II

Total No. of Questions - 24

Total No. of Printed Pages - 3

Regd. No. | | | | | | | | | | |

Part - III

MATHEMATICS, Paper - I (A)

(Algebra, Vector Algebra and Trigonometry)

(English Version)

Time : 3 Hours ***Max. Marks : 75***

Note : This question paper consists of **three** sections *A*, *B* and *C*.

SECTION A

I. **Very short answer** type questions. **10 × 2 = 20**

i) Answer **all** questions.

ii) Each question carries **two** marks.

1. If $f = \{(1,2), (2,-3), (3,-1)\}$, then find :

i) $2f$ ii) $2+f$

2. Find the domain of the real valued function $f(x) = \sqrt{x^2 - 25}$.

3. Let $\bar{a} = 2\bar{i} + 4\bar{j} - 5\bar{k}$, $\bar{b} = \bar{i} + \bar{j} + \bar{k}$ and $\bar{c} = \bar{j} + 2\bar{k}$. Find the unit vector in the opposite direction of $\bar{a} + \bar{b} + \bar{c}$.

4. Find the vector equation of the plane which passes through the points $2\bar{i} + 4\bar{j} + 2\bar{k}$, $2\bar{i} + 3\bar{j} + 5\bar{k}$ and parallel to the vector $3\bar{i} - 2\bar{j} + \bar{k}$.

5. If $\bar{a} = (4, 3, 5)$ is the center of a sphere which passes through the point $(-1, -1, 2)$, then find the cartesian equation of the sphere.

6. Find the period of the function defined by

$f(x) = Sin(x + 2x + ... + nx)$ for all $x \in R$ and $n \in Z^+$.

7. If $3Sin\theta + 4Cos\theta = 5$, then find the value of $4Sin\theta - 3Cos\theta$.

8. If $Sinh\,x = \frac{3}{4}$, find $Cosh(2x)$ and $Sinh(2x)$.

9. Show that in a ΔABC, $b \,.\, Cos^2\frac{C}{2} + c \,.\, Cos^2\frac{B}{2} = s$.

10. If the amplitude of $(z-1)$ is $\frac{\pi}{2}$, then find the locus of z where $z = x + iy$.

SECTION B

II. **Short answer** type questions. **5 × 4 = 20**

i) Attempt **any five** questions.

ii) Each question carries **four** marks.

11. If $\bar{a}, \bar{b}, \bar{c}$ are linearly independent vectors, then show that $\bar{a} - 3\bar{b} + 2\bar{c}$, $2\bar{a} - 4\bar{b} - \bar{c}$ and $3\bar{a} + 2\bar{b} - \bar{c}$ are linearly independent.

12. If $0 \le \alpha, \beta \le \pi$, then show that $Sin(\alpha - \beta) = Sin\alpha\,Cos\,\beta - Cos\alpha\,Sin\,\beta$ by the vector method.

13. If A is not an integral multiple of π, prove that

$$CosA \cdot Cos2A \cdot Cos4A \cdot Cos8A = \frac{Sin16A}{16SinA}.$$

14. Solve the equation $Cot^2x - (\sqrt{3}+1)Cot\,x + \sqrt{3} = 0$ where $\left(0 < x < \frac{\pi}{2}\right)$.

15. Find the value of $Tan\left(Cos^{-1}\frac{4}{5} + Tan^{-1}\frac{2}{3}\right)$.

16. If $Sin\theta = \frac{a}{b+c}$, then show that $Cos\theta = \frac{2\sqrt{bc}}{b+c}Cos\frac{A}{2}$.

17. Show that $16Sin^5\theta = Sin5\theta - 5Sin3\theta + 10Sin\theta$.

SECTION C

III. Long answer type questions. **5 × 7 = 35**

i) Attempt **any five** questions.

ii) Each question carries **seven** marks.

18. Let $f : A \to B$ be a bijection, then show that $fof^{-1} = I_B$ and $f^{-1}of = I_A$.

19. Using mathematical induction, prove that

$$1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots \text{ upto } n \text{ terms} = \frac{n(n+1)^2(n+2)}{12} \quad \forall\, n \in N.$$

20. If $A = (1, -2, -1)$, $B = (4, 0, -3)$, $C = (1, 2, -1)$, $D = (2, -4, -5)$, then find the distance between the lines AB and CD.

21. If $A + B + C = 180^0$, then prove that

$$Cos^2 \frac{A}{2} + Cos^2 \frac{B}{2} + Cos^2 \frac{C}{2} = 2\left(1 + Sin \frac{A}{2}\, Sin \frac{B}{2}\, Sin \frac{C}{2}\right).$$

22. In a ΔABC, show that $r + r_1 + r_2 - r_3 = 4R\, Cos\, C$.

23. On a tower AB of height h, there is a flag staff BC. At a point 'd' meters away from the foot of the tower, AB and BC are making equal angles.

Show that the height of the flag staff is $h\left(\frac{d^2 + h^2}{d^2 - h^2}\right)$ meters.

24. If n is an integer and $z = Cis\theta, \left(\theta \neq (2n+1)\frac{\pi}{2}\right)$, then show that

$$\frac{z^{2n} - 1}{z^{2n} + 1} = i\, Tan\, n\theta.$$