Code No: RR320804 **Set No. 1**

III B.Tech Supplimentary Examinations, Aug/Sep 2008
MATHEMATICAL METHODS FOR CHEMICAL ENGINEERING
(Chemical Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. A tank contains $100ft^3$ of fresh water, $2ft^3$ of brine, having a concentration of 1 pcf of salt, is run into the tank per minute, and the mixture, kept uniform by mixing, runs out at the rate of $1ft^3/min$. What will be the exit brine concentration when the tank contains $150ft^3$ of brine? [16]

2. The reversible reaction $x_1 \rightleftarrows x_2$ occurs isothermally in a batch reactor. The forward and reactions are both first order with rate constant 1 s^{-1} and $2s^{-1}$ respectively. The initial concentrations of x_1, x_2 are 2 g mol / cc and 3 g mol / cc. Determine the equilibrium concentration in the reactor. [16]

3. (a) Find the differential equation of all spheres whose centers lie on the z axis.
   (b) Find the differential equation of all planes which are at a constant distance "a" from the origin. [8+8]

4. (a) Find the maximum and minimum distances form origin to the curve $5x^2 + 6xy + 5y^2 - 8 = 0$
   (b) The temperature T at any point (x,y,z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$ [8+8]

5. (a) A point of application of the force (-2, 4, 7) is displaced from the point (3,-5, 1) to the point (5,9,7) but the force is suddenly halved. When the point of application moves half the distance find the work done.
   (b) Given vectors A = i -3j + 2k and B = 2i + j - k. Find the projection of A X B parallel to 5i - k. [8+8]

6. Find the work done in moving a particle in the force field $F = 3x^2.i+(2xz?y).j+z.k$ along
   (a) The straight line from (0,0,0) to (2,1,3).
   (b) The curve define by $x^2 = 4y, 3x^3 = 8z$ from x= 0 to x=2. Use the concept of line integral. [6+10]

7. An infinitely wide flat plate is maintained at a constant temperature T_o. The plate is immersed in an infinitely wide and thick stream of constant-density fluid originally at temperature T_1. If the origin of coordinates is taken at the leading edge of the plate, a rough approximation to the true velocity distribution is $V_x = \beta y$ (β is a constant) $V_y = 0$ $V_z = 0$.
   Turbulent heat transfer is assumed negligible, and molecular transport of heat is

assumed important only in the y direction. The thermal conductivity of the fluid, k, is assumed constant. Determine the temperature distribution within the fluid and the heat-transfer coefficient between the fluid and the plate. [16]

8. Solve the following differential equations using the laplace transform method.
   (a) $\frac{d^2y}{dt^2} + 4y = 3\ Where\ y(0) = y^1(0) = 1$
   (b) $\frac{dC_A(t)}{dt} + \left(\frac{1}{\tau} + K\right) C_A(t) = \frac{1}{\tau} C_{A0}(\tau)$
   Where K and τ are the constants and $C_A(0) = 0$ and C_{A0} and C_A are the input and output concentrations respectively. [8+8]

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Code No: RR320804 **Set No. 2**

III B.Tech Supplimentary Examinations, Aug/Sep 2008
MATHEMATICAL METHODS FOR CHEMICAL ENGINEERING
(Chemical Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. A tank is initially filled with 100 gallons of salt solution containing 1 lb of slat per gallon. Fresh brine containing 2 lb. of slat per gallon runs into the tank at the rate of 5 gallons per minute and the mixture assumed to be kept uniform by stirring, runs out at the same rate. Find the amount of salt in the tank at any time, and determine how long it will take for this amount to reach 150 lb. [16]

2. A container is maintained at a constant temperature of 800^0F and is fed with a pure gas A at a steady react of 1 lb mole/min, the product gas steam is with drawn from the container at the rate a necessary to keep the total pressure constant at a value of 3 atm. The container contents are vigorously agitated, and the gas mixture is always well mixed. The following irreversible second order gas-phase reaction occurs in the container $2A \rightarrow B$
At a temperature of 800^0F, the reaction-rate constant for the reaction has the numerical value of $1,000 ft^3/lbmole/min$. Both A and B are perfect gases. Because of their low temperature, no reaction occurs in the lines leading to and from the vessel. If, under steady-state conditions, the product stream is to contain $33\frac{1}{3}$ mole % B, how large in cubic feet should be the volume of the reaction container? [16]

3. (a) If x increases at the rate of 2cm/Sec at the instant when x = 3cm and Y = 1 cm at what rate must y be changing in order that the function $2xy - 3x^2y$ shall neither be increasing nor decreasing.
   (b) Find the total differential coefficient of x^2y with respect to x when x and y are connected by the relation $x^2 + xy + y^2 = 1$. [8+8]

4. The temperature T at any point (x,y,z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 + 1$. [16]

5. Given A = i - 2j - 3k B = 2i + j - 3k C = i + j + k Verify
   (a) A . (B X C) = B . (C X A).
   (b) (A X B) X C = (A . C) B - (B . C) A. [8+8]

6. Find the work done in moving a particle in the force field $F = 3x^2.i+(2xz?y).j+z.k$ along
   (a) The straight line from (0,0,0) to (2,1,3).
   (b) The curve define by $x^2 = 4y, 3x^3 = 8z$ from x= 0 to x=2. Use the concept of line integral. [6+10]

7. A slab of porous material is to be dried from both faces at a constant rate of $Wlb/(hr)(ft^2)$. The initial moisture content of the slab is uniform at $C_0 pcf$. The movement of moisture through the slab is in the x direction only, and the rate of movement is proportional to the concentration gradient of the moisture in the slab, dc/dx. If the slab is of thickness l, determine the concentration of moisture within the slab as a function of time and distance. Show that, as the drying time increases, the concentration distribution becomes parabolic. [16]

8. Solve the following differential equation with the laplace transform method.
   (a) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = t$, $Where\, x(0) = x^1(0) = 0$
   (b) Find the lap lace inverse transform of $C_A(s) = \frac{k_p/\tau_p^3}{s(s+1/\tau_p)^3}$ $where\, k_p\, and\, \tau_p\, are\, constants$

[8+8]

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Code No: RR320804 **Set No. 3**

III B.Tech Supplimentary Examinations, Aug/Sep 2008
MATHEMATICAL METHODS FOR CHEMICAL ENGINEERING
(Chemical Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. A tank contains $100 ft^3$ of fresh water, $2 ft^3$ of brine, having a concentration of 1 pcf of salt, is run into the tank per minute, and the mixture, kept uniform by mixing, runs out at the rate of $1 ft^3/min$. What will be the exit brine concentration when the tank contains $150 ft^3$ of brine? [16]

2. Under certain conditions cane sugar in water is converted into dextrose at a rate which is proportional to the amount unconverted at any time. Of the 75 gm. at time t - 0, 8 gm. are converted during the first 30 minutes, find the amount converted in $1\frac{1}{2}$ hours. [16]

3. If the three thermodynamic variables P, V, and T are connected by a relation (P,V,T) = 0 show that :
$\left(\partial p/\partial T\right)_V \left(\partial T/\partial V\right)_P \left(\partial V/\partial P\right)_T = -1$ [16]

4. (a) Find the minimum value of $x^2 + y^2 + z^2$ given that
   i. $xyz = a^3$
   ii. $Ax + bx = cz = P$
   (b) Given $x + y + z = a$. Find the maximum value of $x^m y^n z^p$. [8+8]

5. If A = i - 2j + k, B = 2i - 4j + 6k, Verify that $\nabla(A.B) = A.\nabla B + B.\nabla A + AX(\nabla XB) + BX(\nabla XA)$. [16]

6. (a) Explain the concept of line integral. Define circulation of a vector.
   (b) Evaluate the line integral $\int_c F\,dr$ where $F = 3xyi + y^2 j$ and the space curve c is the Curve in the xy- plane $y = 2x^2$ from (0, 0) to (1, 2). [6+10]

7. A sphere, initially at a uniform temperature T_0, is suddenly placed in a fluid medium whose temperature is maintained constant at a value T_1. The heat-transfer coefficient between the medium and the sphere is constant at a value h. The sphere is isotropic, and the temperature variation of the physical properties of the material forming the sphere may be neglected. Derive the equation relating the temperature of the sphere may be neglected. Derive the equation relating the temperature of the sphere to the radius r and time t. [16]

8. Show that laplace transform of
   (a) $\cos \omega t = \frac{s}{s^2+\omega^2}$
   (b) $e^{-at} \sin \omega t = \frac{\omega}{(s+a)^2+\omega^2}$

(c) $t^n = \frac{n!}{s^{n+1}}$. [6+6+4]

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Code No: RR320804 **Set No. 4**

III B.Tech Supplimentary Examinations, Aug/Sep 2008
MATHEMATICAL METHODS FOR CHEMICAL ENGINEERING
(Chemical Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. Consider a gas containing an entrained mist of nonvolatile tar located inside the cylinder of a reciprocating compressor. Determine the work required to compress the gas adiabatically and reversibly from its present pressure of 0.33 atm to a pressure of 1.0atm. Given molecular wt of gas is 24 and specific heat at constant volume, is constant at $6.2Btu/(lbmole)/^0F$) . The tar is always present as a mist in the ratio 0.2 lb of tar pound of transfer gas. The specific heat of tar is $0.5Btu/(Lb)(^0F)$. The initial cylinder volume is $0.4ft^3$. The initial cylinder pressure is 0.33 atm. The final cylinder pressure is to be 1.0 atm. [16]

2. A countercurrent packed absorption tower is to be used for carrying out the liquid-phase reaction
   $A + B \rightarrow C$
   This reaction is irreversible, and the reaction rate may be expressed as follows:
   $-\frac{dN_A}{dt} = kN_AX_B$
   Components Band C are nonvolatile and never appear in the gas phase. Substance B is introduced into the tower dissolved in a nonvolatile solvent. Compound A is volatile and is introduced into the tower as a vapor carried by an insoluble, inert gas.The rate of transfer of A from the gas phase to the liquid phase is controlled by the gas-phase resistance and is proportional to $K_ga(Y_A - Y_A^*)$.Y_A^* is the equilibrium gas-phase concentration corresponding to the liquid-phase concentration, X_A. Y_A^* is related to X_A by the equation
   $Y_A^* = mX_A$
   Assuming isothermal conditions, develop the differential equation for the decrease in the concentration of A in the gas phase as a function of the tower height z and the following known quantities:
   a = area for mass transfer per unit of tower volume, ft^2/ft^3
   G = inert-gas rate, lb moles/hr
   H = moles of inert solvent held up by packing per unit of tower volume
   k = reaction-rate constant, lb moles/- (lb mole/hr)
   KG = mass-transfer coefficient, lb moles/hr (unit Y_A -Y_B) (ft^2)
   L = inert-solvent rate, lb moles/hr
   m = proportionality constant
   S = tower cross section, ft^2.
   Nomenclature for Variables
   t -time, hr
   z = distance from bottom of tower, ft
   N = moles of a component; N_A refers to moles of substance A, etc.
   Y -gas-phase concentration, moles per mole of inert gas;

Y_A refers to sub-stance A
X = liquid-phase concentration, moles per mole of inert solvent; X_A refers to sub-stance A, etc. [16]

3. (a) If the kinetic energy $K = \omega v^2/2g$. find approximately the change in the kinetic energy as w changes from 49 to 49.5 and v changes form 1600 to 1590.
   (b) Find the possible percentage error in computing the resistance r from the formula $1/r = 1/r_1 + 1/r_2$ if r_1 and r_2 are both in error by 2%. [6+10]

4. P is a function of both x and y as given by $P = Ax + B\frac{xy-a}{y} + \frac{c}{(xy)^{-1}/2}$
   where A, B, and C are constants. Obtain expressions for the values of x and y corresponding to maximum or minimum values of P. [16]

5. (a) Define a vector. Distinguish between vectors and scalars by giving examples.
   (b) Explain the laws of vector addition and subtraction. [8+8]

6. (a) Explain the concept of line integral. Define circulation of a vector.
   (b) Evaluate the line integral $\int_c F\,dr$ where $F = 3xyi + y^2j$ and the space curve c is the Curve in the xy- plane $y = 2x^2$ from (0, 0) to (1, 2). [6+10]

7. Imagine a slab which extends indefinitely in the x direction and so arranged that heat flows in the x direction only. At time t = 0, suppose that the temperature distribution inside the slab is given by T = f(x). Now expose the slab face at x = 0 to a constant-temperature medium at T_1. The heat-transfer coefficient between the medium and the slab surface is constant at a value h. The temperature variation of the physical properties of the material forming the slab may be neglected. Determine the temperature distribution within the slab. [16]

8. Invert the following Laplace transforms
   (a) $\frac{s-1}{s(s+2)}$
   (b) $\frac{1}{(s^2+4)(s^2+1)}$
   (c) $\frac{sA+B}{(s+i\omega)(s-i\omega)}$. [4+6+6]

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