

**MATHEMATICS - I**

(Common to all branches)

Time: 3 hours

Max Marks: 70

Answer any FIVE questions  
All questions carry equal marks

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1 (a) Solve:  $(x^2 - 1)\frac{dy}{dx} + 2xy = 1$ .

(b) Solve :  $x\frac{dy}{dx} + y = \log x$ .

2 (a) Solve by method of variation of parameters:  $(D^2 + a^2)y = \sec ax$ .

(b) Solve by method of variation of parameters:  $y'' + 4y = \tan 2x$ .

3 (a) Verify Rolle's theorem for  $f(x) = x(x + 3)e^{-x/2}$  in  $[-3, 0]$ .

(b) Verify Rolle's theorem for  $f(x) = e^x \sin x$  in  $[0, \pi]$ .

4 (a) Prove that the volume of revolution of  $r^2 = a^2 \cos 2\theta$  about the initial line is

$$\frac{\pi a^3}{6\sqrt{2}} [3 \log(\sqrt{2} + 1) - \sqrt{2}].$$

(b) Determine the volume of the solid generated by revolving the lemniscate  $r = a + b \cos \theta$  ( $a > b$ ) about the initial line.

5 (a) Evaluate  $\int_0^\infty \int_0^{\pi/2} e^{-r^2} r \, d\theta \, dr$ .

(b) Change the order of integration in  $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x \, dy \, dx}{\sqrt{x^2+y^2}}$  and evaluate.

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- 6 (a) Find the Laplace transform of  $f(t) = \begin{cases} t, & 0 < t < 3 \\ 3, & t > 3 \end{cases}$
- (b) Find  $L^{-1} \left\{ \frac{se^{-s/2} + \pi e^{-s}}{s^2 + \pi^2} \right\}$ .
- 7 (a) Using Laplace transform, evaluate  $\int_0^{\infty} \frac{(\cos 6t - \cos 4t)}{t} dt$ .
- (b) Solve the D.E.  $\frac{d^2x}{dt^2} + 9x = \cos 2t$  Using L.T. given that  $x(0) = 1$ ,  $x\left(\frac{\pi}{2}\right) = -1$ .
- 8 (a) Find the values of constants  $\lambda$  and  $\mu$  so that the surfaces  $\lambda x^2 - \mu yz = (\lambda + 2)x$  and  $4x^2y + z^3 = 4$  may intersect orthogonally at the point  $(1, -1, 2)$ .
- (b) Evaluate  $\iint_S \vec{F} \cdot \vec{n} dS$ , where  $\vec{F} = (18z)\mathbf{i} - 12\mathbf{j} + 3y\mathbf{k}$  and  $S$  is the surface of the plane  $2x + 3y + 6z = 12$  in the first octant.

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