

Code No: R10107 / R10

Set No. 1

I B.Tech I Semester Regular Examinations, February 2011

MATHEMATICAL METHODS

(Common to CSE, EEE, CE, EIE, AE, BT & AME)

Time: 3 hours

Max Marks: 75

Answer any FIVE Questions
All Questions carry equal marks

1 (a). Find the rank of the matrix $A = \begin{bmatrix} 1 & 0 & -2 & 3 & 0 \\ 2 & 1 & 0 & 2 & 4 \\ -1 & -2 & 1 & 0 & 1 \\ 2 & 3 & 4 & 9 & \end{bmatrix} ?$

(b). Solve the following system of equations $2p + q + 2r + s = 6$; $6p - 6q + 6r + 12s = 36$
; $4p + 3q + 3r - 3s = 1$; $2p + 2q - r + s = 10$?

[6M + 9M]

2 (a). Find the characteristic roots and characteristic vectors of the

matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} ?$

(b). Using Cayley–Hamilton theorem find A^{-1} if $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} ?$

[7M + 8M]

3. Diagonalise the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and find A^4 using modal matrix P ?

[15M]

4 (a). Using bisection method find an appropriate root of the equation $x - \cos x = 0$ that lies between 0.5 and 1. Carry out 5 steps of approximation?

(b). Find a real root of the equation $\log x - \cos x = 0$ near $x=1$ correct up to 3 decimal places using Newton-Raphson method?

[7M + 8M]

5 (a). Using Newton Forward Difference Formula estimate $y(0.12)$ from the following data

x	0.10	0.15	0.20	0.25	0.30
y	0.656	0.522	0.410	0.316	0.240

- (b). Find the missing term in the following table

x	1	2	3	4	5
y	2	5	7	--	32

[10M + 5M]

- 6 (a). A rod is rotating in a plane. The following table gives the angle
- Θ
- (in radians) through which the rod has turned for various values of time (seconds)

t	0.0	0.2	0.4	0.6	0.8	1.0
Θ	0.00	0.12	0.49	1.12	2.02	3.20

Calculate the angular velocity and the angular acceleration of the rod when $t = 0.3$ seconds?

- (b). Evaluate
- $\int_{0.6}^{2.0} y dx$
- using Trapezoidal rule?

[9M + 6M]

- 7 (a). Solve by Taylor's series expansion the initial value problem
- $\frac{dy}{dx} = y^2 + 1$
- with
- $y(0) = 0$
- to find the values of
- y
- at
- $x = 0(0.2)0.6$
- ?

- (b). Use Milne's method to find
- $y(0.8)$
- for the differential equation
- $\frac{dy}{dx} = (1 + x^2)y$
- and
- $y(0)=1$
- by obtaining the initial values by Taylor's series method?

[7M + 8M]

- 8 (a). Fit a straight line by the method of least squares to the following data.

Age	21	42	38	64	53	61	47
Absence (no.of days)	4	14	10	38	19	34	17

- (b). Fit a curve of the form
- $y = ab^x$
- to the following data

x	2	3	4	5	6	7
y	640	512	410	328	262	210

[7M + 8M]

Code No: R10107 / R10

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Time: 3 hours

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1 (a) Reduce the matrix to echelon form and hence find its rank

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 1 \\ -1 & -1 & 1 & 2 \\ -1 & -1 & 1 & -1 \end{bmatrix} ?$$

(b) Find two non-singular matrices P and Q such that PAQ is in normal form and

hence find the rank of A where A is $\begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$

[7M + 8M]

2 (a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 7 & 0 \\ 2 & 6 & 1 \end{bmatrix}$

(b) For the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ find A^{-1} by using Cayley–Hamilton theorem?

[7M + 8M]

3. Reduce the Q.F $2x^2+2y^2+2z^2-2xy-2yz-2xz$ into canonical form and find its nature, rank, index and signature

[15M]

4 (a) Find the root of the equation $f(x)=x^2-3x+1$ correct up to 3 decimal places starting with $x=1$ by successive approximation

(b) Using Newton-Raphson method, find a real root of the equation $x \sin x + \cos x = 0$ near $x=\pi$ up to 4 decimal places

[7M + 8M]

5 (a) Prove the following

(i) $\Delta \nabla = \Delta - \nabla$ (ii) $\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta} = \Delta + \nabla$ (iii) $\nabla E = E \Delta = \Delta$?

- (b) Using Lagrange's interpolation Formula fit a polynomial to the following data

x	0	1	3	4
f(x)	-12	0	6	12

[6M + 9M]

- 6 (a) A curve is expressed by the following values of x and y, find the slope at the point x = 0.5?

x	0.4	0.5	0.6	0.7	0.8
y	1.58	1.80	2.04	2.33	2.65

- (b) Evaluate
- $\int_0^2 e^{-x^2} dx$
- using Simpson's 1/3 rule by taking h = 0.25?

[6M + 9M]

- 7 (a) Solve the differential equation
- $\frac{dy}{dx} = x + y$
- subject to y (0) = 1 by Picard's method and hence find y(0.2) ?

- (b) By using RK Third order formulae find y(0.25), when
- $\frac{dy}{dx} = 1 + xy$
- and y(0)=1?

[7M + 8M]

- 8 (a) Fit a straight line to the following data and estimate the most probable yield of rice for 40 inches of water.

Water x(inches)	12	18	24	30	36	42	48
Yield y(tons)	5.27	5.68	6.25	7.21	8.02	8.71	8.42

- (b) Fit a curve of the form
- $y = ab^x$
- to the following data

x	0	2	4
y	5.1	10	31.1

[7M + 8M]

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- 1 (a) Reduce the matrix to normal form and hence find its rank

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} ?$$

- (b) Find whether the following equations are consistent, if so, solve them
 $x + y + 2z = 4$; $2x - y + 3z = 9$; $3x - y - z = 2$?

[7M + 8M]

2. State and prove Cayley–Hamilton theorem and Verify Cayley–Hamilton theorem for

$$A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \text{ and hence find } A^{-1} ?$$

[15M]

- 3 (a) Define nature, rank, index and signature of the Quadratics form

- (b) Reduce the Q.F $4x^2+3y^2+z^2-8xy-6yz+4xz$ into canonical form by Diagonalisation?

[4M + 11M]

- 4 (a) By using Regula-Falsi method, find the 4th root of 12 and 32 correct up to 3 decimal places?

- (b) By Newton-Raphson method find a real root of the equation $x^6=x^4+x^3+1$ near 1.5 correct up to 3 decimal places?

[7M + 8M]

- 5 (a) Evaluate $\Delta^2 \left[\frac{5x+12}{x^2+5x+6} \right]$ taking the interval of differencing being one unit?

- (b) Find $y(30)$ using Gauss Forward Differences formula when $y(21) = 18.4708$, $y(25) = 17.8144$, $y(29) = 17.107$, $y(33) = 16.3432$, $y(37) = 15.5154$?

[5M + 10M]

- 6 (a) Find $\frac{dy}{dx}$ from the data near $x = 1.5$ using Central Forward Difference Formula

x	1	1.2	1.4	1.6	1.8	2
y	2.72	3.32	4.06	4.95	6.05	7.39

- (b) Evaluate $\int_0^{\frac{\pi}{2}} e^{\sin x} dx$ correct to 4 decimal places by Simpson's 3/8 rule?
[7M + 8M]

- 7 (a) Apply Milne Predictor Corrector method to find $y(0.8)$, $y(1.0)$ from the equation $\frac{dy}{dx} = y - x^2$, $y(0) = 1$ by obtaining the starting values by Euler's method ?

- (b) Use Adam's method to find $y(1.4)$ from $\frac{dy}{dx} = x^2 y + x^2$ and $y(1)=1$. find the initial values $y(1.1)$, $y(1.2)$ and $y(1.3)$ by Taylor's method?
[7M + 8M]

- 8 (a) To following data relate to results of a fertiliser experiment on crop yields:

Unit of fertiliser used (X)	0	2	4	6	8	10
Units of yield (Y)	110	113	118	119	120	118

Fit a straight line to the above data and estimate the amounts of yield when units of fertiliser used are 3 and 7 respectively.

- (b) Fit second degree parabola to the following data

x	0	1	2	3	4
y	1	5	10	22	38

[8M + 7M]

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Time: 3 hours

Max Marks: 75

Answer any FIVE Questions
All Questions carry equal marks

- 1 (a) Reduce the matrix to normal form and hence find its rank

$$A = \begin{bmatrix} 2 & 1 & 2 & 4 \\ 4 & 2 & 5 & 8 \\ 3 & 1 & 4 & 6 \\ 1 & 1 & 2 & 2 \end{bmatrix} ?$$

- (b) Solve the system of equations

$$x + y + 2z = 4; 3x + y - 3z = -4; 2x - 3y - 5z = -5?$$

[7M + 8M]

- 2 (a) Find the characteristic roots and characteristic vectors of the matrix

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- (b) Verify Cayley–Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence find A^4 ?

[7M + 8M]

3. Reduce the Q.F $2x_1^2 + 7x_2^2 + 5x_3^2 - 8x_1x_2 + 4x_1x_3 - 10x_2x_3$ into canonical form by using Lagrange's reduction and find its nature, rank, index and signature?

[15M]

- 4 (a) Find the root of $x^3 + x^2 + x + 7 = 0$ in the interval $(-2.5, -2.0)$ correct up to 4 decimal places by method of False position ?

- (b) Using Iteration method find a root of the equation $3x = \cos x + 1$ in the interval $(0, 1)$?

[7M + 8M]

- 5 (a) Evaluate $\Delta^2 [3e^x]$, $\Delta \sin(ax + b)$?

- (b) Using Newton's Forward Difference Interpolation formula, find y at x = 8 from the following table

x	0	5	10	15	20	25
y	7	11	14	18	24	32

[4M + 11M]

6. Find the area bounded by the curve $y = x^3 - x + 1$, x axis between $x=0$ and $x = 1.2$ by using (i) Trapezoidal rule (ii) Simpson's 1/3 rule (iii) Simpson's 3/8 rule (iv) Boole's rule

[15M]

- 7 (a) Apply second order RK method to find $y(0.2)$, where $\frac{dy}{dx} = x + \sqrt{y}$ & $y(0) = 1$?

- (b) Use Milne's method to find $y(0.5)$ from $10\frac{dy}{dx} = x^2 + y^2$ and $y(0)=1$. obtain the initial values by Taylor's method?

[7M + 8M]

- 8 (a) Fit a straight line by the method of least squares to the following data.

x	21	42	38	64	53	61	47
y	4	14	10	38	19	34	17

- (b) Fit a curve of the form $y = ae^{bx}$ to the following data

X	2	4	6	8	10
Y	4.077	11.084	30.128	81.897	222.62

[7M + 8M]