\rightarrow$ (2)

(1) × 5 $\Rightarrow$ 30x – 25y = 0 $\rightarrow$(3)

(2) × 5 $\Rightarrow$ 30x – 24y = 30

Subtracting (3) from (4), we get

y = 30

(1) $\Rightarrow$ 6 x – 5 (30) = 0

6x = 150

$$x = \frac{150}{6} = 25$$

Hence the required numbers are 25 and 30

Ans. (c)

164. Let the given numbers be x and y. Then according to the given conditions of the problem.

$$\frac{x+1}{y+1}=\frac{1}{2}$$

$\Rightarrow$ 2x + 2 = y + 1

2x – y = –1 $\rightarrow$ (1)

and $\frac{x-5}{y-5}=\frac{5}{11}$

11x – 55 = 5y – 25

11x – 5y = 30 $\rightarrow$ (2)

$\therefore$ (1) × 11 $\Rightarrow$ 22x – 11y = –11 $\rightarrow$ (3)

(2) × 2 $\Rightarrow$ 22x – 10y = 60 $\rightarrow$ (4)

(3) – (4)$\Rightarrow$ – y = – 71

y = 71

(1) $\Rightarrow$2x – 71 = – 1

2x = 70

x = 35

Hence, the required numbers are 35 and 71

Ans. (b)

165. Let the number to be subtracted be x.

Then according to the problem

$$\frac{27-x}{43-x}=\frac{7}{15}$$

$\Rightarrow$ 15 (27– x) = 7 (43 – x)

$\Rightarrow$ 405 – 15x = 301 – 7x

15x – 7x = 405 – 301

8x = 104

$$x = \frac{104}{8} = 13$$

Hence the required number is 13

Ans. (a)

166. Let the unit digit = x

and ten digits = y

$\therefore$ x+y = 3 $\rightarrow$ (1)

and the number = 10y + x

Reversing the order of digits

Units digit = y

and ten's digit = x

$\therefore$ Number = 10x + y

According to the given condition of the problem

7(10y + x) = 4(10x+y)

70y + 7x = 40x + 4y

70x – 40x + 70y – 4y = 0

– 33x + 66y = 0

– x+2y = 0 $\rightarrow$ 92)

Adding (1) and (2) we get

3y = 3

y = 1

(1) $\Rightarrow x + 1 = 3$

$x = 3 - 1 = 2$

Hence, the required number is 12

167. The committee of six must include atleast 2 ladies.

i.e. two or more ladies. As there are only 3 ladies, the following possibilities arise:

The committee of 6 consists of

(i) 4 men and 2 ladies, (ii) 3 men and 3 ladies.

The number of ways for (i) = ${}^7C_4 + {}^3C_2$

= 35 × 3 = 105

The number of ways for (ii) = ${}^7C_3 \times {}^3C_3$

= 35 × 1

= 35

Hence the total number of ways of forming a committee so as to include atleast two ladies = 105 + 35 = 140

Ans. (a)

168. We have $nCr = \frac{n!}{r!(n-4)!}$

Now substituting for n and r, we get

$${}^{28}C_{2r} = \frac{28!}{(2r)!(28-2r)!}$$

$$24C_{2r-4} = \frac{24!}{(2r-4)!\{24-(2r-4)\}!}$$

$$= \frac{24!}{(2r-4)!\,(28-2r)!}$$

Given $28_{C_{2r}} : 24_{C_{2r-4}} = 225:11$

$$\Rightarrow \frac{28_{C_{2r}}}{24_{C_{2r-4}}} = \frac{28!}{(2r)!(28-2r)!} \div \frac{(2r-4)!\,(28-2r)!}{24!}$$

$$= \frac{28\times27\times26\times25\times24!}{2r(2r-1)(2r-2)(2r-3)(2r-4)!\,(28-2r)!} \times \frac{(2r-4)!\,(28-2r)!}{24!}$$

$$= \frac{28\times27\times26\times25}{2r(2r-1)(2r-2)(2r-3)} = \frac{225}{11}$$

$\Rightarrow$ $2r(2r-1)(2r-2)(2r-3) = \dfrac{11 \times 28 \times 27 \times 26 \times 25}{225}$

$= 11 \times 28 \times 3 \times 26$

$= 11 \times 7 \times 4 \times 3 \times 13 \times 2$

$= 11 \times 12 \times 13 \times 14$

$= 14 \times 13 \times 12 \times 11$

$\therefore$ $2r = 14$

$r = 7$

Ans. (b)

169. Let in the number unit's digit = x

and ten's digit = y

$\therefore$ Number = 10y + x

According to the given conditions of the problem,

$8(x+y) + 1 = 10y + x$

(or) $8x + 8y + 1 = 10y + x$

$\Rightarrow$ $8x - x + 8y - 10y + 1 = 0$

$7x - 2y + 1 = 0 \rightarrow (1)$

and $13(y - x) + 2 = 10y + x$

$13y - 13x + 2 = 10y + x$

$\Rightarrow$ $x + 13x + 10y - 13y - 2 = 0$

$\Rightarrow$ $14x - 3y - 2 = 0 \quad \rightarrow (2)$

$(1) \times 2 \Rightarrow 14x - 4y + 2 = 0 \quad \rightarrow (3)$

$(2) - (3) \Rightarrow y - 4 = 0$

$y = 4$

Put y = 4 in (1), we get

$7x - 8 + 1 = 0$

$7x = 7$

$x = 1$

Hence, the required number is 41

Ans. (b)

170. We want to find out the number of combination of 12 things taken 3 at a time and this is by:

$$^{12}C_3 = \frac{12!}{3!(12-3)!}$$

$$= \frac{12!}{3!\,9!} = \frac{12\times11\times10\times9!}{3!\,9!}$$

$$= \frac{12\times11\times10}{3\times2}$$

= 220

Ans. (c)

172. $\underset{x\to9}{\text{Lt}}\ \frac{\sqrt{x}-3}{x-9} = \underset{x\to9}{\text{Lt}}\ \frac{\sqrt{x}-3}{\sqrt{x}-3}\frac{1}{\sqrt{x}+3}$

App $\text{lt} = \frac{1}{3+3} = \frac{1}{6}$

Ans. (a) $\frac{1}{6}$

173. $\underset{x\to9}{\text{Lt}}\ \frac{\sqrt{x+0}-\sqrt{2a}}{x-0} \Rightarrow \underset{x\to9}{\text{Lt}}\ \frac{\sqrt{x+a}-\sqrt{2a}}{x-0}\times\frac{\left(\sqrt{x+a}-\sqrt{2a}\right)}{\left(\sqrt{x+a}+\sqrt{2a}\right)}$

$\Rightarrow \underset{x\to9}{\text{Lt}}\ \frac{x+a-2a}{(x-a)\left(\sqrt{x+a}+\sqrt{2a}\right)} = \underset{x\to a}{\text{Lt}}\ \frac{(x-a)}{(x-a)\left(\sqrt{x+a}+\sqrt{2a}\right)}$

App Lt

$$\frac{1}{\left(\sqrt{a+a}+\sqrt{2a}\right)} = \frac{1}{2\sqrt{2a}}$$

Ans. (b) $\frac{1}{2\sqrt{2a}}$

174. $\underset{x\to\infty}{\text{Lt}}\ \frac{6+5x^2}{4x+15x^2} \Rightarrow \underset{x\to\infty}{\text{Lt}}\ x^2\frac{\left(5+\frac{6}{x^2}\right)}{x^2\left(15+\frac{4}{x}\right)}$

App Lt.

$\Rightarrow \frac{5+0}{15+0} = \frac{1}{3}$

Ans. (c) $\frac{1}{3}$

ANSWERS

175. $\underset{x\to\infty}{Lt}\frac{a-bx}{x^2} \Rightarrow \underset{x\to\infty}{Lt}\left(\frac{a}{x^2}-\frac{b}{x}\right)$

App Lt.

$\Rightarrow (0 \ldots 0) = 0$

Ans.(a) 0

176. $y=\frac{e^x-e^{-x}}{e^x+e^{-x}} \Rightarrow \frac{dy}{dx} = \frac{(e^x+e^{-x})\frac{d}{dx}(e^x-e^{-x})-(e^x-e^{-x}).\frac{d}{dx}(e^x+e^{-x})}{(e^x+e^{-x})^2}$

$= \frac{(e^x+e^{-x})(e^x+e^{-x})-(e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}$

$= \frac{e^{2x}+e^{-2x}+2-e^{2x}-e^{-2x}+2}{(e^x+e^{-x})^2} = \frac{4}{(e^x+e^{-x})^2}$

Ans. (b) $\frac{4}{(e^x+e^{-x})^2}$

177. $y = \frac{x}{(1+x)^2} \Rightarrow \frac{dy}{dx} = \frac{(1+x)^2\frac{d}{dx}x-x\frac{d}{dx}(1+x)^2}{(1+x)^4}$

$\Rightarrow \frac{(1+x)^2.1-x.2(1+x)}{(1+x)^4} = \frac{1+x^2+2x-2x-2x^2}{(1+x)^4}$

$= \frac{1-x^2}{(1+x)^4} = \frac{1-x}{(1+x)^3}$

Ans. (b) $\frac{1-x}{(1+x)^3}$

178. $y = \sqrt{x+\sqrt{x}}$

$\frac{dy}{dx}=\frac{d}{dt}t^{1/2}.\frac{d}{dx}(x+\sqrt{x})$

$= \frac{1}{2\left(\sqrt{x+\sqrt{x}}\right)}.\left(1+\frac{1}{2\sqrt{x}}\right)$

$$= \frac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}$$

Ans. (a) $\frac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}$

179. y = 7^{x^2+2x}

$$\frac{dy}{dx} = \frac{d}{dt}7^t.\frac{d}{dx}(x^2+2x)$$

$$= 7^{x^2+2x}.\log 7.\ (2x+2)$$

$$\frac{dy}{dx} = 2(x+1).7^{x^2+2}.\log 7$$

Ans. (b) $2(x+1).7^{x^2+2}.\log 7$

180. y = log $\left(x+\sqrt{x^2+a^2}\right)$

$$\frac{dy}{dx} = \frac{d}{dt}\log t.\frac{d}{dx}\left(x+\sqrt{x^2+a^2}\right)$$

$$= \frac{1}{x+\sqrt{x^2+a^2}}.\left[1+\frac{1}{2\sqrt{x^2+a^2}}.2x\right]$$

$$= \frac{\left(\sqrt{x^2+a^2}+x\right)}{\sqrt{x^2+a^2}\left(x+\sqrt{x^2+a^2}\right)}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x^2+a^2}}$$

Ans. (a) $\frac{1}{\sqrt{x^2+a^2}}$

181. Given $(x-y)\,e^{\frac{x}{x-y}} = a$

Differentiate on both sides.

$$(x-y).e^{\frac{x}{x-y}}\left[\frac{(x-y)(1)-x.\left(1-\frac{dy}{dx}\right)}{(x-y)^2}\right]+e^{\frac{x}{x-y}}\left(1-\frac{dy}{dx}\right)=0$$

$$e^{\frac{x}{x-y}}\left[\frac{x-y-x+x\frac{dy}{dx}}{(x-y)}+1-\frac{dy}{dx}\right] = 0$$

$$\frac{-y+x\frac{dy}{dx}}{(x-y)}+1-\frac{dy}{dx}=0$$

$$\frac{-y}{x-y}+\frac{x}{x-y}\,\frac{dy}{dx}+1-\frac{dy}{dx}=0$$

$$\frac{dy}{dx}\left[\frac{x}{x-y}-1\right]=\frac{y}{x-y}-1$$

$$\frac{dy}{dx}\left[\frac{x-x+y}{x-y}\right]=\frac{y-x+y}{x-y}$$

$$\frac{dy}{dx}\left[\frac{y}{x-y}\right]=\frac{2y-x}{x-y}$$

$$\frac{dy}{dx}=\frac{2y-x}{y}$$

$$\therefore\quad y\frac{dy}{dx}+x=y\left[\frac{2y-x}{y}\right]+x$$

$= \quad 2y - x + x$

$$y\frac{dy}{dx}+x=2y$$

Ans. (c)

182. Given demand law $x = \sqrt{10-p^2}$

$$x = (10-p^2)^{1/2}$$

$$\frac{dx}{dp}=\frac{1}{2}(10-p^2)^{\frac{1}{2}-1}\ (-2p)$$

$$\frac{1(-2p)}{2\sqrt{10-p^2}}$$

$$\frac{dx}{dp} = \frac{-p}{\sqrt{10-p^2}}$$

$$|ed| = \frac{p}{x} \cdot \frac{dx}{dp}$$

$$= \frac{p}{(10-p)^{1/2}} \cdot \frac{-p}{2\sqrt{10-p^2}}$$

when p = 2

$$|ed| = \frac{2}{\sqrt{6}} \cdot \frac{-2}{\sqrt{6}}$$

ed $= -2/3$

$$|ed| = \frac{2}{3}$$

Ans. (a)

Alternate

183. $\therefore \int \frac{x^3}{x+1} dx$

$$= \int \left(x^3 - x + 1 - \frac{1}{x+1} \right) dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + x - \log(x+1) + c$$

Ans. (c)

184. $\int \left(\frac{e^{4x} + e^{2x}}{e^{3x}} \right) dx$

$$= \int \left(\frac{e^{4x}}{e^{3x}} + \frac{e^{2x}}{e^{3x}} \right) dx$$

$$= \int e^x dx + \int e^{-x} dx$$

$$= e^x - e^{-x} + c$$

Ans. (b)

185. $\int \frac{x^4+1}{x^2+1} dx = \int \left(x^2 - 1 + \frac{2}{x^2+1} \right) dx$

$= \int x^2\, dx - \int dx + 2\int \frac{1}{x^2+1} dx$

$= \frac{x^3}{3} - x + 2\tan^{-1} x + c$

Ans. (c)

186. Let $I = \int \log(x+1)dx$

$\therefore \quad I = \int \log(x+1).1dx$

Integrating by parts

[Here log (x+1) is to be taken as first function and unity as second function)

I = [log (x+1] integral of '1' – integral of

[d/dx (log (x+1)] + integral of '1']

$= \quad \log(x+1).x - \int \frac{1}{x+1}.x\,dx$

$= \quad x\log(x+1) - \int \frac{x+1-1}{x+1} dx$

$= \quad x\log(x+1) - \int \left(1 - \frac{1}{x+1}\right) dx$

$= \quad x\log(x+1) - [x - \log(x+1)]$

$I = x\log(x+1) - x + \log(x+1) + c$

187. Consider $\frac{1}{\sqrt{x}+\sqrt{1+x}} = \frac{\sqrt{x}-\sqrt{1+x}}{x-(1+x)}$

$= \quad \sqrt{1+x} - \sqrt{x}$

$\therefore \quad I = \int \frac{dx}{\sqrt{x}+\sqrt{1+x}}$

$= \quad \int \sqrt{1+x}\,dx - \int \sqrt{x}\,dx$

$I = I_1 - I_2$

$I_1 = \int \sqrt{1+x}\,dx$

Let z = 1 + x

dz = dx

$\therefore\ I_1 = \int\sqrt{1+x}\ dx = \int\sqrt{z}\ dz$

$= \frac{2}{3}z^{3/2}$

$= \frac{2}{3}(1+x)^{3/2}$

$\therefore\ I = 2/3\ (1+x)^{3/2} - \frac{x^{3/2}}{3/2} + c$

$I = 2/3\ \left\{(1+x)^{3/2} - x^{3/2}\right\} + c$

188. Consider $x^3 + x^2 - 2x = x(x^2 + x - 2)$

$= x(x^2 + 2x - x - 2)$

$= x\ \{x(x+2) - (x+2)\}$

$= x\ (x-1)\ (x+2)$

$\therefore$ We may write $\frac{x^2 - x + 2}{x^3 + x^2 - 2x} = \frac{x^2 - x + 2}{x(x-1)\ (x+2)}$

Let $\frac{x^2 - x + 2}{x(x-1)(x+2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2}$

(or) $x^2 - x + 2 = A(x-1)\ (x+2) + Bx(x+2) + Cx(x-1)$

Substituting x = 1, We find 2 = 3B

B = 2/3

Substituting x = – 2, We find 8 = 6c

i.e. C = 4/3

Substituting x = 0, We find 2 = – 2A

A = – 1

$\therefore\ I = \int\frac{x^2 - x + 2}{x^3 + x^2 - 2x}dx = -\int\frac{dx}{x} + \frac{2}{3}\int\frac{dx}{x-1} + \frac{4}{3}\int\frac{dx}{x+2}$

I = – log x + 2/3 log (x – 1) + 4/3 log (x+2) + log c

Ans. (c)

189. Let $I = \int \frac{1}{3x^2+13x-10}\,dx$

$$= \frac{1}{3}\int \frac{1}{x^2+\frac{13x}{3}-\frac{10}{3}}\,dx$$

$$= \frac{1}{3}\int \frac{1}{\left[x^2+2.\frac{13}{6}x+\left(\frac{13}{6}\right)^2\right]-\left(\frac{13}{6}\right)^2-\frac{10}{3}}\,dx$$

$$= \frac{1}{3}\int \frac{1}{(x+13/6)^2-\frac{289}{36}}\,dx$$

Let t = x+13/6

$\therefore$ dt = dx

$\therefore$ I = 1/3 $\int \frac{1}{t^2-(17/6)^2}\,dt$

$$= \frac{1}{3}.\frac{1}{2(17/6)}\log\left[\frac{t-17/6}{t+17/6}\right]$$

$$= \frac{1}{17}\log\left[\frac{6t-17}{6t+17}\right]$$

$$= \frac{1}{17}\log\left[\frac{6(x+13/6)-17}{6\left(x+\frac{13}{6}\right)+17}\right]$$

$$= \frac{1}{17}\log\left[\frac{3x-2}{3x+15}\right]+c$$

Ans. (b)

190. $\int e^x\{f(x)+f'(x)\}\,dx$

By the method of integration by parts, we may write

$$\int e^x f(x)dx = f(x)\int e^x\,dx - \int\left\{\frac{d}{dx}f(x)\int ex\,dx\right\}dx$$

$$= e^x f(x) - \int e^x f'(x)\,dx$$

Transposing $\int e^x f(x)\,dx + \int e^x f'(x)\,dx = e^x f(x)$

(or) $\int e^x \{f(x) + f'(x)\}\,dx = e^x f(x)$

191. $\int_a^b \frac{\log x}{x} dx$ Let $\log x = t$ $x = a, t = \log a$

$\frac{1}{x} = \frac{dt}{dx}$ $x = b, t = \log b$

$\int_{\log b}^{\log a} t.dt. \Rightarrow \left[\frac{t^2}{2}\right]_{\log a}^{\log b}$

$\Rightarrow \frac{1}{2}\left[(\log b)^2 - (\log a)^2\right] \Rightarrow \frac{1}{2}\left[\log(ab).\log\left(\frac{b}{a}\right)\right]$

Ans. (a) $\frac{1}{2}\left[\log(ab).\log\left(\frac{b}{a}\right)\right]$

192. $\int [f(x) + f(-x)]\,[g(x) - g(-x)]\,dx$

$\Rightarrow \int 0.[g(x) - g(-x)]dx \Rightarrow 0$

Ans. (a) 0

193. $\int_a^b \frac{dx}{(a+b-x)^{2/3}}$ Let $a + b - x = t$ $x = a, t = b$

$-1 = \frac{dt}{dx}$ $x = b, t = a$

$\Rightarrow -\int_b^a t^{-2/3}\,dt \Rightarrow \int_a^b t^{-2/3}\,dt$

$\Rightarrow 3\left[t^{1/3}\right]_a^b \Rightarrow 3\left[b^{1/3} - a^{1/3}\right]$

Ans. (a) $3.\left[b^{1/3} - a^{1/3}\right]$

194. $I = \int_0^2 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{2-x}} dx$ ... (i)

$I = \int_0^2 \frac{\sqrt{2-x}}{\sqrt{2-x} + \sqrt{x}}$ ... (ii) $[f(x) = f(a - x)]$

$$2I = \int_0^2 dx = [x]_0^2 = 2$$

$$\therefore \quad I = \frac{1}{2} \times 2 = 1$$

Ans. (a) 1

195. $I = \int_0^1 \log\left(\frac{1}{x} - 1\right) dx \Rightarrow \int_0^1 \log\left(\frac{1-x}{x}\right) dx$ ... (i)

$I = \int_0^1 \log\left(\frac{1-1+x}{1-x}\right) dx \qquad [f(x) = f(a-x)]$

$I = \int_0^1 \log \frac{x}{1-x} \Rightarrow -\int_0^1 \log\left(\frac{1-x}{x}\right) dx$ ... (ii)

$2I = 0 \qquad \therefore I = 0$

Ans. (c) 0

196. No. of ways in which 7 dept distributed among 3 minister

$= \left(7_{C_3} \times 4_{C_3} \times 1\right) + \left(7_{C_3} \times 4_{C2}\, x\right) \times 3!$

= (120 + 330) × 6 = 1980

Ans. (d) None of these

197. No. of selections of letters

(i) 2 like and 1 different = $3_{C_1} \times 2_{C_1} = 3 \times 2 = 6$

(ii) 3 different = $5_{C_3} = 10$

$\therefore$ Total no. of ways of selection letters = 16

$\therefore$ Total words = 16×3! = 96 words.

Ans. (b) 96

198. No. of ways to form three digit nos. by using (1, 2, 3, 4, 3, 2) are – 42.

Ans. (b) 42

199. $S_n + S_{n-2} - 2.S_{n-1} = \frac{n}{2}[2a + (n-1)d] + \frac{n-2}{2}[2a + (n - 2 - 1)d]$

$\ldots 2.\frac{n-1}{2}[2a + (n - 1 - 1)d]$

$$= \frac{n}{2}[2a+(n-1)d] + \left(\frac{n-2}{2}\right)[2a+(n-3)d] - (n-1)[2a+(n-2)d]$$

$$= an + \frac{(n-1)n}{2}d + a(n-2) + \frac{(n-3)(n-2)}{2}d - 2a(n-1) - (n-1)(n-2)d$$

$$= \quad d\left[\frac{(n-1)}{2}n + \frac{(n-3)(n-2)}{2} - (n-1)(n-2)\right]$$

$$d\left(\frac{n^2 - n + n^2 - 5n^2 + 6 - 2n^2 - 4 + 6n}{2}\right)$$

$$= \quad \frac{2d}{2} = d$$

Ans. (a) d

200. $\frac{A3}{A(n-1)} = \frac{1}{3}$

$\frac{a+3d}{a+(n-1)d} = \frac{1}{3} \Rightarrow (3+3\ d)\ 3 = 3 + (n-1)d$

$\therefore \quad d = \frac{6}{n-10}$

$\therefore \quad 7n = a + (N...1)\ d \Rightarrow 31 = 3 + (n + 2 - 1)\ \frac{6}{n-10}$

$28 = (n+1)\ \frac{6}{n-10} \quad \therefore n = 13$

Ans. (c) 13

Model Test Paper – BOS/CPT – 8

151. The first no. divide by 8 between 100 and 200 is 104

The last no. divide by 8 between 100 and 200 is 200

$\therefore$ The total number divide by 104 and 200 is 13.

All number divide by 8 also divide by 2 is 13

Ans. (b)

152. Sum of 1st n odd number

S = 1+3+5+ ... + (2n – 1)

Since $S = \frac{n}{2}[2a + (n-1)d]$

S = n/2 [2.1 + (n – 1)2]

= n (1+n – 1)

= n(n)

S = n^2

Ans. (a)

153. Let the number be x.

According to the given condition of the problem is

36x = x + 1050

36x = 1050

x = $\frac{1050}{35}$

x = 30

Ans. (b)

154. The formula is $1^3 + 2^3 + 3^3 + ... + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$

$\therefore \quad 1^3 + 2^3 + 3^3 + ... + 12^3 = \left\{\frac{12(12+1)}{2}\right\}^2$

= $[6(13)]^2$

= $[78]^2 = 6084$

Ans. (c)

155. Let S = 1+9+24+46+75+........+t_n

shifting 1 places to the right in the RHS

S = 1+9+24+46+75+...........t_{n-1}+t_n

subtracting term by term,

0=1+8+15+22+29+............+(t_n-t_{n-1}) - t_n

Transposing

t_n = 1+8+15+22+29+ .... to n^{th} term

$= \frac{n}{2}\{2.1+(n-1)7\}$

$= \frac{7}{2}n^2 - \frac{5}{2}n$

Now Sn = $\sum tn = \frac{7}{2}\sum n^2 - \frac{5}{2}\sum n$

$= \frac{7}{2}\frac{n(n+1)(2n+1)}{6} - \frac{5}{2}\frac{n(n+1)}{2}$

$= \frac{n(n+1)}{2}\left\{\frac{7}{6}(2n+1) - \frac{5}{2}\right\}$

$= \frac{n(n+1)}{2}\left(\frac{7}{3}n - \frac{4}{3}\right)$

Sn $= \frac{n(n+1)(7n-4)}{6}$

156. Let the number to be added be x then according to the problem.

$\frac{83+x}{263+x} = \frac{1}{3}$,

3(83+x) = 263 + x

249 + 3x = 263 + x

3x – x = 263 – 249

2x = 14

x = $\frac{14}{2}$ = 7

Hence the required number is 7

Ans. (c)

157. Initially, let the number of employees be 9 and wages per head be Rs. 14. Then, total wages bill = Rs. (9×14) = Rs. 126

Further, the number of employees becomes 8 and the wages per head becomes Rs. 15.

∴ Now total wages bill = Rs. (8×15) = Rs. 120

∴ Ratio of the wages bill = 126:120

= 21:20

Thus the wages bill is decreased in the ratio 21:20

Ans. (c)

ANSWERS

158. Let C gets Rs. = x

Given B = ¼ of C = ¼ (x)

and $A = \frac{2}{3}$ of B = 2/3 (¼x) = $\frac{1}{6}x$

Also, given $\frac{1}{6}x + \frac{1}{4}x + x = 680$

$$\frac{2x + 3x + 12x}{12} = 680$$

$$\frac{17x}{12} = 680$$

$$x = \frac{680 \times 12}{17} = 480$$

Ans. (c)

159. Let us assume that when x is added to each of the four given numbers, they become in proportion.

$\Rightarrow$ 10 + x : 18 + x = 22 + x : 38 + x

$\therefore$ Product of the means = Product of the extremes.

$\therefore$ (10+x) (38+x) = (18+x) (22+x)

$\Rightarrow$ $380+48x + x^2 = 396 + 40x + x^2$

8x = 16

x = 2

Required number = 2

Ans. (a)

160. Let the required numbers be x and y. Since the mean proportional between a and c is given by the relation $b = \sqrt{ac}$

$\therefore$ Mean proportional = $\sqrt{xy}$

According to the question,

$\sqrt{xy} = 24$

xy = 576 $\rightarrow$ (1)

Again suppose that the third proportional to x and y is z. Then

x : y = y : z

$\Rightarrow \quad x : z = y : y$

$xz = y^2$

$\Rightarrow \quad z = \dfrac{y^2}{x}$

According to the question,

$\dfrac{y^2}{x} = 192$

$\Rightarrow \quad y^2 = 192\,x \qquad \rightarrow (2)$

From equation (2), $\quad x = \dfrac{y^2}{192}$

Putting this value of x in equation (1)

$\dfrac{y^2}{192}.y = 576$

$y^3 = 576 \times 192$

$= \quad 24 \times 24 \times 24 \times 8$

$= \quad 24 \times 24 \times 24 \times 2 \times 2 \times 2$

$y = \quad (24 \times 24 \times 24 \times 2 \times 2 \times 2)^{1/3}$

$= \quad 24 \times 2$

$y = 48$

$(1) \Rightarrow xy = 576$

$x\,(48) = 576$

$x = \dfrac{576}{48} = 8$

Hence the required number are 12 and 48.

161. $\displaystyle \mathop{Lt}_{x \to 0} \frac{e^x + e^{-x} - 2}{x} \Rightarrow \left[\frac{e^{x-1}}{x} + \frac{(e^{-x} - 1}{x}\right]$

$\displaystyle \Rightarrow \mathop{Lt}_{x \to 0} \left[\left(\frac{e^x - 1}{x}\right) + \left(\frac{e^{-x} - 1}{x}\right)\right] \qquad \left\{\mathop{Lt}_{x \to 0} \frac{e^x - 1}{x} = 1\right\}$

$1 - 1 = 0$

Ans. (b) 0

162. $\underset{x\to 0}{\text{Lt}} \dfrac{e^{x^{-1}}-1}{e^{x^{-1}}+1} \Rightarrow \underset{x\to 0}{\text{Lt}} \left(\dfrac{e^{1/x}-1}{e^{1/x}+1}\right)$

RHL $\underset{x\to 0^+}{\text{Lt}} \left(\dfrac{e^{1/x}-1}{e^{1/x}+1}\right)$ App Lt = $\dfrac{\infty}{\infty}$

$\rightarrow$ RHL does not exist

LHL $\underset{x\to 0^-}{\text{Lt}} \left(\dfrac{e^{1/x}-1}{e^{1/x}+1}\right)$ App Lt $\dfrac{\infty}{\infty}$

LHL does not exist.

$\rightarrow$ Lt does not exist in f(x)

Ans. (c) does not exist

163. $\underset{x\to 0}{\text{Lt}} \dfrac{3x-|x|}{7x-5|x|}$

RHL $\underset{x\to 0^+}{\text{Lt}} \dfrac{3x-x}{7x-5x} = \dfrac{2x}{2x} = 1$

LHL $\underset{x\to 0^-}{\text{Lt}} \dfrac{3x-(-x)}{7x-5(-x)} = \dfrac{4x}{12x} = \dfrac{1}{3}$

LHL $\neq$ RHS

$\therefore$ f(x) does not exist at x = 0

Ans. (c) does not exist

164. $\underset{x\to 0}{\text{Lt}} \dfrac{e^{ax}-e^{bx}}{x} \Rightarrow \underset{x\to 0}{\text{Lt}} \left[a.\dfrac{\left(e^{ax}-1\right)}{ax} - b\dfrac{\left(e^{bx}-1\right)}{bx}\right]$

$\Rightarrow$ a.1 – b.1 $\Rightarrow$ a – b $\qquad \left\{\underset{x\to 0}{\text{Lt}} \dfrac{e^x-1}{x} = 1\right\}$

Ans. (a) a … b

165. $\underset{x\to 0}{\text{Lt}} \dfrac{e^x-1}{\log(1+x)} \Rightarrow \underset{x\to 0}{\text{Lt}} \left[\left(\dfrac{e^x-1}{x}\right).\dfrac{x}{\log(1+x)}\right]$

$\Rightarrow \underset{x\to 0}{\text{Lt}} \left(\dfrac{e^x-1}{x}\right) . \underset{x\to 0}{\text{Lt}} \dfrac{1}{\dfrac{\log(1+x)}{x}}$

$\Rightarrow$ 1.1 = 1

Ans. (b) 1

166. Given $y = \dfrac{\sqrt{1-x}}{\sqrt{1+x}}$

$$\frac{dy}{dx} = \frac{\left(\sqrt{1+x}\right)\frac{d}{dx}\left(\sqrt{1-x}\right) - \left(\sqrt{1-x}\right)\frac{d}{dx}\left(\sqrt{1+x}\right)}{\left(\sqrt{1+x}\right)^2}$$

$$= \frac{\left(\sqrt{1+x}\right)\left[\frac{1}{2}(1-x)^{\frac{1}{2}-1}(-1)\right] - \left(\sqrt{1-x}\right)\left[\frac{1}{2}(1+x)^{\frac{1}{2}-1}\right]}{(1+x)}$$

$$= \frac{-\frac{1}{2}\frac{\sqrt{1+x}}{\sqrt{1-x}} - \frac{1}{2}\frac{\sqrt{1-x}}{\sqrt{1+x}}}{(1+x)}$$

$$= -\frac{1}{2}\left[\frac{(1+x)+(1-x)}{\sqrt{1-x}\,\sqrt{1+x}} \cdot \frac{(1+x)}{1}\right]$$

$$\frac{dy}{dx} = -\frac{1}{2}\left[\frac{2}{\sqrt{1-x}\sqrt{1+x}} \cdot \frac{1+x}{1}\right] = \frac{-1}{(1+x)^{3/2}\left(\sqrt{1-x}\right)}$$

Ans. (b)

167. Given $y = \dfrac{x}{\sqrt{1+x^2}}$

$$\frac{dy}{dx} = \frac{\sqrt{1+x^2}\,(1) - x.\frac{1}{2\sqrt{1+x^2}}(2x)}{\left(\sqrt{1+x^2}\right)^2}$$

$$= \frac{\sqrt{1+x^2} - \frac{x^2}{\sqrt{1+x^2}}}{\left(1+x^2\right)}$$

$$= \frac{\left(1+x^2\right) - x^2}{\left(1+x^2\right)\left(\sqrt{1+x^2}\right)}$$

$$\frac{dy}{dx} = \frac{1}{\left(1+x^2\right)^{\frac{3}{2}}}$$

$$\therefore \quad x^3 \frac{dy}{dx} = \frac{x^3}{\left(1+x^2\right)^{\frac{3}{2}}} = \left[\frac{x}{\sqrt{1+x^2}}\right]^3$$

$$x^3 \frac{dy}{dx} = [y]^3$$

Ans. (c)

168. Given $x^y = e^{x-4}$

Taking log on both sides

$$\log\left(x^y\right) = \log\left(e^{x-y}\right)$$

y log x = (x – y) log e

y log x = x – y

y (1 + log x) = x

$$y = \frac{x}{1+\log x}$$

Differentiate on both sides

$$\frac{dy}{dx} = \frac{(1+\log x).1 - x\left(\frac{1}{x}\right)}{\left(1+\log\ x\right)^2}$$

$$= \frac{1+\log-1}{\left(1+\log x\right)^2} = \frac{\log x}{\left(1+\log x\right)^2}$$

Ans. (a)

169. Given $y^3 x^5 = (x+y)^8 \quad \rightarrow (1)$

Differentiate on both sides.

$$y^3\left(5x^4\right) + x^5.3y^2 \frac{dy}{dx} = 8(x+y)^7\left[1+\frac{dy}{dx}\right]$$

$$5y^3x^4 + 3x^5y^2 \frac{dy}{dx} = 8(x+y)^7 + 8(x+y)^7 \frac{dy}{dx}$$

$$\frac{dy}{dx}\left[3x^5\ y^2 - 8\ (x+y)^7\right] = 8\ (x+y)^7 - 5y^3x^4$$

$$\frac{dy}{dx}=\frac{8(x+y)^7-5y^3x^4}{3x^5y^2-8(x+y)^7}=\frac{8(x+y)^7-5\frac{(x+y)^8}{x}}{\frac{3}{y}(x+y)^8-8(x+y)^7}\quad \text{Using equation (1)}$$

$$=\frac{(x+y)^7\left[8-\frac{5}{x}(x+y)\right]}{(x+y)^7\left(\frac{3}{y}(x+y)-8\right)}$$

$$=\frac{y[8x-5(x+y)]}{x[3(x+y)-8y]}=\frac{y[8x-5x-5y]}{x[3x+3y-8y]}$$

$$=\frac{y[3x-5y]}{x[3x-5y]}=\frac{y}{x}$$

Ans. (a)

170. Given $y=x^{x^{x\ldots\infty}}$

i.e. $y=x^y$

Taking log on both sides

$\log y=\log\left(x^y\right)$

$\log y=y\log x$

Differentiate on both sides

$$\frac{1}{y}\frac{dy}{dx}=y.\frac{1}{x}+\log x.\frac{dy}{dx}$$

$$\frac{dy}{dx}\left[\frac{1}{y}-\log x\right]=\frac{y}{x}$$

$$\frac{dy}{dx}\left[\frac{1-y\log x}{y}\right]=\frac{y}{x}$$

$$\frac{dy}{dx}=\frac{y^2}{x(1-y\log x)}$$

$$\therefore\quad x.\frac{dy}{dx}=\frac{y^2}{1-y\log x}$$

Ans. (b)

ANSWERS

171. $\int_{-1}^{1}(e^{x} - e^{x})dx = \int_{-1}^{1}(0)\,dx$

$= \quad 0$

Ans. (b) 0

172. $\int_{1}^{e} \frac{1+\log x}{x}\,dx$

Let 1 + log x = t $\qquad$ x = 0, t = 1

$\frac{1}{x} = \frac{dt}{dx}$ $\qquad$ x = e, t = 2

$= \int_{1}^{2} + dt = \left[\frac{t^2}{2}\right]_1^2$

$= \quad 2 - \frac{1}{2} = \frac{3}{2}$

Ans. (a) $\frac{3}{2}$

173. $\int_{0}^{\log 3} \frac{ex}{1+e^{x}}dx$ $\qquad$ If $\quad 1 + e^{x} = t$ $\qquad$ x = 0, t = 2

$e^{x} = \frac{dt}{dx}$ $\qquad$ x = log3, t = 4

$\int_{2}^{4} \frac{1}{-t}dt = [\log t]_2^4 \Rightarrow \log 4 - \log 2$

= log 2

Ans. (b) log 2

174. $\int_{0}^{1} \frac{x}{1+\sqrt{1+x^2}}$ $\qquad$ let $(1 + x^{2}) = t2$ $\qquad$ x = 0, t = 1

$x = 1\ t = \sqrt{2}$

$2x = 2t \frac{dt}{dx} \Rightarrow dx = \frac{t}{x}\,dx$

$\int_{1}^{\sqrt{2}} \frac{t\,dt}{1+t} = \int_{1}^{\sqrt{2}} \left(1 - \frac{1}{1+t}\right)dt$

$$= \left[t - \log(1+t)\right]_1^{\sqrt{2}}$$

$$= \left(\sqrt{2}-1\right) - \left[\log\left(1+\sqrt{2}\right) - \log 1\right]$$

$$= \left(\sqrt{2}-1\right) - \log\left(1+\sqrt{2}\right) + 0$$

$$= \left(\sqrt{2}-1\right) - \log\left(1+\sqrt{2}\right)$$

Ans. (d) None of these

175. $\int_0^1 \frac{dx}{(1+x)(2+x)} \Rightarrow \int_0^1 \left(\frac{1}{1+x} - \frac{1}{2+x}\right) dx$

$$\Rightarrow \left[\log(1+x) - \log(2+x)\right]_0^1 \Rightarrow \left[\log\frac{1+x}{2+x}\right]_0^1$$

$$\Rightarrow \log \frac{2}{3}$$

Ans. (a) $\log \frac{2}{3}$

176. a + ar = 15 $\Rightarrow$ a (1+ r) = 15

$a = ar + ar^2 + ar^3 - \infty$

$a = \frac{ar}{1-r} \Rightarrow 1 - r = r \Rightarrow r = 1/2$

$\therefore\ a = \frac{15 \times 2}{3} = 10$

$\therefore$ Sum of Series $= \frac{a}{1-r} = \frac{10}{1-1/2} = 20$

Ans. (a) 20

177. $a = \frac{1}{1-x} \Rightarrow \frac{1}{a} = 1 - x$

$b = \frac{1}{1-y} \Rightarrow \frac{1}{b} = 1 - y$

$\therefore\ \frac{1}{a} + \frac{1}{b} = 1 - x + 1 - y \Rightarrow 2 - (x+y) = 2.1 = 1$

Ans. (c) 1

ANSWERS

178. $90 = 2000 \times \frac{R}{100} \times \frac{3}{4}$

$\therefore$ R = 6%

Ans. (b) 6%

179. $216 = 5400 \times \frac{6}{100} \times n$

n = 4/6 yrs. = 8 months

Ans. (b) 8 months

180. $I_1 = 10000 \frac{R}{100} \times 2 = 200R$

$I_2 = 6000 \times \frac{R}{100} \times 3 = 1800R$

$I_1 + I_2 = 1900 \Rightarrow 200R + 180R = 1900$

$\therefore$ 380R = 1900 $\therefore$ R = 5%

Ans. (b) 5%

181. If all the observation are equal.

Then standard deviation = 0

Ans. (a) 0

182. If every item is increased by 5 then mean $(\bar{x})$ also increased by 5, but the value of $\sum(x-\bar{x})^2$ remain same.

$\therefore$ Standard deviation will remain same,

Standard deviation = 10

Ans. (c) – 10

183. S.D. $= \sqrt{\frac{\sum d^2}{N}} = \sqrt{\frac{360}{10}} = 6$

Coefficient of variation = $100 \frac{x\,SD}{Am}$

$= \frac{100 \times 6}{40} = 15$

Ans. (a) 15

184. Coefficient of M.D. = $\frac{M.D}{Am} \times 100$

$44 = \frac{5.77 \times 100}{AM}$

A.M. = 13.11

Ans. 13.11

185. The S.D. of two values is equal to half their difference.

S.D. $= \frac{|a-b|}{2}$

The Statement is correct

Ans. (a) True

186. Computation of Correlation Coefficient

	x	y	xy	x^2	y^2
	50	40	2000	2500	1600
	50	40	2000	2500	1600
Total	100	80	4000	5000	3200

$\bar{x} = \frac{100}{2} = 50, \quad \bar{y} = \frac{80}{2} = 40$

Cov (x, y) $= \frac{4000}{2} - (50)(40) = 0$

$\therefore$ r = 0

Ans. (c)

187. Given $r_R = \frac{2}{3}, \quad \sum di^2 = 55$

$\therefore \quad r_R = 1 - \frac{b\sum di^2}{n(n^2-1)}$

$\frac{2}{3} = 1 - \frac{6(55)}{n(n^2-1)}$

$\frac{2}{3} - 1 = \frac{-330}{n(n^2-1)}$

$-\frac{1}{3} = -\frac{330}{n(n^2-1)}$

$n(n^2-1) = 990 = 10\ (10^2-1)$

$\therefore$ n = 10 as n must a positive

Ans. (a)

188. Let us assume that 4x + 3y + 7 = 0 $\rightarrow$ (1)

represent the regression line of x on y and 3x + 4y + 8 = 0 $\rightarrow$ (2) represent the regression line of y on x.

(1) 4x = –7 – 3y

$x = -\frac{7}{4} - \frac{3}{4}y$

$\therefore$ bxy = $-\frac{3}{4}$

(2) 4y = – 8 – 3x

$y = -2 - \frac{3}{4}x$

$\therefore$ byx = $-\frac{3}{4}$

$\therefore$ r^2 = byx. bxy = $\left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right) = \frac{9}{16}$

$\therefore$ r = $\sqrt{\frac{9}{16}} = \pm\frac{3}{4} = -\frac{3}{4} = -0.75$

(We take the sign of n as negative since both the regression coefficient are negative).

Ans. (c)

189. Ans. (d) Refer Properties

190. Given byx = 1.2 $\rightarrow$ (1)

$U = \frac{x-100}{2}$

$\Rightarrow$ x = 100 + 2U

$\Rightarrow$ $\bar{x} = 100 + 2\bar{U}$

and $v = \frac{y-200}{3}$

$\Rightarrow$ y = 200 + 3v

$\Rightarrow$ $\bar{y} = 200 + 3\bar{v}$

$$byx = \frac{\sum(x-\bar{x})(y-\bar{y})}{E(x-\bar{x})^2}$$

$$= \frac{\sum\left[2\left(U-\bar{U}\right)3\left(v-\bar{v}\right)\right]}{\sum\left[2(U-\bar{U})\right]^2}$$

$$= \quad \frac{2\times3}{4}\frac{\sum\left(U-\bar{U}\right)\left(V-\bar{V}\right)}{\sum(U-\bar{U})^2}$$

= 3/2 bvu

$\Rightarrow$ bvu = 2/3 byx = 2/3 × 1.2 = 0.8

Ans. (b)

191. Let A, is First bag is selected

A2 is second bag is selected.

B: In a draw of 2 balls, one is red and the other is black.

The required probability

$P = P(A_1 \cap B) + P(A_2 \cap B)$

$= \quad P(A_1)\, P(B/A_1) + P(A_2).\, P(B/A_2)$

Since there are two bags, the selection of each being equally likely.

$\therefore \quad P(A_1) = P(A_2) = 1/2$

$P(B/A_1)$ = Probability of drawing one red and one black ball in a draw of 2 balls from the 1st bag

$$= \quad \frac{5C_1 \times 3C_1}{8C_2} = \frac{15}{28}$$

$P(B/A_2)$ = Probability of drawing one red and one black ball in a draw of 2 balls from the 2nd bags.

$$= \frac{4C_1 \times 5C_1}{9C_2} = \frac{5}{9}$$

(1) $\Rightarrow$ p = 1/2 × $\frac{15}{28} + \frac{1}{2} \times \frac{5}{9}$

$$p. = \frac{15}{56} + \frac{5}{18} = \frac{135+140}{504} = \frac{275}{504}$$

Ans. (a)

ANSWERS

192. Let A1 is first purse is selected.

A2 is second purse is selected.

Let B: In a draw of one coin, one coin must be silver

The required probabilities.

$P = P\ (A_1 \cap B) + P\ (A_2 \cap B)$

$= \quad P(A_1).P(B/A_1) + P(A_2).P(B/A_2) \rightarrow (1)$

Since there are two purse, the selection of each being equally likely

$\therefore \quad P(A_1) = \frac{1}{2}, P(A_2) = \frac{1}{2}$

$P(B/A_1)$ = Probability of drawing one silver coin from the first purse.

$= \quad \frac{3C_1}{7C_1} = 3/7$

$P(B/A_2)$ = Probability of drawing one silver coin from the second purse.

$= \quad \frac{4C_1}{7C_1} = \frac{4}{7}$

Substituting (1) $\Rightarrow$ p=1/2 $\left(\frac{3}{7}\right) + \frac{1}{2}\left(\frac{4}{7}\right)$

$= \quad \frac{3}{14} + \frac{4}{14} = \frac{7}{14} = \frac{1}{2}$

Ans. (a)

193. When two tosses of unbiased dice the total sample space.

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

n(s) = 36

In the above sample space, let x be the number of sines getting from the experiment.

Let x = 0, means no sin. = number of times = 25

x = 1, means no sin. = number of times = 10

x = 2, means no sin. = number of times = $\frac{01}{36}$

∴ Expected table is:

x	0	1	2
p(x)	25	10	1

∴ The required probability = Mean = Expected Value

$$= \sum \frac{x\ p(x)}{n(s)}$$

$$= \frac{0\times 25+1\times 10+2\times 1}{36}$$

$$= \frac{12}{36}=\frac{1}{3}$$

Ans. (a)

194. The experiment of throwing three dice is theoretically same as that of throwing a die thrice.

Let E be the event of throwing six in a throw of die.

∴ P(E) = 1/6 and P ($\overline{E}$) = 1 – P(E)

= 1–1/6= 5/6

Let x denotes the random variable "number of Sixes".

∴ The possible values of x are 0, 1, 2, 3

∴ P(x=2) = P($E_1E_2\overline{E}_3$ or $E_1\overline{E}_2E_3$ or $\overline{E}_1E_2E_3$)

= $P(E_1)\ .\ P(E_2)\ P(\overline{E}_3) + P(E_1)\ P(\overline{E}_2)\ P(E_3) + P(\overline{E}_1) + P(E_2)\ P(E_3)$

$$= \frac{1}{6}.\frac{1}{6}.\frac{5}{6}+\frac{1}{6}.\frac{5}{6}.\frac{1}{6}+\frac{5}{6}.\frac{1}{6}.\frac{1}{6}$$

$$P(x=2) = \frac{15}{216}$$

Ans. (c)

195. let A and B denote the events that the Chartered Accountant is selected in firms X and Y respectively. Then in the usual notations, we are given.

P(A) = 0.7

P($\overline{A}$) = 1– P(A) = 1=0.7=0.3

P($\overline{B}$) = 0.5

$\therefore$ P(B) = 1 – P(B) = 1 – 0.5 = 0.5

and P $(\overline{A} \cup \overline{B})$ = 0.6

By De – Morgan's law

$\overline{(A \cap B)} = \overline{A} \cup \overline{B}$

$\therefore$ P $(A \cup B) = 1 - P\ \overline{(A \cap B)}$

= $1 - P(\overline{A} \cup \overline{B})$

= 1 – 0.6

= 0.4

The probability that the Chartered Accountant will be selected in one of the two firms X or Y is given by:

P (AUB) = P(A) + P(B) – P $(A \cap B)$

= 0.7 + 0.5 – 0.4

= 0.8

Ans. (a)

196. $\int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx$

$\mathbf{I} = \log(\log x) \int dx - \int \left[\frac{d}{dx} [\log(\log x)] \right] .x\,dx + \int \frac{1}{(\log x)^2} = dx$

$= \log(\log x).x - \int \frac{1}{(\log x)} .dx + \int \frac{1}{(\log x)^2} dx$

$x. \log(\log x) - \left[\frac{1}{(\log x)} \int dx + \left(\frac{1}{(\log x)^2} . \frac{x}{x} dx \right) \right] + \int \frac{1}{(\log x)^2} = dx$

$\Rightarrow x \log(\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2} + \int \frac{1}{(\log x)^2} dx$

$\Rightarrow x. \log(\log x) - \frac{x}{\log x} + c$

Ans. (a) $x. \log(\log x) - \frac{x}{\log x} + c$

197. 'is equal to' Satisfies Reflexive, Symnetric and transitive Relation

$\therefore$ This is Equivalence Relation.

Ans. (d) Equivalence Relation.

198. f(x) = x^2+2

$\therefore\ f(-x) = (-x)^2 + 2$

= $x^2 + 2$

$f(-x) = f(x)$

$\therefore$ f(x) is even function

Ans. (b) even function.

199. f(x) = $12^{1+x} = 12.12^{x}$ $0 \leq x < 9$

Range = 12×12^{0}, 12×12^{1} .....12×12^{9}

$\therefore$ Range = $12 \leq f(x) < 12^{10}$

Ans. (a) $12 \leq f(x) \leq 12^{10}$

200. 'Is greater than' over the set of real number is not satisfied Reflexive and Symmetric relation it only satisfied transitive Relation

Ans. (a) Transitive relation.

Model Test Paper – BOS/CPT – 9

151. Given $\frac{x}{x+y} = \frac{17}{23}$

$\Rightarrow 23x = 17x + 17y$

$23x - 17x = 17y$

$6x = 17y$

$x = \frac{17}{6}y$

Now, $\frac{x+y}{x-y} = \frac{\frac{17}{6}y + y}{\frac{17}{6}y - y}$

$= \frac{17y+6y}{6} \times \frac{6}{17y-6y}$

$= \frac{23y}{11y} = \frac{23}{11}$

$\therefore \frac{x+y}{x-y} = \frac{23}{11}$

152. Given $\sqrt{1+\frac{25}{144}} = 1+\frac{x}{12}$

Squaring on both sides:

$1+\frac{25}{144} = \left(1+\frac{x}{12}\right)^2$

$\frac{144+25}{144} = 1+\frac{x^2}{144}+\frac{2x}{12}$

$\frac{169}{144} = \frac{144+x^2+24x}{144}$

$\therefore x^2+24x-25=0$

$(x+25)(x-1)=0$

$x=-25,\ x=1$

x = 1 ($\therefore$ negative neglected)

Ans. (a)

153. Given $(4)^3 \times (\sqrt{2})^8 = 2^n$

i.e. $((2)^2)^3 \times \left((2)^{\frac{1}{2}}\right)^8 = 2^n$

$2^6 \times 2^4 = 2^n$

$2^{10} = 2^n$

$\therefore$ n = 10

Ans. (a)

154. Let total number of men went to a hotel = x

Given, A man Spent Rupees = Total number of men

= x

$\therefore$ Given Data = x + x = 15625

$x^2 = 15625$

$x = \sqrt{15625} = 125$

Ans. (b)

155. Given A + B + C = 1000 $\rightarrow$ (1)

A + C = 400 $\rightarrow$ (2)

B + C = 700 $\rightarrow$ (3)

(2) $\Rightarrow$ A = 400 – C

(3) $\Rightarrow$ B = 700 – C

(1) $\Rightarrow$ 400 – C + 700 – C + C = 1000

C = 100

Ans. (a)

156. Given $\log\left(\frac{a-b}{2}\right) = 1/2 (\log a + \log b)$

$\therefore \quad 2 \log \frac{a-b}{2} = \log a + \log b$

$\log\left(\frac{a-b}{2}\right)^2 = \log ab$

$\Rightarrow \quad \left(\frac{a-b}{2}\right)^2 = ab$

$\left(\frac{a-b}{4}\right)^2 = ab$

$(a - b)^2 = 4ab$

$a^2 + b^2 - 2ab = 4ab$

$a^2 + b^2 = 6ab$

Ans. (a)

157. Given $\log_{10} x = 4$

$\therefore$ $x = 10^4$

Ans. (c) x = 10000

158. log 225 = log (9×25)

= log 9 + log 25

= $\log 3^2 + \log 5^2$

= $2\log 3 + 2\log 5$

= $2\log 3 + 2\log 10/2$

= $2\log 3 + 2\log 10 - 2\log 2$

= $2 \times 0.477 + 2 - 2\,(0.301)$

$\log 225 = 2.352$

Ans. (a)

159. Let $2^{100} = x$

Taking log on both sides.

$\log 2^{100} = \log x$

$100 \log 2 = \log x$

$\log x = 100 \times 0.3010$

$\log x = 30.1000$

$\therefore$ the no. of digits in 2^{100} is 31

Ans. (b)

160. Given $nP3 = \dfrac{n!}{(n-3)!} = 60$

$\therefore n(n-1)(n-2) = 60 = 5 \times 4 \times 3$

$\therefore \quad n = 5$

Ans. (c)

161. $\lim\limits_{x\to 0} \dfrac{a^x + b^x - 2}{x} \Rightarrow \lim\limits_{x\to 0} \dfrac{(a^x - 1) + (b^x - 1)}{x}$

$\Rightarrow \lim\limits_{x\to 0} \dfrac{a^x - 1}{x} + \lim\limits_{x\to 0} \dfrac{b^x - 1}{x} \qquad \left[\lim\limits_{x\to 0} \dfrac{a^x - 1}{x} = \log_e^a\right]$

App Lt

= $\log a + \log b = \log(ab)$

Ans. (a) $\log(ab)$

162. $\lim\limits_{x\to 0} \dfrac{10^x - 5^x - 2^x + 1}{x}$

$\Rightarrow \lim\limits_{x\to 0} \left[\dfrac{(10^x - 1)}{x} - \dfrac{(5^x - 1)}{x} - \dfrac{(2^x - 1)}{x}\right] \qquad \left\{\lim\limits_{x\to 0} \dfrac{a^x - 1}{x} = \log_e^a\right\}$

App Lt

= $\log 10 - \log 5 - \log 2$

= $\log\left(\frac{10}{5\times 2}\right) = \log 1 \Rightarrow 0$

Ans. (b) 0

163. $\underset{x\to 0}{\text{Lt}} \frac{10^x - 5^x - 2^x - 1}{x^2} \Rightarrow \underset{x\to 0}{\text{Lt}} \frac{[5^x \times 2^x - 5^x - 2^x - 1]}{x^2} = \frac{5^x(2^x - 1) - 1(2^x - 1)}{x^2}$

$\underset{x\to 0}{\text{Lt}} \frac{(2^x - 1)(5^x - 1)}{x^2} \Rightarrow \underset{x\to 0}{\text{Lt}} \frac{(2^x - 1)}{x} \times \underset{x\to 0}{\text{Lt}} \frac{(5^x - 1)}{x}$

$\Rightarrow$ $\log 2 \times \log 5$

Ans. (a) $\log 5 \times \log 2$

164. $\underset{x\to 0}{\text{Lt}} \frac{e^{5x} - e^{3x} - e^{2x} + 1}{x}$

$\Rightarrow \underset{x\to 0}{\text{Lt}} \left[5\frac{(e^{5x} - 1)}{5x} - 3.\frac{(e^{3x} - 1)}{3^x} - 2\frac{(e^{2x} - 1)}{2^x}\right]$

App Lt

5.1 – 3.1 – 2.1

= 5 – 3 – 2 = 0 $\left\{\underset{x\to 0}{\text{Lt}} \frac{e^x - 1}{x} = 1\right\}$

Ans. (b) 0

165. $\underset{x\to 0}{\text{Lt}} \frac{e^{5x} - e^{3x} - e^{2x} - 1}{x^2} = \underset{x\to 0}{\text{Lt}} \frac{(e^{3x}.e^{2x} - e^{3x} - e^{2x} - 1)}{x^2}$

$\underset{x\to 0}{\text{Lt}} \frac{\left\lfloor e^{3x}(e^{2x} - 1) - 1(e^{2x} - 1)\right\rfloor}{x^2} \Rightarrow \underset{x\to 0}{\text{Lt}} \frac{\left\lfloor (e^{3x} - 1)\,(e^{2x} - 1)\right\rfloor}{x^2}$

$\underset{x\to 0}{\text{Lt}} \left(\frac{e^{3x} - 1}{x}\right) \times \underset{x\to 0}{\text{Lt}} \left(\frac{e^{2x} - 1}{x}\right)$

App Lt

3 × 2 = 6

Ans. (a) 6

ANSWERS

166. Given $\int \frac{1}{\sqrt{x^2 + a^2}} \, dx$

Let $\sqrt{x^2 + a^2} = z - x$

$\therefore \quad z = x + \sqrt{x^2 + a^2}$

$$\frac{dz}{dx} = 1 + \frac{1}{2\sqrt{x^2 + a^2}}(2x)\, dx$$

$$= \frac{\sqrt{x^2 + a^2} + x}{\sqrt{x^2 + a^2}} = \frac{z}{\sqrt{x^2 + a^2}}$$

$$\therefore \quad \frac{dz}{z} = \frac{dx}{\sqrt{x^2 + a^2}}$$

$$\therefore \quad \int \frac{dx}{\sqrt{x^2 + a^2}} = \int \frac{dz}{z} = \log z + c$$

$$= \log\left(x + \sqrt{x^2 + a^2}\right) + c$$

167. $\int \frac{1}{\sqrt{x^2 - a^2}} dx$

Let $\sqrt{x^2 - a^2} = z - x$

$\therefore \quad z = x + \sqrt{x^2 - a^2}$

$$\frac{dz}{dx} = 1 + \frac{1}{2\sqrt{x^2 - a^2}}(2x) = 1 + \frac{x}{\sqrt{x^2 - a^2}}$$

$$= \frac{\sqrt{x^2 - a^2} + x}{\sqrt{x^2 - a^2}} = \frac{z}{\sqrt{x^2 - a^2}}$$

$$\frac{dz}{z} = \frac{dx}{\sqrt{x^2 - a^2}}$$

$$\therefore \quad \int \frac{1}{\sqrt{x^2 - a^2}} dx = \int \frac{1}{2} dz + c$$

$$= \log(z) = \log\left(x + \sqrt{x^2 - a^2}\right) + c$$

Ans. (c)

168. Let t = 3x

$\therefore$ dt = 3 dx

$\therefore \; I = \int \frac{1}{t^2-1}\frac{dt}{3} = \frac{1}{3}\int \frac{1}{t^2-1^2}\,dt$

$= \frac{1}{3}\frac{1}{2+1}\log\left(\frac{t-1}{t+1}\right) + c$

$= \frac{1}{6}\log\left(\frac{3x-1}{3x+1}\right) + c$

Ans. (b)

169. Let $I = \int \frac{x-1}{\sqrt{x^2+1}}\,dx$

$\therefore \; I = \int \left[\frac{x}{\sqrt{x^2+1}} - \frac{1}{\sqrt{x^2+1}}\right]dx$

$= \int \frac{x}{\sqrt{x^2+1}}\,dx - \int \frac{1}{\sqrt{x^2+1}}\,dx$

$I = I_1 - I_2$ (say)

$I_1 = \int \frac{x}{\sqrt{x^2+1}}\,dx$

Let $t = x^2 + 1$

$\therefore$ dt = 2x dx

$\therefore \; I_1 = \int \frac{1}{\sqrt{t}}\frac{dt}{2}$

$= \frac{1}{2}\int t^{-1/2}\,dt$

$= \frac{1}{2}\frac{t^{1/2}}{1/2} = \sqrt{t} = \sqrt{x^2+1}$

$I_2 = \int \frac{1}{\sqrt{x^2+1}}\,dx$

$= \log\left(x + \sqrt{x^2+1}\right)$

$\therefore$ $I = I_1 - I_2$

$$I = \sqrt{x^2+1} - \log\left(x+\sqrt{x^2+1}\right) + c$$

Ans. (a)

170. Let $I = \int\left(1-x^2\right)\log x\ dx$

$$\therefore I = \int \log x\ (1-x^2)\ dx$$

[Here (log x) i s to be take n as first function and (1 – x^2) as second function]

Integrating by parts:

$$I = \log x\left(x-\frac{x^3}{3}\right) - \int\frac{1}{x}\left(x-\frac{x^3}{3}\right)dx$$

$$= \left(x-\frac{x^2}{3}\right)x\log x - \int\left(x-\frac{x^2}{3}\right)dx$$

$$= \left(1-\frac{x^2}{3}\right)x\log x - \int\left(x-\frac{x^3}{3*3}\right) + c$$

$$I = \left(1-\frac{x^2}{3}\right)x\log x - \left(x-\frac{x^3}{9}\right) + c$$

Ans. (c)

171. Among 4 doctors, 4 officers and 1 doctor who is also an officer committee of 3 can be form in such manner.

(i) 1 doctor, 1 officer, 1 doctor who is also officer $=4_{C_1} \times 4_{c_1} \times 1 = 16$

(ii) 2 doctor and doctor – officer = $4_{C_2} \times 1 = 6$

(iii) 2 officer and doctor – officer = ${}^4C_2 \times 1 = 6$

(iv) 2 doctor and 1 officer = $4_{C_2} \times 4_{C_1} = 24$

(v) 1 doctor and 2 officer = $4_{C_1} \times 4_{C_2} = 24$

Total no. of ways = 16 + 6 + 6 + 24 + 24 = 76

Ans. (a) 76

172. Elector can vote for one or more vacancies in such manner …

(i) For 3 vacancies – $5_{C_3} = 10$

(ii) For 2 vacancies – $5_{C_2} = 10$

(iii) For 1 vacancy – $5_{C_1} = 5$

∴ Total ways = 10 + 10 + 5 = 25

Ans. (c) 25

173. No. of ways in which 12 different thing distributed in 4 groups.

$$= \frac{12!}{(3!)^4} = 15400$$

Ans. (a) 15,400

174. Factor of 420 is = {2, 3, 4, 5, 6, 7, 10, 12, 14, 15, 20, 21, 30, 28, 35, 42, 60, 84, 105, 210, 140, 420}

No. of factor of 420 = 22

Ans. (b) 22.

175. Five balls are kept in 3 boxes as no box will empty

$$= \left(5_{C_1} \times 4_{c_1} \times 3_{c_3}\right) \times 3!$$

= (5 × 4 × 1) × 6 = 120 ways.

Ans. (b) 120 ways.

176. 243 + 324 + 432 + – n terms

$3^5.1 + 3^4.4 + 3^3.4^2$ + – n terms

∴ $a = 3^5$ $r = 4/3$

$$Sn = \frac{3^5.\left[\left(\frac{4}{3}\right)^n - 1\right]}{\left(\frac{4}{3} - 1\right)} = 3^5.3 \left[\left(\frac{4}{3}\right)^n - 1\right]$$

$$= 3^6\left[\frac{4^n}{3^n} - 1\right]$$

Ans. (a) $3^6\left[\frac{4^n}{3^n} - 1\right]$

177. $S_8 = 5.S_4 \Rightarrow \frac{a\left[r^8 - 1\right]}{r - 1} = \frac{5.a\left[r^4 - 1\right]}{r - 1}$

$\Rightarrow \left(r^4 + 1\right) = 5$

$r^4 = 4 = \left(\sqrt{2}\right)^4$

$\therefore$ r = ± $\sqrt{2}$

Ans. (c) ± $\sqrt{2}$

178. 4+44+444 ... n terms

$= \frac{4}{9}[9+99+999+.....\text{n terms}]$

$= \frac{4}{9}[(10 - 1)+(100 - 1) + (1000 - 1) + ... \text{n terms})$

$= \frac{4}{9}\left[\frac{10(10^n-1)}{10-1}-n\right] \Rightarrow \frac{4}{9}\left[\frac{10(10^n-1)}{9}-n\right]$

Ans. (a) $\frac{4}{9}\left[\frac{10(10^n-1)}{9}-n\right]$

179. $\frac{a+b}{2}=15$ and $\sqrt{ab}$ = 9 $\therefore$ ab = 81

a = (30 – b) & (30 – b)b = 01

$\therefore$ $b^2 - 30b + 81 = -0$ $\therefore$ b = 27, 3 and a = 3, 27

$\therefore$ Nos are 27, 3

Ans. (a) 27, 3

180. Product of n Gm between two No. is equal to n th Power of single Gm between two nos. This statement is correct.

Ans. (a) True

181. The weighted arithmetic mean of first a natural numbers whose weights are equal to the corresponding number is equal to

$\frac{2n+1}{3}$

Ans. (a) $\frac{2n+1}{3}$

182. $\bar{x} = \frac{w_1x_1+w_2x_2+w_3x_3}{(x_1+x_2+x_3)}$

$110 = \frac{100\times5 = 125\times5 + w_3\times5}{15}$

$1650 = 500 + 625 + 5.w_3$

$w_3 = \frac{525}{5} = 105$ kg.

Ans. (b) 105 Kgs.

183. $\sum x - n \times 2.5 = 50$

$\sum x - 2.5n = 50 \rightarrow$ (i)

$\sum x - 3.5n = -50 \rightarrow$ (ii)

[Eg. (i) – Eg. (ii)]

$1.0n = 100$

$\therefore$ n = 100 $\therefore$ $\Sigma x = 300$

$\therefore$ mean = $\frac{\Sigma x}{n} = \frac{300}{100} = 3$

Ans. (a) 100, 3

184. The most reliable value is mean.

Ans. (a) Mean

185. In which Central Value arranging is required – Median

Ans. (c) Median

186. There are 365 days in a normal year (without leap year)

No. 365 = 7 × 52 + 1

$\therefore$ In a year will contain at least 52 Tuesday

The possible remaining one Tuesday

Let A be the event of getting 53 Tuesday in the year.

$\therefore$ P (A) = 1/7

Ans. (b)

187. Given two unbiased dice are thrown, then the simple space are:

S = { (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

$\therefore$ n(S) = 36

Sample space of sum of the faces is not less than 10

A = {(4,6), (5,5), (5,6), (6,4), (6,5), (6,6)}

n(A) = 6

$\therefore$ Required probability = $\frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

Ans. (a)

188. Let A be the person travels by a plane

$\therefore$ P(A) = $\frac{1}{5}$

Let B be the person travels by a train

$\therefore$ P (B) = $\frac{2}{3}$

$\therefore$ Probability of his travelling neither by plane nor by train.

P(AB) = P(A). P(B) (Since A and B are mutually exclusive conditional probability)

= $\left(\frac{1}{5}\right)\left(\frac{2}{3}\right) = \frac{2}{15}$

Ans. (b)

189. Let A denote the event of drawing a diamond and B denote the event of drawing a King from a pack of Cards. Then we have P(A) = $\frac{13}{52} = \frac{1}{4}$

and P(B) = $\frac{4}{52} = \frac{1}{13}$

P(AUB) = P(A) + P(B) – P(A $\cap$ B)

= $\frac{1}{4} + \frac{1}{13} - P(A \cap B) \rightarrow (1)$

There is only one case favourable to the event A $\cap$ B vize, king of diamond.

Hence, P (A $\cap$ B) = $\frac{1}{52}$

$\therefore$ (1) $\Rightarrow$ P(AUB) = $\frac{1}{4} + \frac{1}{13} - \frac{1}{52} = \frac{13+4-1}{52} = \frac{16}{52} = \frac{4}{13}$

Ans. (c)

190. Let us define the events:

E_1 : A solves the problem

E_2 : B solves the problem

then we are given

$$P(E_1) = \frac{6}{6+9} = \frac{6}{15} = \frac{2}{5}$$

and $P(E_2) = \frac{10}{10+12} = \frac{5}{11}$

Assuming that A and B try to solve the problem independently. E_1 and E_2 are independent.

$$\therefore P(E_1 \cap E_2) = P(E_1).P(E_2) = \frac{2}{5} \times \frac{5}{11} = \frac{2}{11}$$

The problem will be solved if at least one of the students A and B solves the problem.

Hence, the probability of the problem being solved is given by

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \frac{2}{5} + \frac{5}{11} - \frac{2}{11}$$

$$= \frac{22 + 25 - 10}{55} = \frac{37}{55} = 0.673$$

Ans. (a)

192. Given Mean of Binomial distribution = μ = np = 3 $\rightarrow$ (1)

and Variance of Binomial distribution = σ^2 = npq =.2 $\rightarrow$ (2)

$$\frac{(2)}{(1)} \Rightarrow \frac{npq}{np} = \frac{2}{3}$$

$\therefore$ q = 2/3

$\therefore$ p + q = 1

$$p = 1 - q = 1 - \frac{2}{3} = \frac{1}{3}$$

(1) $\Rightarrow$ n(1/3) = 3

= n = 9

$\therefore$ p=1/3, q = 2/3, n=9

By the Binomial distribution $p(x) = nC_x \, p^x \, q^{n-x}$. The probability that the variate takes values less than or equal to 2 i.e. $p(x \leq 2) = p(x = 0) + P(x = 1) + P(x = 2)$.

= $9C_0 (1/3)^0 (2/3)^{9-0}$

+ $9C_1 (1/3)^1 (2/3)^{9-1}$

+ $9C_2 (1/3)^2 (2/3)^{9-2}$

= $9C_0 (1) (2/3)^9$

+ $9C_1 (1/3)^1 (2/3)^8 + 9C_2 (1/3)^2 (2/3)^7$

= $(2/3)^9 + 3 (2/3)^8 + 4 (2/3)^7$

$P(x \leq 2) = 0.3767$

Ans. (a)

193. Exhaustive cases: 2 digits can be selected out of 9 digits 1 through 9 in $9C_2$ ways.

$\therefore$ Exhaustive number of cases = $9C_2 = \frac{9\times8}{1\times2} = 36$

Favoarable number of cases. Among the digits 1 through 9.

Even digits are: 2, 4, 6 and 8 i.e. 4 in all

Odd digits are 1, 3, 5, 7 and 9 i.e. 5 in all.

The sum of the two digits drawn will be even if

(i) Either both the selected digits are even (or)

(ii) both the selected digits are odd.

Two even digits can be selected out of the 4 even digits in $4C_2$ ways and two odd digits can be selected out of the 5 odd digits in $5C_2$ ways.

Hence, the favourable number of cases that the sum of the two selected digits in even.

= $4C_2 + 5C_2$

= $\frac{4\times3}{1\times2} + \frac{5\times4}{1\times2}$

= 6+10 = 16

$\therefore$ P (sum of the two selected digits is even)

= $\frac{\text{Number of favourable cases}}{\text{Exhaustive number of cases}}$

= $\frac{16}{36} = \frac{4}{9}$

and P(Both selected digits are odd) = $\frac{5C_2}{9C_2} = \frac{10}{36} = \frac{5}{18}$

Ans. (e)

194. Let S be the sample space of the experiment

$\therefore$ S = {(1,1), (1,2), ... (6,5), (6,6)}

Let A = event of getting sum 6

and B = event of getting 4 at least once.

$\therefore$ A = {(1,5), (2,4), (3,3), (4,2), (5,1)}

and B = {(4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (1,4), (2,4), (3,4), (5,4), (6,4)}

$\therefore$ P(A) = 5/36 and P(B) = $\frac{11}{36}$

Also, AB = {(4,2), (2,4)}

$\therefore$ P(AB) = $\frac{2}{36}$

$\therefore$ The required probability

= Probability of getting 4 on at least one die given that sum is 6

$= P(B|A) = \frac{P(BA)}{P(A)} = \frac{P(AB)}{P(A)}$

$= \frac{2/36}{5/36} = \frac{2}{5}$

Ans. (b)

196. Standard Error of mean = $\frac{\sigma}{\sqrt{n}}$

$= \frac{12.6}{\sqrt{36}} = \frac{12.6}{6}$

Standard Error of mean = 2.1

Ans. (a) 2.1

197. Standard Error of Mean without replacement

$= \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$

$= \frac{12.6}{\sqrt{36}}\sqrt{\frac{101-36}{101-1}} = 2.1 \times \sqrt{0.65}$

SE = 2.1 × 0.806 = 1.69

Ans. (b) 1.69

198. $P = \frac{5}{25} = \frac{1}{5}$

$Q = 1 - \frac{1}{5} = \frac{4}{5}$

$n = 5$

S.E. of proportion of defectives = $\sqrt{\frac{PQ}{n}}$

$= \sqrt{\frac{1}{5} \times \frac{4}{5} \times \frac{1}{5}} = \sqrt{0.032}$

SE = 0.1088

Ans. (b) 0.1088

199. n = 2, N = 4

Total number of possible sample of size of with replacement = $4^2 = 16$

Ans. (a) 16

200. n = 2, N = 4

Total number of possible sample of size without replacement = $N_{C_n} = 4_{C_2} = 6$

Ans. (b) 6

Model Test Paper – BOS/CPT – 10

151. The value of $3^3 + 4^3 + 5^3 + ... + 11^3$

$= \left[\frac{11(11+1)}{2}\right]^2 - \left[1^3 + 2^3\right]$

$= (11 \times 6)^2 - (1 + 8)$

$= (66)^2 - (9)$

$= 4356 - 9 = 4347$

Ans. (c)

152. Let the two numbers are x and y

Given $x + y = 75 \rightarrow (1)$

$x - y = 20 \rightarrow (2)$

(1)+(2) $\Rightarrow 2x = 95$

$$x = \frac{95}{2}$$

$\therefore$ (1) $\Rightarrow = 75 - \frac{95}{2}$

$$= \frac{55}{2}$$

$\therefore$ The difference of their squares = $x^2 - y^2$

$$= \left(\frac{95}{2}\right)^2 - \left(\frac{55}{2}\right)^2$$

$$= \frac{9025}{4} - \frac{3025}{4}$$

$$= \frac{6000}{4} = 1500$$

Ans. (a)

153. Let the two numbers are x and y

Given $x + y = 13 \rightarrow (1)$

and $x^2 + y^2 = 85 \rightarrow (2)$

(1) $\Rightarrow$ $y = 13 - x$

Substitute $y = 13 - x$ in (2)

$$x^2 + (13 - x)^2 = 85$$

$$x^2 + 169 + x^2 - 26x - 85 = 0$$

$$2x^2 - 26x + 84 = 0$$

$$x^2 - 13x + 42 = 0$$

$$(x - 7)(x - 6) = 0$$

$x = 7, x = 6$

When $x = 7$ (1) $\Rightarrow y = 13 - 7 = 6$

When $x = 6$ (1) $\Rightarrow y = 13 - 6 = 7$

$\therefore$ The number (7, 6)

Ans. (a)

ANSWERS

154. Let the two consecutive members are x and x – 1

Given $x^2 - (x-1)^2 = 37$

$x^2 - (x^2 + 1 - 2x) = 37$

$x^2 - x^2 - 1 + 2x = 37$

2x = 38

x = 19

$\therefore$ x – 1 = 19 – 1 = 18

Ans. (a)

155. Let the number be x.

Given condition (1) $\frac{x}{x+3} \rightarrow (1)$

Given condition (2) $\Rightarrow \frac{x+7}{x+3-2} = 2$

x + 7 = 2 (x+1)

= 2x + 2

x = 5

$\therefore (1) \Rightarrow \frac{5}{5+3} = \frac{5}{8}$

156. Given $\log_x \sqrt{3} = \frac{1}{6}$

$x^{\frac{1}{6}} = \sqrt{3}$

$x = \left(\sqrt{3}\right)^6$

$= \left(\sqrt{3}\right)^2 \left(\sqrt{3}\right)^2 \left(\sqrt{3}\right)^2$

$= 3 \times 3 \times 3$

x = 27

Ans. (b)

157. Let y = $a^{\log_a x}$

Taking log on both sides

log y = log $\left[a^{\log_a x}\right]$

= $\log_a x . \log a$

$\log y = \log x$ [$\because \log_b^a . \log_c^b = \log_c^a$ By properties]

Taking Exponent on both sides.

$e^{\log y} = e^{\log x}$

$y^{\log e} = x^{\log x}$

y = x

Ans. (a)

158. Let y = $3^{2-\log_3 6}$

= $3^2 . 3^{-\log_3 6}$

= $9.3^{-[\log_3 (3x2)]}$

= $9.3^{-[\log_3 3 + \log_3 2]}$

= $9.3^{-[1+\log_3 2]}$

= $9.3^{-1} . 3^{-\log_3 2}$

= $3.3^{\log_3 (1/2)}$

= 3.(1/2) [$\because a^{\log_a x} = x$ properties]

$\therefore$ y = 3/2

Ans. (b)

159. log 30 = log (2×3×5)

= log 2 + log 3 + log 5

= 0.3010 + 0.4771 + 0.6990

= 1.4771

Ans. (c)

160. $\log_{10} 124.5 + \log_{10} 379 = \log_{10}(12.45 \times 10)$

+ $\log_{10}(3.79 \times 100)$

= $\log_{10} 12.45 + \log_{10} 10$

+ $\log_{10} 3.79 + \log_{10} 100$

= 1.0952 + 0.5786 +2

$\therefore$ $\log_{10} 124.5 + \log_{10} 379 = 4.6738$

Ans. (b)

ANSWERS

161. $n_{P_5} : n_{P_3} = 2:1 \Rightarrow \frac{n!}{(n-5)!} \div \frac{n!}{(n-3)!} = \frac{2}{1}$

$\Rightarrow \frac{(n-3)!}{(n-5)!} = \frac{2}{1} \Rightarrow (n-3)(n-4) = 2.1$

$\therefore$ n – 3 = 2 $\Rightarrow$ n = 5

Ans. (b) 5

162. No. of ways to enter into room = 10

No. of ways to came out from a different door = 9

$\therefore$ Total Ways = 10.9 = 90 ways

Ans. (a) 90

163. Five digit Nos. by using digit (1, 2, 3, 4, 6) = 5! = 120

Four digit nos. by using digit (1, 2, 3, 4, 6) = $5P_4$

= 120

$\therefore$ Total Nos. greater than 1000 = 120 + 120

= 240

Ans. (c) 240

164. Total Nos. of 6 digit (greater than 1 lakh)

by using (1, 1, 1, 2, 2, 3) are = $\frac{6!}{3!\ 2!}$

= 60

Ans. (a) 60

165. No. of ways in which 17 billiard can be arranged.

If 7 are black, 6 red and 4 white are

= $\frac{17!}{7!\ 6!\ 4!}$ = 4084080

Ans. (b) 4084080

166. Let $I = \int \frac{xe^x}{(x+1)^2} dx$

$= \int \frac{(x+1-1)e^x}{(x+1)^2} dx$

$$= \int \frac{(x+1)e^x}{(x+1)^2} dx - \int \frac{e^x}{(x+1)^2} dx$$

$$= \int \frac{e^x}{(x+1)} dx - \int \frac{e^x}{(x+1)^2} dx$$

$$= I_1 - I_2$$

$$I = \int \frac{e^x \ dx}{(x-1)}$$

Integrating by parts.

$$I_1 = \frac{1}{1+x} e^x - \int \left(-\frac{1}{(x+1)^2} \right) e^x \ dx$$

$$\therefore I = \frac{1}{(1+x)} e^x + \int \frac{1}{(x+1)^2} e^x \ dx - \int \frac{e^x}{(x+1)^2} dx$$

$$I = \frac{1}{(1+x)} e^x$$

Ans. (b)

167. $\int e^x \frac{(x-1)}{(x+1)^3} dx = \int \frac{x+1-2}{(x+1)^3} e^x \ dx$

$$= \int e^x \left\{ \frac{1}{(x+1)^2} - \frac{2}{(x+1)^3} \right\} dx$$

$$= \int e^x \{ f(x) + f'(x) \} dx$$

Where $f(x) = \frac{1}{(x+1)^2}$

$$= e^x \ f(x) \ + c$$

$$\int \frac{e^x (x-1)}{(x+1)^3} dx = \frac{ex}{(x+1)^2} + c$$

Ans. (c)

168. See the text book example page No. 9.28

Ans. (a)

169. $\int \frac{dx}{x^2 - a^2} = \int \frac{dx}{(x-a)(x+a)}$

$$\int \frac{1}{2a}\left(\frac{1}{x-a} - \frac{1}{x+a}\right) dx$$

$$= \frac{1}{2a}\left[\int \frac{dx}{x-a} - \int \frac{dx}{x+a}\right]$$

$$= \frac{1}{2a}[\log(x-a) - \log(x+a)]$$

$$\int \frac{dy}{x^2 - a^2} = \frac{1}{2a}\log\left[\frac{x-a}{x+a}\right]$$

170. $\int \frac{1}{a^2 - x^2} dx = \int \frac{dx}{(a+x)(a-x)}$

$$= \int\left(\frac{-1}{2a}\right)\left[\frac{1}{a+x} - \frac{1}{a-x}\right] dx$$

$$= -\frac{1}{2a}\left[\int \frac{1}{a+x} dx - \int \frac{1}{a-x} dx\right]$$

$$= -\frac{1}{2a}[\log(a+x) - \log(a-x)]$$

$$= \frac{-1}{2a}\log\left[\frac{a+x}{a-x}\right]$$

171. $e^{x-y} + \log xy + xy = 0$

d.wr.t.u.

$$e^{x-y}\left(1 - \frac{dy}{dx}\right) + \frac{1}{xy}\left[x\frac{dy}{dx} + y.1\right] + \left[x\frac{dy}{dx} + y.1\right] = 0$$

$$e^{x-y} - e^{x-y}.\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} + \frac{1}{x} + x\frac{dy}{dx} + y = 0$$

$$\left(-e^{x-y}+\frac{1}{y}+x\right)\frac{dy}{dx} = -e^{x-y}-\frac{1}{x}-y$$

$$\frac{1}{y}(xy+1-y.e^{x.y})\frac{dy}{dx} = -\frac{1}{x}(x.e^{x-y}+1+xy)$$

$$\therefore \frac{dy}{dx}=\frac{-y}{x}.\left(\frac{x.e^{x-y}+1+xy}{1+xy-y.e^{x-y}}\right)$$

Ans. (d) None of these

172. $y = x^{\log(\log x)}$

$\log y = \log(\log x) . \log x$

$$\frac{1}{y}.\frac{dy}{dx} = \log(\log x).\ \frac{1}{x} + (\log x).\ \frac{1}{(\log x)}.\frac{1}{x}$$

$$\therefore \quad \frac{dy}{dx}=\frac{y}{x}\ [\log(\log x)+1]$$

Ans. (a) $\frac{y}{x}\ [\log(\log x)+1]$

173. $y = x + \cfrac{1}{x+\cfrac{1}{x}}$

$$y = \frac{x^3+x+x}{x^2+1}=\frac{x^3+2x}{x^2+1}$$

$$\frac{dy}{dx}=\frac{(x^2+1)(3x^2+2)-(x^3+2x)(2x)}{(x^2+1)^2}$$

$$= \frac{3x^4+3x^2+2x^2+2-2x^4-4x^2}{(x^2+1)^2}$$

$$\frac{dy}{dx}=\frac{x^4+x^2+2}{(x^2+1)^2}$$

Ans. (a) $\frac{x^4+x^2+2}{(x^2+1)^2}$

174. $\sqrt{\frac{y}{x}} + \sqrt{\frac{x}{y}} = 6$

$\frac{y+x}{\sqrt{xy}} = 6 \Rightarrow x+y = 6\sqrt{xy}$

$\therefore 1+\frac{dy}{dx} = \frac{6}{2\sqrt{xy}}\left(x\frac{dy}{dx}+y.1\right)$

$\left(1-3\sqrt{\frac{x}{y}}\right)\frac{dy}{dx} = 3\sqrt{\frac{y}{x}}-1$

$\frac{dy}{dx} = \frac{3\sqrt{\frac{y}{x}}-1}{1-3\sqrt{\frac{x}{y}}} \Rightarrow \frac{\left(3\sqrt{y}-\sqrt{x}\right)\sqrt{y}}{\left(\sqrt{y}-3\sqrt{x}\right)\sqrt{x}}$

$\frac{dy}{dx} = \frac{3y-\sqrt{xy}}{\sqrt{xy}-3x} = \frac{3y-\left(\frac{x+y}{6}\right)}{\left(\frac{x+y}{6}\right)-3x}$

$\frac{dy}{dx} = \frac{17y-x}{y-17x} = \frac{x-17y}{17x-y}$

Ans. (c) $\frac{x-17y}{17x-y}$

175. $^{47}C_4 + \sum_{i=0}^{3} 50-i_{C_3}$

$\Rightarrow \quad 47_{C_4} + 50_{C_3} + 49_{C_3} + 48_{C_3} + 47_{C_3}$

$\Rightarrow$ 178365 + 19600 + 18424 + 17296 + 16215

$\Rightarrow$ 249900

Ans. (a) 249900

176. a = 100

$S_6 = 5.\ S_6$

$\frac{6}{2}[2a+(6-1)d] = 5.\frac{6}{2}[2(a+6d)+(6-1)d]$

3 [200 + 5d] = 15 [200+12d + 5d]

$200+5d = 1000 + 85d$

$80d = -800 \Rightarrow d = -10$

Ans. (a) – 10

177. $S_n = 3n^2 + n$

$\therefore S_1 = 4$ $\therefore a_1 = 4$

$S_2 = 14$ $a_2 = 14 - 4 = 10$ $\therefore d = 6$

$S_3 = 30$ $a_3 = 30 - 14 = 16$

$\therefore Tp = a + (p - 1) d$

$= 4 + (p - 1) 6$

$Tp = (6p - 2)$

Ans. (b) (6p – 2)

178. $S_m = S_n$

$$\frac{m}{2}[2a+(m-1)d] = \frac{n}{2}[2a+(n-1)d]$$

$$2ma - 2na = (n^2 - n)d - (m^2 - m)d$$

$$2a(m - n) = (n^2 - n - m^2 - m)d$$

$$2a(m - n) = -(m - n)(m+n - 1)d$$

$$\therefore 2a = -(m+n-1)d$$

$$S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d]$$

$$\frac{m+n}{2}[(-m+n-1)d + (m+n-1)d]$$

$$S_{m+n} = 0$$

Ans. (a) 0

179. $-\frac{9}{4}, -2, -\frac{7}{4} ..... 0$

$$a = -\frac{9}{4} \quad d = -2 + \frac{9}{4} = \frac{1}{4}$$

$$0 = -\frac{9}{4} + (n-1)\frac{1}{4} \Rightarrow \frac{9}{4} = (n-1)\frac{1}{4}$$

$$\Rightarrow 9 = n - 1 \Rightarrow n = 10$$

Ans. (b) 10th term

180. $6.T_6 = 15.T_{15}$

$\therefore$ 6 (a+5d) = 15 (a+14d)

2a + 10d = 5a + 70d

3a = –60d $\therefore$ a = –20d

T_{21} = a + 20d

= –20 d + 20d

T_{21} = 0

Ans. (c) 0

181. The average of n numbers is x.

If any no.is multiplied to each of datas. Then average will also multiplied by such no.

$\therefore$ New average = (n + 1) x

Ans. (c) (n + 1) x.

182. $Av = \frac{\sum x}{n} \Rightarrow 1.5 = \frac{\sum x}{\delta}$

$= \sum x$ = 12 kg. (increased weight)

$\therefore$ Weight of New person = 65 + 12 = 77 kg.

Ans. (c) 77 Kg.

183. If passes students = x

$\therefore 35 = \frac{39x + 15(120 - n)}{120}$

4200 = 39x + 1800 – 15x

2400 = 24x

$\therefore$ x = 100

Passed Students = 100

Ans. (a) 100

184. $\frac{\sum x}{17} = 45$

$\sum x$ = 765

Total of first 9 numbers = 9 × 51 = 459

Total of last 9 numbers = 9 × 36 = 324

$\therefore$ Value of 9th number = (459 + 324) – 765

= 18

Ans. (c) 18

185. $\sum x = 11 \times 30 = 330$

Total of first five numbers = 5 × 25 = 125

Total of last five numbers = 5 × 28 = 140

∴ Value of 6th number = (125 + 140) ~ 330 = 65

Ans. (b) 65

186. Ans. (a) Refer properties

187. Ans. (b) Refer Properties.

188. Let A be the hearts playing cards in a part. Let B be the club playing cards in a part.

∴ $P(A) = \frac{13}{52}$

$P(B) = \frac{13}{52}$

Here A and B are mutually exclusive.

∴ $P(A \cup B) = P(A) + P(B) = \frac{13}{52} + \frac{13}{52} = \frac{26}{52} = \frac{1}{2}$

189. Total number of balls in the bag = 6 + 4 = 10

Since three balls are drawn out of 10 balls in $^{10}C_3$ ways

∴ Exhaustive number of Cases = $10\,C_3$ = 120

The number of favourable cases two balls are blue and balls is red

= $6C_2 \text{ x } 4C_1$

= 60

∴ Probability of 2 balls are blue and 1 is red = $\frac{6c_2 \times 4c_1}{10c_3}$

= $\frac{60}{120} = ½$

Ans. (c)

190. There are 366 days in a leap year.

Now 366 = 7 x 52 + 2

∴ The leap year will contain at least 52 Mondays. The possible combination for the remaining two days are:

(i) Sunday and Monday

(ii) Monday and Tuesday

(iii) Tuesday and Wednesday

(iv) Wednesday and Thursday

(v) Thursday and Friday

(vi) Friday and Saturday

(vii) Saturday and Sunday

Let A be the event of getting 53 Mondays in the leap year. Therefore, only those combinations will be favourable to the event A which contain "Monday"

$\therefore$ The combination (i) and (ii) are favourable to the happening of A

$\therefore$ P(A) = 2/7

Ans. (a)

191. Given P(A) = 1/3

P(B) = 3/4

and A and B are independent events

$$P(A \cup B) = 1 - P(A \cap B)^{1}$$

$$= \quad 1 - [P(A^{1}).P(B^{1})]$$

$$= \quad 1 - \{[1 - P(A)] \; [1 - P(B)]\} \qquad \text{[A and B are independent]}$$

$$= \quad 1 - \left[\left(1 - \frac{1}{3}\right)\left(1 - \frac{3}{4}\right) \right]$$

$$= \quad 1 - \left\{ \left(\frac{2}{3}\right)\left(\frac{1}{4}\right) \right\}$$

$$P \;\; P(A \cup B) = 1 - \left[\frac{1}{6}\right] = \frac{5}{6}$$

Ans. (b)

192. Out of given 4 letters, there are two letters are vowel (O,E). Let A be the first letter is vowel.

P(A) = 2/4

Let B be the second letter is vowel

P(B) = 1/3

Here A and B are independent

P(AB) = P(A).P(B)

= $\frac{2}{4} \cdot \frac{1}{3}$ = 1/6

Ans. (a)

193. Out of given 4 letters, there are two letters are vowel (O, E)

Let A be the first letter is vowel.

i.e. P(A) = 2/4

Let B be the second letters is vowel.

P(B) = 1/3

Here A and B are Mutually executive

$\therefore$ $P(A \cup B)$ = P(A) + P (B)

= $\frac{2}{4}+\frac{1}{3}=\frac{6+4}{12}=\frac{10}{12}=\frac{5}{6}$

Ans. (a)

194. Let A be the first letter selected M from the 'HOME'.

B be the second letter selected M from the 'HOME'

P(A) = 1/4, P(B) = 1/4

A and B are Mutually exclusive

$\therefore$ P(AUB) = P(A) + P(B)

= $\frac{1}{4}+\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$

Ans. (b)

195. By addition thereon

P(A or B) = P(A) + P(B)

0.65 = [1 – P('not A)] + p

= [1 – 0.65] + p

$\therefore$ p = 0.65 – 0.35

p = 0.30

Ans. (c)

197. Since f(x) is a Polynomial.

& a_1, a_2, a_3 are in AP

$\therefore$ $f(a_1)$, $f(a_2)$, $f(a_3)$ also in AP

$\therefore$ $f'(a_1), f'(a_2), f'(a_3)$ also in AP

Ans. (a) AP

198. $A = P\left[\frac{(1+i)^n}{i} - 1\right]$

$20{,}000 = P\left[\frac{(1.04)^{10}}{0.04} - 1\right]$

After solving we get

P = 2470 (Approx)

Ans. (a) 2470

199. byx = 1.2 & bxy = – 0.5

This is wrong because bxy and byx have same sign.

Ans. (b) false.

200. The mean of poison distribution is 1.6 and variance is 2. This is wrong because P – d will greater than 2

Ans. (b) false.