QUESTION BOOKLET CODE **D**

Test Paper Code: CA

Time: 3 Hours

Maximum Marks: 300

INSTRUCTIONS

A. General:

1. This Question Booklet is your Question Paper.
2. This Question Booklet contains **44** pages and has 100 questions.
3. Answer ALL questions.
4. The Question Booklet **Code** is printed on the right-hand top corner of this page.
5. The Question Booklet contains blank spaces for your rough work. No additional sheets will be provided for rough work.
6. **Clip board, log tables, slide rule, calculator, cellular phone, pager and electronic gadgets in any form are NOT allowed.**
7. Write your **Name** and **Roll Number** in the space provided at the bottom.
8. All answers are to be marked only on the machine gradable Objective Response Sheet **(ORS)** provided inside this booklet, as per the instructions therein.
9. The Question Booklet along with the Objective Response Sheet **(ORS)** must be handed over to the Invigilator before leaving the examination hall.

B. Filling-in the ORS:

10. Write your Roll Number in the boxes provided on the upper left-hand-side of the **ORS** and darken the appropriate bubble under each digit of your Roll Number using a **HB pencil**.
11. Ensure that the **code** on the **Question Booklet** and the **code** on the **ORS** are the same. If the codes do not match, report to the Invigilator immediately.
12. On the lower-left-hand-side of the **ORS**, write your Name, Roll Number, Name of the Test Paper, Name of the Test Centre and put your signature in the appropriate box with ball-point pen. Do not write these anywhere else.

C. Marking of Answers on the ORS:

13. Each question has **4 choices** for its answer: (A), (B), (C) and (D). Only **ONE** of them is the correct answer.
14. On the right-hand-side of **ORS**, for each question number, darken with a **HB Pencil,** ONLY one bubble corresponding to what you consider to be the most appropriate answer, from among the four choices.
15. There will be **negative marking** for wrong answers.

MARKING SCHEME:

(a) **For each question, you will be awarded 3 (three) marks, if you have darkened only one bubble corresponding to the correct answer.**

(b) **In case you have not darkened any bubble for a question, you will be awarded 0 (zero) mark for that question.**

(c) **In all other cases, you will be awarded –1 (minus one) mark for the question.**

Name .... Roll Number ☐☐☐☐☐☐☐

DO NOT BREAK THE SEALS ON THIS BOOKLET. AWAIT INSTRUCTIONS FROM THE INVIGILATOR.

SEAL

SEAL

D

Special Instructions / Useful Data

$\mathbb{N}$ denotes the set of natural numbers.

$\mathbb{Z}$ denotes the set of integers.

$\mathbb{Q}$ denotes the set of rational numbers.

$\mathbb{R}$ denotes the set of real numbers.

P^T denotes the transpose of a matrix P.

$\bar{x}$ denotes the complement of a Boolean variable x.

f' denotes the derivative of a function f.

$\frac{\partial z}{\partial x}$ denotes the partial derivative of z with respect to x.

$E(X)$ denotes the expected value of a random variable X.

$\mathbb{R}^3 = \{\mathbf{x}^T = (x_1, x_2, x_3) : x_1, x_2, x_3 \in \mathbb{R}\}$

$\mathbb{Z}_n = \{0, 1, 2, \cdots, n-1\}$

$\oplus_n$ denotes addition modulo n.

All **bold faced** vectors are column vectors.

For all **C programs** assume that all standard library functions are accessible.

1. Consider the following declaration in C

```
struct student {
        char name[12] ;
        float gradepoint ;
};
struct student MCA[5] ;
```

The number of bytes needed to store the array *MCA* is

(A) 16

(B) 25

(C) 70

(D) 80

2. Which of the following is a valid C directive?

(A) # include <stdio.h>;

(B) # include <stdio.h>

(C) include <stdio.h>;

(D) include <stdio.h>

3. Which of the following is NOT a Random Access Storage Device?

(A) Magnetic Tape

(B) Hard Disk

(C) Floppy Disk

(D) CD

4. Let $x = 0.125E + 01$, $y = (1.01)_2$ and $z = (1.2)_3$. Which of the following is TRUE?

(A) x, y and z are equal

(B) Only x and y are equal

(C) Only x and z are equal

(D) All x, y and z are different

5. 10's complement of the decimal number 56789 is

(A) 01234

(B) 12345

(C) 43210

(D) 43211

6. The largest natural number whose base 7 representation has exactly four digits, is

(A) 2400

(B) 6666

(C) 7777

(D) 2401

7. Consider the following program segment

```
{ int x,  i,  j ;
        x = 0 ;
        for (i = 0; i < 19; i ++)
         for (j = i + 1; j < 20; j ++)
           x ++; }
```

The value of x after executing the segment is

(A) 171

(B) 190

(C) 342

(D) 380

8. Consider the following C statements

```
P :  for (i = 0; i < 8; i+ = 3) {printf ("*");}
Q :  for (i = 4; i > 0; i- = 2) {printf ("*");}
R :  for (i = 0; i <= 9; i+ = 3) {printf ("*");}
S :  for (i = 0; i < 7; i ++) {if (i %3 == 0) printf ("*");}
```

Which one of the following is a TRUE statement?

(A) P, Q, R and S give the same output

(B) P and S give the same output

(C) Q and R give the same output

(D) P, Q and S give the same output

9. Let x, y and z be Boolean variables. The number of possible values for the expression

$$xy + \bar{z}\,x$$

is

(A) 1

(B) 2

(C) 4

(D) 8

10. The binary equivalent of the hexadecimal number $A52C$ is

(A) 1010101101100

(B) 1010010100101100

(C) 1010111000101100

(D) 1010010100101101

11. The decimal value of $(21)_8 \times (101)_{16}$ lies in the interval

(A) 3000 – 3499

(B) 3500 – 3999

(C) 4000 – 4499

(D) 4500 – 4999

12. Consider the following program segment

```
{ int n = 1;
  float x, term;
  float sum = 1;
  term = 1;
  while (n < 51)
  {
      term *= -x*x/(n*(n+1));
      sum += term ;
      n += 2;
  }
}
```

For a given x the value of sum approximates the function

(A) $\sin x$

(B) $\cos x$

(C) e^{-x}

(D) e^{-x^2}

13. Let $f : \mathrm{N} \to \mathrm{N}$ be defined as

$$f(n) = \begin{cases} 1, & \text{if } n = 1 \text{ or } n = 2 \\ f(n-1) + f(n-2), & \text{otherwise.} \end{cases}$$

What is the value of $f(10)$?

(A) 34

(B) 45

(C) 55

(D) 89

14. Let X and Y be 4 bit registers with initial contents as 1011 and 1001, respectively. The following sequence of operations are performed on the two registers :

$$Y \leftarrow X \oplus Y$$
$$X \leftarrow X \oplus Y$$
$$Y \leftarrow X \oplus Y$$

where $\ominus$ denotes XOR operation. The final contents of the two registers are

(A) $X = 1001$, $Y = 1011$

(B) $X = 1011$, $Y = 1001$

(C) $X = 1011$, $Y = 1011$

(D) $X = 1001$, $Y = 1001$

15. The Boolean expression

$$(x+y)(y+\overline{z})(z+\overline{x})$$

is equal to

(A) xyz

(B) $xy\overline{z}$

(C) $(\overline{x}+z)y$

(D) $(x+\overline{z})y$

16. Let x and y be independent Boolean variables, each taking values 0 or 1 with probabilities 0.5 and 0.5, respectively. The probability that

$$x + y(\overline{x}+\overline{y}) = 1$$

is

(A) 0

(B) 0.25

(C) 0.5

(D) 0.75

17. The unit place of the number 27^{82} is

(A) 1

(B) 3

(C) 7

(D) 9

18. Consider the following C program

```
void main ( )
{
   int i, s ;
   for (i = 0; ; i + +)
    { s = s + i / (i – 2) ;
     if (i > 5) break ;
    }
}
```

Which one of the following is a TRUE statement?

(A) There is a syntax error

(B) There is a type mismatch error

(C) There is a runtime error

(D) There is no runtime error

19.

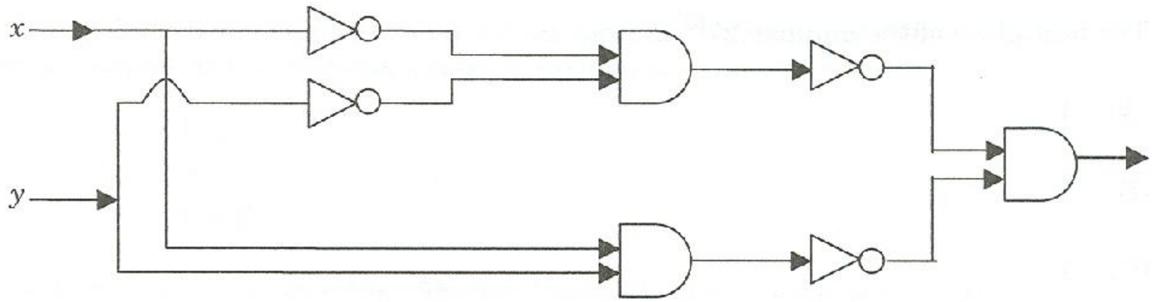

Figure 1

The logic circuit diagram given in Figure 1 is equivalent to

(A) AND gate

(B) OR gate

(C) NAND gate

(D) XOR gate

20.

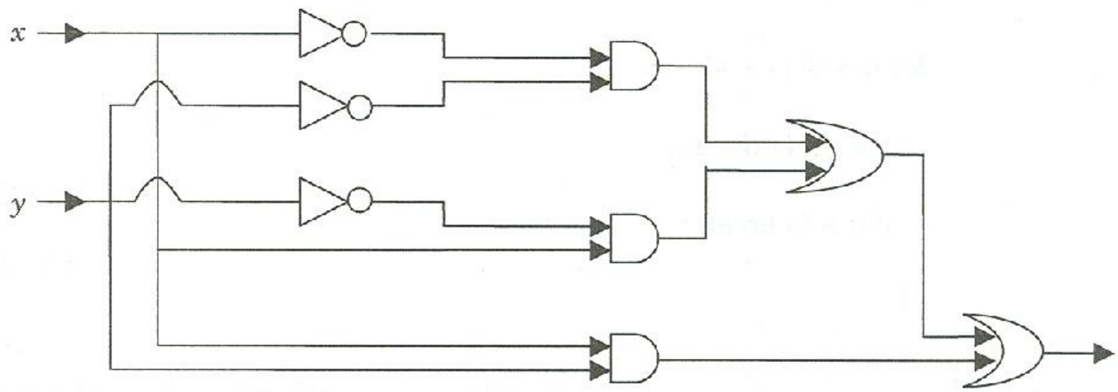

Figure 2

The logic circuit diagram shown in Figure 2 is equivalent to the Boolean expression

(A) $x + y$

(B) $x + \overline{y}$

(C) $\overline{x} + y$

(D) $\overline{x} + \overline{y}$

21. Consider the following C program segment

```
int gradepoint ;
char ch;
switch (ch) {
  case 'A' :   {gradepoint = 10 ;}
  case 'B' :   {gradepoint = 8 ; break ;  }
  case 'C' :   {gradepoint = 6 ; }
  default  :   {gradepoint = 0 ; } }
```

Executing the program segment for *ch* = 'A', 'B', 'C' *gradepoints* are respectively

(A) 10, 8, 6

(B) 10, 8, 0

(C) 8, 8, 6

(D) 8, 8, 0

22. What is the output of the following C program?

```
void fun (int * p)
{ int i, sum = 0 ;
  for (i = 2; i < 4; ++i)
        sum += * (p + i);
  printf ("%d", sum);
}
void main ( )
{ int a[5] = {10, 20, 30, 40, 50};
  fun (a + 1);
}
```

(A) 90

(B) 120

(C) 130

(D) 140

23. Which of the following is an 8-bit processor?

(A) Intel 80286

(B) Intel 8086

(C) Intel 8085

(D) Intel Pentium II

24. The maximum number of characters that can be encoded in a fixed length encoding scheme with n bits is

(A) 2^n

(B) $n!$

(C) n^2

(D) n

25. BIOS is the acronym for

(A) Binary Input Output Source

(B) Basic Input Output Support

(C) Binary Input Output System

(D) Basic Input Output System

26. What is the sum of the interior angles of an n vertex simple polygon?

(A) $(n-2)\pi$

(B) $\frac{(n+3)\pi}{6}$

(C) $\frac{(n+1)\pi}{4}$

(D) $\frac{n\pi}{3}$

27. For $a, b \in \mathbb{Z}$, define a relation aRb if $ab \geq 0$. Then the relation R is

(A) symmetric, reflexive and transitive

(B) symmetric and reflexive but NOT transitive

(C) symmetric and transitive but NOT reflexive

(D) reflexive and transitive but NOT symmetric

28. A student computes the sum of squares of the first 40 natural numbers and gives an incorrect answer 22019. By mistake, the student forgot to add the square of one of the numbers. The missed number is

(A) 5

(B) 7

(C) 9

(D) 11

29. Which of the following is NOT a Software?

(A) Adobe

(B) Browser

(C) Compiler

(D) Device Driver

30. For which of the following combinations, a JK Flip-Flop will enter into the complement of the present state?

(A) J = 0, K = 0

(B) J = 0, K = 1

(C) J = 1, K = 0

(D) J = 1, K = 1

31. For which of the following combinations an SR Flip-Flop is set to 1?

(A) S = 0, R = 0

(B) S = 0, R = 1

(C) S = 1, R = 0

(D) S = 1, R = 1

32. If $\sin x + \cos x = \alpha$ then $\sin(2x)$ is

(A) $1-\alpha^2$

(B) $\alpha^2 - 1$

(C) $1+\alpha^2$

(D) α^2

33. The next term in the series 191, 211, 232, 254, ---- is

(A) 267

(B) 276

(C) 277

(D) 287

34. The number of ways in which 4 boys and 5 girls can sit in a row so that there is a girl between any two boys is

(A) 4! 5!

(B) 3 (4! 5!)

(C) 5 (4! 5!)

(D) 15 (4! 5!)

35. The number of all functions $f:\{1,2,\cdots,n\}\to\{1,2,\cdots,m\}$ is

(A) $m(m-1)\cdots(m-n+1)$

(B) $n(n-1)\cdots(n-m+1)$

(C) m^n

(D) n^m

D

36. Consider the following program

```
void swap (int a, int b)
{ int temp ;
 temp = a ;
a = b ;
b = a ;
}
void main ( )
{ int x, y;
  x = 2;  y = 3;
  swap (x, y);
  printf ("x = %d  y = %d \n", x, y);
}
```

The output of the program is

(A) $x = 2 \quad y = 2$

(B) $x = 2 \quad y = 3$

(C) $x = 3 \quad y = 2$

(D) $x = 3 \quad y = 3$

37. Match the file extensions in **List 1** with the corresponding applications in **List 2**

List 1		**List 2**	
1.	mp3	P.	image
2.	xls	Q.	music
3.	jpeg	R.	database
4.	mdb	S.	spread sheet

(A) (1, Q), (2, S), (3, R), (4, P)

(B) (1, Q), (2, S), (3, P), (4, R)

(C) (1, Q), (2, P), (3, S), (4, R)

(D) (1, Q), (2, R), (3, P), (4, S)

38. Match the items of **List 1** with the items of **List 2**

List 1		**List 2**	
1.	Operating Systems	P.	Pentium
2.	Application Software	Q.	Linux
3.	Processor	R.	Router
4.	Network	S.	Anti Virus

(A) (1, Q), (2, S), (3, P), (4, R)

(B) (1, Q), (2, R), (3, P), (4, S)

(C) (1, P), (2, S), (3, Q), (4, R)

(D) (1, P), (2, R), (3, S), (4, Q)

39. The value of

$$3+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ddots}}}}$$

is

(A) $2-\sqrt{2}$

(B) $3-\sqrt{2}$

(C) $2+\sqrt{2}$

(D) $3+\sqrt{2}$

40. Which of the following diseases is NOT caused by mosquito bite?

(A) Dengue

(B) Encephalitis

(C) Malaria

(D) Typhoid

41. Which country won the 2006 FIFA World Cup?

(A) Argentina

(B) France

(C) Germany

(D) Italy

42. Who is the father of Bhishma in the Mahabharata?

(A) Bharat

(B) Devavrata

(C) Parashar

(D) Shantanu

43. Who among the following is NOT a Nobel Laureate?

(A) Amartya Sen

(B) J.C. Bose

(C) Muhammad Yunus

(D) S. Chandrasekhar

44. Let p and q be distinct primes and H be a proper subgroup of the additive group of integers. Suppose $S = H \cap \{p, q, p+q, pq, p^q, q^p\}$ has exactly three elements. Then S is

(A) $\{pq, p^q, q^p\}$

(B) $\{p+q, pq, p^q\}$

(C) (p, pq, p^q)

(D) $\{p, p+q, pq\}$

45. Consider the dihedral group $D_4 = \{e, r, r^2, r^3, f, rf, r^2f, r^3f\}$ with $r^4 = e = f^2$ and $rf = fr^{-1}$. The product $r^3fr^{-1}f^{-1}r^3fr$ corresponds to

(A) f

(B) rf

(C) r^2f

(D) r^3f

46. The point on the sphere $x^2 + y^2 + z^2 = 1$ farthest from the point $(1, -2, 1)$ is

(A) $\left(\frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

(B) $\left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

(C) $\left(\frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

(D) $\left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

47. The general solution of the differential equation

$$y''' + y'' - y' - y = 0$$

is

(A) $(c_1 + xc_2 + x^2c_3)e^x$

(B) $(c_1 + xc_2 + x^2c_3)e^{-x}$

(C) $c_1e^x + (c_2 + xc_3)e^{-x}$

(D) $(c_1 + xc_2)e^x + c_3e^x$

48. The solution of the differential equation

$$\left(x^2y+xy^2\right)dx+\left(\frac{x^3}{3}+x^2y+\sin y\right)dy=0$$

is

(A) $\frac{x^3y}{3}+\frac{x^2y^2}{2}-\cos y=c$

(B) $\frac{x^3y}{3}+\frac{x^2y^2}{2}+\cos y=c$

(C) $\frac{x^3}{3}+\frac{x^2y^3}{6}-\cos y=c$

(D) $\frac{x^3}{3}+\frac{x^2y^3}{6}+\cos y=c$

49. Let D be the region in the first quadrant lying between $x^2+y^2=1$ and $x^2+y^2=4$. The value of the integral

$$\iint_D \sin\left(x^2+y^2\right)dx\,dy$$

is

(A) $\frac{\pi}{4}(\cos 1-\cos 2)$

(B) $\frac{\pi}{4}(\cos 1-\cos 4)$

(C) $\frac{\pi}{2}(\cos 1-\cos 2)$

(D) $\frac{\pi}{2}(\cos 1-\cos 4)$

50. If the line $y = mx$, $0 \le x \le 2$ is rotated about the line $y = -1$, then the area of the generated surface is

(A) $4\pi(1+m)\sqrt{1+m}$

(B) $4\pi\left(1+m^2\right)\sqrt{1+m}$

(C) $4\pi\left(1+\sqrt{m}\right)\sqrt{1+m^2}$

(D) $4\pi(1+m)\sqrt{1+m^2}$

51. Let f be an increasing, differentiable function. If the curve $y = f(x)$ passes through $(1, 1)$ and has length

$$L = \int_1^2 \sqrt{\left(1 + \frac{1}{4x^2}\right)}\, dx,\ 1 \le x \le 2,$$

then the curve is

(A) $y = \ln\left(\sqrt{x}\right) - 1$

(B) $y = 1 - \ln\left(\sqrt{x}\right)$

(C) $y = \ln\left(1 + \sqrt{x}\right)$

(D) $y = 1 + \ln\left(\sqrt{x}\right)$

52. Let

$$U = \left\{ \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} : a \in \mathbb{R} \right\} \text{ and } V = \left\{ \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} : a \in \mathbb{R} \right\}.$$

The angle between U and V is

(A) 0

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{3}$

53. Let

$$f(x)=x^3+x^2-x+15 \text{ and } g(x)=x^3+2x^2-x+15.$$

Then, over $\mathbb{Q}$

(A) f is irreducible and g is reducible

(B) f is reducible and g is irreducible

(C) Both f and g are reducible

(D) Both f and g are irreducible

54. Let P be a 3×3 matrix such that for some $\mathbf{c}$, the linear system $P\mathbf{x}=\mathbf{c}$ has infinite number of solutions. Which one of the following is TRUE?

(A) The linear system $P\mathbf{x}=\mathbf{b}$ has infinite number of solutions for all $\mathbf{b}$

(B) Rank $(P)=3$

(C) Rank $(P)\neq1$

(D) Rank $(P)\leq2$

55. Let X, Y, Z be independent Poisson variables, such that $E(X)=E(Y)$ and $E(Z)=2E(X)$. If $P(X=5, Y=4)$ is equal to $P(Z=8)$, then $E(X)$ is

(A) $\dfrac{2^6}{21}$

(B) $\dfrac{2^6}{7}$

(C) $\dfrac{2^7}{7}$

(D) $\dfrac{2^7}{21}$

56. Let X be a binomial random variable with parameters n and p. If the mean and the standard deviation of X are 3 and $\frac{3}{2}$, respectively, then what is the value of (n, p)?

(A) $\left(4, \frac{3}{4}\right)$

(B) $\left(6, \frac{1}{2}\right)$

(C) $\left(9, \frac{1}{3}\right)$

(D) $\left(12, \frac{1}{4}\right)$

57. Two letters are chosen one after another without replacement from the English alphabet. What is the probability that the second letter chosen is a vowel?

(A) $\frac{4}{25}$

(B) $\frac{5}{26}$

(C) $\frac{5}{25}$

(D) $\frac{1}{5}\frac{1}{26}$

58. Let

$$f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 5.$$

Then the local maximum and the local minimum of the function f are at the points

(A) $(-2, 0)$ and $(-2, 2)$, respectively

(B) $(-2, 0)$ and $(0, 2)$, respectively

(C) $(0, 2)$ and $(-2, 0)$, respectively

(D) $(0, 2)$ and $(0, 0)$, respectively

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(C) $\left(9, \frac{1}{3}\right)$

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(B) $(-2, 0)$ and $(0, 2)$, respectively

(C) $(0, 2)$ and $(-2, 0)$, respectively

(D) $(0, 2)$ and $(0, 0)$, respectively

59. If

$$\lim_{h\to 0}\frac{1}{h}\int_{\alpha}^{\alpha+h} e^{-t^2}\,dt = 1,$$

then the value of α is

(A) -1

(B) 0

(C) 1

(D) 2

60. Consider the equations

$$\sin(\cos x) = x \qquad (1)$$

and

$$\cos(\sin x) = -x \qquad (2)$$

for $x \geq 0$. Then

(A) Only Equation (1) has a solution

(B) Only Equation (2) has a solution

(C) Both Equations (1) and (2) have solutions

(D) Neither Equation (1) nor Equation (2) has a solution

61. The area of the parallelogram with sides

$$\mathbf{x} = \vec{i} + \vec{j} + \vec{k} \text{ and } \mathbf{y} = -\vec{i} + \vec{j}$$

is

(A) $\sqrt{6}$

(B) $2\sqrt{3}$

(C) $3\sqrt{2}$

(D) 6

62. Let

$$P = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

The eigenvectors corresponding to the eigenvalues i and $-i$ are respectively

(A) $\begin{pmatrix} 1 \\ i \end{pmatrix}$ and $\begin{pmatrix} -1 \\ i \end{pmatrix}$

(B) $\begin{pmatrix} 1 \\ i \end{pmatrix}$ and $\begin{pmatrix} i \\ -i \end{pmatrix}$

(C) $\begin{pmatrix} -1 \\ i \end{pmatrix}$ and $\begin{pmatrix} i \\ -i \end{pmatrix}$

(D) $\begin{pmatrix} i \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ i \end{pmatrix}$

63. Let P be a 2×2 matrix such that $P^{102} = \mathbf{0}$. Then

(A) $P^2 = \mathbf{0}$

(B) $(I-P)^2 = \mathbf{0}$

(C) $(I+P)^2 = \mathbf{0}$

(D) $P = \mathbf{0}$

64. For $n \geq 5$, the expression

$$1 + 2x + 3x^2 + 4x^3 + \cdots + nx^{n-1}, \quad x \neq 1,$$

is equal to

(A) $\dfrac{nx^n(1-x) - x^n + 1}{(1-x)^2}$

(B) $\dfrac{nx^n(x-1) - x^n + 1}{(1-x)^2}$

(C) $\dfrac{nx^n(x-1) + x^n - 1}{(1-x)^2}$

(D) $\dfrac{nx^n}{(1-x)^2}$

65. The integral

$$\int_0^{\frac{\pi}{2}} \min(\sin x, \cos x)\, dx$$

equals

(A) $\sqrt{2}-2$

(B) $2-\sqrt{2}$

(C) $2\sqrt{2}$

(D) $2+\sqrt{2}$

66. Let

$$F(x)=\int_0^x (t-1)(t-2)(t-3)(t-4)\, dt, \ \ 0 \le x \le 5.$$

Then F has local minimum at the points

(A) $\{0, 2, 4\}$

(B) $\{1, 3, 5\}$

(C) $\{0, 3, 4\}$

(D) $\{3, 4, 5\}$

67. Let $G=\{n \in \mathbb{Z}: 1 \le n \le 55, \ \gcd(n, 56)=1\}$ be a multiplicative group modulo 56. Consider the sets

$$S_1=\{1, 9, 17, 25, 33, 41\} \text{ and } S_2=\{1, 15, 29, 43\}.$$

Which one of the following is TRUE?

(A) S_1 is a subgroup of G but S_2 is NOT a subgroup of G

(B) S_1 is NOT a subgroup of G but S_2 is a subgroup of G

(C) Both S_1 and S_2 are subgroups of G

(D) Neither S_1 nor S_2 is a subgroup of G

68. Let $\sigma = (125)\,(36)$ and $\tau = (1456)\,(23)$ be two elements of the permutation group on 6 symbols. Then the product $\sigma \circ \tau$, where $\sigma \circ \tau(i) = \sigma(\tau(i))$, is

(A) $(14)\,(26)\,(35)$

(B) $(13)\,(26)\,(45)$

(C) $(14)\,(25)\,(36)$

(D) $(13)\,(24)\,(56)$

69. The number of group homomorphisms from the group $(Z_{18}, \oplus_{18})$ to the group $(Z_{30}, \oplus_{30})$ is

(A) 3

(B) 4

(C) 5

(D) 6

70. Which of the following pair of linear programming constraints is equivalent to the inequality $|x_1 - x_2| \leq a$?

(A) $x_1 - x_2 \leq a,\; x_2 - x_1 \leq a$

(B) $x_1 - x_2 \leq a, x_2 - x_1 \leq -a$

(C) $x_1 - x_2 \leq -a,\; x_2 - x_1 \leq -a$

(D) $x_1 - x_2 \leq -a,\; x_2 - x_1 \leq a$

71. If the Primal Linear Programming Problem is unbounded then which of the following is TRUE?

(A) Dual problem is unbounded

(B) Dual problem has a single bounded optimal solution

(C) Dual problem has multiple bounded optimal solutions

(D) Dual problem is infeasible

72. Consider the following Primal Linear Programming Problem :

Maximize $\mathbf{c}^T\mathbf{x}$

Subject to $P\mathbf{x} = \mathbf{b}$
$\mathbf{x} \geq \mathbf{0}$

The Dual Linear Programming Problem is

(A) Minimize $\mathbf{y}^T\mathbf{b}$ Subject to : $P^T\mathbf{y} = \mathbf{c}$, $\mathbf{y}$ unrestricted

(B) Minimize $\mathbf{y}^T\mathbf{b}$ Subject to : $P^T\mathbf{y} \geq \mathbf{c}$, $\mathbf{y}$ unrestricted

(C) Minimize $\mathbf{y}^T\mathbf{b}$ Subject to : $P^T\mathbf{y} = \mathbf{c}$, $\mathbf{y} \geq \mathbf{0}$

(D) Minimize $\mathbf{y}^T\mathbf{b}$ Subject to : $P^T\mathbf{y} \geq \mathbf{c}$, $\mathbf{y} \geq \mathbf{0}$

73. For $\lambda > 0$, the value of the integral

$$\int_0^\infty e^{-\lambda x^2}\,dx$$

equals

(A) $\frac{1}{2}\sqrt{\frac{\pi}{\lambda}}$

(B) $\sqrt{\frac{\pi}{2\lambda}}$

(C) $\sqrt{\frac{2\pi}{\lambda}}$

(D) $2\sqrt{\frac{\pi}{\lambda}}$

74. Consider the function

$$f(x, y) = (x+y)^2 - (x+y) + 1.$$

The absolute maximum value and the absolute minimum value of the function on the unit square $\{(x, y): 0 \le x \le 1,\ 0 \le y \le 1\}$, respectively are

(A) 3 and $\frac{3}{2}$

(B) $\frac{3}{2}$ and $\frac{3}{4}$

(C) 3 and $\frac{3}{4}$

(D) 2 and $\frac{3}{4}$

75. Let P be an $n \times n$ idempotent matrix, that is, $P^2 = P$. Which of the following is FALSE?

(A) P^T is idempotent

(B) The possible eigenvalues of P are 0 or 1

(C) The nondiagonal entries of P can be zero

(D) There are infinite number of $n \times n$ nonsingular matrices that are idempotent

76. Let

$$P = \begin{pmatrix} 1 & 0 & 5 \\ 1 & 2 & 5 \\ 1 & 3 & 1 \end{pmatrix}.$$

Then $8P^{-1}$ is equal to

(A) $\begin{pmatrix} 13 & -4 & -1 \\ -15 & 4 & 3 \\ 10 & 0 & -2 \end{pmatrix}$

(B) $\begin{pmatrix} 13 & -15 & 10 \\ -4 & 4 & 0 \\ -1 & 3 & -2 \end{pmatrix}$

(C) $\begin{pmatrix} 13 & 10 & -15 \\ -4 & 0 & 4 \\ -1 & -2 & 3 \end{pmatrix}$

(D) $\begin{pmatrix} 13 & -4 & -1 \\ 10 & 0 & -2 \\ -15 & 4 & 3 \end{pmatrix}$

77. A cow is tied with a pole by a 100 meter long rope. What is the probability that at some point of time the cow is at least 60 meters away from the pole?

(A) $\frac{9}{25}$

(B) $\frac{13}{25}$

(C) $\frac{16}{25}$

(D) $\frac{18}{25}$

78. Consider the following Linear Programming Problem :

$$\text{Maximize } 3x_1 + 8x_2$$

$$\begin{aligned} \text{Subject to } \quad 2x_1 + 5x_2 &\leq 10 \\ 6x_1 + x_2 &\leq 6 \\ x_1,\ x_2 &\geq 0 \end{aligned}$$

The optimal value of the objective function is

(A) 0

(B) 3

(C) $\dfrac{111}{7}$

(D) 16

79. Let

$$f(x,\ y) = xy^2 + yx^2.$$

Suppose the directional derivative of f in the direction of the unit vector (u_1, u_2) at the point $(1, -1)$ is 1. Then among the following, (u_1, u_2) is

(A) $(-1, 0)$

(B) $(0, 1)$

(C) $(1, 0)$

(D) $\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$

80. Let θ, $0 \le \theta \le \pi$ be the angle between the planes

$$x - y + z = 3 \text{ and } 2x - z = 4.$$

The value of θ is

(A) $\cos^{-1}\left(\frac{1}{5}\right)$

(B) $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

(C) $\cos^{-1}\left(\frac{1}{\sqrt{15}}\right)$

(D) $\cos^{-1}\left(\frac{3}{\sqrt{15}}\right)$

81. Let $y(x) = x\sin x$ be one of the solution of an n^{th} order linear differential equation with constant coefficients. Then the minimum value of n is

(A) 1

(B) 2

(C) 3

(D) 4

82. The solution of the initial value problem

$$xy' - y = 0$$

with $y(1) = 1$ is

(A) $y(x) = x$

(B) $y(x) = \frac{1}{x}$

(C) $y(x) = 2x - 1$

(D) $y(x) = \frac{1}{2x - 1}$

83. Let

$$\mathbf{x} = \vec{i} + \vec{j} + \vec{k},\ \mathbf{y} = \alpha\vec{i} + \vec{k} \text{ and } \mathbf{z} = \vec{i} + \alpha\vec{j}.$$

Then the volume of the parallelopiped with sides $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$ is

(A) $1+\alpha+\alpha^2$

(B) $1+\alpha-\alpha^2$

(C) $1-\alpha+\alpha^2$

(D) $\alpha^2+\alpha-1$

84. If Ω denotes the region bounded by the X-axis and the lines $y = x$ and $x = 1$, then the value of the integral

$$\iint_{\Omega} \frac{\cos(2x)}{x} dx\, dy$$

is

(A) $\frac{\sin 2}{2}$

(B) $\frac{\cos 2}{2}$

(C) $\cos 2$

(D) $\sin 2$

85. The spheres

$$x^2+y^2+z^2=1 \text{ and } x^2+(y-\sqrt{3})^2+z^2=4$$

intersect at an angle

(A) 0

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{3}$

86. The function f defined on $\mathbb{R}$ by

$$f(x) = 3^x + 4^x - 5^x$$

has

(A) exactly one zero

(B) exactly two zeros

(C) exactly three zeros

(D) infinitely many zeros

87. Consider the alternating group $A_4 = \{\sigma \in S_4 : \sigma \text{ is an even permutation}\}$. Which of the following is FALSE?

(A) A_4 has 12 elements

(B) A_4 has exactly one subgroup of order 4

(C) A_4 has a subgroup of order 6

(D) Number of 3 cycles in A_4 is 8

88. Let $G = \{1, 2, \cdots, p-1\}$ be the group with respect to multiplication modulo p. If the inverse of 110 in G is 4, then p is of the form

(A) $5n+1$

(B) $5n+2$

(C) $5n+3$

(D) $5n+4$

89. Let G be a group with respect to multiplication. If $x = \alpha\sqrt{2} + \beta\sqrt{3} \in G$ then x^{-1} is

(A) $\dfrac{\alpha\sqrt{2} + \beta\sqrt{3}}{2\alpha^2 + 3\beta^2}$

(B) $\dfrac{\alpha\sqrt{2} - \beta\sqrt{3}}{2\alpha^2 - 3\beta^2}$

(C) $\dfrac{\alpha\sqrt{2} + \beta\sqrt{3}}{2\alpha^2 - 3\beta^2}$

(D) $\dfrac{\alpha\sqrt{2} - \beta\sqrt{3}}{2\alpha^2 + 3\beta^2}$

90. Consider

$$f(x) = 1 + xe^{-x}.$$

The Newton-Raphson iterative scheme for finding a root of $f(x) = 0$ is

(A) $x_{n+1} = \dfrac{1 + x_n^2 e^{-x_n}}{(x_n - 1)e^{-x_n}}$

(B) $x_{n+1} = \dfrac{x_n^2 e^{-x_n} + x_n\left(1 + e^{-x_n}\right) - 1}{1 + x_n e^{-x_n}}$

(C) $x_{n+1} = \dfrac{x_n^2 e^{-x_n} + x_n\left(1 - e^{-x_n}\right) + 1}{1 + x_n e^{-x_n}}$

(D) $x_{n+1} = \dfrac{1 + x_n^2 e^{-x_n}}{1 + x_n e^{-x_n}}$

91. Consider

x	-1	0	1	2	3
$f(x)$	1	5	3	1	5

Applying Simpson's one third rule, the value of the integral

$$\int_{-1}^{3} f(x)\,dx$$

is

(A) 10

(B) 12

(C) $\frac{41}{3}$

(D) 15

92. The slope of the tangent line to the curve

$$x = a(t - \sin t),\ y = a(1 - \cos t),\ t \in \mathbb{R},$$

at $t = \frac{\pi}{2}$ is

(A) -1

(B) 0

(C) 1

(D) ∞

93. Let

$$f(x) = x^3 - x^2 + 1,\ \ 0 \le x \le 1.$$

Then the absolute minimum value of $f(x)$ is

(A) $\frac{14}{27}$

(B) $\frac{5}{9}$

(C) $\frac{23}{27}$

(D) 1

94. Suppose

$$z = x\sin\left(\frac{x}{y}\right) + y\sin\left(\frac{y}{x}\right),\ xy \ne 0.$$

Then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is equal to

(A) $-z$

(B) 0

(C) z

(D) $2z$

95. Which of the following is FALSE?

(A) A unique interpolating polynomial of degree n is obtained from the given values at fixed $n+1$ points

(B) The Lagrange interpolation formula can be applied to equispaced points

(C) The Newton's forward difference interpolation formula can be applied to non-equispaced points

(D) The trapezoidal rule gives exact value of the integral for linear functions

96. The maximum absolute error that occurs in rounding off a number after 6 places of decimal is

(A) 5×10^{-8}

(B) 10^{-7}

(C) 5×10^{-7}

(D) 5×10^{-6}

97. Let

$$f(x) = 2x^3 - x^2 + 2x - 5 .$$

Consider the following statements about the roots of $f(x) = 0$

P : At least one root is positive.

Q : At least one root is negative.

R : There is a root between $x = 1$ and $x = 2$.

Which one of the following is TRUE?

(A) P, Q and R are valid statements

(B) P and Q are valid statements but R is NOT a valid statement

(C) P and R are valid statements but Q is NOT a valid statement

(D) P is a valid statement but Q and R are NOT valid statements

98. Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$, $\mathbf{v} \neq \mathbf{0}$. Which of the following is FALSE?

(A) $\left| \mathbf{u} \cdot \frac{\mathbf{v}}{\|\mathbf{v}\|} \right|$ is the length of the projection of $\mathbf{u}$ along $\mathbf{v}$

(B) If $\mathbf{u} \cdot \mathbf{w} = \mathbf{v} \cdot \mathbf{w}$ for all $\mathbf{w} \in \mathbb{R}^3$, then $\mathbf{u} = \mathbf{v}$

(C) $\mathbf{u} \cdot \mathbf{v} = \frac{1}{2}\left(\|\mathbf{u} + \mathbf{v}\|^2 - \|\mathbf{u} - \mathbf{v}\|^2 \right)$

(D) $\|\mathbf{u} + \mathbf{v}\|^2 + \|\mathbf{u} - \mathbf{v}\|^2 = 2\left(\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 \right)$

99. Let

$$P = \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix}.$$

Then

(A) P has two linearly independent eigenvectors

(B) P has an eigenvector

(C) P is nonsingular

(D) There exists a nonsingular matrix S such that $S^{-1}PS$ is a diagonal matrix

100. Let V be the vector space of all polynomials with real coefficients. If W is the vector subspace of V generated by

$$1-x,\ x^2-x,\ x^2-1 \text{ and } x^2-3x+2,$$

then the dimension of W is

(A) 1

(B) 2

(C) 3

(D) 4

Space for rough work