Code No: RR220202 **Set No. 1**

II B.Tech II Semester Supplimentary Examinations, Apr/May 2008
MATHEMATICS-III
(Common to Electrical & Electronic Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Electronics & Telematics, Metallurgy & Material Technology, Aeronautical Engineering and Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Evaluate $\int\limits_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$.
   (b) prove that $\Gamma(n+\frac{1}{2}) = \frac{\sqrt{\pi}}{4n}\frac{\Gamma(2n+1)}{\Gamma(n+1)}$
   (c) If m>0, n>0, then prove that $\frac{1}{n}\beta(m, n+1) = \frac{1}{m}\beta(n+1, m) = \frac{\beta(m,n)}{m+n}$ [5+5+6]

2. (a) Prove that $\int\limits_{-1}^{1} (x^2 - 1)\, P_{n+1}\, \mathrm{P'}_n\, dx = \frac{2n(n+1)}{(2n+1)(2n+3))}$
   (b) Prove that $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{\sin x}{x} - \cos x\right]$ [8+8]

3. (a) Test for analyticity at the origin for f(z) $= \frac{x^3y(y-ix)}{x^6+y^2}$ for z $\neq$ 0
   $= 0$ for z = 0.
   (b) Find all values of z which satisfy (i) e^z = 1+i (ii) sinz =2. [8+8]

4. (a) Evaluate $\int\limits_C \frac{z^2-2z-2}{(z^2+1)^{2z}}\, dz$ where C is $|\, z - i\,| = 1/2$ using Cauchy's integral formula
   (b) Evaluate $\int\limits_{(0,0)}^{(1,1)} (3x^2 + 4xy + ix^2) dz$ along y=x^2.
   (c) Evaluate $\int\limits_c \frac{e^{2z}\, dz}{(z^2+\Pi^2)^3}$ where C is $|\, z\,| = 4$ using Cauchy's integral formula. [5+5+6]

5. (a) Find the Laurent series expansion of the function $\frac{z^2-1}{z^2+5z+6}$ about z = 0 in the region $2 < |z| < 3$
   (b) Expand $f(z) = \frac{a}{(2z+1)^3}$ about (i) z = 0 (ii) z = 2. [8+8]

6. (a) Find the poles and the corresponding residues of the function $\frac{1}{(z^2-1)^3}$
   (b) Evaluate $\int\limits_c \frac{(4-3z)}{z(z-1)(z-2)} dz$ where c is |z| = $\frac{3}{2}$ by residues theorem. [8+8]

7. (a) Evaluate $\int\limits_0^{2\pi} \frac{\sin^2\theta\, d\theta}{a^+ b\cos\theta}$ using residue theorem.

(b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$ using residue theorem. [8+8]

8. (a) show that the function w=4/z transforms the straight line x=a in the z-plane into a circle in the w-plane

   (b) Find the bilinear transformation which maps the points z=∞ , i,0 onto the points w=0,1,∞ [8+8]

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Code No: RR220202 **Set No. 2**

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MATHEMATICS-III
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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Evaluate $\int_1^1 \frac{x^2 dx}{\sqrt{1-x^5}}$ interms of β function.
   (b) Prove that $\int_0^1 (1-x^n)^{1/n} dx = \frac{1}{n}\frac{\left[\Gamma\left(\frac{1}{n}\right)\right]^2}{2\Gamma(2/n)}$
   (c) Prove that $\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{2}{n}\right)\Gamma\left(\frac{3}{n}\right)$.........$\Gamma\left(\frac{n-1}{n}\right) = \frac{(2\pi)^{\frac{n-1}{2}}}{n^{1/2}}$ [5+5+6]

2. (a) Show that the coefficient of t^n in the power series expansion of $e^{\frac{x}{2}(t-\frac{1}{t})}$is $J_n(x)$.
   (b) Prove that $\int_{-1}^1 xP_n(x)P_{n-1}(x) = \frac{2}{(4n^2-1)}$. [8+8]

3. (a) Find the analytic function f(z) = u + iv if u-v= e^x(cos y-sin y)
   (b) Find all principal values of $(1+i\sqrt{3})^{1+i\sqrt{3}}$ [8+8]

4. (a) Evaluate using Cauchy's Integral Formula $\int_c \frac{(z+1)\ dz}{z^3-4z}$ where c is $|\,z+2\,| = 3/2$
   (b) Evaluate $\int_C z^3 dz$ where c is the curve x=t,y=t^2
   (c) Evaluate $\int_C \frac{e^{3z}\ dz}{(z+i)^4}$ where c is $|\,z\,| = 3$ using Cauchy's integral formula [5+5+6]

5. (a) Find the Laurent series expansion of the function $\frac{z^2-1}{z^2+5z+6}$ about z = 0 in the region $2 < |z| < 3$
   (b) Expand $f(z) = \frac{a}{(2z+1)^3}$ about (i) z = 0 (ii) z = 2. [8+8]

6. (a) Find the poles of the function $\frac{e^{iz}}{(z^2+1)}$ and corresponding residues.
   (b) Evaluate $\int_c \frac{z}{(z-1)(z-2)^2}$ dz Where **c** is the circle $|\ z - 2\ | = \frac{1}{2}$ using residue theorm. [8+8]

7. (a) Show that $\int_0^\pi \frac{a d\theta}{a^2+\sin^2\theta} = \frac{\pi}{\sqrt{1+a^2}}$, (a > 0) using residue theorem.
   (b) Apply the calculus of residues to evaluate $\int_{-\infty}^{\infty} \frac{cosx}{(x^2+a^2)(x^2+b^2)} dx$, a > b > 0. [8+8]

8. (a) Discuss the transformation w=cos z.
   (b) Find the bilinear transformation which maps the points (l, i, -l) into the points (o,1,∞). [8+8]

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Code No: RR220202 **Set No. 3**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Evaluate $\int_{1}^{1} \frac{x^2 dx}{\sqrt{1-x^5}}$ interms of β function.
   (b) Prove that $\int_{0}^{1} (1-x^n)^{1/n} dx = \frac{1}{n}\frac{\left[\Gamma\left(\frac{1}{n}\right)\right]^2}{2\Gamma(2/n)}$
   (c) Prove that $\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{2}{n}\right)\Gamma\left(\frac{3}{n}\right)$.........$\Gamma\left(\frac{n-1}{n}\right) = \frac{(2\pi)^{\frac{n-1}{2}}}{n^{1/2}}$ [5+5+6]

2. (a) Prove that $P_n(0)$=0 for n odd and $P_n(0) = \frac{(-1)^{\frac{n}{2}} n!}{2^n\left(\frac{n}{2}!\right)^2}$ if n is even.
   (b) Prove that $J_2 - J_0 = 2\, J_0''$ [8+8]

3. (a) If f(z) is an analytic function, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.
   (b) If tan log (x+iy) = a + i b where $a^2 + b^2 \neq 1$ prove that tan log $(x^2 + y^2) = \frac{2a}{1-a^2-b^2}$ [8+8]

4. (a) Evaluate $\int_{c} \frac{Cos\, z - \sin z \;\; dz}{(z+i)^3}$ with c: $|\, z\,| = 2$ using Cauchy's integral formula
   (b) Evaluate $\int_{1-i}^{2+i} (2x+1+iy)dz$ along (1-i) to (2+i) using Cauchy's integral formula. [8+8]

5. (a) For the function f(z)= $\frac{2z^3+1}{z\,(z+1)}$ find Taylor's series valid in the neighbourhood of z=1
   (b) Find Laurent's series for f(z) = $\frac{1}{z^2\,(1-z)}$ and find the region of convergence [8+8]

6. (a) Find the poles and corresponding residue at each pole of the function $\frac{z^2}{(z-1)^2(z+2)}$.
   (b) Evaluate $\int_{C} \frac{z-dz}{(z^2+1)}$ where c is |z+1| = 1 by residue theorem. [8+8]

7. (a) Evaluate $\int_{0}^{2\pi} \frac{d\theta}{(5-3cos\theta)^2}$ using residue theorem.
   (b) Evaluate $\int_{0}^{\infty} \frac{\sin\ mx}{x} dx$ using residue theorem. [8+8]

8. (a) Find the image of the region in the z-plane between the lines y=0 and y=Π/2 under the transformation $\omega = e^z$

   (b) Find the image of the line x=4 in z-plane under the transformation $w=z^2$

[8+8]

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Code No: RR220202 **Set No. 4**

II B.Tech II Semester Supplimentary Examinations, Apr/May 2008
MATHEMATICS-III
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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Evaluate $4\int_0^\infty \frac{x^2dx}{1+x^4}$ using $\beta - \Gamma$ functions
   (b) Prove that $\beta\left(m+\frac{1}{2}, m+\frac{1}{2}\right) = \frac{\pi}{m,2^{4m-1}}$
   (c) Evaluate $\int_0^2 (8-x^3)^{1/3}dx$ using $\beta - \Gamma$ functions [5+5+6]

2. Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & if\ m \neq n \\ \frac{2}{2n+1} & if\ m = n \end{cases}$ [16]

3. (a) If f(z) is an analytic function, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.
   (b) If tan log (x+iy) = a + i b where $a^2 + b^2 \neq 1$ prove that tan log ($x^2 + y^2$) = $\frac{2a}{1-a^2-b^2}$ [8+8]

4. (a) Evaluate $\int_c \frac{ze^z\, dz}{(z+a)^3}$ where c is any simple closed curve enclosing the point z = -a using Cauchy's integral formula.
   (b) Evaluate $\int x^2 + ixy$ from A(1,1) to B(2,8) along x=t y=t^3
   (c) Evaluate $\int_C \left[\frac{e^z}{z^3} + \frac{z^4}{(z+i)^2}\right]dz$ where c: $|\,z\,| = 2$ Using Cauchy's integral theorem [5+5+6]

5. (a) Expand f(z) = $\frac{e^{2z}}{(z-1)^3}$ about z=1 as a Laurent series. Also find the region of convergence.
   (b) Find the Taylor series for $\frac{z}{z+2}$ about z=1, and find the region of convergence [8+8]

6. (a) Find the poles and residues at each pole tanhz.
   (b) Evaluate $\int_C \frac{z^3\ dz}{(3-1)^2(z-3)}$ where c is $|\,z\,| = 2$ by residue theorem. [8+8]

7. (a) State and prove Rouche's theorem
   (b) Evaluate $\int_0^{2\pi} \frac{\sin 3\theta\, d\theta}{5-3\cos\theta}$ using residue theorem. [8+8]

8. (a) Find the image of the straight lines x=0; y=0; x=1 and y=1 under the transformation $w=z^2$.

   (b) Show that the relation $w = \frac{5-4z}{4z-2}$ transforms the circle $|z| = 1$ into a circle of radius unity in the w-plane. [8+8]

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