12. Show that the equations of common tangents to the circles $x^2 + y^2 = 2a^2$ and the parabola $y^2 = 8ax$ are $y = \pm (x + 2a)$.

13. Find the eccentricity, coordinates of foci, length of latus rectum and the equations of directrices of the ellipse $9x^2 + 16y^2 - 36x + 32y - 92 = 0$.

14. Show that the points with polar coordinates $(0, 0)$, $\left(3, \frac{\pi}{2}\right)$ and $\left(3, \frac{\pi}{6}\right)$ form an equilateral triangle.

15. Evaluate $\int x \, Cos^{-1} x \, dx, \; x \in (-1, 1)$.

16. Solve $\sqrt{1 + x^2} \, dx + \sqrt{1 + y^2} \, dy = 0$.

17. Solve $\frac{dy}{dx} - y \tan x = e^x \sec x$.

SECTION - C **5 × 7 = 35**

LONG ANSWER TYPE QUESTIONS

Attempt **any 5** questions. Each question carries 7 marks.

18. Show that the circles $x^2 + y^2 - 6x - 2y + 1 = 0$, $x^2 + y^2 + 2x - 8y + 13 = 0$ touch each other. Find the point of contact and the equation of common tangent at their point of contact.

19. Find the coordinates of the limiting points of the coaxial system to which the circles $x^2 + y^2 + 10x - 4y - 1 = 0$ and $x^2 + y^2 + 5x + y + 4 = 0$ are two members.

20. Show that the poles w.r.t. the parabola $y^2 = 4ax$ of the tangents to the rectangular hyperbola $x^2 - y^2 = a^2$ lies on the ellipse $4x^2 + y^2 = 4a^2$.

21. If $y = \frac{Sinh^{-1} x}{\sqrt{1 + x^2}}$ then show that $(1 + x^2) y_2 + 3xy_1 + y = 0$ and hence deduce that $(1 + x^2) y_{n+2} + (2n + 3) xy_{n+1} + (n + 1)^2 y_n = 0$

22. Solve $\int (6x + 5) \sqrt{6 - 2x^2 + x} \, dx$.

23. Show that $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx = \frac{\pi}{2\sqrt{2}} \log(\sqrt{2} + 1)$.

24. Find the approximate value of π from $\int_0^1 \frac{1}{1 + x^2} dx$ using Simpson's rule by dividing $[0, 1]$ into 4 equal parts.