1 RATIONAL NUMBERS

THEORY

1.1 INTRODUCTION

We know that for two given integers p and q, their sum p + q, difference p – q and product pq is always integer. But the system of integers suffered from the defect that division is not always possible within the system. For example, to problems such as 3 ÷ 5 or – 4 ÷ 3 there was no answer. That is to say no integer could be found to fill in the blank 5 × ....= 3 or 3 × ....= –7. Therefore, need was felt to go beyond integers and construct a new number system which include integers and in which all division could be carried out. The numbers that were created were called Rational Numbers.

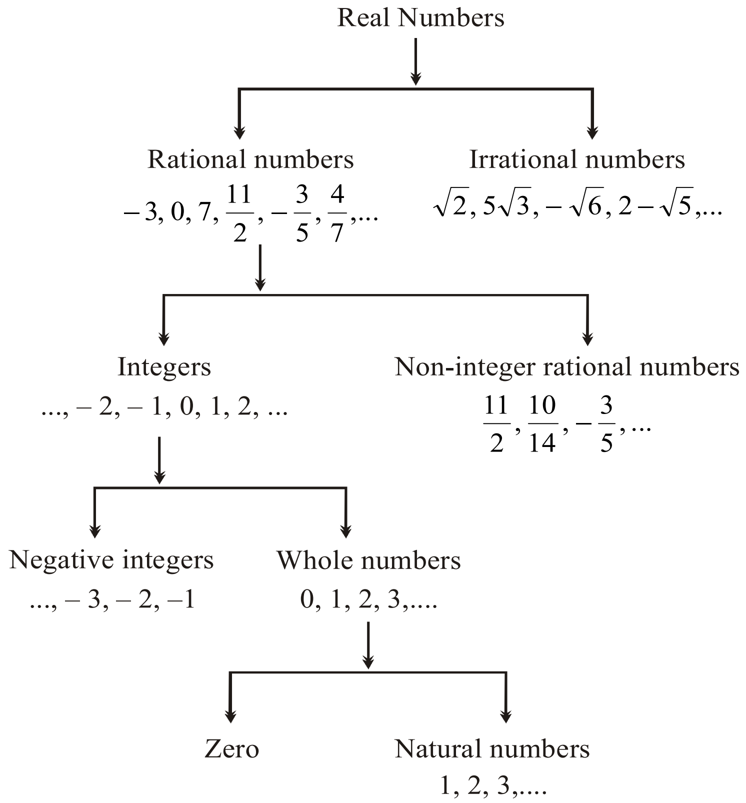
The word 'rational' is derived from the word ratio.

Definition: A rational number is any number that can be written in the form p/q where p and q are integers and $q \neq 0$.

For example, $\frac{5}{6}, -\frac{6}{11}, \frac{8}{-9}$ are rational numbers.

1.2 NUMBERS

In Hindu Arabic system we use ten symbols 0,1, 2, 3, 4, 5, 6, 7, 8, 9 called digits to represent any number. A group of figures denoting a number is called a numeral.



1.2.1 Defining Various Types of Numbers

(a) **Natural numbers:** Counting numbers are called natural numbers.
thus N = {1,2,3,... ... ... } is the set of natural numbers.

(b) **Whole Numbers :** All counting numbers together with zero form the set of whole numbers.
Thus W = {0,1,2,3 .... } is the set of whole numbers.
Every natural number is a whole number but 0 is a whole number which is not a natural number.

(c) **Integers**: The set I of all natural numbers 0 and negatives of counting numbers is the set of all integers.
Thus I = {............ –3, –2, –1, 0, 1, 2, 3, ... } is the set of all integers.

(d) **Rational Numbers**: The numbers of the form p/q where p and q are integers and $q \neq 0$ are known as rational numbers e.g. $\{\frac{3}{5}, \frac{9}{7}, \frac{-2}{3}, \frac{0}{1}$ etc}
Thus Q = { p/q : p and q are integers & $q \neq 0$} is the set of all rational numbers. Every integer is a rational number.

(e) **Terminating and Repeating decimals :** Every rational number has a particular characteristic that is, when expressed in the decimal form, it is expressible either in terminating decimals or in repeating decimals.

$\frac{1}{2} = 0.5$ $\frac{1}{3} = 0.333 = 0.\bar{3}$

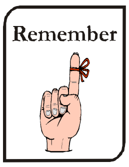
(f) **Irrational Number**: All numbers when expressed in decimal form which are in non-terminating and non repeating form are known as irrational numbers e.g. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π etc.

(g) **Real Numbers**: The totality of all rational and all irrational numbers forms the set R of all real numbers. Thus every natural, every whole number, every integer, every rational number and every irrational number is a real number.

(h) **Even and odd numbers :** Integers divisible by 2 are known as even integers while those which are not divisible by 2 are known as odd integers.
Thus, –6, –4, –2, 0, 2, 4, 6 .... are even integers.
and : –5, –3, –1, 1, 3, 5 .... are odd integers.

(i) **Prime Numbers :** A number greater than 1 is called a prime number if it has exactly two factors namely 1 and itself. For example 2, 3, 5, 7, 11 etc.

(j) **Composite Numbers :** Numbers greater than 1 which are not primes are known as composite numbers e.g. 4, 6, 8 .... are all composite numbers.



(i) ***1 is neither prime nor Composite.***
(ii) ***2 is the only even number which is prime.***
(iii) ***There are 25 prime numbers between 1 & 100.***
(iv) ***The sum (or difference) of a rational number and an irrational number is irrational.***
(v) ***The product of a rational and an irrational number is irrational (Except 0).***

1.2.2 Properties of Rational Number

(A) CLOSURE PROPERTY :

(i) Addition :

We take an example,

$$\frac{1}{4} + \left(-\frac{3}{2}\right) = \frac{1+(-6)}{4} = \frac{-5}{4}$$ which is a rational number.

If a & b are two rational number then a+b is also a rational number. This property is known as closure property for addition of rational numbers.

(ii) Subtraction :

Subtraction is inverse of addition. So to subtract a rational number we add its additive inverse. For example,

$$-\frac{2}{5} - \left(-\frac{4}{9}\right) = \frac{-2}{5} + \frac{4}{9} \qquad \left\{\text{additive inverse of } \frac{-4}{9} \text{ is } \frac{4}{9}\right\}$$

Thus : If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers then :

$$\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \left(-\frac{c}{d}\right)$$

The difference of any two rational numbers a & b, i.e. a–b, is a rational numbers.

for e.g.

$$\frac{1}{2} - \frac{4}{9} = \frac{1}{2} + \left(-\frac{4}{9}\right) = \frac{9-8}{18} = \frac{1}{18}$$ a rational number.

This property is known as **closure property for subtraction** of rational numbers.

(iii) Multiplication :

If a and b are two rational numbers then a × b is also a rational number :

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

for e.g. $\frac{2}{3} \times \frac{5}{7} = \frac{2\times 5}{3 \times 7} = \frac{10}{21}$ which is a rational number.

Hence this is a closure property for multiplication of rational numbers.

(iv) Division:

If a & b are two rational numbers and b $\neq$ 0 then a ÷ b is always a rational number.

for e.g. $\frac{2}{3} \div -\frac{4}{9} = \frac{2}{3} \times \frac{9}{-4} = -\frac{3}{2}$ is a rational number.

Hence this is a closure property for Division of rational numbers.

For any rational number a, a ÷ 0 is not defined.

So rational numbers are not closed under division.

However if we exclude zero then the collection of all other rational numbers is closed under division.

(B) COMMUTATIVE PROPERTY :

(i) Addition : Addition is commutative for rational numbers.

If a and b are any two rational numbers then a + b = b + a.

This property is known as commutative property for addition of rational numbers.

If a and b are any two rational numbers then a+b is also a rational number.

For example,

$\frac{1}{2} + \frac{3}{4} = \frac{2+3}{4} = \frac{5}{4}$ which is a rational number.

$\frac{-2}{3} + \frac{-1}{5} = \frac{-10+(-3)}{15} = \frac{-13}{15}$ which is a rational number.

(ii) Subtraction is not commutative

It can be explanined as follows:

$a - b \neq b - a$

e.g. $\frac{2}{3} - \frac{5}{4} \neq \frac{5}{4} - \frac{2}{3}$

both are not equal hence subtraction is *not* commutative for rational numbers.

(iii) Multiplication is commutative for rational number :

In general :

$a \times b = b \times a$ for any rational numbers.

$\frac{3}{5} \times \frac{4}{9} = \frac{4}{9} \times \frac{3}{5} = \frac{12}{35}$

Both are equal hence multiplication is commutative for rational numbers.

(iv) Division is not commutative for rational numbers :

$$\frac{-a}{b} \div \frac{c}{d} \neq \frac{c}{d} \div \left(\frac{-a}{b}\right)$$

The expression on both sides are not equal.

for .e.g. $\frac{-5}{4} \div \frac{3}{7} \neq \frac{3}{7} \div \frac{-5}{4} \Rightarrow \frac{-35}{12} \neq \frac{12}{-35}$

Hence division is not commutative for rational numbers.

(C) ASSOCIATIVE PROPERTY :

(i) Addition is associative:

e.g. a, b, c are three rational numbers then :

$a + (b + c) = (a + b) + c$

This property is known as **associative property** for addition of rational numbers.

(ii) Subtraction is not associative for rational number :

$a - (b + c) \neq (a - b) + c$

(iii) Multiplication is associative for rational number:

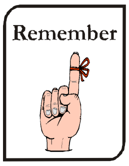
For any three rational numbers a, b, c

$a \times (b \times c) = (a \times b) \times c$

so multiplication is associative for rational numbers.

(iv) Division is not associative :

$$\frac{a}{b} \div \left(\frac{c}{d} \div \frac{e}{f}\right) \neq \left(\frac{a}{b} \div \frac{c}{d}\right) \div \frac{e}{f}$$



Properties of Rational Number

Closure :

Operation	Whole Numbers	Integers	Rational Numbers
Addition	Closed under addition	Closed under addition	Closed under addition
Subtraction	Not closed under subtraction	Closed under subtraction	Closed under subtraction
Multiplication	Closed under multiplication	Closed under multiplication	Closed under multiplication
Division	Not closed under division	Not closed under division	Not closed under division

Commutativity:

Operation	Whole Numbers	Integers	Rational Numbers
Addition	Commutative	Commutative	Commutative
Subtraction	Not Commutative	Not Commutative	Not Commutative
Multiplication	Commutative	Commutative	Commutative
Division	Not Commutative	Not Commutative	Not Commutative

Associativity :

Opeation	Whole Numbers	Integers	Rational Nunbers
Addition	Associative	Associative	Associative
Subtraction	Associative	Associative	Associative
Multiplication	Associative	Associative	Associative
Division	Not associative	Not associative	Not associative

1.3 THE ROLE OF ZERO (0)

(i) Addition of 0 to a rational number

If C is a rational number then:

$$C + 0 = 0 + C = C.$$

Zero is called the identity for the addition of rational number.

If $\frac{p}{q}$ is a rational number then $0 \times \frac{p}{q} = 0 = \frac{p}{q} \times 0$

It follows that the product of a rational number and zero is always zero.

1.4 NEGATIVE OF A NUMBER

If x be any rational number then –x is also called a rational number such that $x + (-x) = 0 = (-x) + x$.
Here $-x$ is called the negative of x or **additive inverse of** $\boldsymbol{x}$

1.5 RECIPROCAL OR MULTIPLICATIVE INVERSE

If a/b be a rational number then b/a is called multiplicative inverse if :

$$\frac{a}{b} \times \frac{b}{a} = 1$$

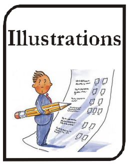


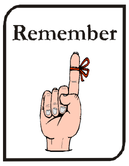
Illustration 1

Is $\frac{8}{9}$ the multiplicative inverse of $-1\frac{1}{8}$? Why or why not?

Solution :

$$-1\frac{1}{8} = \frac{-9}{8} \qquad \therefore \qquad \frac{8}{9}\times\left(-1\frac{1}{8}\right) = \frac{8}{9}\times\frac{-9}{8} = 1 \neq 1$$

$\therefore$ $\frac{8}{9}$ is not the multiplicate inverse of $-1\frac{1}{8}$.



(i) ***Zero has no reciprocal.***

(ii) ***Reciprocal of 1 is 1.***

(iii) ***Reciprocal of –1 is –1.***

1.6 DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION OF RATIONAL NUMBERS

If a, b, c are any three rational numbers then :

$a \times (b + c) = a \times b + a \times c$

This property is illustrated by the following example:

$$\frac{2}{3} \times \left(\frac{5}{6}+\frac{7}{8}\right) = \left(\frac{2}{3}\times\frac{5}{6}\right) + \left(\frac{2}{3}\times\frac{7}{8}\right)$$

1.7 DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER SUBTRACTION OF RATIONAL NUMBERS

Since $b - c = b + (-c)$

$a \times (b - c) = a \times b - a \times c$

The distributive property of multiplication over subtraction is illustrated by the following example:

$$\frac{2}{3}\left(\frac{5}{6}-\frac{1}{3}\right) = \frac{2}{3}\times\frac{5}{6}-\frac{2}{3}\times\frac{1}{3}$$

1.8 ABSOLUTE VALUE OF A RATIONAL NUMBER

Absolute value of a rational number is its numerical value (value without signs)

For example, $\left|-\frac{3}{5}\right| = \frac{3}{5}$ & $\left|\frac{7}{9}\right| = \frac{7}{9}$

Properties:

The absolute value of the sum of two rational numbers is always less than or equal to the sum of the absolute values of the given numbers.

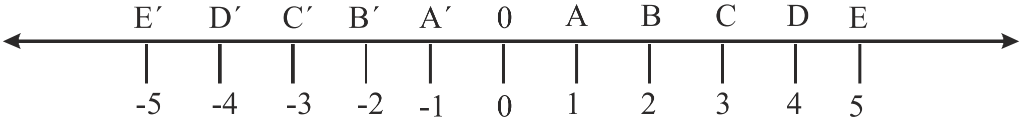
$$|x + y| \leq |x| + |y|$$

The absolute value of the product of two rational numbers is equal to the product of the absolute values of the given numbers.

$$|x \times y| = |x| \times |y|$$

1.9 REPRESENTATION OF RATIONAL NUMBERS ON THE NUMBER LINE

(i) Draw any line. Take a point O on it. Call it 0 (zero) . Set off equal distances on right as well as on the left of 0. Each such distance is of unit length. Clearly points A, B, C, D etc. represents the integers 1, 2, 3, 4 ..... etc. respectively & the points A′, B′, C′, D′ represents the integers –1, –2, –3, –4 respectively.



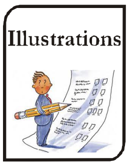


Illustration 2

Represent 13/5 and –13/5 on number line

Solution

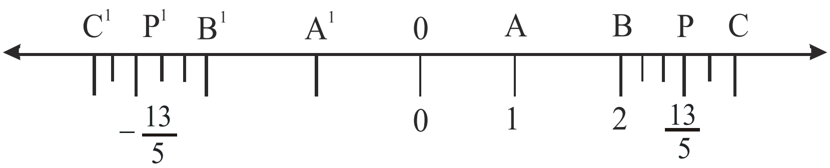
Draw a line. Take a point O on it. Let it be represented by 0.

Now $\frac{13}{5} = 2\frac{3}{5} = 2 + \frac{3}{5}$

$-\frac{13}{5} = -\left[2 + \frac{3}{5}\right]$

From O set off unit distances OA, AB and BC clearly, the points A, B, C represents, 1, 2, 3 respectively. Now take 2 units OA and AB and divide the third unit BC into 5 equal parts. Take 3 parts out of these 5 parts to reach at point P. Then point P represents rational number $\frac{13}{5}$.

Similarly on left side P′ represents $-\left[2 + \frac{3}{5}\right]$



1.10 RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

If x and y are two rational number. Such that $x < y$ then $\frac{1}{2}(x + y)$ is a rational number lying between x and y.

Illustration 3

Give three rational numbers lying between $\frac{1}{3}$ and $\frac{1}{2}$.

Solution :

The rational number $= \frac{1}{2} \times \left(\frac{1}{3} + \frac{1}{2}\right)$ lies between $\frac{1}{3}$ and $\frac{1}{2}$.

Now, $\frac{1}{2} \times \left(\frac{1}{3} + \frac{1}{2}\right) = \frac{1}{2} \times \left(\frac{2+3}{6}\right) = \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$

Therefore, $\frac{1}{3} < \frac{5}{12} < \frac{1}{2}$.

Let us now find a rational number between $\frac{1}{3}$ and $\frac{5}{12}$.

We know that $\frac{1}{2}\left(\frac{1}{3} + \frac{5}{12}\right)$ is one such number.

Also, $\frac{1}{2}\left(\frac{1}{3} + \frac{5}{12}\right) = \frac{1}{2}\left(\frac{4}{12} + \frac{5}{12}\right) = \frac{1}{2} \times \frac{9}{12} = \frac{9}{24} = \frac{3}{8}$ $\quad \therefore \quad \frac{1}{3} < \frac{3}{8} < \frac{5}{12} < \frac{1}{2}$

Now let us find a rational number between $\frac{5}{12}$ and $\frac{1}{2}$.

One such number is

$$\frac{1}{2}\left(\frac{5}{12} + \frac{1}{2}\right) = \frac{1}{2}\left(\frac{5}{12} + \frac{6}{12}\right) = \frac{1}{2} \times \frac{11}{12} = \frac{11}{24}$$

$$\therefore \quad \frac{1}{3} < \frac{3}{8} < \frac{5}{12} < \frac{11}{24} < \frac{1}{2}$$

Hence, $\frac{3}{8}, \frac{5}{12}, \frac{11}{24}$ are the required three rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$.

1.11 REPRESENTING THE SQUARE ROOT OF A POSITIVE NUMBER ON THE NUMBER LINE

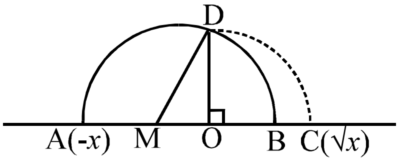
Let x be a positive real number. We will now locate $\sqrt{x}$ on the number line.

Step. 1 : Mark –x on the number line. Let this point be represented by A. Mark 1 unit on the number line. Let this be represented by B.

Step. 2 : Locate the midpoint M of AB.

Step. 3 : With M as the centre and MA or MB as radius draw a semicircle. Since diameter AB= (x+1) units, MA= MB = ½ (x+1) units.

Step. 4 : Draw OD perpendicular to AB meeting the semicircle in D. Join MD. Note the Δ DMO is a right triangle with MD = ½ (x+1) units and MO = [½ (x+1)-1) units. = ½ (x-1) units.



Step 5 : Using the Pythagorean theorem, we obtain :

$$OD^2 = MD^2 - MO^2$$

$$= \frac{1}{4}(x+1)^2 - \frac{1}{4}(x-1)^2$$

$$= \frac{1}{4}(4x) = x \quad = OD = \sqrt{x}$$

With O as the centre and OD as the radius, draw an arc to meet the number line at C. The point C represents $\sqrt{x}$.

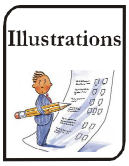
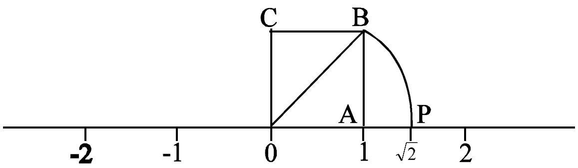


Illustration 4 : Locate $\sqrt{2}$ on the number line.

Sol.

Step 1 : Draw the number line with O representing the number 0 and A representing the number 1.

Step 2 : Construct a square OABC with each side equal to 1 unit.



By the Pythagorean theorem :

$$OB^2 = OA^2 + AB^2$$

$$= 1^2 + 1^2$$

$$= 1 + 1 = 2$$

$$OB = \sqrt{2}$$

Step 3 : With O as centre and OB as radius, draw an arc to meet the number line at point P.

Since OP = OB = $\sqrt{2}$, the point P represents $\sqrt{2}$ on the number line.

POINTS TO REMEMBER

- Rational numbers are **closed** under **addition, subtraction, multiplication** and **division**.
- Rational numbers are **commutative** under **addition** and **multiplication**.
- Rational numbers are **associative** under **addition** and **multiplication**.
- In rational numbers, there exists an **identity element 'zero'** under addition. i.e. $a + 0 = 0 + a = a$
- In rational numbers, there exists an **identity element 'one'** under multiplication. i.e. $a \times 1 = 1 \times a = a$
- Given a rational numbers $\frac{p}{q}$, there exists and additive inverse $-\frac{p}{q}$ such that $\frac{p}{q} + \left(-\frac{p}{q}\right) = 0$.
- Given a rational numbers $\frac{p}{q}$, there exists and multiplicative inverse $\frac{q}{p}$ such that $\frac{p}{q} \times \frac{q}{p} = 1$.
- In rational numbers, multiplication, distributes over addition and subtraction.
- Every rational numbers can be represented on the number line.
- On the number line, a rational number on the right is always greater than the number on the left.
- Between two rational numbers there lie an infinite number of rational numbers.
- If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$.
- If $\frac{a}{b}$ and $\frac{c}{d}$ $(\neq 0)$ are two rational numbers, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$.
- If a and b are two rational numbers, then
  (i) $(a + b)$ is always a rational number (closure property)
  (ii) $(a - b)$ is always a rational number (closure property)
  (iii) $(a + b) = (b + a)$ is always a rational number (commutative property of addition)

SOLVED EXAMPLES

Example 1 :

Find the absolute value of :

(i) $\frac{8}{3}$ **(ii)** $\frac{-5}{4}$ **(iii)** $\left(\frac{1}{2}-\frac{3}{4}\right)$ **(iv)** $\frac{-1}{2}\times\frac{3}{7}$

(v) $\frac{-4}{5}\div\frac{16}{25}$

Solution :

(i) $\left|\frac{8}{3}\right| = \frac{8}{3}$ (ii) $\left|\frac{-5}{4}\right| = \frac{5}{4}$

(iii) $\left|\frac{1}{2}-\frac{3}{4}\right| = \left|\frac{2-3}{4}\right| = \left|\frac{-1}{4}\right| = \frac{1}{4}$ (iv) $\left|\frac{-1}{2}\times\frac{3}{7}\right| = \left|\frac{-3}{14}\right| = \frac{3}{14}$

(v) $= \left|\frac{-4}{5}\times\frac{25}{16}\right| = \left|\frac{-5}{4}\right| = \frac{5}{4}$

Example 2 :

Verify : $\left|\frac{-1}{8}\times\frac{5}{11}\right| = \left|\frac{-1}{8}\right|\times\left|\frac{5}{11}\right|$

Solution :

L.H.S. $= \left|\frac{-1}{8}\times\frac{5}{11}\right| = \left|\frac{-5}{88}\right| = \left|\frac{5}{88}\right|$

R.H.S. $= \left|\frac{-1}{8}\right|\times\left|\frac{5}{11}\right| = \frac{1}{8}\times\frac{5}{11} = \frac{5}{88}$

$\because$ $\frac{5}{88} = \frac{5}{88}$

Hence, $\left|\frac{-1}{8}\times\frac{5}{11}\right| = \left|\frac{-1}{8}\right|\times\left|\frac{5}{11}\right|$

Example 3 :

Subtract $\frac{-5}{12}$ **from** $\frac{-5}{8}$

Solution :

$\frac{-5}{8}-\left(\frac{-5}{12}\right) = \frac{-5}{8}+\frac{5}{12} = \frac{-5\times3+2\times5}{24} = \frac{-15+10}{24}$ [L.C.M. of 8 and 12 = 24]

$= \frac{-5}{24}$

Example 4 :

Simplify : $\left(\frac{-3}{2}\times\frac{4}{5}\right)+\left(\frac{9}{5}\times\frac{-10}{3}\right)-\left(\frac{1}{2}\times\frac{3}{4}\right)$

Solution :

$$\left(\frac{-3}{2}\times\frac{4}{5}\right)+\left(\frac{9}{5}\times\frac{-10}{3}\right)-\left(\frac{1}{2}\times\frac{3}{4}\right)=\left(\frac{-3\times4}{2\times5}\right)+\left[\frac{9\times(-10)}{5\times3}\right]-\left(\frac{1\times3}{2\times4}\right)$$

$$=\frac{-6}{5}+\frac{-6}{1}-\frac{3}{8}=\frac{-48-240-15}{40}=\frac{-303}{40}$$

Example 5 :

Divide : (i) $\frac{7}{8}$ **by** $\frac{3}{5}$ **(ii)** $\frac{7}{18}$ **by** $\frac{4}{-9}$

Solution :

(i) $\frac{7}{8}\div\frac{3}{5}=\frac{7}{8}\times\frac{5}{3}=\frac{7\times5}{8\times3}=\frac{35}{24}$

(ii) $\frac{7}{18}\div\frac{4}{-9}=\frac{7}{18}\times\frac{-9}{4}=\frac{7\times(-9)}{18\times4}=\frac{-7}{8}$

Example 6 :

Verify the commutative property for addition of rational numbers with the help of the following rational numbers.

(i) $\frac{7}{8},\frac{3}{5}$ **(ii)** $\frac{5}{-6},\frac{2}{3}$

Solution :

(i) Consider, $\frac{7}{8}+\frac{3}{5}=\frac{3}{5}+\frac{7}{8}$ $\Rightarrow$ $\frac{35+24}{40}=\frac{24+35}{40}$

$\Rightarrow$ $\frac{59}{40}=\frac{59}{40}$ $\therefore$ $\frac{7}{8}+\frac{3}{5}=\frac{3}{5}+\frac{7}{8}$

Hence, commutative property for addition of rational numbers is verified.

(ii) Consider, $\frac{5}{-6}+\frac{2}{3}=\frac{2}{3}+\frac{5}{-6}$

$\Rightarrow$ $\frac{-5}{6}+\frac{2}{3}=\frac{2}{3}+\frac{-5}{6}\left[\frac{5}{-6}=\frac{-5}{6}\right]$

$\Rightarrow$ $\frac{-5+4}{6}=\frac{4+(-5)}{6}$ $\Rightarrow$ $\frac{-1}{6}=\frac{-1}{6}$

$\therefore$ $\frac{5}{-6}+\frac{2}{3}=\frac{2}{3}+\frac{5}{-6}$

Hence, commutative property for addition of rational numbers is verified.

Example 7 :

State the property used in each of the following :

(i) $\frac{-4}{7}+\frac{5}{21}=\frac{5}{21}+\frac{-4}{7}$

(ii) $\frac{-1}{2}+\frac{-5}{21}=\frac{-5}{21}+\frac{-1}{2}$

(iii) $\frac{3}{4}+\left(\frac{7}{2}+\frac{-3}{8}\right)=\left(\frac{3}{4}+\frac{7}{2}\right)+\frac{-3}{8}$

(iv) $\left(\frac{-11}{12}+\frac{-5}{6}\right)+4=\frac{-11}{12}+\left(\frac{-5}{6}+4\right)$

(v) $\frac{3}{10}+\left(\frac{-11}{15}+\frac{10}{-9}\right)=\left(\frac{3}{10}+\frac{-11}{15}\right)+\frac{10}{-9}$

(vi) $\frac{3}{-7}+\left(\frac{-5}{8}+\frac{9}{4}\right)=\left(\frac{3}{-7}+\frac{-5}{8}\right)+\frac{9}{4}$

Solution :

(i) Commutative (ii) Commutative (iii) Associative

(iv) Associative (v) Associative (vi) Associative

Example 8 :

Insert five rational numbers between x and | x | where $x=-\frac{17}{20}$.

Solution :

We have $x=-\frac{17}{20}$ $\Rightarrow$ $|x|=\left|-\frac{17}{20}\right|=\frac{17}{20}$

Now, we have to find 5 rational numbers, between $\frac{-17}{20}$ and $\frac{17}{20}$.

As –6, –5, –4, –3, –2 lie between –17 and 17.

Therefore, $-\frac{6}{20},-\frac{5}{20},-\frac{4}{20},-\frac{3}{20}$ and $-\frac{2}{20}$ lie between $-\frac{17}{20}$ and $\frac{17}{20}$.

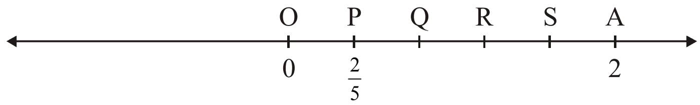
Example 9 :

Represent (a) $\frac{2}{5}$ (b) $\frac{-7}{3}$ on the number line.

Solution :

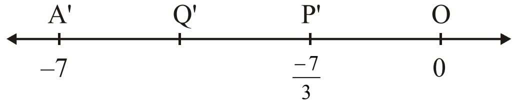
(a) Draw a number line. Mark a point O to represent 0 and another point A to represent the distance 2 units. Divide OA into 5 equal parts (equal to the denominator of $\frac{2}{5}$), at P, Q, R and S (figure)

The point P represents the rational number $\frac{2}{5}$.



(b) Draw a number line. Mark a point O to represent 0 and a point A' at a distance of 7 units on the left of O to represent –7. Divide OA' into 3 equal parts at P' and Q'.

The point P' represents $\frac{-7}{3}$ (figure).

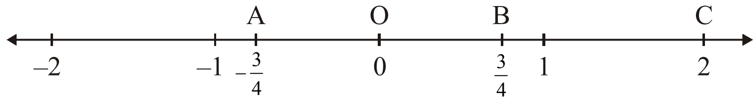


Example 10 :

Find two rational numbers whose absolute value is $\frac{3}{4}$.

Solution :

Draw a number line and mark on it two points A and B whose distance from O is $\frac{3}{4}$ units. (figure)



Now A represents the number $\frac{-3}{4}$ and B represents the number $\frac{3}{4}$. The absolute value of both the numbers $\left(\text{i.e. } \frac{-3}{4} \text{ and } \frac{3}{4}\right)$ is $\frac{3}{4}$. Thus there are two numbers $\frac{-3}{4}$ and $\frac{3}{4}$ whose asbolute value is $\frac{3}{4}$.

Example 11 :

Reshma spent Rs. $18\frac{3}{4}$ on food and bought a pen for Rs. $10\frac{1}{2}$. How much did she spend?

Solution :

Total amount spent = Rs. $18\frac{3}{4}$ and $10\frac{1}{2}$ = Rs. $\left(\frac{75}{4}+\frac{21}{2}\right)$ = Rs. $\frac{75+42}{4}$ = Rs. $\frac{117}{4}$ = Rs. $29\frac{1}{4}$

Example 12 :

Use a short method to find $\frac{1}{2}+\left(\frac{-3}{5}\right)+\frac{3}{2}$.

Solution :

$$\frac{1}{2}+\left(\frac{-3}{5}\right)+\frac{3}{2} = \left(\frac{1}{2}+\frac{3}{2}\right)+\left(\frac{-3}{5}\right) = \frac{1+3}{2}+\left(\frac{-3}{5}\right) = \frac{4}{2}+\left(\frac{-3}{5}\right) = \frac{20-6}{10} = \frac{14}{10} = \frac{7}{5} = 1\frac{2}{5}$$

Example 13:

Simplify : $-2\frac{2}{3}\times 1\frac{1}{4}$

Solution :

$$-2\frac{2}{3} = \frac{-8}{3}$$ (Convert to the form $\frac{p}{q}$)

$$1\frac{1}{4} = \frac{5}{4}$$

$$\therefore \quad -2\frac{2}{3}\times 1\frac{1}{4} = \frac{-8}{3}\times\frac{5}{4} = \frac{-10}{3} = -3\frac{1}{3}$$

Example 14 :

Ruchi bought $2\frac{1}{2}$ kg potatoes at Rs.10 per kg and $1\frac{3}{8}$ kg tomatoes at Rs.$16\frac{8}{11}$ per kg. How much money did she give to the shopkeeper?

Solution :

Cost of 1 kg potatoes = Rs.10

Cost of $2\frac{1}{2}$ kg potatoes = Rs. $10\times\frac{5}{2}$ $\left(2\frac{1}{2}=\frac{5}{2}\right)$

= Rs.25

Cost of 1 kg of tomatoes = Rs. $16\frac{8}{11}$ = Rs. $\frac{184}{11}$

Cost of $1\frac{3}{8}$ kg of tomatoes = Rs. $\frac{184}{11}\times\frac{11}{8}$ $\left(1\frac{3}{8}=\frac{11}{8}\right)$

= Rs.23

Total amount paid = Rs. (25 + 23) = Rs.48

Example 15 :

Is 0.3 the multiplicative inverse of $3\frac{1}{3}$? Why or why not?

Solution :

$3\frac{1}{3}=\frac{10}{3}$ $\therefore$ $0.3\times 3\frac{1}{3}=\frac{3}{10}\times\frac{10}{3}=1$

$\therefore$ 0.3 is the multiplicate inverse of $3\frac{1}{3}$.

Example 16 :

Find 29 rational numbers between $\frac{-2}{5}$ and $\frac{1}{5}$.

Solution :

$\frac{-2}{5}=\frac{-2\times10}{5\times10}=\frac{-20}{50}$ and $\frac{1}{5}=\frac{1\times10}{5\times10}=\frac{10}{50}$

Now the 29 integers between –20 and 10 are –19, –18, –17, ..., 8, 9.

The cooresponding 29 rational between $\frac{-2}{5}$ and $\frac{1}{5}$ are $\frac{-19}{50},\frac{-18}{50},\frac{-17}{50},......,\frac{8}{50},\frac{9}{50}$.

Example 17 :

Insert 5 rational numbers between $-\frac{1}{3}$ and $\frac{1}{2}$.

Solution :

Convert to equivalent rational numbers having same denominators

$\frac{-1}{3} = \frac{-1\times 4}{3\times 4} = \frac{-4}{12}$ and $\frac{1}{2} = \frac{1\times 6}{2\times 6} = \frac{6}{12}$

The integers between –4 and 6 are –3, –2, – 1, 0, 1, 2, 3, 4, 5

The corresponding rational numbers are

$\frac{-3}{12}, \frac{-2}{12}, \frac{-1}{12}, \frac{0}{12}, \frac{1}{12}, \frac{2}{12}, \frac{3}{12}, \frac{4}{12}, \frac{5}{12}$

Example 18 :

Classify the following numbers as rational or irrational.

(i) $\sqrt{225}$ (ii) 7.478047800478000......

Solution :

(i) rational (ii) irrational

Example 19 :

A heap of coconuts is divided into groups of 2, 3 and 5. Each time one coconut is left over. Find the least number of coconuts in the heap.

Solution :

Required least number of coconuts = (L.C.M. of 2, 3 and 5)
+1 = 2 × 3 × 5 + 1 = 30 + 1 = 31

Example 20 :

Prove that $2+\sqrt{3}$ is an irrational number.

Solution :

Let us suppose if possible that $2+\sqrt{3}$ is a rational number.

Then $2+\sqrt{3} = \frac{p}{q}$, where p and q both are integers.

Now, $\sqrt{3} = \frac{p}{q} - \frac{2}{1}$ $\Rightarrow$ $\sqrt{3} = \frac{p-2q}{q}$

Since $\frac{p-2q}{q}$ is a rational number but $\sqrt{3}$ is an irrational number, that condradicts our assumption.

So $2+\sqrt{3}$ is an irrational number.

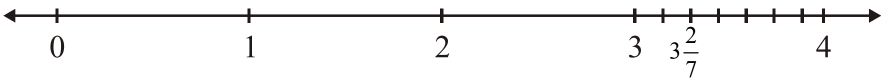
Example 21:

Represent $3\frac{2}{7}$ on the number line.

Solution :

In order to represent $3\frac{2}{7}$ on the number line, take 3 units lengths between 0 and 3 and divide the unit length between 3 and 4 into seven equal parts and take the end of 2nd part on it.

This point represents the rational number $3\frac{2}{7}$.



Example 22 :

Express 1.272727..... = $1.\overline{27}$ in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

Solution :

Let x = 1.272727...... Since two digits are repeating, we multiply x by 100 to get

$$100x = 127.2727.......$$

So, $100x = 126 + 1.272727...... = 126 + x$

Therefore, $100x - x = 126$, i.e. $99x = 126$ i.e.

$$x = \frac{126}{99} = \frac{14}{11}$$

We can check the reverse that $\frac{14}{11} = 1.\overline{27}$

Example 23 :

Express $0.12\overline{3}$ in $\frac{p}{q}$ form.

Solution :

Let $x = 1.2\overline{3}$ i.e. x = 1.2333 .....(i)

Multiply both sides of (i) by 100

$100x = 12.333$ .....(ii)

Multiply both sides of (ii) by 10

$1000x = 123.333$ .....(iii)

Subtract (ii) and (iii)

$$\begin{array}{r} 1000x = 123.333 \\ 100x = 12.333 \\ \hline 900x = 111.0 \\ \hline \end{array}$$

$$\Rightarrow \quad x = \frac{111}{900} = \frac{3\times 37}{900} = \frac{37}{300}$$

Example 24 :

Express $0.\overline{34} + 0.3\overline{4}$ as a single decimal.

Solution :

$0.6\overline{87}$

CONCEPT APPLICATION LEVEL - I [NCERT Questions]

EXERCISE 1

Q.1 **(i)** $-\frac{2}{3}\times\frac{3}{5}+\frac{5}{2}-\frac{3}{5}\times\frac{1}{6}$ **(ii)** $\frac{2}{5}\times\left(-\frac{3}{7}\right)-\frac{1}{6}\times\frac{3}{2}+\frac{1}{14}\times\frac{2}{5}$

Sol. (i) $-\frac{2}{3}\times\frac{3}{5}+\frac{5}{2}-\frac{3}{5}\times\frac{1}{6}$

$$=\frac{3}{5}\times\frac{-2}{3}+\frac{5}{2}-\frac{3}{5}\times\frac{1}{6}$$ [by commutativity]

$$=\frac{3}{5}\times\frac{-2}{3}-\frac{3}{5}\times\frac{1}{6}+\frac{5}{2}$$ [by associativity]

$$=\frac{3}{5}\times\left(\frac{-2}{3}-\frac{1}{6}\right)+\frac{5}{2}$$ [by distributivity]

$$=\frac{3}{5}\times\left(\frac{-4-1}{6}\right)+\frac{5}{2}$$

(ii) $\frac{2}{5}\times\left(-\frac{3}{7}\right)-\frac{1}{6}\times\frac{3}{2}+\frac{1}{14}\times\frac{2}{5}=\frac{2}{5}\times\left(-\frac{3}{7}\right)-\frac{1}{6}\times\frac{3}{2}+\frac{2}{5}\times\frac{1}{14}$ [by commutativity]

$$=\frac{2}{5}\times\left(-\frac{3}{7}\right)+\frac{2}{5}\times\frac{1}{14}-\frac{1}{6}\times\frac{3}{2}$$ [by associativity]

$$=\frac{2}{5}\times\left\{\left(-\frac{3}{7}\right)+\frac{1}{14}\right\}-\frac{1}{6}\times\frac{3}{2}$$ [by distributivity]

$$=\frac{2}{5}\times\left\{\frac{(-6)+1}{14}\right\}-\frac{1}{6}\times\frac{3}{2}$$

$$=\frac{2}{5}\times\left\{\frac{-5}{14}\right\}-\frac{1}{6}\times\frac{3}{2}=\frac{-1}{7}-\frac{1}{4}=\frac{-4-7}{28}=\frac{-11}{28}$$

Q.2 **Write the additive inverse of each of the following :**

(i) $\frac{2}{8}$ **(ii)** $\frac{-5}{9}$ **(iii)** $\frac{-6}{-5}$ **(iv)** $\frac{2}{-9}$ **(v)** $\frac{19}{-6}$

Sol. (i) $\frac{2}{8}$, $\therefore$ Additive inverse of $\frac{2}{8}$ is $\frac{-2}{8}$.

(ii) $\frac{-5}{9}$, $\therefore$ Additive inverse of $\frac{-5}{9}$ is $\frac{5}{9}$

(iii) $\frac{-6}{-5}$, $\frac{-6}{-5}=\frac{6}{5}$ $\therefore$ Additive inverse of $\frac{-6}{-5}$ is $\frac{-6}{5}$

(iv) $\frac{2}{-9}$, $\therefore$ Additive inverse of $\frac{2}{-9}$ is $\frac{2}{9}$

(v) $\frac{19}{-6}$, $\therefore$ Additive inverse of $\frac{19}{-6}$ is $\frac{19}{6}$

Q.3 Verify that – (– x) = x for.

(i) $x = \frac{11}{15}$ **(ii)** $x = -\frac{13}{17}$

Sol. (i) $x = \frac{11}{15}$

LHS $= -(-x)$

$= -\left(-\frac{11}{15}\right) = \frac{11}{15} = x = \text{RHS}$

(ii) $x = -\frac{13}{17}$

LHS $= -(-x)$

$= -\left\{-\left(\frac{-13}{17}\right)\right\} = -\frac{13}{17} = x = \text{RHS}$

Q.4 Find the multiplicative inverse the following :

(i) -13 **(ii)** $\frac{-13}{19}$ **(iii)** $\frac{1}{5}$ **(iv)** $\frac{-5}{8} \times \frac{-3}{7}$

(v) $-1 \times \frac{-2}{5}$ **(vi)** -1

Sol. (i) The multiplicative inverse of -13 is $\frac{-1}{13}$

(ii) The multiplicative inverse of $\frac{-13}{19}$ is $\frac{-19}{13}$

(iii) The multiplicative inverse of $\frac{1}{5}$ is 5.

(iv) $\frac{-5}{8} \times \frac{-3}{7} = \frac{(-5)\times(-3)}{8\times 7} = \frac{15}{56}$

Therefore, the multiplicative inverse of $\frac{-5}{8} \times \frac{-3}{7}$ is $\frac{56}{15}$

(v) $-1 \times \frac{-2}{5} = \frac{(-1)\times(-2)}{5} = \frac{2}{5}$

Therefore, the multiplicative inverse of $-1 \times \frac{-2}{5}$ is $\frac{5}{2}$

(vi) The multiplicative inverse of -1 is -1. [$\because$ $(-1) \times (-1) = 1$]

Q.5 Name the property under multiplication used in each of the following :

(i) $\frac{-4}{5} \times 1 = 1 \times \frac{-4}{5} = -\frac{4}{5}$ **(ii)** $-\frac{13}{17} \times \frac{-2}{7} = \frac{-2}{7} \times \frac{-13}{17}$

(iii) $\frac{-19}{17} \times \frac{17}{-19} = 1$

Sol. (i) 1 is the multiplicative identity (ii) Commutativity (iii) Multiplicative inverse.

Q.6 Multiply $\frac{6}{13}$ by the reciprocal of $\frac{-7}{16}$.

Sol. Reciprocal of $\frac{-7}{16}$ is $\frac{-16}{7}$. Now, $\frac{6}{13} \times \frac{-16}{7} = \frac{6 \times (-16)}{13 \times 7} = \frac{-96}{91}$

Q.7 Tell what property allows you to compute : $\frac{1}{3} \times \left(6 \times \frac{4}{3}\right)$ as $\left(\frac{1}{3} \times 6\right) \times \frac{4}{3}$.

Sol. Associativity.

Q.8 Is $\frac{8}{9}$ the multiplicative inverse of $-1\frac{1}{8}$? Why or why not ?

Sol. $-1\frac{1}{8} = -\frac{9}{8}$

Now, $\frac{8}{9} \times \frac{-9}{8} = -1 \neq 1$

So, No ? $\frac{8}{9}$ is not the multiplicative inverse of $-1\frac{1}{8}\left(= -\frac{9}{8}\right)$ because the product of $\frac{8}{9}$ and $-1\frac{1}{8}\left(= -\frac{9}{8}\right)$ is not 1.

Q.9 Is 0.3 the multiplicative inverse of $3\frac{1}{3}$? Why or why not ?

Sol. $0.3 = \frac{3}{10}$ $\Rightarrow$ $3\frac{1}{3} = \frac{10}{3}$

yes, 0.3 is the multiplicative inverse of $\frac{10}{3}$ becuase $\frac{3}{10} \times \frac{10}{3} = \frac{3 \times 10}{10 \times 3} = \frac{30}{30} = 1$

Q.10 Write :

(i) The rational number that does not have a reciprocal.
(ii) The rational numbers that are equal to their reciprocals.
(iii) The rational number that is equal to its negative.

Sol. (i) The rational number '0' does not have a reciprocal.
(ii) The rational numbers 1 and (– 1) are equal to their reciprocals respectively.
(iii) The rational number 0 is equal to its negative.

Q.11 Fill in the blanks :

(i) Zero has .... reciprocal.

(ii) The numbers .... and .... are their own reciprocals.

(iii) The reciprocal of – 5 is ....

(iv) Reciprocal of $\frac{1}{x}$, where $x \neq 0$ is ....

(v) The product of two rational numbers is always a ....

(vi) The reciprocal of a positive rational number is ....

Sol. (i) Zero has no reciprocal.

(ii) The numbers 1 and – 1 are their own reciprocals.

(iii) The reciprocal of – 5 is $-\frac{1}{5}$.

(iv) Reciprocal of $\frac{1}{x}$, where $x \neq 0$ is x.

(v) The product of two rational numbers is always a rational number.

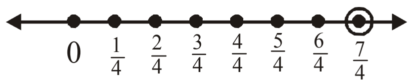
(vi) The reciprocal of a positive rational number is positive.

EXERCISE - 2

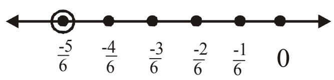
Q.1 Represent these numbers on the number line.

(i) $\frac{7}{4}$ (ii) $\frac{-5}{6}$

Sol. : (i) $\frac{7}{4}$, We make 7 marking s of distance $\frac{1}{4}$ each the right of 0 and starting from 0. The seventh marking represents $\frac{7}{4}$

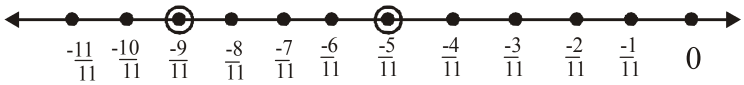


(ii) $\frac{-5}{6}$, We make 5 markings of distance $\frac{1}{6}$ each on the left of 0 and starting from 0. The fifth marking represents $-\frac{5}{6}$.



Q.2 Represent $\frac{-2}{11}, \frac{-5}{11}, \frac{-9}{11}$ on a number line.

Sol.: We make 9 markings of distance $\frac{1}{11}$ each on the left of 0 and starting from 0. The second marking represents $-\frac{2}{11}$; the fifth marking represents $-\frac{5}{11}$ and the ninth marking represents $-\frac{9}{11}$.



Q.3 Write five rational numbers which are smaller than 2.

Sol.: Five rational numbers which are smaller than 2 are 1, $\frac{1}{2}$, 0, – 1, $-\frac{1}{2}$

Q.4 Find ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$.

Sol. $\frac{-2}{5} = \frac{-2\times 2}{5\times 2} = \frac{-4}{10}$

Converting them into rational numbers with the same denominators

$$\frac{1}{2} = \frac{1\times 5}{2\times 5} = \frac{5}{10}$$

Now, $\frac{-4}{10} = \frac{-4\times 2}{10\times 2} = \frac{-8}{20}$

Multiplying the numberator and denominator both by 2

$$\frac{5}{10} = \frac{5\times 2}{10\times 2} = \frac{10}{20}$$

Multiplying the numerator and denominator both by 2

Thus, ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$ may be taken as :

$$\frac{-7}{20}, \frac{-6}{20}, \frac{-5}{20}, \frac{-4}{20}, \frac{-3}{20}, \frac{-2}{20}, \frac{-1}{20}, 0, \frac{1}{20}, \frac{2}{20}$$

Q.5 Find five rational numbers between:

(i) $\frac{2}{3}$ and $\frac{4}{5}$ (ii) $\frac{-3}{2}$ and $\frac{5}{3}$ (iii) $\frac{1}{4}$ and $\frac{1}{2}$

Sol. (i) $\frac{2}{3} = \frac{2\times 5}{3\times 5} = \frac{10}{15}$

$$\frac{4}{5} = \frac{4\times 3}{5\times 3} = \frac{12}{15}$$

Converting them into rational numbers with the same denominators

Now, $\frac{10}{15} = \frac{10\times 4}{15\times 4} = \frac{40}{60}$; $\frac{12\times 4}{15\times 4} = \frac{48}{60}$

Therefore, five rational numbers between $\frac{2}{3}$ and $\frac{4}{5}$ may be taken as

$$\frac{41}{60}, \frac{42}{60}, \frac{43}{60}, \frac{44}{60}, \frac{45}{60}$$

(ii) $\frac{-3}{2} = \frac{-3\times 3}{2\times 3} = \frac{-9}{6}$

$$\frac{5}{3} = \frac{5\times 2}{3\times 2} = \frac{10}{6}$$

Converting them to rational number with the same denominators.

Therefore, five rational numbers between $\frac{-3}{2}$ and $\frac{5}{3}$ may be taken as $\frac{-8}{6}, \frac{-7}{6}, 0, \frac{1}{6}, \frac{2}{6}$

(iii) $\frac{1}{4} = \frac{1}{4}$

$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}$

Converting them into rational numbers with the same denominators.

Now, $\frac{1}{4} = \frac{1 \times 8}{4 \times 8} = \frac{8}{32}$

Multiplying the numerator and denominator both by 8.

Now, $\frac{1}{4} = \frac{1 \times 16}{2 \times 16} = \frac{16}{32}$

Multiplying the numerator and denominator both by 16.

Thus, five rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$ may be taken as

$\frac{9}{32}, \frac{10}{32}, \frac{11}{32}, \frac{12}{32}, \frac{13}{32}$

Q.6 Write five rational numbers greater than – 2.

Sol. Five rational numbers greater than –2 are : $\frac{-3}{2}, -1, \frac{-1}{2}, 0, \frac{1}{2}$

Q.7 Find ten rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$.

Sol. $\frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{12}{20}$

$\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$

Converting them into rational numbers with the same denominators.

Now, $\frac{12}{20} = \frac{12 \times 8}{20 \times 8} = \frac{96}{160}$

Multiplying the numerator and denominator both by 8

$\frac{15}{20} = \frac{15 \times 8}{20 \times 8} = \frac{120}{160}$

Multiplying the numerator and denominator both by 8 Thus, ten rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$ may be taken as :

$\frac{97}{160}, \frac{98}{160}, \frac{99}{160}, \frac{100}{160}, \frac{101}{160}, \frac{102}{160}, \frac{103}{160}, \frac{104}{160}, \frac{105}{160}, \frac{106}{160}$

CONCEPT APPLICATION LEVEL - II

SECTION - A

- **FILL IN THE BLANKS**

Q.1 $\frac{-4}{7}+0=$ ....

Q.2 $\left(\frac{-8}{13}\right)\times$ .... $=\left(\frac{-8}{13}\right)$

Q.3 .... $\times\frac{-17}{47}=0$

Q.4 $\frac{-4}{3}\times\left[\frac{1}{2}+\left(\frac{7}{5}\right)\right]=\left(\frac{-4}{3}\times\text{...........}\right)+\left(\frac{-4}{3}\times\text{...........}\right)$

Q.5 $\frac{-4}{5}\times\left(\frac{5}{7}\times\frac{-8}{9}\right)=\left(\frac{-4}{5}\times\text{..........}\right)\times\frac{-8}{9}$

Q.6 $\frac{2}{5}\div\frac{2}{5}=$ ....

Q.7 $\frac{-11}{15}\div\left(\text{...................}\right)=-1$

Q.8 $\frac{4}{9}\div$ .... $=\frac{4}{9}$

Q.9 Write the rational numbers which are their own reciprocals ....

Q.10 Is subtraction of rational numbers commutative? ....

Q.11 A rational number between x and y is ....

Q.12 $\frac{1}{5}$ lies to the left of 0 on a number line. Is this a true statement? ....

Q.13 How many rational numbers are there altogether between 1 and 2?....................

Q.14 Additive inverse of –1 is ....

Q.15 Put suitable word in the sentence below:

$\frac{35}{50}$ has .... decimal expansion.

Q.16 Number 206.006 0006 00006 is ....

Q.17 $\sqrt{3}-(\sqrt{8}+\sqrt{5})$ is ....

Q.18 $(3-\sqrt{4})^2$ is a ....

Q.19 If a is a rational number and b ($b\neq 0$) is an irrational number, then ab is necessarily, ...

Q.20 For given positive integers a and b, there exists unique integers q and r satisfying $a = bq + r$, $0\leq r < b$ is called ....

Q.21 Every composite number can be expressed as a product of primes, which is unique, apart from the order is which prime factors occur, is called ....

Q.22 For any rational number $\frac{p}{q}$ with terminating decimal representation, the prime factorisation of q is of the form ...., where n and m are non-negative integers.

Q.23 Decimal representation of a rational number can not be ....

Q.24 A number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a ....

Q.25 The rational number .... is the additive identity for rational numbers.

SECTION - B

- **MULTIPLE CHOICE QUESTIONS**

Q.1 The value of $(0.\overline{6} + 0.\overline{7} + 0.\overline{8})$ is

(A) $\frac{21}{10}$ (B) $\frac{19}{9}$ (C) $\frac{7}{3}$ (D) None of these

Q.2 If $1 \leq p \leq 10$, then number of prime numbers are there which are of the form $10p + 1$, is

(A) 10 (B) 7 (C) 6 (D) None of these

Q.3 The absolute value of $|x - 6| + |6 - x|$, when $0 < x < 6$ is

(A) 6x (B) 12 (C) $2(6 - x)$ (D) None of these

Q.4 Ismail wanted to type 150 natural number. The number of times he had to press the numbered keys, is

(A) 332 (B) 342 (C) 352 (D) None of these

Q.5 $\left(\frac{3}{-5} + \frac{2}{-8}\right) + \text{....................} = \frac{3}{-5} + \left(\frac{4}{-7} + \frac{2}{-8}\right)$

(A) $\frac{2}{-7}$ (B) $\frac{2}{7}$ (C) $\frac{4}{-7}$ (D) $\frac{4}{7}$

Q.6 $\left(8 + \frac{-6}{17}\right) + \left(\frac{-4}{17}\right) = \left(\text{..................}\right) + \left(\frac{-6}{17} + \frac{-4}{17}\right)$

(A) $\frac{8}{17}$ (B) 8 (C) $\frac{7}{17}$ (D) 7

Q.7 The rational number $0.\overline{3}$ can also be written as ....

(A) $\frac{3}{10}$ (B) $\frac{33}{100}$ (C) $\frac{1}{3}$ (D) 333

Q.8 A rational number between $\frac{1}{5}$ and $\frac{2}{5}$ is ....

(A) $\frac{3}{5}$ (B) $\frac{30}{100}$ (C) $\frac{32}{6}$ (D) $\frac{20}{15}$

Q.9 The sum of $-\frac{1}{9}$ and $-\frac{1}{9}$ is

(A) 0 (B) 1 (C) $\frac{2}{9}$ (D) $\frac{-2}{9}$

Q.10 The multiplicative inverse of $\frac{1}{6}$ is __________.

(A) –6 (B) 6 (C) $-\frac{1}{6}$ (D) 1

Q.11 The rational number equivalent to $\frac{9}{-18}$ is

(A) $\frac{18}{162}$ (B) $\frac{-81}{162}$ (C) $-\frac{1}{9}$ (D) $\frac{1}{9}$

Q.12 0 reduced by $\frac{1}{2}$ is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 2 (D) –2

Q.13 If x and y are rational numbers then $|x+y|$ is

(A) $|x+y| \le |x|+|y|$ (B) $|x+y| = |x|+|y|$

(C) $|x+y| < |x|+|y|$ (D) $|x+y| \ge |x|+|y|$

Q.14 $-\left|\frac{3}{4}-\frac{2}{3}\right|$ is equal to

(A) $\frac{1}{12}$ (B) $-\frac{1}{12}$ (C) $-\frac{17}{12}$ (D) $\frac{17}{12}$

Q.15 Which of the following statements is false?

(A) $\left|\frac{-5}{3}\right|$ lies on the right of 0 on the number line.

(B) $-|-x| = x$ for all rational numbers.

(C) $\frac{-7}{17}$ lies on the left of 0 on the number line

(D) Every whole number is a rational number.

Q.16 The additive inverse of $\frac{-a}{b}$ is

(A) $\frac{b}{a}$ (B) $\frac{a}{-b}$ (C) $\frac{a}{b}$ (D) $\frac{-b}{a}$

Q.17 0 is

(A) Positive rational number (B) Negative rational number

(C) Either positive or negative rational number (D) Neither positive nor negative rational number

Q.18 If the sum of two rational numbers is –6 and one of them is $\frac{-7}{2}$, then the other number is

(A) $\frac{-5}{2}$ (B) $\frac{5}{2}$ (C) $\frac{-19}{2}$ (D) $\frac{19}{2}$

Q.19 Which of the following statements is true?

(A) $\left(\frac{7}{9}-\frac{11}{12}\right)+\frac{2}{3}=\frac{7}{9}-\left(\frac{11}{12}+\frac{2}{3}\right)$ (B) $\left(\frac{8}{15}+\frac{6}{5}\right)-\frac{5}{12}=\frac{8}{15}+\left(\frac{6}{5}-\frac{5}{12}\right)$

(C) $8-\left(2\frac{3}{5}+2\frac{5}{12}\right)=8-2\frac{3}{5}+2\frac{5}{12}$ (D) $\frac{5}{2}-0=0-\frac{5}{2}$

Q.20 The product of the additive inverse and the multiplicative inverse of -3 is

(A) 1 (B) 0 (C) –1 (D) –9

Q.21 Which property of multiplication is illustrated by $\frac{-2}{3}\times\left(\frac{5}{8}+\frac{-3}{7}\right)=\left(\frac{-2}{3}\times\frac{5}{8}\right)+\left(\frac{-2}{3}\times\frac{-3}{7}\right)$

(A) Commutative (B) Distributive (C) Associative (D) None of these

Q.22 A rational number between $\frac{1}{3}$ and $\frac{1}{4}$ is

(A) 0.09 (B) $\frac{7}{24}$ (C) $\frac{1}{24}$ (D) $\frac{-1}{24}$

Q.23 The difference between the greatest and the least of $\frac{-5}{9}, \frac{2}{9}, \frac{-4}{9}$ is

(A) $\frac{-1}{3}$ (B) $\frac{-2}{9}$ (C) –1 (D) $\frac{7}{9}$

Q.24 What should be added to $\frac{-3}{4}$ to get '–1'?

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) 1 (D) $-\frac{3}{4}$

Q.25 What should be subtracted form $-\frac{3}{4}$ to get '–1'?

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) 1 (D) $-\frac{3}{4}$

Q.26 Which of the following is the multiplicative identity for rational numbers?
(A) 1 (B) –1 (C) 0 (D) None of these

Q.27 Which of the following is neither positive nor a negative rational number ?
(A) 1 (B) 0
(C) Such a rational number does not-exist (D) None of these

Q.28 Which of the following rational numbers lies between 0 and –1?

(A) 0 (B) –1 (C) $\frac{-2}{3}$ (D) $\frac{2}{3}$

Q.29 Which of the following is the reciprocal of the reciprocal of a rational number ?
(A) –1 (B) 1 (C) 0 (D) The rational number itself

Q.30 A train goes 80 km in one hour. How much distance will it cover in 45 minutes?
(A) 70 km (B) 60 km (C) 50 km (D) 40 km

Q.31 A man has Rs.100 with him. He bought $3\frac{1}{2}$ litres of milk at Rs. $16\frac{1}{2}$ per litre. How much money is left with him?

(A) Rs. $42\frac{1}{4}$ (B) Rs. $42\frac{1}{3}$ (C) Rs. $44\frac{1}{4}$ (D) Rs. $44\frac{1}{3}$

Q.32 Praneeta bought $3\frac{1}{2}$ m ribbon at Rs. $5\frac{3}{7}$ per metre, $4\frac{3}{4}$ m cloth at Rs. $27\frac{1}{2}$ per metre. How much money did she spend?

(A) Rs. $140\frac{5}{8}$ (B) Rs. $149\frac{5}{8}$ (C) Rs. $145\frac{5}{8}$ (D) Rs. $140\frac{3}{8}$

Q.33 If $-\frac{8}{17}+\frac{4}{5}=\frac{4}{5}+x$, then x is

(A) $\frac{4}{5}$ (B) $\frac{8}{17}$ (C) $\frac{8}{5}$ (D) $-\frac{8}{17}$

Q.34 What should be added to $\frac{1}{3}+\frac{1}{5}+\frac{7}{15}$ to get sum 0 ?

(A) $-\frac{1}{3}$ (B) -1 (C) $-\frac{1}{5}$ (D) $-\frac{7}{15}$

Q.35 The property $x \times (y + z) = x \times y + x \times z$ is known as
(A) commutative property (B) closure property
(C) associative property (D) distributive property

Q.36 The number $\frac{11}{3}$ on the number line will be represented between which two consecutive odd natural numbers?
(A) 1 and 2 (B) 1 and 3 (C) 3 and 4 (D) 3 and 5

Q.37 If $\left(-\frac{4}{9}\right)\div p=\frac{8}{15}$, then p is

(A) $\frac{15}{8}$ (B) $-\frac{5}{6}$ (C) $-\frac{6}{5}$ (D) $-\frac{4}{9}$

Q.38 If x is a rational number, such that $x \times x = x$, then x is

(A) x (B) x^2 (C) 1 (D) $\frac{1}{x}$

Q.39 The sum of two rational numbers is $\frac{3}{7}$, if one of the numbers is $-\frac{3}{10}$, then other number is

(A) $\frac{5}{7}$ (B) $\frac{51}{10}$ (C) $\frac{51}{70}$ (D) $\frac{51}{7}$

Q.40 What number should be subtracted from $-\frac{5}{4}$ to get additive identity?

(A) $\frac{5}{4}$ (B) $-\frac{5}{4}$ (C) $\frac{6}{7}$ (D) $-\frac{6}{7}$

Q.41 What number should be added to $-\frac{5}{4}$ to get its multiplicative inverse?

(A) $\frac{4}{5}$ (B) $-\frac{4}{5}$ (C) $\frac{20}{9}$ (D) $\frac{9}{20}$

Q.42 What should be added to $\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{5}\right)$ to get 1?

(A) $-\frac{31}{30}$ (B) $\frac{31}{30}$ (C) $\frac{1}{30}$ (D) $-\frac{1}{30}$

Q.43 On dividing the sum of $\frac{18}{5}$ and $-\frac{7}{15}$ by their difference we get

(A) $\frac{47}{61}$ (B) $\frac{61}{47}$ (C) $\frac{47}{15}$ (D) $\frac{61}{15}$

Q.44 Between two rational numbers –2 and 2, which whole numbers are there?
(A) –1, 0 (B) 0, 1 (C) 1, 2 (D) –2, –1

Q.45 What should be subtracted from $\frac{-5}{9}$ to get $\frac{1}{6}$?

(A) $\frac{7}{3}$ (B) $\frac{6}{3}$ (C) $\frac{-13}{18}$ (D) $\frac{-12}{18}$

Q.46 Write the multiplicative inverse of $\frac{-6}{5}\times\frac{2}{-3}$

(A) 1 (B) $\frac{5}{4}$ (C) $\frac{4}{5}$ (D) 0

Q.47 Write the additive inverse of $\frac{-5}{6}+\frac{2}{3}$

(A) $\frac{1}{6}$ (B) $\frac{-1}{6}$ (C) 6 (D) –6

Q.48 Using distributive property, evaluate $\frac{-5}{3}\times\frac{5}{7}-\frac{4}{7}\times\frac{5}{3}$

(A) $\frac{15}{7}$ (B) $-\frac{15}{7}$ (C) $\frac{45}{21}$ (D) $\frac{15}{21}$

Q.49 The product of two rational numbers is $\frac{-56}{25}$. If one number is $\frac{-8}{15}$, find the other.

(A) $\frac{42}{10}$ (B) $\frac{21}{4}$ (C) $\frac{42}{5}$ (D) $\frac{21}{10}$

Q.50 Divide the sum of $\frac{-2}{5}$ and $\frac{5}{4}$ by their difference.

(A) $\frac{-7}{12}$ (B) $\frac{-17}{33}$ (C) $\frac{22}{20}$ (D) $\frac{-33}{20}$

Q.51 What number should be added to $\frac{-3}{8}$ to get $\frac{7}{9}$?

(A) $\frac{83}{72}$ (B) $\frac{29}{72}$ (C) $\frac{-17}{18}$ (D) $\frac{17}{18}$

Q.52 Subtract the sum of $\frac{-5}{8}$ and $\frac{7}{10}$ from the sum of $\frac{3}{-5}$ and $\frac{8}{15}$.

(A) $\frac{170}{1200}$ (B) $\frac{-17}{120}$ (C) $\frac{14}{119}$ (D) $\frac{180}{1200}$

Q.53 A piece of wire $\frac{15}{4}$ m long is broken into pieces. One piece is $2\frac{1}{2}$ m long. Find the length of the other piece

(A) $\frac{6}{7}$ m (B) $\frac{5}{9}$ m (C) $\frac{5}{4}$ m (D) $\frac{5}{2}$ m

Q.54 By what number should we multiply $\frac{-12}{13}$ to get $\frac{4}{39}$?

(A) $\frac{3}{27}$ (B) $\frac{4}{9}$ (C) $-\frac{5}{9}$ (D) $-\frac{1}{9}$

Q.55 Divide the sum of $\frac{11}{7}$ and $\frac{-7}{5}$ by their product.

(A) $\frac{1}{11}$ (B) $\frac{-6}{77}$ (C) $\frac{-4}{35}$ (D) $\frac{-11}{5}$

Q.56 Divide the sum of $\frac{-9}{4}$ and $\frac{-8}{3}$ by the difference of $\frac{13}{8}$ and $\frac{-7}{16}$.

(A) $\frac{-236}{99}$ (B) $\frac{21}{9}$ (C) $\frac{-27}{11}$ (D) $\frac{5}{8}$

Q.57 The cost of $5\frac{2}{7}$ metres of cloth is Rs. $28\frac{1}{3}$. What is the cost of 1 metre of cloth?

(A) Rs. $10\frac{1}{10}$ (B) $4\frac{51}{111}$ (C) Rs. $\frac{595}{111}$ (D) $\frac{695}{111}$

Q.58 Find the area of a square piece of land whose each side measures $6\frac{1}{4}$ m.

(A) $\frac{625}{16}m^2$ (B) $\frac{25}{4}m^2$ (C) $\frac{605}{16}m^2$ (D) $\frac{1205}{32}m^2$

Q.59 The area of a rectangle is $45\frac{1}{2}$ m^2. If its length is $3\frac{1}{4}$ m, what is its breadth?

(A) $\frac{15}{2}$ m (B) 7 m (C) 15 m (D) 14 m

SECTION - C

- **Match the Following :**

Q.1

	Column I		**Column II**
(A)	An irrational number between $\sqrt{2}$ and $\sqrt{3}$ is	(p)	$\frac{53}{125}$
(B)	Value of 0.424 is	(q)	$2-\sqrt{3}$
(C)	If $\sqrt{3}=1.732$, then value of $(2+\sqrt{3})$	(r)	$\frac{\sqrt{2}+\sqrt{3}}{2}$
(D)	Rationalising factor of $(2+\sqrt{3})$ is	(s)	3.732

Q.2

	Column I		**Column II**
(A)	The sum of two irrational numbers is not always	(p)	a rational number
(B)	Average of two rational is always	(q)	non-terminating and non-repeating
(C)	Decimal representation of $\sqrt{3}$	(r)	an irrational number
(D)	Between any two rational numbers, number of rational number is	(s)	5
(E)	If ab = 60 and HCF of a and b = 12, then LCM of a and b is	(t)	infinite

Q.3

	Column I		**Column II**
(A)	Distributive property of multiplication over addition is	(p)	rational numbers
(B)	A rational number which lies between any two rational numbers a and b is	(q)	For any three rational numbers a, b and c; we have a(b + c) = ab + ac
(C)	All integers are	(r)	irrational
(D)	Square root of all positive prime numbers are	(s)	$\frac{a+b}{2}$

Q.4

	Column I		Column II
(A)	$\frac{551}{2^3 \times 5^6 \times 7^9}$	(p)	is a non-terminating but repeating decimal representation
(B)	$\frac{422}{2^3 \times 5^4}$	(q)	is an irrational number
(C)	$\frac{2}{\sqrt{3}}$	(r)	is a terminating decimal representation
(D)	$\sqrt{5} - 4$	(s)	is a rational number
		(t)	is non-terminating and non-reccurring decimal representation

SECTION - D

- **Assertion and Reason**

Direction : *Each of these question contains an Assertion followed by Reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(A) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.

(B) If both **Assertion** and **Reason** are correct, but Reason is **not the correct explanation** of Assertion.

(C) If **Assertion** is **correct** but **Reason** is **incorrect.**

(D) If **Assertion** is **incorrect** but **Reason** is **correct**.

Q.1 **Assertion:** (3, 5) and (17, 19) are twin prime.

Reason : A pair of primes which differ by 2 are called twin primes.

Q.2 **Assertion:** Sum of two irrational number $(2-\sqrt{5})$ and $(2+\sqrt{5})$ is also an irrational number.

Reason : Sum of two irrational number need not be an irrational number.

Q.3 **Assertion:** $5\sqrt{3}$ is an irrational number.

Reason : For any two given integers a and b there exist unique integers q and r satisfying a = bq + r; $0 \le r < b$.

ANSWER KEY

CONCEPT APPLICATION LEVEL - II

SECTION - A

Q.1	$\frac{-4}{7}$	Q.2	1	Q.3	0	Q.4	$\frac{1}{2}, \frac{7}{5}$	Q.5	$\frac{5}{7}$	Q.6	1
Q.7	$\frac{11}{15}$	Q.8	1	Q.9	1 and –1	Q.10	No	Q.11	$\frac{x+y}{2}$	Q.12	No

Q.13 infinite Q.14 1 Q.15 terminating Q.16 an irrational number

Q.17 an irrational number Q.18 rational number Q.19 an irrational number

Q.20 Euclid's Division Lemma Q.21 fundamental theorem of arithmetic

Q.22 $2^n 5^m$ Q.23 non-terminating, non repeating

Q.24 rational number Q.25 zero

SECTION - B

Q.1	C	Q.2	C	Q.3	C	Q.4	B	Q.5	C	Q.6	B	Q.7	C
Q.8	B	Q.9	D	Q.10	B	Q.11	B	Q.12	B	Q.13	A	Q.14	B
Q.15	B	Q.16	C	Q.17	D	Q.18	A	Q.19	B	Q.20	C	Q.21	B
Q.22	B	Q.23	D	Q.24	B	Q.25	A	Q.26	A	Q.27	B	Q.28	C
Q.29	D	Q.30	B	Q.31	A	Q.32	B	Q.33	D	Q.34	B	Q.35	D
Q.36	D	Q.37	B	Q.38	C	Q.39	C	Q.40	B	Q.41	D	Q.42	D
Q.43	A	Q.44	B	Q.45	C	Q.46	B	Q.47	A	Q.48	B	Q.49	A
Q.50	B	Q.51	A	Q.52	B	Q.53	C	Q.54	D	Q.55	B	Q.56	A
Q.57	C	Q.58	A	Q.59	D								

SECTION - C

Q.1 (A)- r; (B)-p; (C)-s; (D)-q

Q.2 (A)- r; (B)-p; (C)-q; (D)-t; (E)-s

Q.3 (A)- q; (B)-s; (C)-p; (D)-r

Q.4 (A)- p,s ; (B)-r,s; (C)-q,t; (D)-q,t

SECTION - D

Q.1 A Q.2 D Q.3 B