15 VISUALISING SOLID SHAPE

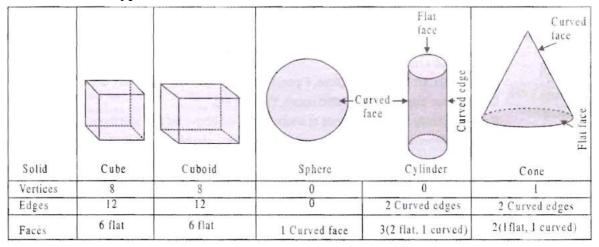
15.1 INTRODUCTION

We live in a 3-dimensional (3D) world but we constantly want to represent what we see in 3D and 2-dimensions (2D). In previous classes we have read a lot about 3-dimensional shapes. Here, we shall revise what we have read and learn more about 3D objects.

The figures having only two dimensions are called plane figures or 2-dimensional (2D) figure. For example, triangles, quadrilaterals etc., are plane figures or 2-dimensional figures.

The figures having three dimensions (length, breadth and height) are called solid figures or 3-dimensional (3D) figures. For example, cube, cuboid, cone, pyramid etc., are solid figures or 3-dimensional figures.

15.2 VARIOUS TYPES OF SOLIDS



Some Common Types of Solids :

15.2.1 Prisms

A prims is a solid formed by joining two congruent plane shapes together with straight line. The two congruent shapes called the bases, are parallel to each other.

For example : A rectangular prism is formed by joining two identical rectangles together with straight lines.

Solids	Rectangular Prism (cuboid)	Square Prism (cube)	Triangular Prism	Pentagonal Prism	Hexagonal Prism
Vertices	8	8	6	10	12
Edges	12	12	9	15	18
Faces	6 (All rectangles)	6 (All squares)	5 [2 triangles 3 rectangles]	7 2 Pentagons 5 rectangles	8 2 hexagon 6 rectangles

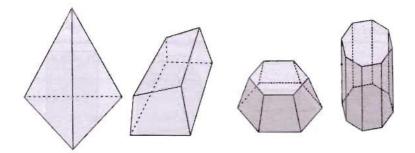
15.2.2 Pyramids

A pyramid is any three-dimensional solid where the upper surfaces are triangular and converge at one point. It has one base (usually a polygon). A pyramid is named according to the shape of its base. **For example :** A pyramid with pentagonal base is a pentagonal pyramid.

Solids	Triangular Pyramid	Rectangle Pyramid	Pentagonal Pyramid	Hexagonal Pyramid
Vertices	4	5	6	7
Edges	6	8	10	12
Faces	4 (All Triangles)	5 [4 Triangles [1 Rectangle]	6 [5 Triangles] 1 Pentagon]	7 [6 Triangles] 1 Hexagon]

15.2.3 Polyhedron

Polyhendron is a geometric solid with faces and straight edges. A polyhedron may be classified according to the number of its faces, as shown below.

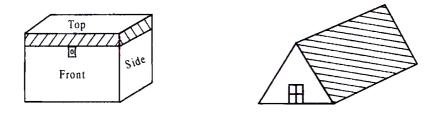


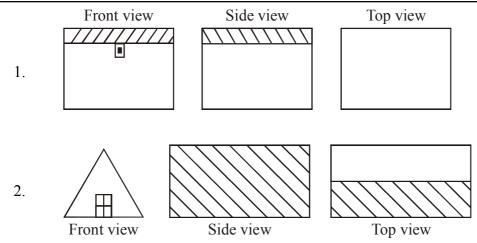
15.3 DIFFERENT VIEW OF SOLIDS

Every solid has four different views according to our looking at them.

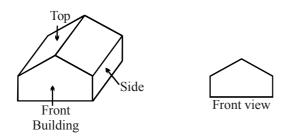
- (1) Top view
- (2) Side view
- (3) Front view
- (4) View from any angle.

We shall understand it by a few examples : We take a few 3-D object.

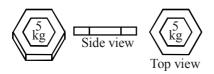




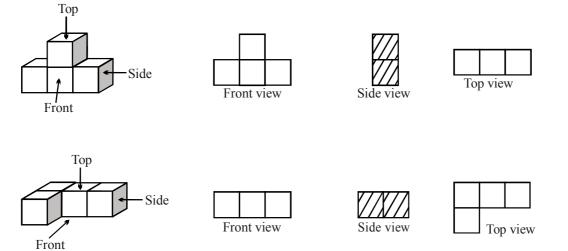
For example, the given building can have following views.



Similarly, a 5 kg weight can have the following views.



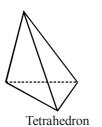
Similarly, we can get different views of figures made by joining cubes.

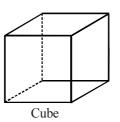


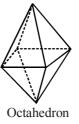
15.4 EULER'S FORMULA

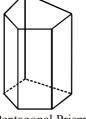
Leonhard Euler was Swiss mathematician. One of his most remarkable observation gave a relationship among the number of vertices (V), edges (E) and faces (F) of a polyhedron.

To understand Euler's formula we study the following figures :







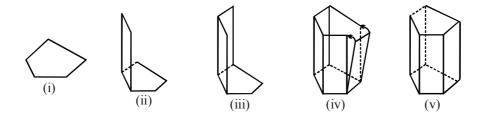


Pentagonal Prism

	F	V	$\mathbf{F} + \mathbf{V}$	E	F + V — E
Tetrahedron	4	4	8	6	2
Cube	6	8	14	12	2
Octahedron	8	6	14	12	2
Pentagonal Prism	7	10	17	15	2

15.4.1 Explanation

Let us build the prism by successively adding faces to the base of the prism shown above.



- (i) First put the base.
- (ii) Now add the second face. In adding a second face there are two vertices, and one edge is common with the first so that the number of new edges is one more than the new vertices. But we have added one face.

 \therefore V – E + F = 1 is true for two faces joined along one edge.

- (iii) Add a new face. The number of new edges is one more than the number of new vertices and one face is added.
- (iv) The formula V E + F = 1 remains true for each added face.
- (v) When the last face is added, no new edges or vertices are added but one face is added and the formula becomes : V E + F = 2 or F + V = E + 2

Remember	1.	The figures having only two dimensions are called plane figures or 2-dimensional figures.
(CE)	2.	The figures having three dimensions (length, breadth and height) are called solid figures or 3-dimensional figures.
	3.	Prisms are solid figures which have a uniform cross-section. A prism is named according to its cross-section.
	4.	A pyramid has a plane figure for a base and all other sides are triangles meeting at one point. A pyramid is named according to the shape of its non-triangular face. If all its faces are triangular, then it is called a triangular pyramid.
	5.	A solid shape made up of polygonal regions is called a polyhedron.
	6.	The concept of convex polyhedron is similar to the concept of convex polygons.
	7.	A polyhedron is said to be regular, if its faces are made up of regular polygons and the same number of faces meet at each vertex.
	8.	For every simple polyhedron $F - E + V = 2$, where F, E and V denote the number of faces, edges and vertices respectively of the polyhedron. This is called Euler's formula.

SOLVED EXAMPLE

Example 1:

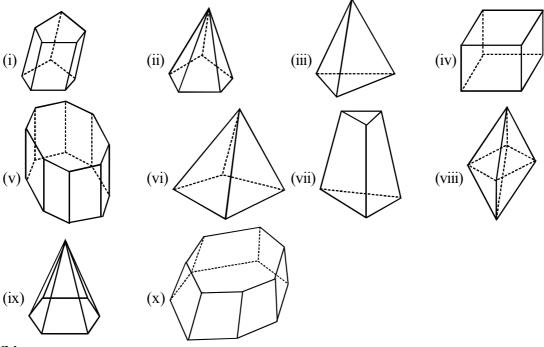
How many f	faces does each of the foll	owing solid figures have?	

(i) Cylinder	(ii) Cone	(iii) Square	prism	(iv) Regular Octahedron
(v) Octagonal	pyramid			
Solution :				
(i) 3	(ii) 2	(iii)6	(iv) 8	(v) 9

Example 2 :

Copy and complete the table by referring to the diagrams given below, where :

F represents the number of faces of a solid, E represents the number of edges and V represents the number of vertices, In each case verify the Euler's formula : F + V = E + 2



Solution :

Solid	F	Ε	V	F + V	E + 2
(i)	7	15	10	17	17
(ii)	6	10	6	12	12
(iii)	4	6	4	8	8
(iv)	6	12	8	14	14
(v)	10	24	16	26	26
(vi)	5	8	5	10	10
(vii)	5	9	6	11	11
(viii)	8	12	6	14	14
(ix)	7	12	7	14	14
(x)	9	21	14	23	23

Example 3 :

Why the following solids are not polyhedron?

(ii) A cone

(i) A sphere

(iii)Acylinder

Solution :

Since, a polyhedron is a solid shape bounded by polygons. However, (i) a sphere (ii) a cone and (iii) a cylinder are not polyhendron because they are made of polygons. i.e., their faces are not polygons.

Example 4 :

A polyhedron is having 8 vertices and 12 edges. How many faces of it are there?

Solution :

Number of vertices (V) = 8 Number of edge (E) = 12 Let the number of faces = F Now, using Euler's formula F + V = E + 2We have $F + 8 = 12 + 2 \implies F + 8 = 14 \implies F = 14 - 8$ $\Rightarrow F = 6$

Thus, the required number of faces = 6.

Example 5 :

An icosahedron is having 20 triangular faces and 12 vertices. Find the number of its edges.

Solution :

Here : Number of vertices (V) = 12 Let the number fo edges be E. \therefore Using Euler's formula we have $F + V = E + 2 \implies 20 + 12 = E + 2 \implies 32 = E + 2$ $\Rightarrow E = 32 - 2 = 30$ Thus, the required number of edges = 30.

Example 6 :

What is the least number of planes that can enclose a solid? Name the simplest regular polyhedron and verify Euler's formula for it.

Solution :

At least 4 planes can form to enclose a solid. Tetrahedron is the simple polyhedron. Following figure represents a simplest solid, called tetrahedron.

A tetrahedron has :

4 triangular faces, i.e., F = 44 vertices, i.e., V = 46 edges, i.e., E = 6Now, substituting the value of F, V and E in Euler's formula, i.e, F + V = E + 2We have 4 + 4 = 6 + 2 8 = 8, which is true. Thus, Euler's formula is verified for a tetrahedron.

Example 7 :

Can a polyhedron have 12 faces, 22 edges and 17 vertices?

Solution :

Euler's formula for polyhedron : (No. of faces) + (No. of vertices) = (No. of edges) + 2 L.H.S = 12 + 17 = 29R.H.S = 22 + 2 = 24 \therefore L.H.S \neq R.H.S Thus Faderle Formula for a increase such as a face and a second

Thus Euler's Formula for given number of faces, edges and vertices is not satisfied. Therefore there can not be polyhedron of the given number of faces, edges and vertices.

Example 8:

Can a polyhedron have 14 faces, 20 edges and 8 vertices?

Solution :

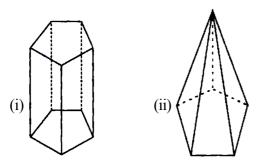
Euler's formula for polyhedron :

(No. of faces) + (No. of vertices) = (No. of edges) + 2 L.H.S = 14 + 8 = 22R.H.S = 20 + 2 = 22 \therefore L.H.S = R.H.S

Thus Euler's Formula for given number of faces, edges and vertices is satisfied. Therefore there can be polyhedron of the given number of faces, edges and vertices.

Example 9:

Verify Euler's formula for the given figures.



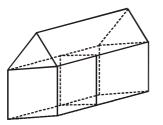
Solution :

(i) Euler's formula for polyhedron : (No. of faces) + (No. of vertices) = (No. of edges) + 2L.H.S = 7 + 10 = 17R.H.S = 15 + 2 = 17L.H.S = R.H.S.... Hence Euler's Formula is verified. (ii) Euler's formula for polyhedron : (No. of faces) + (No. of vertices) = (No. of edges) + 2L.H.S = 6 + 6 = 12R.H.S = 10 + 2 = 12L.H.S = R.H.S.... Hence Euler's Formula is verified.

Picture based questions

Example 10:

The model was build using 3 solids.



Name the solids.

Solution :

	(i)	Triangular Prism	(ii)	Cube	(iii)	Cube
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Example 11:

How many vertices, faces and edges has (a) an octagonal pyramid (b) a hexagonal prism

Solution :

(a)	In octagonal pyramid		
	Number of vertices $= 8 + 1 = 9$		
	Number of faces $= 8 + 1 = 9$		
	Number of edges $= V + F - E = 2$		
	= 9 + 9 - E = 2	\Rightarrow	E=16
(b)	In hexagonal prism,		
	Number of vertices = $2 \times 6 = 12$		
	Number of faces $= 6 + 2 = 8$		
	Number of edges = $V + F - E = 2$		
	= 12 + 8 - E = 2	\Rightarrow	E = 18

Example 12 :

A polyhedron has 30 edges and 20 vertices. How many faces does this polyhedron have ? **Solution :**

Here, E = 30, V = 20 $\therefore V + F - E = 2$ 20 + F - 30 = 2 F = 2 - 20 + 30 = 12Hence the net hedron has 12 for

Hence the polyhedron has 12 faces.

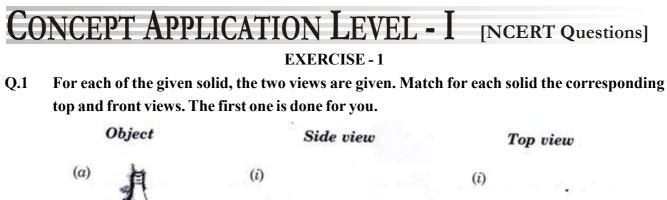
Example 13:

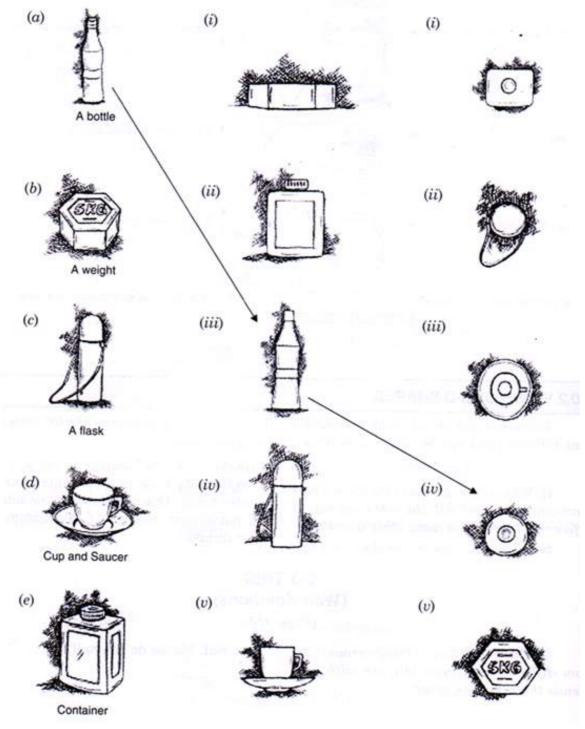
In the above example, the solid is a prism on a 10 sides polygon. How many edges does this polyhedron have ?

Sol. Here, F = 20, V = 12

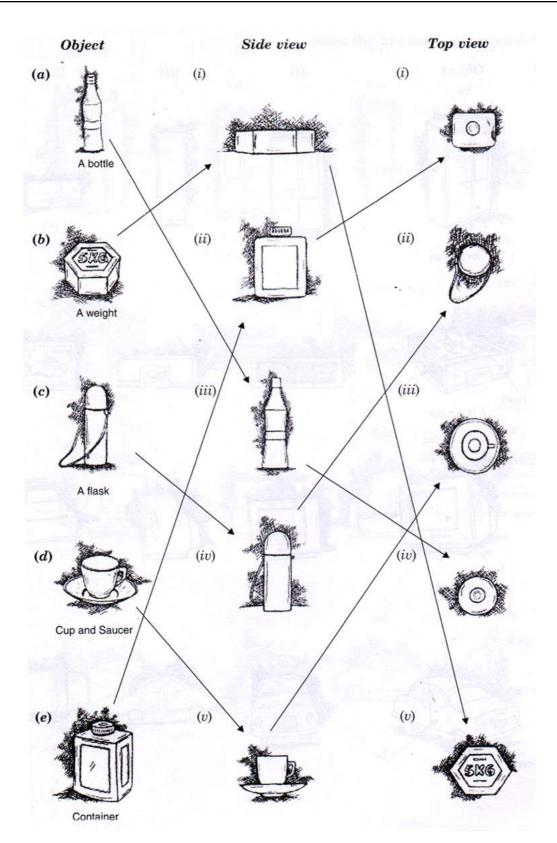
 $\therefore \quad V + F - E = 2$ or $12 + 20 \quad E = 2$ $\therefore \quad E = 12 + 20 - 2 = 30$

Hence the polyhedron has 30 edges.

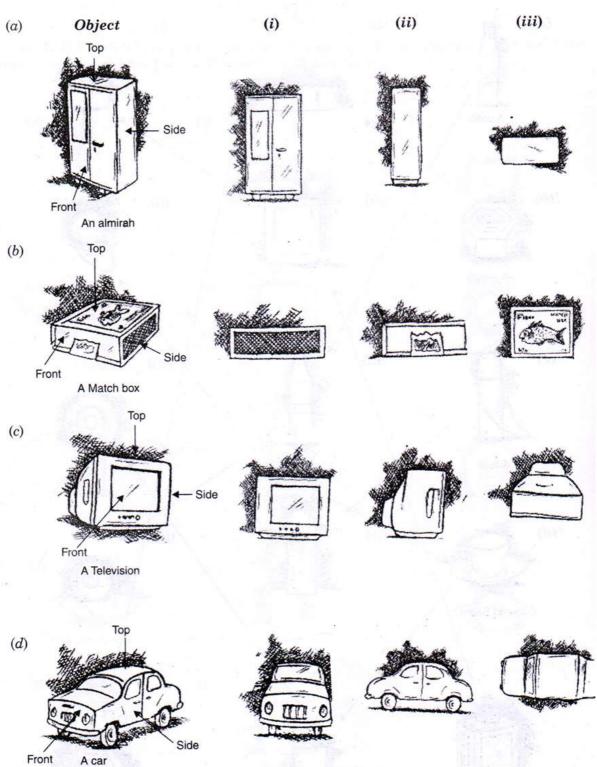


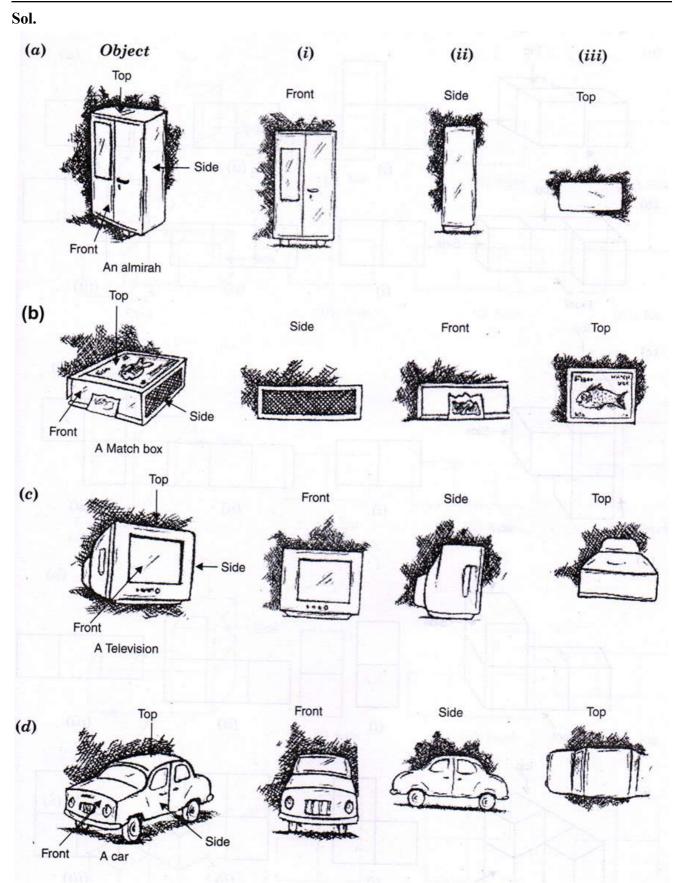


Sol.

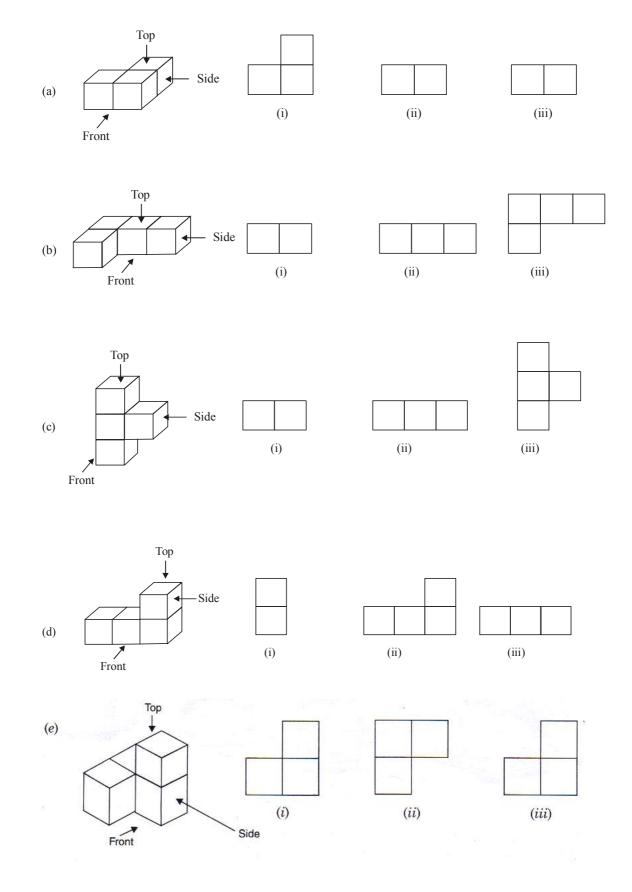


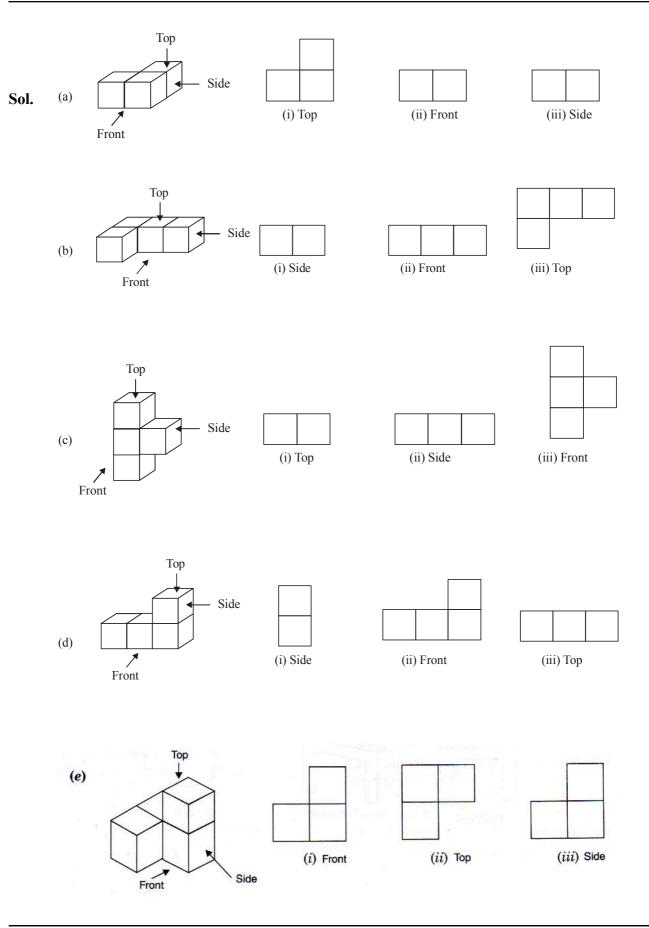
Q.2 For each of the given solid, the three views are given. Identify for each solid the corresponding top, front and side views.





Q.3 For each given solid, identify the top view, front view and side view.





Q.4 Draw the front view, side view and top view of the given objects.

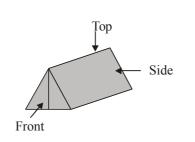
(a) A military tent

A table

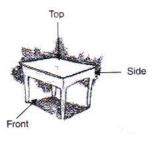
(b)

(d)

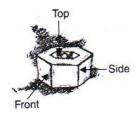
(f)

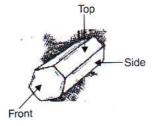






A hexagonal block

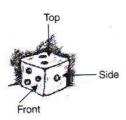




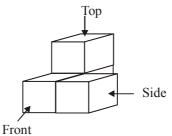
(e)

A dice

(c)







Sol.

(b)



Front view

Side view

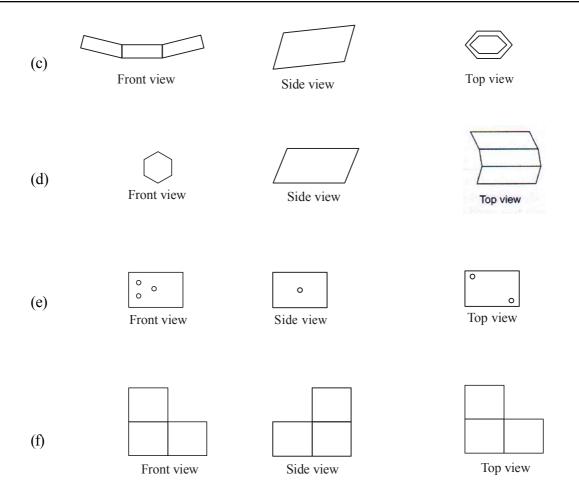




Top view



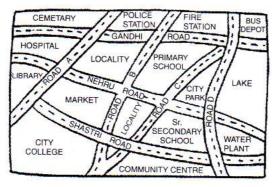
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EXERCISE - 2

- Q.1 Look at the given map of a city. Answer the following
 - (a) Colour the map as follows : Blue-water, Red-fire station, Orange-Library, Yellow-Schools, Green-Park, Pink-College, Purple-Hospital, Brown-Cemetery.
 - (b) Mark a green 'X' at the intersection of Road 'C' and Nehru Road, Green 'Y' at the intersection of Gandhi Road and Road A.
 - (c) In red, draw a short street route from Library to the bus depot.
 - (d) Which is further east, the city park or the market ?



(e) Which is further south, the primary school or the Sr. Secondary School?

Sol. (a) Please colour yourself. (d) City park (b) Please mark yourself

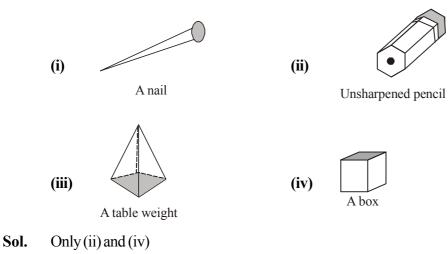
(c) Please draw yourself

) City park (e) Secondary school

EXERCISE - 3

Q.1	.1 Can a polyhedron have for its faces		
	(i) 3 triangles ?	(ii) 4 triangles ?	(iii) a square and four triangles ?
Sol.	(i) No	(ii) Yes	(iii) Yes

- Q.2 Is it possible to have a polyhedron with any given number of faces ? (Hint : Think of a pyramid).
- Sol. Possible, only if the number of faces are greater than or equal to 4.
- Q.3 Which are prisms among the following?



Q.4 (i) How are prisms and cylinders alike?

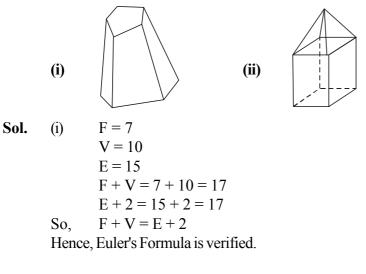
(ii) How are pyramids and cones alike ?

- Sol. (i) A prism becomes a cylinder as the number of sides of its base becomes larger and larger.
 - (ii) A pyramid becomes a cone as the number of sides of its base becomes larger and larger.

Q.5 Is a square prism same as a cube ? Explain.

Sol. No. It can be a cuboid also.

Q.6 Verify Euler's formula for these solids.



(ii)
$$F = 9$$

 $V = 9$
 $E = 16$
 $F + V = 9 + 9 = 18$
 $E + 2 = 16 + 2 = 18$
So, $F + V = E + 2$

Hence, Euler's Formula is verified.

Q.7 Using Euler's formula find the unknown.

	(i)	(ii)	(iii)
Faces	?	5	20
Vertices	6	?	12
Edges	12	9	?

Sol. (i) F + V = E + 2

	\Rightarrow	F + 6 = 12 + 2
	\Rightarrow	F + 6 = 14
	\Rightarrow	F = 14 - 6 = 8
(ii)	F + V	V = E + 2
	\Rightarrow	5 + V = 9 + 2
	\Rightarrow	5 + V = 11
	\Rightarrow	V = 11 - 5 = 6

(iii) F + V = E + 2 $\Rightarrow 20 + 12 = E + 2$ $\Rightarrow 32 = E + 2$ $\Rightarrow E = 32 - 2$ $\Rightarrow E = 30$

Q.8 Can a polyhedron have 10 faces, 20 edges and 15 vertices ?

- Sol. Here F = 10E = 20
 - E = 20V = 15
 - So, F + V = 10 + 15 = 25
 - E + 2 = 20 + 2 = 22
 - $\therefore \qquad F + V \neq E + 2$
 - \therefore Such a polyhedron is not possible

DO THIS

Q.1 Tabulate the number of faces, edges and vertices for the following polyhedrons : (Here 'V' stands for number of vertices. 'F' stands for number of faces and 'E' stands for number of edges).

Solid	F	V	Ε	$\mathbf{F} + \mathbf{V}$	E + 2
Cuboid					
Triangular pyramid					
Triangular prism					
Pyramid with square base					
Prism with square base					

What do you infer from the last two columns ? In each case, do you find F + V = E + 2, i.e., F + V - E = 2? This relationship is called Euler's formula. In fact this formula is true for any polyhedron.

Sol.

Solid	F	V	Ε	$\mathbf{F} + \mathbf{V}$	E + 2
Cuboid	6	8	12	14	14
Triangular pyramid	4	4	6	8	8
Triangular prism	5	6	9	11	11
Pyramid with square base	5	5	8	10	10
Prism with square base	6	8	12	14	14

CONCEPT APPLICATION LEVEL - II SECTION - A

► FILL IN THE BLANKS

- Q.1 A cuboid is called a _____.
- Q.2 A regular tetrahedron has _____ faces.
- Q.3 A hexagonal prism has ______ edges.
- Q.4 An octagon pyramid has _____ vertices.
- Q.5 A regular octahedron is formed when _____ pyramid with _____ triangles as lateral faces are joined.

___.

- Q.6 A solid figure which has only one vertex is _____.
- Q.7 An iron almirah looks like a _____.
- Q.8 Total faces in a pyramid which has eight edges are _____.
- Q.9 A solid whose surface is made up of polygonal faces is called a _____.
- Q.10 Name the solid figure which has 6 vertices, 12 edges and 8 triangular faces _____.
- Q.11 Solids are shown on paper by their _____ representations.
- Q.12 If all corners of a polygon are joined to a point not lying in its plane. We get a _____.
- Q.13 The side faces of a pyramid form its _____.
- Q.14 The end of which a prism may be supposed to stand is called ______ of the prism.
- Q.15 The perpendicular distance between the ends of a prism is its _____
- Q.16 The straight line joining the centres of the ends of a prism is called the _____ of the prism.
- Q.17 A pyramid is called a quadrilateral pyramid if its base is _____.
- Q.18 A tetrahedron has vertices.
- Q.19 Each face of a tetrahedron is an _____ triangle.
- Q.20 A cylinder has _____ faces.
- Q.21 An octahedron has ______ faces ______ vertices ______ edges.
- Q.22 A pyramid on n sided polygon has ______ faces ______ vertices ______ edges.
- Q.23 A prism on n sides polygon has ______ faces ______ vertices ______ edges.
- Q.24 A regular prism has all its ______ equal.

MULTIPE CHOICE OUESTIONS

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SECTION-B

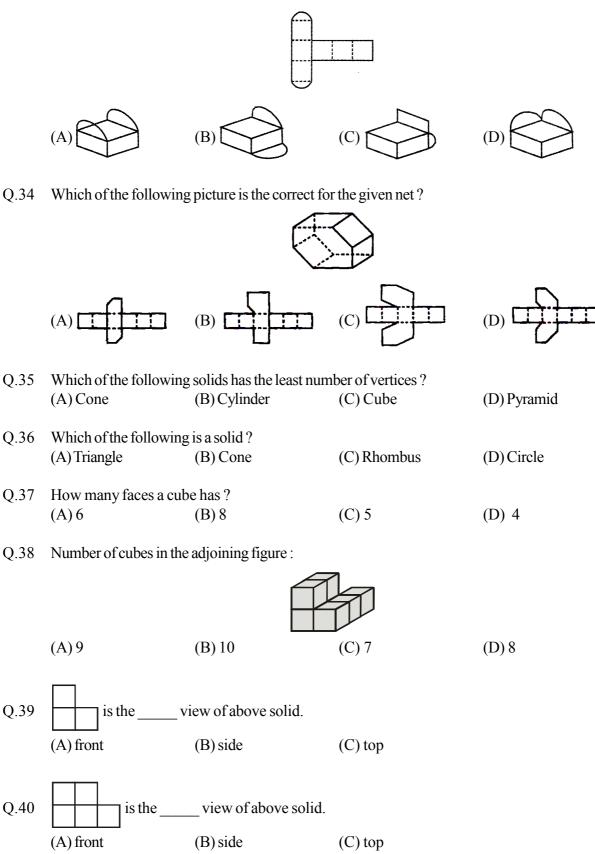
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Q.1	The top view of a cub	01d 1s a :								
	(A) Square	(B) Rectangle	(C) Parallelogram	(D) None of these						
Q.2	A solid cone is a : (A)2-dimensional figu (C) Either 2-dimension	re al or 3-dimensional figur	(B) 3-dimensional figu e (D) None of these	re						
Q.3	The odd one in the foll (A) Sphere	owing is : (B) Cylinder	(C) Circle	(D) Cone						
Q.4	A 3-dimensional figur (A) Sphere	e which does not have a (B) Cylinder	ny vertex and any flat fac (C) Cone	ce is a : (D) None of these						

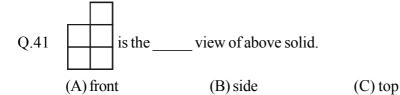
Q.5	The top view of the given figure is :		
	(A) (B)	(C)	(D) None of these
Q.6	The name of the pyramid whose base is (A) Hexagonal pyramid (B) Tetrahedro	1 20	id (D) None of these
Q.7	The solid which is not a polyhedron is : (A) Pyramid (B) Prism	(C) Cuboid	(D) Cylinder
Q.8	If a polyhedron has 12 vertices and 8 fa (A) 12 (B) 14	ices, then the number of edges (C) 16	of the polyhedron is : (D) 18
Q.9	The correct form of Euler's formula (wh (A) $V + E - F = 2$ (B) $F + V - E$	•	al meanings) is : (D) $V + E - F = 1$
Q.10	Which one of the following is the possible(A) 5, 9, 7(B) 8, 18, 12	e number of faces, edges and ver (C) 8, 12, 7	tices respectively of a polyhedron? (D) None of these
Q.11	The number of edge in a pyramid with (A) 4 (B) 6	square base is : (C) 8	(D) 10
Q.12	Which is a two dimensional figure ?(A) Circle(B) Cylinder	(C) Sphere	(D) Tetrahedron
Q.13	Which is a three dimensional figure ?(A) Rhombus(B) Quadrilater	al (C) Cone	(D) A line segment
Q.14	How many plane faces does a cylinder (A) One (B) Two	has? (C) Three	(D) None
Q.15	Flat surface of a three dimensional figur (A) Edge (B) Vertex	e is called : (C) Surface	(D) Corner
Q.16	The number of vertices in a cone is : (A) 1 (B) 2	(C) 6	(D) 8
Q.17	Solids with lines segments as their edge (A) Square (B) Polygons	es are called : (C) Polyhedrons	(D) Cylinders
Q.18	If $E = 5$, $V = 3$ then the value of F is : (A) 6 (B) 4	(C) 7	(D) 2
Q.19	Which of the following solids has maxim (A) Cylinder (B) Cuboid	num number of vertices ? (C) Cone	(D)Tetrahedron

СН-15:	VISUALISING SOLID SHAPE	MATHEMATICS / CLASS-VIII
Q.20	If polyhedron has six faces and eight vertic (A) 12 (B) 10	ces, find the number of edges. (C) 11 (D) 13
Q.21	A polyhedron has sixteen vertices and two (A) 12 (B) 10	enty four edges. How many faces does it have? (C) 11 (D) 13
Q.22	A polyhedron has seven vertices and ten f (A) 15 (B) 20	Paces. How many edges does it have ?(C) 22(D) 25
Q.23	A solid has forty faces, sixty edges. How r (A) 15 (B) 20	many vertices does it have? (C) 22 (D) 25
Q.24	Which of the following is the number of fa (A) 1 (B) 2	ces of a hemisphere ? (C) many (D) none of these
Q.25	Which of the following is a triangular pyran (A) Rectangular pyramid (C) Tetrahedron	nid having all faces as equilateral triangular ? (B) Square pyramid (D) None of these
Q.26	Which of the following is the number of ve (A) 0 (B) 1	ertices of sphere? (C) 2 (D) 4
Q.27	Which of the following can be other name (A)A triangular prism (B)A rectangular	-
Q.28	If the base of a prism is a polygon of 'n' sides, (A) $n+2$ (B) $n+1$	then which of the following is the number of faces of the prism? (C) n (D) $n - 1$
Q.29	Which of the following is the base of a tetr (A) a square (B) a rectangle	rahedron ? (C) a square antiprism (D) a cuboctanedron
Q.30	Which of the following is the other name of (A) a tetrahedron (C) a squareantiprism	of a cube ? (B) a regular hexahedron (D) a cuboctanedron
Q.31	Which of the following nets matches that	of a cube ?
		(C) (D) (D)
Q.32	Which of these nets matches that of a cylin	nder?
	\wedge	Θ

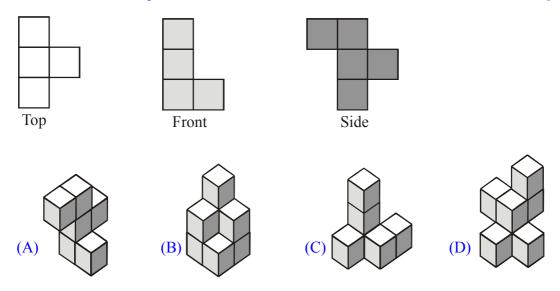


Q.33 Which of the following picture is the correct for the given net?





Q.42 The given figure shows 3 different views of a three-dimensional figure constructed from cubes. Which could be the correct option? [IMO-2016]



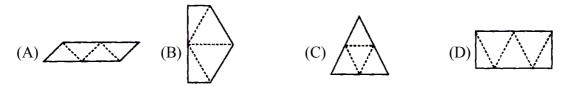
SECTION-C

MORE THAN ONE CORRECT ANSWER

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Q.1

- Q.1 Which of the following(s) represents the Euler's formula? (A) F + V - E = 2 (B) F + V = 2 + E (C) F + V - 2 = E (D) E + F = V
- Q.2 Given below are 4 nets. Which of them is the correct net of an equilateral triangular pyramid?



SECTION -D

MATCH THE COLUMN Match the following: **Column**A **Column B** Number of faces of a cuboid (a) (i) 2 Number of vertices in a tetrahedron 6 (b) (ii) Number of faces of a shape 4 (c) (iii) Number of faces of a hemisphere 1 (d) (iv)

ANSWER KEY

CONCEPT APPLICATION LEVEL - II

SECTION-A

Q.1	regualr prism	Q.2	4	Q.3	18	Q.4	9
Q.5	square, equilateral	Q.6	cone	Q.7	cuboid	Q.8	5
Q.9	polyhedron	Q.10	regular octahe	dron		Q.11	Two dimensional
Q.12	Pyramid	Q.13	Lateral surface	e Q.14	base	Q.15	height
Q.16	axis	Q.17	quadrilateral	Q.18	4	Q.19	isosceles
Q.20	3	Q.21	8,6,12	Q.22	n+1, n+1, 2	n	
Q.23	n+2, 2n, 3n	Q.24	side				

SECTION-B

Q.1	В	Q.2	В	Q.3	С	Q.4	А	Q.5	С	Q.6	С	Q.7	D
Q.8	D	Q.9	В	Q.10	В	Q.11	С	Q.12	А	Q.13	С	Q.14	В
Q.15	С	Q.16	А	Q.17	С	Q.18	В	Q.19	В	Q.20	А	Q.21	В
Q.22	А	Q.23	С	Q.24	В	Q.25	В	Q.26	А	Q.27	D	Q.28	А
Q.29	D	Q.30	В	Q.31	В	Q.32	А	Q.33	А	Q.34	D	Q.35	В
Q.36	В	Q.37	А	Q.38	С	Q.39	А	Q.40	В	Q.41	С	Q.42	Α

SECTION-C

Q.1 ABC Q.2 ABC

SECTION-D

Q.1 (a)-ii, (b)-(iii), (c)-(iv), (d)-(i)