Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA- Ist SEMESTER–EXAMINATION – MAY/JUNE - 2012

Subject code: 2610003 Date: 31/05/2012

Subject Name: Discrete Mathematics for Computer Science (DMCS)

Time: 02:30 pm – 05:00 pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define: **07**

i) Join irreducible elements.

ii) Atoms of a Boolean algebra.

Determine Join-irreducible elements and atoms of following Boolean algebra.

i) (S_{210}, D)

ii) $\langle P(S), \cap, \cup, ', \Phi, S\rangle$ where S = {a, b, c}

Also draw the Hasse Diagram.

(b) Define Lower bound and greatest lower bound. Let P = < 3, 5, 9, 15, 24, 45}, D> be a poset. Draw the Hasse diagram. Find **07**

i) the maximal element. & minimal element.

ii) The greatest and least element.

iii) the lower bounds of {3, 5}, if any & the upper bound of { 9, 15 } , if any

iv) GLB of {15, 45} & LUB of {3, 9, 15}.

Q.2 **(a)** State the importance & purpose of Discrete Mathematical Structures with its application to computers science. **07**

(b) i) Let P(x) be the statement " $x = x^2$ ".If the domain consists of the integers, **02**

what are the truth values of $\forall x$ P(x)and $\exists$ x P(x) **05**

ii) Define: Logical Equivalence of the statement formula. Without constructing truth table show that ($\neg p \wedge (\neg q \wedge r)) \vee (q \wedge r) \vee(p \wedge r) \equiv r$

OR

(b) i) Define: Disjoint sets. If $A_1 = \{\{1, 2\}.\{3\}\}$, $A_2 = \{\{1\},\{2,3\}\}$ and $A_3 = \{\{1, 2, 3\}\}$, then show that A1, A2, and A3 are mutually disjoint. **03**

ii) Define law of Modus Ponen and Law of Hypothetical Syllogism with an example. **04**

Q.3 **(a)** i) Define: Equivalence relation. If I be the set of integers and if R be defined **04**

by " a R b iff a – b is an even integer" where a, b ϵ I, then show that the relation R is an equivalence relation.

ii) Define giving example for each term **03**

1. Sublattice
2. Complemented lattice
3. Modular lattice

(b) i) Define: Maximal Compatibility Block. Let the compatibility relation on a set {1, 2, 3, 4, 5, 6} be given by following matrix. Construct the graph and find the maximum compatibility blocks **04**

2	1				
3	1	1			
4	1	1	1		
5	0	1	0	0	
6	0	0	1	0	1
	1	2	3	4	5

ii) State the absorption law for lattice. Verify it for (S_{45}, D) by taking any two elements. **03**

OR

Q.3 **(a)** i) Find the value of Boolean Expression. **04**

$\alpha(x_1, x_2, x_3, x_4) = [x_1 * (x_2 \oplus x_1') * (x_3 * x_4' * x_2')] \oplus (x_1 * x_4)$ where

$x_1 = 5, x_2 = 6, x_3 = 15, x_4 = 3$ in Boolean algebra $<S_{30}$, gcd, lcm, ', 30> and $n' = 30/n$.

03

.ii) Prove the Boolean identities

a) $(a * b) \oplus (a * b') = a$

b) $a * (a' \oplus b) = a * b$

(b) i) Use the Quine-Mccluskey algorithm to find the prime implicants and also obtain a minimal expression for function: $f(a,b,c,d)=\Sigma(1,2,5,6, 13, 14, 15)$ **04**

ii) Obtain the sum of product canonical form of Boolean expression in three variables x_1, x_2, x_3 for $(x_1 \oplus x_2) * x_3$ **03**

Q.4 **(a)** **07**

Define: Group and Abelian group. Show that in a group (G, $*$), if for a, b ϵ G, $(a * b)^2 = a^2 * b^2$, then (G, $*$) is an Abelian group. Prove that the set {1, -1, i, -i} form an Abelian multiplicative group (G, x) where i is an imaginary no. $i = \sqrt{-1}$.

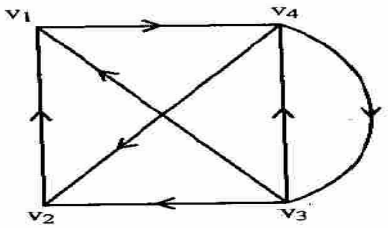
(b) Define: Group Homomorphism, Group Isomorphism and Kernel of the homomorphism. Prove that G: $(Z_4, +4) \longrightarrow (Z_5^*, x_5)$ is isomorphism. **07**

OR

Q.4 **(a)** Define Subgroup of a group Find all subgroups of cyclic group of order 12 with generator 'a'. Also find order of generators of G. **07**

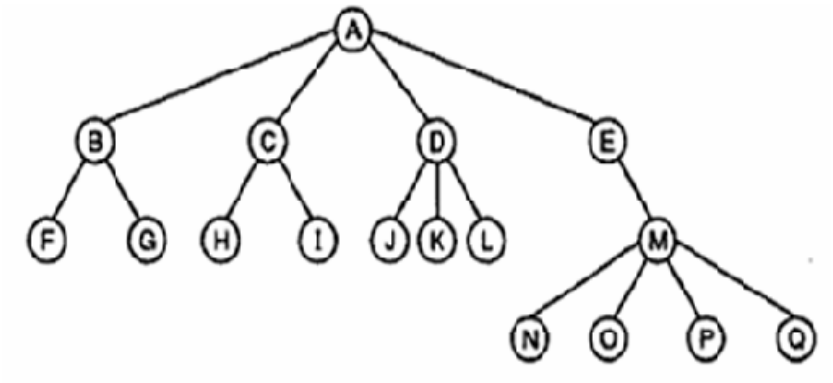
(b) Define symmetric group (S_3, $\Diamond$). Write composition table of all permutations defined on the symbols 1, 2, & 3 Determine all the proper subgroups of (S_3, $\Diamond$). Which subgroup is normal subgroup? **07**

Q.5 **(a)** Define adjacency matrix of a graph and obtain the adjacency matrix (A) for the following graph. What do transpose of adjacency matrix (A^T) indicate? Draw its graph. State the indegree and outdegree of all the vertices. Find A^2 and interpret in detail by stating the results. **07**



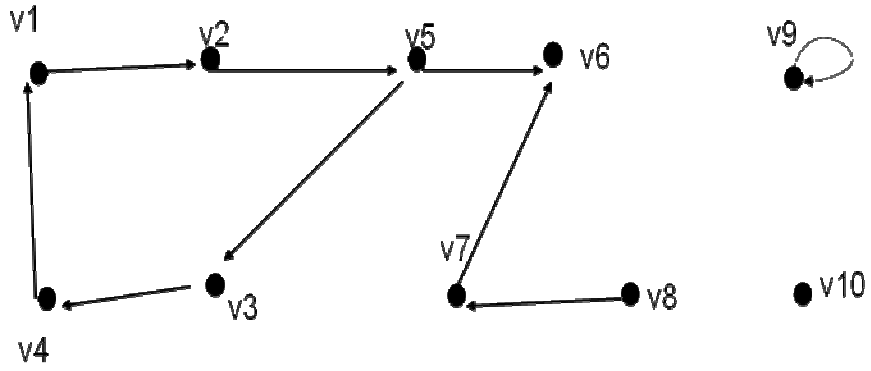
(b) i) Define Forest with an example **02**

ii) Define Binary tree. Convert the given tree into the Binary tree. **05**



OR

Q.5 **(a)** Define node base of a diagraph. State its properties. Find all node base of the diagraph given below: **07**



(b) Define rooted tree, level of a vertex, leaf, descendents and ancestor of a vertex with a suitable example. Prove that a full m-ary tree with i internal vertex has n = mi +1 vertices **07**

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