Code No: R059210401 **Set No. 1**

II B.Tech I Semester Regular Examinations, November 2007
PROBABILITY THEORY AND STOCHASTIC PROCESS
(Common to Electronics & Communication Engineering, Electronics & Telematics and Electronics & Computer Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Discuss Joint and conditional probability.
   (b) When are two events said to be mutually exclusive? Explain with an example?
   (c) Determine the probability of the card being either red or a king when one card is drawn from a regular deck of 52 cards. [6+6+4]

2. (a) Define rayleigh density and distribution function and explain them with their plots.
   (b) Define and explain the guassian random variable in brief?
   (c) Determine whether the following is a valid distrbution function. F(x) = 1-exp(-x/2) for x ⇒ 0 and 0 elsewhere. [5+5+6]

3. (a) State and prove properties of characteristic function of a random variable X
   (b) Let X be a random variable defined by the density function
   $f_X(x) = \begin{cases} \frac{5}{4}(1-x^4) & 0 < x \leq 1 \\ 0 & elsewhere \end{cases}$. Find E[X] ,E[$X^2$] and variance. [8+8]

4. The joint space for two random variables X and Y and corresponding probabilities are shown in table

   Find and Plot

   (a) $F_{XY}(x,y)$
   (b) marginal distribution functions of X and Y.
   (c) Find P(0.5 < X < 1.5),
   (d) Find P(X $\leq$ 1, Y $\leq$ 2) and
   (e) Find P(1 < X $\leq$ 2, Y $\leq$ 3).

X, Y	1,1	2,2	3,3	4,4
P	0.05	0.35	0.45	0.15

[3+4+3+3+3]

5. (a) Show that the variance of a weighted sum of uncorrected random variables equals the weighted sum of the variances of the random variables.
   (b) Two random variables X and Y have joint characteristic function
   ϕX, Y(ω_1,ω_2) = exp($-2\omega_1^2 - 8\omega_2^2$).
      i. Show that X and Y are zero mean random variables.

ii. are X and Y are correlated. [8+8]

6. Let X(t) be a stationary continuous random process that is differentiable. Denote its time derivative by $\dot{X}$(t).

(a) Show that $E\left[\overset{\bullet}{\times}\ (t)\right] = 0$.

(b) Find $R_{\times\dot{\times}}\ (\tau)$ in terms of $R_{\times\times}\ (\tau)$sss [8+8]

7. (a) Derive the expression for PSD and ACF of band pass white noise and plot them

(b) Define various types of noise and explain. [8+8]

8. (a) Define the following random processes

i. Band Pass

ii. Band limited

iii. Narrow band. [3×2 = 6]

(b) A Random process X(t) is applied to a network with impulse response h(t) = u(t) exp (-bt)

where b > 0 is ω constant. The Cross correlation of X(t) with the output Y (t) is known to have the same form:

$R_{XY}\ (\tau)$ = u(τ)τ exp (-bY)

i. Find the Auto correlation of Y(t)

ii. What is the average power in Y(t). [6+4]

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Code No: R059210401 **Set No. 2**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
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1. (a) With an example define and explain the following:
   i. Equality likely events
   ii. Exhaustive events.
   iii. Mutually exclusive events.

   (b) In an experiment of picking up a resistor with same likelihood of being picked up for the events; A as "draw a 47 Ω resistor", B as "draw a resistor with 5% tolerance" and C as "draw a 100 Ω resistor" from a box containing 100 resistors having resistance and tolerance as shown below. Determine joint probabilities and conditional probabilities. [6+10]

   Table 1

   Number of resistor in a box having given resistance and tolerance.

Resistance(Ω)	Tolerance		
	5%	10%	Total
22	10	14	24
47	28	16	44
100	24	8	32
Total	62	38	100

2. (a) What is binomial density function? Find the equation for binomial distrbution function.

   (b) What do you mean by continuous and discrete random variable? Discuss the condition for a function to be a random variable. [6+10]

3. (a) Define moment generating function.

   (b) State properties of moment generating function.

   (c) Find the moment generating function about origin of the Poisson distribution. [3+4+9]

4. (a) Define conditional distribution and density function of two random variables X and Y

   (b) The joint probability density function of two random variables X and Y is given by

   $f(x,y) = \begin{cases} a(2x+y^2) & 0 \le x \le 2 \ , \quad 2 \le y \le 4 \\ 0 & elsewhere \end{cases}$. Find

i. value of 'a'

ii. $P(X \leq 1, Y > 3)$. [8+8]

5. (a) let X_i, i = 1,2,3,4 be four zero mean Gaussian random variables. Use the joint characteristic function to show that E $\{X_1\ X_2\ X_3\ X_4\}$ = E$[X_1\ X_2]$ E$[X_3\ X_4]$ + E$[X_1X_3]$E$[X_2X_4]$ + E$[X_2X_3]$ E$[X_1X_4]$

   (b) Show that two random variables X_1 and X_2 with joint pdf.
   $f_{X1X2}(X_1, X_2) = 1/16\ |X_1| < 4, 2 < X_2 < 4$ are independent and orthogonal.[8+8]

6. A random process Y(t) = X(t)- X(t $+\tau$) is defined in terms of a process X(t) that is at least wide sense stationary.

   (a) Show that mean value of Y(t) is 0 even if X(t) has a non Zero mean value.

   (b) Show that $\sigma Y^2 = 2[R_{XX}(0) - R_{XX}(\tau)]$

   (c) If Y(t) = X(t) +X(t + τ) find E[Y(t)] and σY^2. [5+5+6]

7. (a) If the PSD of X(t) is Sxx(ω). Find the PSD of $\frac{dx(t)}{dt}$

   (b) Prove that S_{xx} (ω) = S_{xx} (-ω)

   (c) If $R(\tau) = ae^{|by|}$. Find the spectral density function, where a and b are constants. [5+5+6]

8. (a) A Signal x(t) = u(t) exp (-αt) is applied to a network having an impulse response h(t)= ω u(t) exp (-ω t). Here α & ω are real positive constants. Find the network response? (6M)

   (b) Two systems have transfer functions $H_1(\ \omega)$ & $H_2(\ \omega)$. Show the transfer function H(ω) of the cascade of the two is H(ω) =$H_1(\ \omega)$ H_2 (ω).

   (c) For cascade of N systems with transfer functions $H_n(\omega)$, n=1,2,.... .N show that H(ω) = $\pi H_n(\omega)$. [6+6+4]

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Code No: R059210401 **Set No. 3**

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1. (a) Define probability based on set theory and fundamental axioms.
   (b) When two dice are thrown, find the probability of getting the sums of 10 or 11. [8+8]

2. (a) Define cumulative probability distribution function. And discuss distribution function specific properties.
   (b) The random variable X has the discrete variable in the set $\{-1, -0.5, 0.7, 1.5, 3\}$ the corresponding probabilities are assumed to be $\{0.1, 0.2, 0.1, 0.4, 0.2\}$. plot its distribution function and state is it a discrete or continuous ditribution function. [8+8]

3. (a) Explain the concept of a transformation of a random variable X
   (b) A Gaussian random variable X having a mean value of zero and variance one is transformed to an another random variable Y by a square law transformation. Find the density function of Y. [8+8]

4. Discrete random variables X and Y have a joint distribution function
   $F_{XY}(x, y) = 0.1u(x + 4)u(y - 1) + 0.15u(x + 3)u(y + 5) + 0.17u(x + 1)u(y - 3)+$
   $0.05u(x)u(y - 1) + 0.18u(x - 2)u(y + 2) + 0.23u(x - 3)u(y - 4)+$
   $0.12u(x - 4)u(y + 3)$
   Find
   (a) Sketch $F_{XY}(x, y)$
   (b) marginal distribution functions of X and Y.
   (c) P($-1 <$ X $\leq 4, -3 <$ Y ≤ 3) and
   (d) Find P(X < 1, Y ≤ 2). [4+6+3+3]

5. (a) let Y = X_1 + X_2 + ...+X_N be the sum of N statistically independent random variables X_i, i=1,2............. N. If Xi are identically distributed then find density of Y, f_y(y).
   (b) Consider random variables Y_1 and Y_2 related to arbitrary random variables X and Y by the coordinate rotation. Y_1=X Cos θ + Y Sin θ Y_2 = -X Sin θ + Y Cos θ
      i. Find the covariance of Y_1 and Y_2, C_{Y1Y2}
      ii. For what value of θ, the random variables Y_1 and Y_2 uncorrelated. [8+8]

6. (a) Define cross correlation function of two random processes X(t) and Y(t) and state the properties of cross correlation function.

   (b) let two random processes X(t) and Y(t) be defined by
   X(t) = A cos ω_0t + B sin ω_0t
   Y(t) = B cos ω_0t - A sin ω_0t
   Where A and B are random variables and ω_0 is a constant. Assume A and B are uncorrelated, zero mean random variables with same variance. Find the cross correlation function R_{XY} (t,t+τ) and show that X(t) and Y(t) are jointly wide sense stationary. [6+10]

7. (a) If the PSD of X(t) is Sxx(ω). Find the PSD of $\frac{dx(t)}{dt}$

   (b) Prove that S_{xx} (ω) = S_{xx} (-ω)

   (c) If $R(\tau) = ae^{|by|}$. Find the spectral density function, where a and b are constants. [5+5+6]

8. (a) A Stationary random process X(t) having an Auto Correlation function $R_{XX}\,\tau = 2e^{-4|\tau|}$ is applied to the network shown in figure 8a find
      i. S_{XX} (ω)
      ii. IH(ω)I^2
      iii. S_{YY}(ω). [4+4+2]

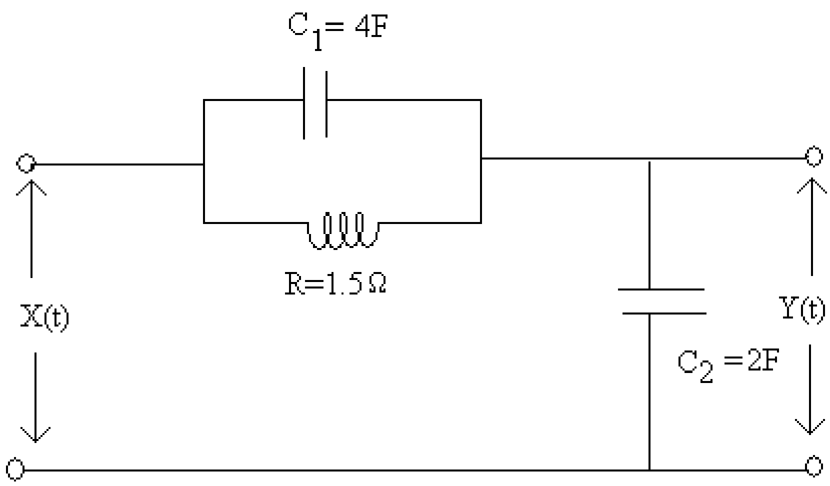

Figure 8a

   (b) Write short notes on different types of noises. [6]

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1. (a) Define and explain the following with an example:
   i. Equally likely events
   ii. Exhaustive events
   iii. Mutually exclusive events
   (b) Give the classical definition of probability.
   (c) Find the probability of three half-rupee coins falling all heads up when tossed simultaneously. [6+4+6]

2. (a) What is poisson random variable? Explain in brief.
   (b) What is binomial density and distrbution function?
   (c) Assume automobile arrives at a gasoline station are poisson and occur at an average rate of 50/hr. The station has only one gasoline pump. If all cars are assumed to require one minute to obtain fuel. What is the probability that a waiting line will occur at the pump? [5+5+6]

3. (a) Define moment generating function.
   (b) State properties of moment generating function.
   (c) Find the moment generating function about origin of the Poisson distribution. [3+4+9]

4. Given the function $f(x, y) = \begin{cases} (x^2 + y^2)/8\pi & x^2 + y^2 < b \\ 0 & elsewhere \end{cases}$
   (a) Find the constant 'b' so that this is a valid joint density function.
   (b) Find $P(0.5b < X^2 + Y^2 < 0.8b)$. [7+9]

5. Three statistically independent random variables X_1,X_2 and X_3 have mean values $\bar{X}_1 = 3$, $\bar{X}_2 = 6$ and $\bar{X}_3 = -2$. Find the mean values of the following functions.
   (a) $g(X_1,X_2,X_3) = X_1 + 3X_2 + 4X_3$
   (b) $g(X_1,X_2,X_3) = X_1\ X_2\ X_3$
   (c) $g(X_1,X_2,X_3) = -2X_1,X_2 - 3X_1\ X_3 + 4X_2\ X_3$
   (d) $g\ (X_1,X_2,X_3) = X_1+X_2+X_3$. [16]

6. Statistically independent zero mean random processes X(t) and Y(t) have auto correlations functions
$R_{XY}(\tau) = \text{e}^{-|\tau|}$ and
$\text{R}_{\text{YY}}(\tau) = \cos(2\Pi\tau)$ respectively.
   (a) find the auto correlation function of the sum W_1(t) = X(t) + Y(t)
   (b) find the auto correlation function of difference W_2(t) = X(t) - Y(t)
   (c) Find the cross correlation function of W_1(t) and W_2(t). [5+5+6]

7. (a) Find the ACF of the following PSD's
      i. $S_{XX}(\omega) = \frac{157+12\omega^2}{(16+\omega^2)(9+\omega^2)}$
      ii. $S_{XX}(\omega) = \frac{8}{(9+\omega^2)^2}$
   (b) State and Prove wiener-Khinchin relations. [8+8]

8. A random noise X(t) having power spectrum $S_{XX}(\omega) = \frac{3}{49+\omega^2}$ is applied to a to a network for which $h(t) = u(t)t^2 \exp(-7t)$. The network response is denoted by Y(t)
   (a) What is the average power is X(t)
   (b) Find the power spectrum of Y(t)
   (c) Find average power of Y(t). [5+6+5]

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