

MATHS

Q. 1. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80 . Then which one of the following gives possible values a and b?

- i. a = 1, b = 6
- ii. a = 3, b = 4
- iii. a = 0, b = 7
- iv. a = 5, b = 2

Sol.

$$\text{Mean} = \frac{\sum x}{n} = 6$$

$$\text{Variance} = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n} \right)^2 = 6.8$$

$$\Rightarrow \frac{a^2 + b^2 + 64 + 25 + 100}{5} = 36 + 6.8$$

$$\Rightarrow a^2 + b^2 + 189 - 180 = 34$$

$$\Rightarrow a^2 + b^2 = 25$$

Possible values of a and b is given by (2)

Q. 2. The vector $\vec{a} = \alpha \hat{i} + 2\hat{j} + \beta \hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β ?

- i. $\alpha = 2, \beta = 1$
- ii. $\alpha = 1, \beta = 1$
- iii. $\alpha = 2, \beta = 2$
- iv. $\alpha = 1, \beta = 2$

Sol.

As $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar

$$\therefore [\vec{a} \vec{b} \vec{c}] = 0$$

$$\text{Or, } \alpha + \beta = 2 \quad (\text{i})$$

Also $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$

$$\therefore \vec{a} = \lambda (\vec{b} + \vec{c})$$

$$\text{or, } \vec{a} = \lambda \left(\frac{\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{2}} \right) \quad (\text{ii})$$

$$\text{But } \vec{a} = \alpha \vec{i} + 2\vec{j} + \beta \vec{k}$$

$$\text{Hence } \lambda = \sqrt{2} \text{ and } \alpha = 1, \beta = 1$$

Which also satisfy (i)

$\therefore$ Correct answer is (2)

Q. 3.

The non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$

Then the angle between $\vec{a}$ and $\vec{c}$ is

- i. $\frac{\pi}{2}$
- ii. π
- iii. 0
- iv. $\frac{\pi}{4}$

Sol. The sign of $\vec{a}$ and $\vec{c}$ are opposite. Hence they are parallel but directions are opposite.
Therefore angle between $\vec{a}$ and $\vec{c}$ is π

$\therefore$ correct answer is (2)

Q. 4. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the

point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then

- i. $a = 6, b = 4$
- ii. $a = 8, b = 2$
- iii. $a = 2, b = 8$
- iv. $a = 4, b = 6$

Sol. Equation of line through (5, 1, a) and (3, b, 1) is

$$\frac{x-5}{-2} = \frac{y-1}{b-1} = \frac{z-a}{1-a} = \lambda$$

any point on (i) is

$$\{5-2\lambda, 1+(b-1)\lambda, a+(1-a)\lambda\} \quad (ii)$$

$$\text{As } \left(0, \frac{17}{2}, -\frac{13}{2}\right) \text{ lies on (i)}$$

$$5-2\lambda=0 \Rightarrow \lambda = \frac{5}{2} \quad (iii)$$

$$1+(b-1) \times \frac{5}{2} = \frac{17}{2}$$

$$\text{or, } 2+5b-5=17$$

$$\text{or, } b=4$$

$$\text{and } a+(1-a) \times \frac{5}{2} = -\frac{13}{2}$$

$$\text{or, } 2a+5-5a=-13$$

$$\text{or, } a=6$$

$\therefore$ Correct answer is (1)

Q. 5. If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ intersect at a point, then the integer k is equal to

- i. 2
- ii. 2
- iii. 5
- iv. 5

Sol. As the given lines intersect

$$\therefore \begin{vmatrix} 2-1 & 3-2 & 1-3 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0$$

$$\text{Or, } \begin{vmatrix} 1 & 1 & 2 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix} = 0$$

$$\text{or, } k = -5, \frac{5}{2}$$

Integer is -5 only

$\therefore$ Correct answer is (3)

Q. 6. The differential of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is

- i. $(y-2)^2 y'^2 = 25 - (y-2)^2$
- ii. $(x-2)^2 y'^2 = 25 - (y-2)^2$
- iii. $(x-2) y'^2 = 25 - (y-2)^2$
- iv. $(y-2) y'^2 = 25 - (y-2)^2$

Sol. The required equation of circle is

$$(x-a)^2 + (y-2)^2 = 25 \quad (i)$$

differentiating we get

$$2(x-a) + 2(y-2)y' = 0$$

$$\text{or, } a = x + (y-2)y' \quad (ii)$$

putting a in (i)

$$(x - x - (y-2)y')^2 + (y-2)^2 = 25$$

$$\text{or, } (y-2)^2 y'^2 = 25 - (y-2)^2$$

∴ The correct answer is (1)

Q. 7. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz, y = az + cx$ and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to

- i. 0
- ii. 1
- iii. 2
- iv. -1

Sol.

$$x = cy + bz \Rightarrow x - cy - bz = 0 \quad (i)$$

$$y = az + bx \Rightarrow bx - y + az = 0 \quad (ii)$$

$$z = bx + ay \Rightarrow bx + ay - z = 0 \quad (iii)$$

Eliminating x, y, z from (i), (ii) and (iii) we get

$$\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$$\text{or, } a^2 + b^2 + c^2 + 2abc = 1.$$

∴ The correct answer is (2)

Q. 8. Let A be a square matrix all of whose entries are integers. Then which one of the following is true?

- i. If $\det A = \pm 1$, then A^{-1} exists and all its entries are integers
- ii. If $\det A = \pm 1$, then A^{-1} need not exist
- iii. If $\det A = \pm 1$, then A^{-1} exist but all its entries are not necessarily integers
- iv. If $\det A = \pm 1$, then A^{-1} exist and all its entries are non-integer

Sol. The obvious answer is (1).

Q. 9. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is

- i. 3
- ii. 2
- iii. 1
- iv. 4

Sol.

Let the roots of $x^2 - 6x + a = 0$

be α and 4β and that of $x^2 - cx + 6 = 0$ be α and 3β

$$\therefore \alpha + 4\beta = 6 \quad (i)$$

$$4\alpha\beta = a \quad (ii)$$

$$\alpha + 3\beta = c \quad (iii)$$

$$3\alpha\beta = 6 \quad (iv)$$

Using (ii) & (iv)

$$\frac{4}{3} = \frac{a}{6} \Rightarrow a = 8$$

Then $x^2 - 6x + a = 0$

reduces to

$$x^2 - 6x + 8 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 32}}{2}$$

$$= \frac{6 \pm 2}{2} = 4, 2$$

$$\therefore \alpha = 2, \beta = 1$$

$\therefore$ Correct answer is (2)

Q. 10. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?

- i. $6.8.{}^7C_4$
- ii. $7.{}^6C_4.{}^8C_4$
- iii. $8.{}^6C_4.{}^7C_4$
- iv. $6.7.{}^8C_4$

Sol. $M = 1, I = 4, P = 2$

These letters can be arranged by

$$\frac{(1+4+2)!}{1!4!2!} = 7.{}^6C_4 \text{ ways}$$

The remaining 8 gaps can be filled by 4 S by 8C_4 ways

: *Total no. of ways* = $7.{}^6C_4.{}^8C_4$

: *Correct answer is (2)*

Q. 11.

Let $I = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$. Then which one of the following is true?

- i. $I < \frac{2}{3}$ and $J > 2$
- ii. $I < \frac{2}{3}$ and $J < 2$
- iii. $I > \frac{2}{3}$ and $J > 2$
- iv. $I < \frac{2}{3}$ and $J > 2$

Sol.

We Know $\frac{\sin x}{x} < 1$, when $x \in (0, 1)$

$$\therefore \frac{\sin x}{\sqrt{x}} < \sqrt{x}$$

$$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}} dx < \int_0^1 \sqrt{x} dx$$

$$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}} dx < \frac{2}{3}$$

Also, $\cos x < 1$, when $x \in (0, 1)$

$$\therefore \frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$$

$$\Rightarrow \int_0^1 \frac{\cos x}{\sqrt{x}} dx < \int_0^1 \frac{1}{\sqrt{x}} dx$$

$$\int_0^1 \frac{\cos x}{\sqrt{x}} dx < 2$$

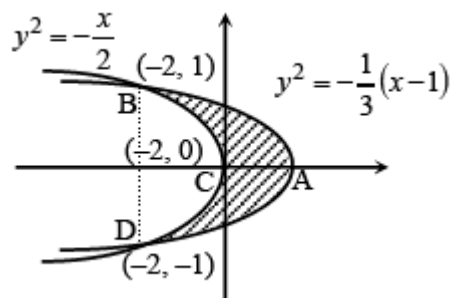
$$\therefore I < \frac{2}{3} \text{ and } J < 2$$

$\therefore$ Correct answer is (4)

Q. 12. The area of the plane region bounded by the curve $x + 2y^2 = 0$ and $3y^2 = 1$ is equal to

- i. $\frac{2}{3}$
- ii. $\frac{4}{3}$
- iii. $\frac{5}{3}$
- iv. $\frac{1}{3}$

Sol.



$$x + 2y^2 = 0 \Rightarrow y^2 = -\frac{x}{2}$$

$$x + 3y^2 = 1 \Rightarrow y^2 = -\frac{1}{3}(x-1)$$

$$\therefore -\frac{x}{2} = -\frac{1}{3}(x-1)$$

$$\text{or, } -\frac{x}{2} = -\frac{x}{3} + \frac{1}{3}$$

$$\text{or, } \frac{x}{3} - \frac{x}{2} = \frac{1}{3}$$

$$\text{or, } -\frac{x}{6} = \frac{1}{3}$$

$$\text{or, } x = -2$$

$$\therefore y^2 = 1 \Rightarrow y = \pm 1$$

Area of the region BCA

$$= \left| \int_0^1 \{(-2y^2) - (1-3y^2)\} dy \right|$$

$$= \left| \int_0^1 (y^2 - 1) dy \right|$$

$$= \left| \left[\frac{y^3}{3} - y \right]_0^1 \right|$$

$$= \left| \frac{1}{3} - 1 \right| = \frac{2}{3}$$

Hence area of the region bounded by the curve is equal to $2 \times \frac{2}{3} = \frac{4}{3}$

$\therefore$ Correct answer is (2)