Code No: R05010102 **Set No. 1**

I B.Tech Supplimentary Examinations, Aug/Sep 2008
MATHEMATICS-I
(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering and Automobile Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the following series for convergence or divergence.
$\frac{1}{2} + \frac{\sqrt{2}}{3} + \frac{\sqrt{3}}{8} + \ldots\ldots + \frac{\sqrt{n}}{n^2-1}$ [5]
   (b) Test whether the following series is absolutely convergent.
$\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^2+1}$ [5]
   (c) State and prove Generalilzed mean value theorem. [6]

2. (a) Find the point with in a triangle such that the sum of the square of its distances from the three vertices is a minimum.
   (b) Find the envelope of the family of straight lines $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1, \theta$ being the parameter. [8+8]

3. (a) In the evolute of the parabola $y^2 = 4ax$, show that the length of the curve from its cusp x = 2a to the point where it meets the parabola $y^2 = 4ax$ is $2a(3\sqrt{3} - 1)$
   (b) Find the length of the arc of the curve $y = \log\left[\frac{e^x-1}{e^x+1}\right]$ from x = 1 to x = 2 [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constant $\sin\sqrt{x} + e^{1/y} = c$.
   (b) Solve the differential equation: $dr + (2r\cot\theta + \sin 2\theta)\, d\theta = 0$.
   (c) The rate at which the population of the city increases at any time is proportional to the population at that time. If there were 1,30,000 people in 1950 and 1,60,000 in 1980. What is the anticipated population in 2010? [3+7+6]

5. (a) Solve the differential equation: $(D^2 + 4)y = \sin t + (1/3)\sin 3t + (1/5)\sin 5t$, $y(0) = 1$, $y'(0) = 3/35$.
   (b) Solve the differential equation: $(D^2 + 1)y = x\sin x$ by variation of parameters method. [8+8]

6. (a) Find $L\left[\frac{\sin^2 t}{t}\right]$

   (b) Find $L^{-1}\left[\frac{s+2}{s^2-4s+13}\right]$

   (c) Evaluate the integral $\int\int\int xy^2z \, dxdydz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$. [5+5+6]

7. (a) Find a and b such that the surfaces $ax^2 - byz = (a + 2)x$ and $4ax^2y + z^3 = 4$ cut orthogonally at (1, -1, 2).

   (b) Show that $\overline{F} = (2xy + z^3)\,i + x^2\,j + 3\,x\,z^2\,k$ is a conservative force field. Find the scalar potential and the work done by F in moving an object in this field from (1, –2, 1) to (3,1,4). [8+8]

8. Verify Green's theorem for $\int (x^2 - \cos by)\,dx + (y + \sin x)$
   where C is the rectangle bounded by (0,0) $(\pi, 0)$ $(\pi, 1)$ and (0,1). [16]

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Code No: R05010102 **Set No. 2**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the series $\frac{\sum (n!)^2}{(2n)!}$. [5]
   (b) Find whether the series $\frac{1}{\sqrt{1^2+1}} - \frac{x}{\sqrt{2^2+1}} + \frac{x^2}{\sqrt{3^2+1}} - \frac{x^3}{\sqrt{4^2+1}} + \ldots\ldots\ldots +$ converges absolutely / conditionally. [5]
   (c) Verify Cauchy's mean value theorem for f(x)=x^3, g(x)=x^2 in [1,2]. [6]
2. (a) Show that the functions u = x+y+z , v = $x^2+y^2+z^2-2xy-2zx-2yz$ and w = $x^3+y^3+z^3-3xyz$ are functionally related. Find the relation between them.
   (b) Find the centre of curvature at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$ of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$. Find also the equation of the circle of curvature at that point. [8+8]
3. (a) Trace the Cissoid of Diocles : y^2 (2a–x) = x^3.
   (b) Show that the surface area of the spherical zone contained between two parallel planes of distance 'h' units apart is 2πah, where 'a' is the radius of the sphere. [8+8]
4. (a) Form the differential equation by eliminating the arbitrary constant x tan(y/x) = c.
   (b) Find the orthogonal trajectories to $x^2 - y^2 = a^2$.
   (c) Radium decomposes at a rate proportional to the amount present at that time. If a fraction 'p' of the original amount disappears in 1 year how much amount will remain at the end of 21 years. [3+7+6]
5. (a) Solve the differential equation: $(D^2 + 1)y = e^{-x} + x^3 + e^x$ sinx.
   (b) Solve $(D^2 + 4)y = sec2x$ by the method of variation of parameters. [8+8]
6. (a) Find the Laplace transformation of $e^{2t} + 4t^3 - 2$ sin3t + 3cos3t.
   (b) Find $L^{-1}\left[\frac{s^3-3s^2+6s-4}{(s^2-2s+2)^2}\right]$

(c) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dxdy$ [5+6+5]

7. (a) Show that curl $(r^n\bar{r}) = 0$
   (b) Evaluate $\iint_S \bar{F}.\bar{n}$ ds where $\bar{F}$=zi+xj+3y^2zk where S is the surface of the cylinder x^2+y^2=1 in the first octant between z=0 and z=2 [8+8]

8. Verify Stokes theorem for the function F = x^2 i + xy j integrated round the square whose sides are x = 0, y = 0, x = a and y = a in the plane z = 0. [16]

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Code No: R05010102 **Set No. 3**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the series $\sum \frac{n^4}{n!}$. [5]
   (b) Find the whether the series $\frac{1}{6} - \frac{1.3}{6.8} + \frac{1.3.5}{6.8.10} - \frac{1.3.5.7}{6.8.10.12} + ...$ converges absolutely or conditionally. [6]
   (c) Verify Rolle's theorem for $f(x) = x^{2n-1}$ (a - x)^{2n} in (0,a). [5]

2. (a) Expand f(x,y) = e^{y}log(1+x) in powers of x and y.
   (b) Find the envelope of the family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where a^2 + b^2 = k^2, k being a constant. [8+8]

3. (a) Trace the Cissoid of Diocles : y^2 (2a–x) = x^3.
   (b) Show that the surface area of the spherical zone contained between two parallel planes of distance 'h' units apart is 2πah, where 'a' is the radius of the sphere. [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constant sin $\sqrt{x}$ + e$^{1/y}$ = c.
   (b) Solve the differential equation: dr + (2r cotθ + sin2θ) dθ = 0.
   (c) The rate at which the population of the city increases at any time is proportional to the population at that time. If there were 1,30,000 people in 1950 and 1,60,000 in 1980. What is the anticipated population in 2010? [3+7+6]

5. (a) Solve the differential equation: y$''$ - 4y$'$ + 3y = 4e^{3x}, y(0) = - 1, y$'$(0) = 3.
   (b) Solve the differential equation: $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\,cos\,\log(1+x)$ [8+8]

6. (a) Find the Laplace transformation of e ^{2t} + 4t^3 − 2 sin3t + 3cos3t.
   (b) Find $L^{-1}\left[\frac{s^3-3s^2+6s-4}{(s^2-2s+2)^2}\right]$

(c) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dxdy$ [5+6+5]

7. (a) Find a unit normal vector to the surface $x^3+y^3+ z^3=3$ at the print (1, –2,–1)
   (b) If $\bar{F} = (x^2 - y)i + (2xz - y)j + z^2k$, evaluate $\int \bar{F}.dt$ along the straight line joining the points (0,0,0) to (1,2,4) [8+8]

8. (a) Apply Green's theorem to prove that the area enclosed by a plane curve is $\frac{1}{2}\oint_C$ (x dy - y dx). Hence find the area of an ellipse whose semi major and minor axes are of lengths a and b.
   (b) Evaluate $\iint_S (y^2z^2\, i + z^2x^2\, j + z^2y^2\, k) \,.\, N\, ds$
   where S is the part of the unit sphere above the xy- plane. [8+8]

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Code No: R05010102 **Set No. 4**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Test the convergence of the following series $\sum \frac{1}{(\log\ \log n)^n}$ [5]
   (b) Find the interval of convergence of the series
   $x + \frac{1}{2}.\frac{x^3}{3} + \frac{1}{2}.\frac{3}{4}.\frac{x^5}{5} + \frac{1.3.5}{2.4.6}.\frac{x^7}{7} + ...$ [5]
   (c) Show that $\log\,(1 + e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + ...$ and hence deduce that $\frac{e^x}{e^x+1} = \frac{1}{2} + \frac{x}{4} - \frac{x^3}{48} + ...$ [6]

2. (a) Expand sin (x+y) in powers of (x+y).
   (b) show that the evolute of cycloid x = a(θ − sinθ), y = a(1 − cosθ) is another cycloid x = a(θ + sinθ), y = - a(θ − sinθ) [8+8]

3. (a) Trace the curve : r = a (1 + cos θ).
   (b) Find the length of the arc of the curve x = e^θsinθ; y = e^θcosθ from $\theta = 0$ to $\theta = \pi/2$. [8+8]

4. (a) Obtain the differential equation of the co-axial circles of the system $x^2+y^2+2ax + c^2 = 0$ where c is a constant and a is a variable parameter.
   (b) Solve the differential equation: $\frac{dy}{dx} = \frac{x-y\cos x}{1+\sin x}$
   (c) Find the orthogonal trajectories of the co-axial curves $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, λ being a parameter [3+7+6]

5. (a) Solve the differential equation: $(D^2$-1)y= $x$sin$x$ + x^2 e^x.
   (b) Solve the differential equation: (x^2D^2+xD+4)y=log x cos (2log$x$). [8+8]

6. (a) Find $L\left[\frac{\sin^2 t}{t}\right]$
   (b) Find $L^{-1}\left[\frac{s+2}{s^2-4s+13}\right]$
   (c) Evaluate the integral $\int\int\int$xy^{2}z dxdydz taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$. [5+5+6]

7. (a) If ϕ=xy+yzzx and $\bar{F} = x^2yi + y^2zj + z^2xk$, then find $\bar{F}$ grad ϕ and $\bar{F}$ × grad ϕ at (3,–1,2)

   (b) Show that $\bar{F}$=(y^2cosx+z^2)i + (2ysinx–4)j+$3xz^2k$ is irrotational and find its scalar potential [8+8]

8. Verify the Stokes theorem for F = y i + z j + x k and surface is the part of the sphere $x^2 + y^2 + z^2 = 1$ above the xy plane. [16]

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