









8

UNDERSTANDING QUADRILATERALS

8.1 POLYGONS

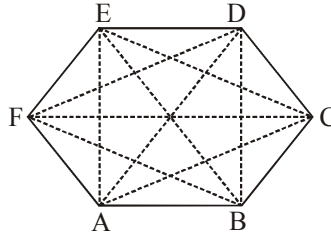
A simple closed curve made up of only the line segments is called a polygon. The simplest polygon is a triangle which is made up of 3 line segments. Let us classify the polygons according to the number of sides (or vertices) they have :

Number of sides or vertices	Name of the polygon	Shape
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
7	Heptagon	
8	Octagon	
9	Nonagon	
10	Decagon	
⋮	⋮	
n	n-gon	

8.1.1 Diagonals of a Polygon

A diagonal is a line segment connecting two non-consecutive vertices of a polygon.

Thus, in the figure ABCDEF is a polygon and each of the line segments AC, AD, AE, BD, BE, BF, CE, CF, DF is a diagonal of the polygon.

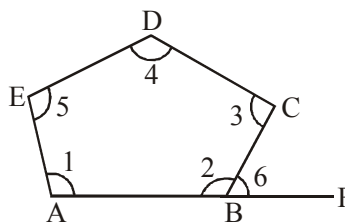


8.1.2 Interior and Exterior Angle of a Polygon

An angle formed by two consecutive sides of a polygon is called an interior angle or simply an angle of the polygon.

In the figure $\angle 1, \angle 2, \angle 3, \angle 4$ and $\angle 5$ are interior angles of the polygon (pentagon) ABCDE.

If we produce a side of a polygon, an exterior angle is formed.



In the figure, the side AB has been produced to F to form the exterior angle CBF marked as $\angle 6$.

$$\angle ABC + \angle CBF = 180^\circ \quad \text{[Linear pair]}$$

or $\angle 2 + \angle 6 = 180^\circ$

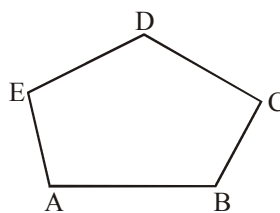
Thus in a polygon.

$$\text{Interior angle} + \text{Exterior angle} = 180^\circ.$$

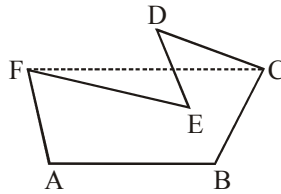
8.1.3 Convex and Concave Polygons

A polygon in which measure of each angle is less than 180° , is called a convex polygon.

In the figure, ABCDE is a convex polygon.



A polygon in which atleast on angle is greater then 180° is called a concave polygon.
In the figure, ABCDEF is a concave polygon.

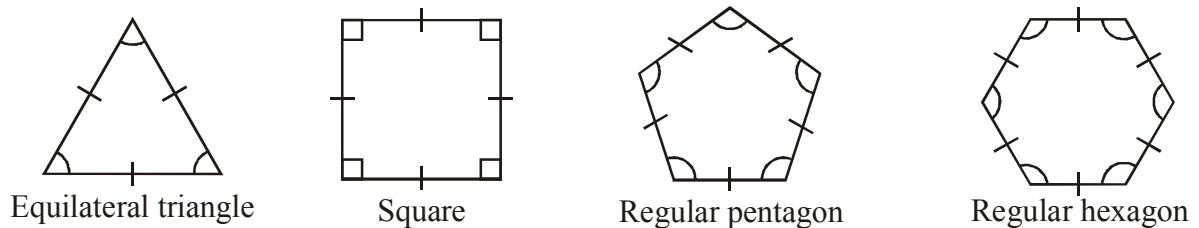


We also observe that in a convex polygon no portions of its diagonals lie in its exteriors. However, in a concave polygon some portion of atleast one diagonal lies in its exterior.

8.1.4 Regular and Irregular Polygons

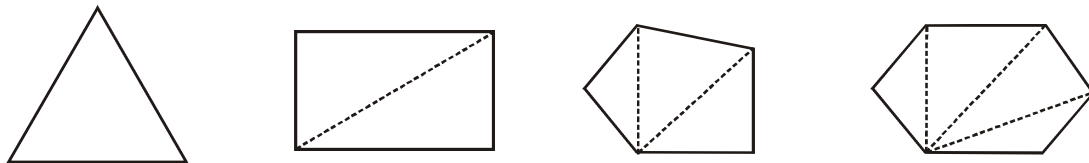
A polygon having all sides equal and all angles equal is called a **regular polygon**. For example, a square is a regular polygon, because it has sides of equal measure and angles of equal measure. A rhombus has sides of equal length but its angles are not equal. Hence, it is not a regular polygon.

Following polygon are regular :



8.2 ANGLE AND PROPERTY

Observe the following figures, Each figure is divided into triangles.



A triangle can be divided into one triangle, a quadrilateral into 2 triangles, a pentagon into 3 triangles, a hexagon into 4 triangles.

Thus, a polygon of n sides can be divided into $(n - 2)$ triangles.

$$\text{Sum of the angles of a triangle} = 180^\circ = (3 - 2) \times 180^\circ$$

$$\text{Sum of the angles of a quadrilateral} = 360^\circ = (4 - 2) \times 180^\circ$$

$$\text{Sum of the angles of a pentagon} = 540^\circ = (5 - 2) \times 180^\circ$$

$$\text{Sum of the angles of a hexagon} = 720^\circ = (6 - 2) \times 180^\circ$$

So, sum of the angles of a polygon of n -sides $= (n - 2) \times 180^\circ = (n - 2) \times 2 \times 90^\circ$

$$= (2n - 4) \times 90^\circ = (2n - 4) \text{ right angles}$$

Thus, Sum of the angles of a polygon of n -sides $= (2n - 4) \text{ right angles}$

$$= (n - 2) \times 180^\circ$$

8.2.1 Sum of the Measures of the Exterior Angles of a Polygon

We know that an exterior angle and the adjacent interior angle of a polygon form a linear pair.

$$\text{Interior angle} + \text{Exterior angle} = 180^\circ$$

If the polygon has n sides (or vertices), then

$$\text{Sum of all interior angles} + \text{Sum of all exterior angle} = n \times 180^\circ = (2n) \text{ right angles}$$

$$\text{or } (2n - 4) \text{ right angles} + \text{sum of all exterior angles} = (2n) \text{ right angles}$$

$$\begin{aligned} \text{or } \text{sum of all exterior angles} &= (2n) \text{ right angles} - (2n - 4) \text{ right angles} \\ &= 4 \text{ right angles} \\ &= 4 \times 90^\circ = 360^\circ \end{aligned}$$

Hence, Sum of all exterior angles of a polygon = 360°

Note : For a regular polygon of n -sides

$$(i) \quad \text{Each exterior angle} = \frac{360^\circ}{n}$$

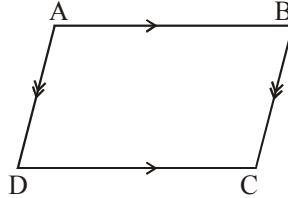
$$(ii) \quad n = \frac{360^\circ}{\text{each exterior angle}}$$

8.3 SPECIAL QUADRILATERALS

1. PARALLELOGRAM

A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel.

In the given figure, $AB \parallel DC$, $AD \parallel BC$, therefore ABCD is a parallelogram.



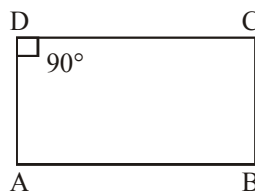
PROPERTIES :-

- (i) The opposite sides of a parallelogram are parallel.
- (ii) The opposite sides of a parallelogram are equal.
- (iii) The opposite angles of a parallelogram are equal.
- (iv) The adjacent angles of a parallelogram are supplementary.
- (v) The diagonals of a parallelogram bisect each other; but they are not equal.

2. RECTANGLE

A rectangle is a parallelogram, whose one angle is a right angle.

In the given figure, ABCD is a parallelogram in which $\angle A = 90^\circ$, hence ABCD is a rectangle.



PROPERTIES : –

A rectangle is a parallelogram in which each angle is a right angle.

In rectangle,

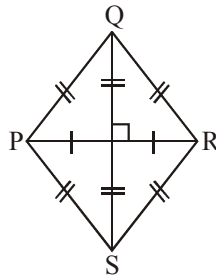
- (i) The opposite sides are equal and parallel,
- (ii) Opposite angles are equal,
- (iii) Diagonals are equal
- (iv) Diagonals bisect each other.

Note : A rectangle is an equiangular figure but not an equilateral one.

3. RHOMBUS

A rhombus is a parallelogram having a pair of adjacent sides are equal.

In the figure, ABCD is a parallelogram in which $AB = AD$. Hence, ABCD is a rhombus.

**PROPERTIES : –**

A rhombus is a parallelogram in which all four sides are equal.

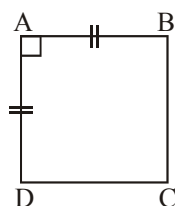
In a rhombus,

- (i) Opposite sides are parallel,
- (ii) All sides are equal,
- (iii) Opposite angles are equal
- (iv) Diagonals bisect each other at right angles.

4. SQUARE

A square is a parallelogram having a pair of adjacent sides are equal and one angle a right angle.

In the given figure, ABCD is a parallelogram with $AB = AD$ and $\angle A = 90^\circ$. Hence, it is a square.



PROPERTIES : –

A parallelogram having all of its sides equal and measure of each angle being 90° , is called a square.

In a square,

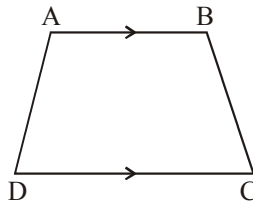
- (i) All four sides are equal,
- (ii) Opposite sides are parallel,
- (iii) Each angle being equal to 90° ,
- (iv) The diagonal are equal,
- (v) The diagonals bisect each other at right angles.

Note : A square is an equilateral and equiangular quadrilateral. Therefore it is called a regular polygon.

5. TRAPEZIUM

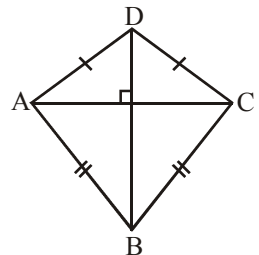
A trapezium is a quadrilateral with only one pair of opposite sides parallel.

In the given figure, ABCD is a trapezium in which $AB \parallel DC$.

**5. KITE**

It is a quadrilateral in which two pairs of adjacent sides are equal.

- (i) $AD = DC$ and $AB = BC$
- (ii) Diagonals are perpendicular but not bisect
- (iii) Only one diagonal divide the figure into two congruent triangles.



SOLVED EXAMPLE

Example 1 :

Find the sum of the angles of a polygon of :

(i) 7 sides

(ii) 9 sides

(iii) 10 sides

Solution :

We know that the sum of the angles of a polygon of n-sides = $(2n - 4)$ right angles

(i) Here $n = 7$

$$\therefore \text{Required sum} = (2 \times 7 - 4) \times 90^\circ = 10 \times 90^\circ = 900^\circ$$

(ii) Here $n = 9$

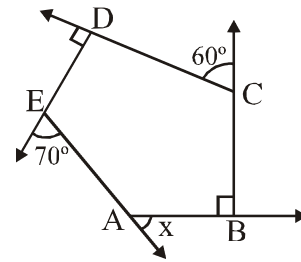
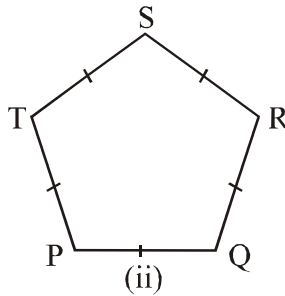
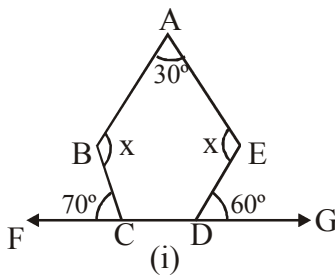
$$\therefore \text{Required sum} = (2 \times 9 - 4) \times 90^\circ = 14 \times 90^\circ = 1260^\circ$$

(iii) Here $n = 10$

$$\therefore \text{Required sum} = (2 \times 10 - 4) \times 90^\circ = 16 \times 90^\circ = 1440^\circ$$

Example 2 :

Find the angles measure x in the following figures.

**Solution :**

$$\angle BCF + \angle BCD = 180^\circ$$

$$\Rightarrow \angle BCD = 180^\circ - 70^\circ = 110^\circ$$

$$\text{Similarly, } \angle EDC = 180^\circ - 60^\circ = 120^\circ$$

Now, ABCDE is a pentagon

So, sum of the interior angles of

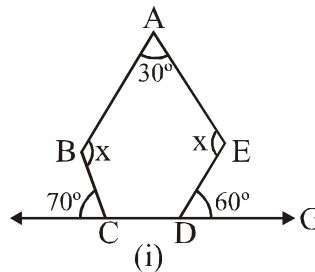
$$ABCDE = (2 \times 5 - 4) \times 90^\circ = 540^\circ$$

$$\angle A + \angle B + \angle C + \angle D + \angle E = 540^\circ$$

$$\Rightarrow 30^\circ + x + 110^\circ + 120^\circ + x = 540^\circ$$

$$\Rightarrow 2x = 540^\circ - 260^\circ = 280^\circ$$

$$x = \frac{280^\circ}{2} = 140^\circ$$

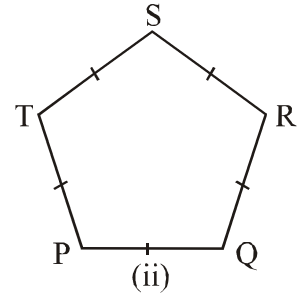


- (ii) The given figure is a regular pentagon.
Sum of the interior angles of the pentagon

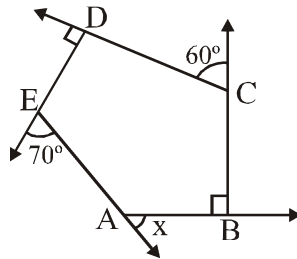
$$= (2 \times 5 - 4) \times 90^\circ = 540^\circ$$

$$\therefore x + x + x + x + x = 540^\circ$$

$$\Rightarrow x = \frac{540^\circ}{5} = 108^\circ$$



- (iii) Exterior angle B = 90°



We know that the sum of all exterior angles of a polygon = 360°

$$\therefore x + 90^\circ + 60^\circ + 90^\circ + 70^\circ = 360^\circ$$

$$\Rightarrow x = 360^\circ - 310^\circ = 50^\circ$$

Example 3 :

Find the measure of each exterior angles of regular polygon of 15 sides.

Solution :

We know that for a regular polygon of n-sides, each exterior angle = $\frac{360^\circ}{n}$

$$\therefore \text{required angle} = \frac{360^\circ}{15} = 24^\circ$$

Example 4 :

How many sides does a regular polygon have, if each of its interior angles is 165° ?

Solution :

Let the regular polygon has n sides.

$$\text{Then, } n \times 165^\circ = (2n - 4) \times 90^\circ$$

$$\Rightarrow 165^\circ n = 180^\circ n - 360^\circ$$

$$\Rightarrow 180^\circ n - 165^\circ n = 360^\circ$$

$$\Rightarrow 15^\circ n = 360^\circ$$

$$\Rightarrow n = \frac{360^\circ}{15} = 24$$

Hence, the regular polygon has 24 sides.

Example 5 :

Is it possible to have a quadrilateral whose angles are of measures 105° , 165° , 55° and 45° ?

Solution :

Sum of the given angles

$$= 105^\circ + 165^\circ + 55^\circ + 45^\circ = 370^\circ$$

As we know that the sum of the angles of a quadrilateral is 360° , hence, the given angles cannot be the angles of a quadrilateral.

Example 6 :

In a quadrilateral ABCD, $A = 40^\circ$, $B = 60^\circ$, $C = 60^\circ$. Find D. Is this quadrilateral a convex or a concave. ?

Solution :

In quadrilateral ABCD,

$$A + B + C + D = 360^\circ$$

[Sum of the angles of a quadrilateral is 360°]

$$\Rightarrow 40^\circ + 60^\circ + 60^\circ + \angle D = 360^\circ$$

$$\Rightarrow 160^\circ + \angle D = 360^\circ$$

$$\Rightarrow \angle D = 360^\circ - 160^\circ$$

$$\Rightarrow \angle D = 200^\circ$$

Since one angle of a quadrilateral ABCD is greater than 180° , therefore it is a concave quadrilateral.

Example 7 :

The angles of a quadrilateral are in the ratio of 3 : 4 : 5 : 6. Find its angles.

Solution :

Suppose the measure of four angles be $3x$, $4x$, $5x$ and $6x$.

$$3x + 4x + 5x + 6x = 360^\circ \quad [\text{Angle sum property of a quadrilateral}]$$

$$\Rightarrow 18x = 360^\circ$$

$$\Rightarrow x = \frac{360^\circ}{18}$$

$$\Rightarrow x = 20^\circ$$

$$\text{So, } 3x = 3 \times 20^\circ = 60^\circ$$

$$4x = 4 \times 20^\circ = 80^\circ$$

$$5x = 5 \times 20^\circ = 100^\circ$$

$$6x = 6 \times 20^\circ = 120^\circ$$

\therefore The angles of the quadrilateral are 60° , 80° , 100° and 120° .

Example 8 :

Three angles of a quadrilateral are equal. Fourth angle is 150° . Find the measure of each equal angle.

Solution :

Let each equal angle be x° and fourth angle = 150°

$$\therefore 3x + 150 = 360 \quad [\text{Sum of the angles of a quadrilateral} = 360]$$

$$\Rightarrow 3x = 360^\circ - 150^\circ$$

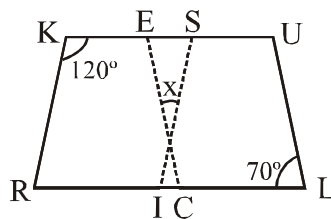
$$\Rightarrow 3x = 210^\circ$$

$$\Rightarrow x = \frac{210^\circ}{3} = 70^\circ$$

Measure of equal angle is 70° .

Example 9 :

In the figure, both RISK and CLUE are parallelograms. Find the value of x .

**Solution :**

RISK is a parallelogram.

$$\therefore \angle RKS + \angle KSI = 180^\circ \quad [\text{Adjacent angles of a parallelogram are supplementary}]$$

$$\Rightarrow \angle KSI = 180^\circ - 120^\circ = 60$$

Again, CLUE is a parallelogram

$$\therefore \angle CLU = \angle CEU \quad [\text{Opposite angles of a parallelogram are equal}]$$

$$\Rightarrow \angle CEU = 70^\circ$$

Now, $x = 180^\circ - (\angle KSI + \angle CEU)$ [Angle sum property of a triangle]

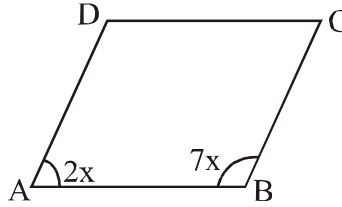
$$\Rightarrow x = 180^\circ - (60^\circ + 70^\circ) = 50^\circ.$$

Example 10 :

Adjacent angles of a parallelogram are in the ratio of 2 : 7. Find their values.

Solution :

Let the adjacent angles of the parallelogram ABCD be $2x$ and $7x$.



$$\Rightarrow 2x + 7x = 180^\circ \quad \text{[Adjacent angles of a parallelogram are supplementary]}$$

$$\Rightarrow 9x = 180^\circ$$

$$\Rightarrow x = \frac{180^\circ}{9} = 20^\circ$$

Also, $2x = 2 \times 20^\circ = 40^\circ$

$$7x = 7 \times 20^\circ = 140^\circ$$

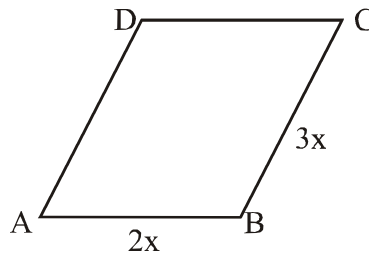
\therefore The adjacent angles of the parallelogram are 40° and 140° .

Example 11 :

Lengths of two sides of a parallelogram are in the ratio of 2 : 3. Find the sides of the parallelogram if its perimeter is 120 cm.

Solution :

Let ABCD be the parallelogram in which $AB = 2x$ cm and $BC = 3x$ cm.



Since opposite sides of a parallelogram are equal, therefore,

$AB = DC = 2x$ cm and $BC = AD = 3x$ cm.

Perimeter of the parallelogram

$$= AB + BC + CD + AD$$

$$= (2x + 3x + 2x + 3x) \text{ cm}$$

$$= 10x \text{ cm}$$

also, perimeter of ABCD = 120 cm [Given]

$$\therefore 10x = 120$$

$$\Rightarrow x = 12$$

Length of the sides = 2×12 cm = 24 cm and 3×12 cm = 36 cm

Hence, the length of the two sides = 24 cm and 36 cm.

Example 12 :

In a parallelogram show that any two adjacent angles are supplementary.

Solution :

In the given figure, PQRS is a parallelogram.

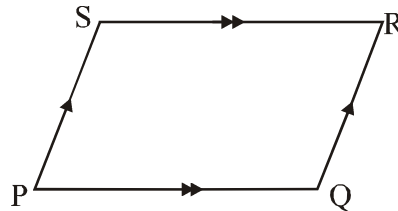
We have to show that

$$\angle P + \angle Q = 180^\circ,$$

$$\angle Q + \angle R = 180^\circ,$$

$$\angle R + \angle S = 180^\circ,$$

$$\angle S + \angle P = 180^\circ$$



In parallelogram PQRS, we have PS and QR are parallel to each other and transversal PQ intersects them at P and Q.

$$\therefore \angle P + \angle Q = 180^\circ$$

[\because interior angles on the same side of the transversal are supplementary]

Similarly, we can prove for other pairs of adjacent angles.

Example 13 :

The diagonals of a quadrilateral are 8cm and 6cm. If the diagonals bisect each other at right angles, find the length of the sides of the quadrilateral.

Solution :

Consider the given figure, in which ABCD is a quadrilateral and diagonals AC and BD bisect each other at O, such that $\angle AOB = 90^\circ$.

$$AC = 8 \text{ cm} \quad \Rightarrow \quad AO = 4 \text{ cm}$$

$$BD = 6 \text{ cm} \quad \Rightarrow \quad BO = 3 \text{ cm}$$

In $\triangle AOB$, using Pythagoras theorem,

$$AB^2 = AO^2 + OB^2$$

$$\Rightarrow AB^2 = 4^2 + 3^2 = 16 + 9 = 25$$

$$\Rightarrow AB = 5 \text{ cm}$$

Applying, Pythagoras theorem in $\triangle AOD$, $\triangle DOC$ and $\triangle COB$, we can show that

$$AD = DC = BC = 5 \text{ cm}.$$

Example 14 :

The diagonals of a rectangle PQRS intersect at O. If $\angle ROQ = 60^\circ$, find $\angle OSP$.

Solution :

$$\angle ROQ = \angle SOP$$

$$= 60^\circ \dots (i)$$

[Vertically opposite angles]

Also, $PR = SQ$

$$\frac{1}{2} PR = PO = OR, \text{ and } \frac{1}{2} QS = QO = OS$$

$$\Rightarrow PO = OS$$

$$\Rightarrow \angle OPS = \angle OSP \dots (ii)$$

[Angles opposite to equal sides are equal]

$$\text{Also, } \angle PSO + \angle OPS + \angle SOP = 180$$

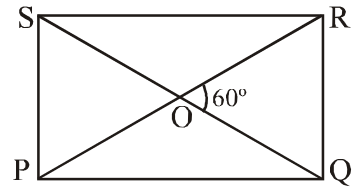
[Angle sum property of a triangle]

$$\angle PSO + \angle OPS + 60^\circ = 180^\circ$$

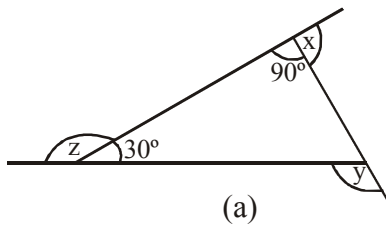
[By (i) and (ii)]

$$\Rightarrow 2\angle PSO = 180^\circ - 60^\circ = 120^\circ$$

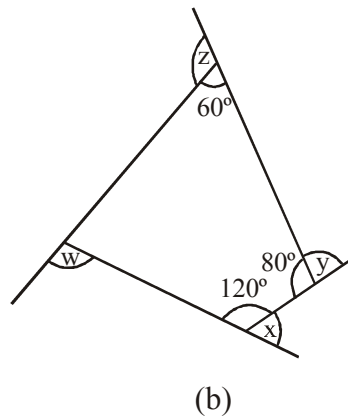
$$\angle PSO = \frac{120^\circ}{2} = 60^\circ$$



Example 15 :



(a) Find $x + y + z$.



(b) Find $x + y + z + w$.

Solution :

$$(a) \because x + 90^\circ = 180^\circ$$

$$x = 180^\circ - 90^\circ = 90^\circ$$

$$y = 30^\circ + 90^\circ = 120^\circ$$

[Sum of interior opposite angles = exterior angle]

$$z = 180^\circ - 30^\circ = 150^\circ$$

$$\text{Now, } x + y + z = 90^\circ + 120^\circ + 150^\circ = 360^\circ$$

(b) ∴ The sum of interior angles of a quadrilateral = 360

$$\angle 1 + 120^\circ + 80^\circ + 60^\circ = 360^\circ$$

or $\angle 1 + 260^\circ = 360^\circ$

or $\angle 1 = 360^\circ - 260^\circ = 100^\circ$

Now,

$$\Rightarrow x + 120^\circ = 180^\circ$$

[Linear pair]

$$\therefore x = 180^\circ - 120^\circ = 60^\circ$$

$$\Rightarrow y + 80^\circ = 180^\circ$$

[Linear pair]

$$\therefore y = 180^\circ - 80^\circ = 100^\circ$$

$$\Rightarrow z + 60^\circ = 180^\circ$$

[Linear pair]

$$\therefore z = 180^\circ - 60^\circ = 120^\circ$$

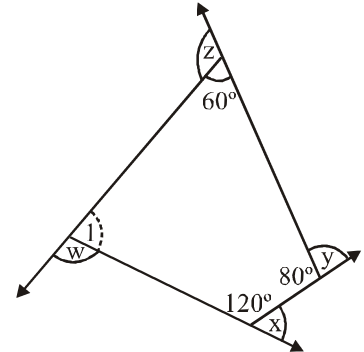
$$\Rightarrow w + 100^\circ = 180^\circ$$

[Linear pair]

$$\therefore w = 180^\circ - 100^\circ = 80^\circ$$

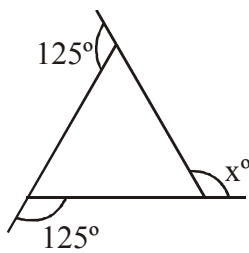
Thus,

$$x + y + z + w = 60^\circ + 100^\circ + 120^\circ + 80^\circ = 360^\circ$$

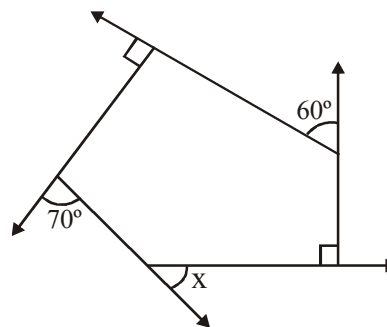


Example 16 :

Find the measure of x in the figure.



(a)



(b)

Solution :

(a)

∴ Sum of all the exterior angles of a quadrilateral = 360°

$$\therefore x + 125^\circ + 125^\circ = 360^\circ$$

or $x + 250^\circ = 360^\circ$

or $x = 360^\circ - 260^\circ = 110^\circ$

(b)

$$\therefore x + 90^\circ + 60^\circ + 90^\circ + 70^\circ = 360^\circ$$

$$x + 310^\circ = 360^\circ$$

or $x = 360^\circ - 310^\circ = 50^\circ$

Example 17 :

How many sides does a regular polygon have if the measure of an exterior angle is 24° ?

Solution :

For a regular polygon, measure of each angle is equal.

$$\therefore \text{Sum of all the exterior angles} = 360^\circ$$

$$\text{Measure of an exterior angle} = 24^\circ$$

$$\therefore \text{Number of angles} = \frac{360^\circ}{24^\circ} = 15$$

Thus, there are 15 sides of the polygon.

Example 18 :

How many sides does a regular polygon have if each of its interior angles is 165° ?

Solution :

The given polygon is a regular polygon.

$$\text{Each interior angle} = 165^\circ$$

$$\therefore \text{Each exterior angle} = 180^\circ - 165^\circ = 15^\circ$$

$$\therefore \text{Number of sides} = \frac{360^\circ}{15^\circ} = 24$$

Thus, there are 24 sides of the polygon.

Example 19 :

(a) Is it possible to have a regular polygon with measure of each exterior angle is 22° ?

(b) Can it be an interior angle of a regular polygon ? Why ?

Solution :

$$(a) \text{ Each exterior angle} = 22^\circ$$

$$\therefore \text{Number of sides} = \frac{360^\circ}{22^\circ} = \frac{180^\circ}{11^\circ}$$

If it is a regular polygon, then its number of sides must be a whole number.

Here, $\frac{180}{11}$ is not a whole number.

$$\therefore 22^\circ \text{ cannot be an exterior angle of a regular polygon.}$$

(b) If 22° is an interior angle, then $180^\circ - 22^\circ$, i.e., 158° is exterior angle.

$$\therefore \text{Number of sides} = \frac{360^\circ}{158^\circ} = \frac{180^\circ}{79^\circ}$$

Which is not a whole number.

Thus, 22° cannot be an interior angle of a regular polygon.

Example 20 :

- (a) What is the minimum interior angle possible for a regular polygon ? Why ?
 (b) What is the maximum exterior angle possible for a regular polygon ?

Solution :

- (a) The minimum number of sides of a polygon = 3
 The regular polygon of 3 sides is an equilateral.
 Each interior angle of an equilateral triangle = 60°
 Hence, the minimum possible interior angle of a polygon = 60°
- (b) The sum of an exterior angle and its corresponding interior angle is 180° .
 And minimum interior angle of a regular polygon = 60°
 \therefore The maximum exterior angle of a regular polygon = $180^\circ - 60^\circ = 120^\circ$.

Example 21:

Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.

Solution :

Let ABCD be a parallelogram such that adjacent angles $A = B$.

Since $\angle A + \angle B = 180^\circ$

$$\therefore \angle A = \angle B = \frac{180^\circ}{2} = 90^\circ$$

Since, opposite angles of a parallelogram are equal.

$$\angle A = \angle C = 90^\circ$$

and $\angle B = \angle D = 90^\circ$

Thus, $\angle A = 90^\circ$, $\angle B = 90^\circ$, $\angle C = 90^\circ$ and $\angle D = 90^\circ$.

Example 22 :

Find the measure of P and S if $\overline{SP} \parallel \overline{RQ}$ in figure, is there more than one method to find $\angle P$?).

Solution :

PQRS is a trapezium such that $\overline{SP} \parallel \overline{RQ}$ and PQ is a transversal.

$$\angle P + \angle Q = 180^\circ \quad [\text{Interior opposite angles are supplementary}]$$

or $\angle P + 130^\circ = 180^\circ$

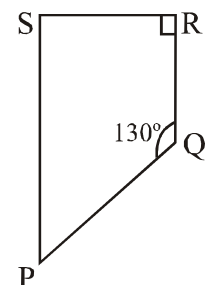
$$\angle P = 180^\circ - 130^\circ = 50^\circ$$

Again $\overline{SP} \parallel \overline{RQ}$ and \overline{RS} is a transversal.

$$\therefore \angle S + \angle R = 180^\circ$$

or $\angle S + 90^\circ = 180^\circ$

$$\therefore \angle S = 180^\circ - 90^\circ = 90^\circ$$



Yes, using the angle sum property of a quadrilateral, we can find mP when mR is known.

$$\begin{aligned} \therefore \quad \angle P + \angle Q + \angle R + \angle S &= 360^\circ \\ \text{or} \quad \angle P + 130^\circ + 90^\circ + 90^\circ &= 360^\circ \\ \text{or} \quad \angle P &= 360^\circ - 130^\circ - 90^\circ - 90^\circ = 50^\circ \end{aligned}$$

Example 23 :

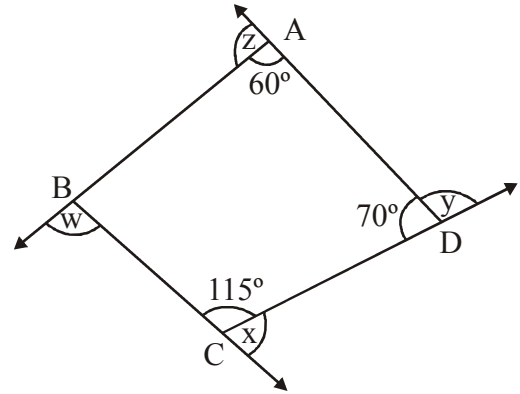
In the adjoining figure, find $x + y + z + w$

Solution :

Since, the sum of the measures of interior angles of a quadrilateral is 360° .

Also, $115^\circ + 70^\circ + 60^\circ = 245^\circ$

$$\begin{aligned} \therefore \quad 245^\circ + \angle ABC &= 360^\circ \\ \angle ABC &= 360^\circ - 115^\circ \\ x &= \text{ext. } \angle BCD \\ &= 180^\circ - \angle BCD \\ &= 180^\circ - 115^\circ = 65^\circ \\ y &= 180^\circ - 70^\circ = 110^\circ \\ z &= 180^\circ - 60^\circ = 120^\circ \\ w &= 180^\circ - 115^\circ = 65^\circ \\ x + y + z + w &= 65^\circ + 110^\circ + 120^\circ + 65^\circ \\ &= 360^\circ \end{aligned}$$



Example 24 :

In a quadrilateral ABCD, the angles A, B, C and D are in the ratio 1 : 2 : 3 : 4. Find the measure of each angle of quadrilateral.

Solution :

$$\begin{aligned} \therefore \quad \angle A : \angle B : \angle C : \angle D &= 1 : 2 : 3 : 4 \\ \therefore \quad \text{Let us suppose that} \\ \angle A &= 1x^\circ, \angle B = 2x^\circ \\ \angle C &= 3x^\circ, \angle D = 4x^\circ \end{aligned}$$

Since, $\angle A + \angle B + \angle C + \angle D = 360^\circ$
 $x + 2x + 3x + 4x = 360^\circ$

$$10x = 360^\circ \Rightarrow x = \frac{360^\circ}{10} = 36^\circ$$

$$\begin{aligned} \angle A &= x^\circ = 36^\circ \\ \angle B &= 2x^\circ = 2 \times 36^\circ = 72^\circ \\ \angle C &= 3x^\circ = 3 \times 36^\circ = 108^\circ \\ \angle D &= 4x^\circ = 4 \times 36^\circ = 144^\circ \end{aligned}$$

Thus, the measure of the angle of the quad. are $36^\circ, 72^\circ, 108^\circ$ and 144° .

Example 25 :

Two regular polygons are such that the ratio of the measures their interior angles is 4 : 3 and the ratio between their number of sides is 2 : 1. Find the number of sides of each polygon.

Solution :

Let $2n$ and n be the number of sides of the regular polygons.

$$\therefore \text{Their interior angles are } \left[\frac{2(2n)-4}{2n} \times 90 \right]^\circ \text{ and } \left[\frac{2n-4}{n} \times 90 \right]^\circ$$

Since the ratio of the interior angles is 4 : 3

$$\therefore \frac{\left[\frac{2(2n)-4}{2n} \times 90 \right]}{\left[\frac{(2n-4)}{n} \times 90 \right]} = \frac{4}{3}$$

$$\Rightarrow \frac{n}{2n} \times \frac{[2(2n)-4]}{[2n-4]} = \frac{4}{3} \qquad \Rightarrow \quad \frac{1}{2} \times \frac{4n-4}{2n-4} = \frac{4}{3}$$

$$\Rightarrow \frac{1}{2} \times \frac{4(n-1)}{2(n-2)} = \frac{4}{3} \qquad \Rightarrow \quad \frac{n-1}{n-2} = \frac{4}{3}$$

$$\Rightarrow 3(n-1) = 4(n-2) \qquad \Rightarrow \quad 3n-3 = 4n-8$$

$$\Rightarrow 3n-4n = -8+3 \qquad \Rightarrow \quad -n = -5$$

$$\Rightarrow n = 5$$

$$\therefore 2n = 2 \times 5 = 10$$

Thus the number of sides of the polygons are 10 and 5 respectively.

Example 26 :

The exterior angle of a regular polygon is one-fifth of its interior angle. How many sides has the polygon ?

Solution :

Let the angle of the polygon 'n'.

$$\therefore \text{Exterior angle of the polygon} = \left[\frac{360}{n} \right]^\circ$$

$$\text{And interior angle of the polygon} = \left[\frac{2n-4}{n} \times 90^\circ \right]$$

$$\text{Since, Exterior angle} = \frac{1}{5} [\text{Interior angle}]$$

$$\Rightarrow \frac{360}{n} = \frac{1}{5} \left[\frac{2n-4}{n} \times 90^\circ \right] \qquad \Rightarrow \quad \frac{360}{n} = \frac{2n-4}{n} \times 18$$

$$\Rightarrow \frac{1}{n} \times \frac{n}{2n-4} = \frac{18}{360} \qquad \Rightarrow \quad 2n-4 = 20$$

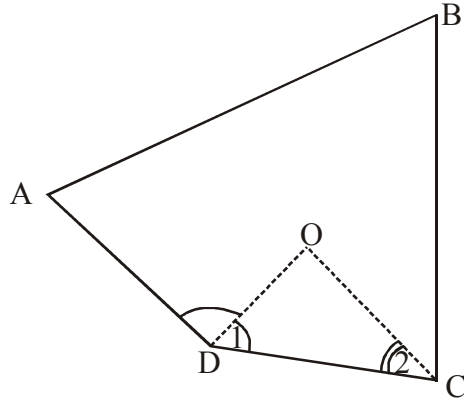
$$\Rightarrow 2n = 20 + 4 = 24 \qquad \Rightarrow \quad n = \frac{24}{2} = 12$$

Thus, the polygon is having 12 sides.

Example 27 :

In a quadrilateral ABCD, DO and CO are the bisectors of $\angle D$ and $\angle C$ respectively. Prove

$$\text{that } \angle COD = \frac{1}{2} [\angle A + \angle B]$$

**Solution :**

In $\triangle COD$, we have

$$\angle COD + \angle 1 + \angle 2 = 180^\circ$$

$$\Rightarrow \angle COD = 180^\circ - [\angle 1 + \angle 2]$$

$$\Rightarrow \angle COD = 180^\circ - \left[\frac{1}{2} \angle D + \frac{1}{2} \angle C \right]$$

But $\angle A + \angle B + \angle C + \angle D = 360^\circ$

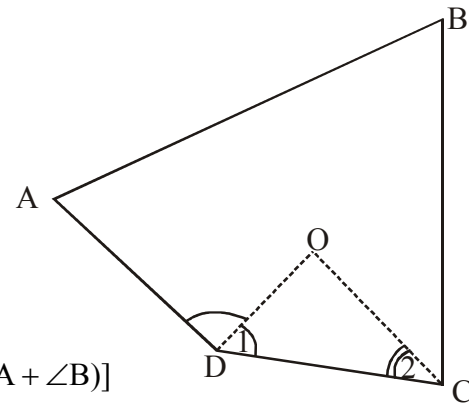
$$\Rightarrow \angle C + \angle D = 360^\circ - (\angle A + \angle B)$$

$$\therefore \angle COD = 180^\circ - \frac{1}{2} [360^\circ - (\angle A + \angle B)]$$

$$= 180^\circ - \frac{1}{2} [360^\circ] + \frac{1}{2} [\angle A + \angle B]$$

$$= 180^\circ - 180^\circ + \frac{1}{2} (\angle A + \angle B) = \frac{1}{2} (\angle A + \angle B)$$

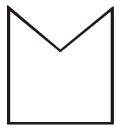
Thus,
$$\angle COD = \frac{1}{2} [\angle A + \angle B]$$



CONCEPT APPLICATION LEVEL - I [NCERT Questions]

EXERCISE - 1

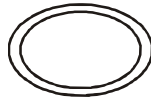
Q.1 Given here are some figures :



(1)



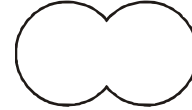
(2)



(3)



(4)



(5)



(6)



(7)



(8)

Classify each of them on the basis of the following :

(a) Simple curve

(b) Simple closed curve

(c) Polygon

(d) Convex polygon

(e) Concave Polygon

Sol. (a) 1, 2, 5, 6, 7

(b) 1, 2, 5, 6, 7

(c) 1, 2, 4

(d) 2

(e) 1, 4

Q.2 How many diagonals does each of the following have?

(a) A convex quadrilateral

(b) A regular hexagon

(c) A triangle

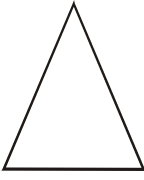
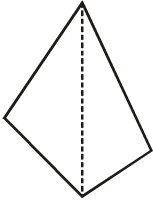
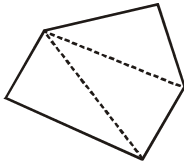
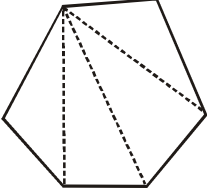
Sol. (a) 2 (b) 9 (c) 0

Q.3 What is the sum of the measures of the angles of a convex quadrilateral? Will this property hold if the quadrilateral is not convex?

Sol. The sum of the measures of the angles of a convex quadrilateral is 360° .

Yes! this property will hold if the quadrilateral is not convex.

Q.4 Examine the table. (Each figure is divided into triangles and the sum of the angles deduced from that.)

Figure				
Side	3	4	5	6
Angle sum	180°	$2 \times 180^\circ$ $= (4 - 2) \times 180^\circ$	$3 \times 180^\circ$ $= (5 - 2) \times 180^\circ$	$4 \times 180^\circ$ $= (6 - 2) \times 180^\circ$

What can you say about the angle sum of a convex polygon with number of sides?

- (a) 7 (b) 8 (c) 10 (d) n

Sol. (a) 7 Angle sum $= (7 - 2) \times 180^\circ = 5 \times 180^\circ = 900^\circ$
 (b) 8 Angle sum $= (8 - 2) \times 180^\circ = 6 \times 180^\circ = 1080^\circ$
 (c) 10 Angle sum $= (10 - 2) \times 180^\circ = 8 \times 180^\circ = 1440^\circ$
 (d) n Angle sum $= (n - 2) \times 180^\circ$

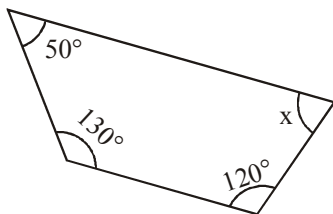
Q.5 What is a regular polygon? State the name of a regular polygon of

- (i) 3 sides (ii) 4 sides (iii) 6 sides

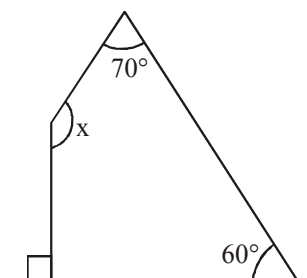
Sol. A polygon, which is both 'equilateral' and 'equiangular', is called a regular polygon.

- (i) 3 sides : The name of the regular polygon is equilateral triangle.
 (ii) 4 sides : The name of the regular polygon is square.
 (iii) 6 sides : The name of the regular polygon is regular hexagon.

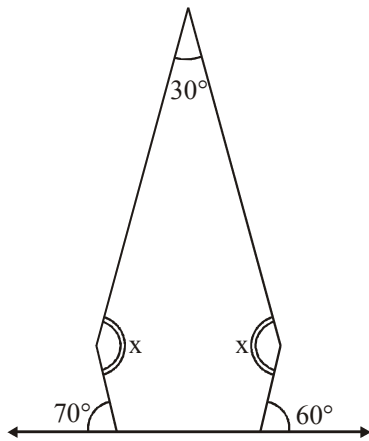
Q.6 Find the angle measure x in the following figures.



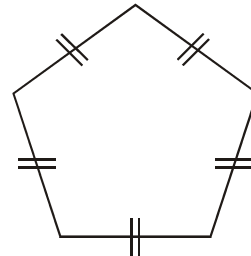
(a)



(b)



(c)



(d)

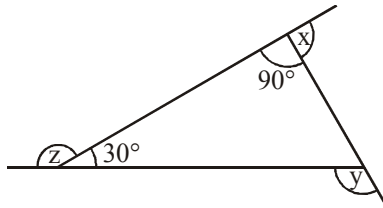
Sol. (a) $x + 50^\circ + 130^\circ + 120^\circ = 360^\circ$ [By angle sum property of a quadrilateral]
 $\Rightarrow x + 300^\circ = 360^\circ$
 $\Rightarrow x = 360^\circ - 300^\circ \Rightarrow x = 60^\circ$

(b) $x + (180^\circ - 90^\circ) + 60^\circ + 70^\circ = 360^\circ$
 [By linear pair property and angle sum property of a quadrilateral]
 $\Rightarrow x + 220^\circ = 360^\circ$
 $\Rightarrow x = 360^\circ - 220^\circ \Rightarrow x = 140^\circ$

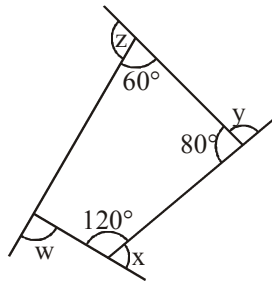
(c) $x + 30^\circ + x + (180^\circ - 70^\circ) + (180^\circ - 60^\circ) = (5 - 2) \times 180^\circ$
 [By linear pair property and angle sum property of a pentagon]
 $\Rightarrow 2x + 30^\circ + 110^\circ + 120^\circ = 540^\circ$
 $\Rightarrow 2x + 260^\circ = 540^\circ$
 $\Rightarrow 2x = 540^\circ - 260^\circ$
 $\Rightarrow 2x = 280^\circ$
 $\Rightarrow x = \frac{280^\circ}{2}$
 $\Rightarrow x = 140^\circ$

(d) $x + x + x + x + x = (5 - 2) \times 180^\circ$
 [By angle sum property of a regular pentagon]
 $\Rightarrow 5x = 540^\circ$
 $\Rightarrow x = \frac{540^\circ}{5}$
 $\Rightarrow x = 108^\circ$

Q.7 (a) Find $x + y + z$.



(b) Find $x + y + z + w$.

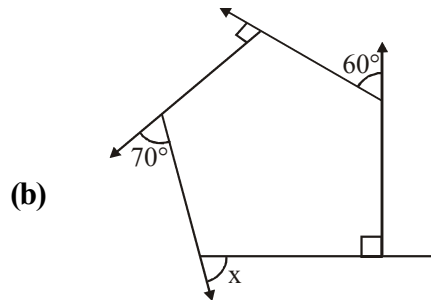
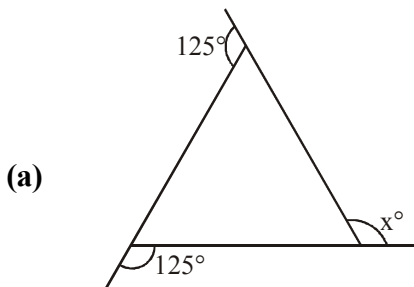


Sol. (a) By linear pair property and angle sum property of a triangle,
 $(180^\circ - x) + (180^\circ - z) + (180^\circ - y) = 180^\circ$
 $\Rightarrow x + y + z = 360^\circ$

(b) By linear pair property and angle sum property of a quadrilateral,
 $(180^\circ - x) + (180^\circ - y) + (180^\circ - z) + (180^\circ - w) = 360^\circ$
 $\Rightarrow x + y + z + w = 360^\circ$

EXERCISE - 2

Q.1 Find x in the following figures.



Sol. (a) $x^\circ + 125^\circ + 125^\circ = 360^\circ$
 $[\because \text{The sum of the measures of the exterior angles of any polygon is } 360^\circ]$
 $\Rightarrow x + 250^\circ = 360^\circ$
 $\Rightarrow x = 360^\circ - 250^\circ$
 $\Rightarrow x = 110^\circ$

(b) $x^\circ + 70^\circ + 60^\circ + (90^\circ + 90^\circ) = 360^\circ$
 $[\because \text{The sum of the measures of the exterior angles of any polygon is } 360^\circ]$
 $\Rightarrow x + 310^\circ = 360^\circ$
 $\Rightarrow x = 360^\circ - 310^\circ$
 $\Rightarrow x = 50^\circ$

Q.2 Find the measure of each exterior angle of a regular polygon of

- (i) 9 sides (ii) 15 sides

Sol. (i) 9 sides

$$\text{Size of each exterior angle} = \frac{360^\circ}{9} = 40^\circ$$

- (ii) 15 sides

$$\text{Size of each exterior angle} = \frac{360^\circ}{15} = 24^\circ$$

Q.3 How many sides does a regular polygon have if the measure of an exterior angle is 24° ?

Sol. Let the number of sides be n . Then, $n(24^\circ) = 360^\circ$

$$\Rightarrow n = \frac{360^\circ}{24^\circ} = 15$$

Hence, the number of sides is 15.

Q.4 How many sides does a regular polygon have if each of its interior angles is 165° ?

Sol. \therefore Each interior angle = 165°

\therefore Each exterior angle = $180^\circ - 165^\circ = 15^\circ$ [Linear pair property]

Let the number of sides be n . Then,

$$n(15^\circ) = 360^\circ$$

$$\Rightarrow n = \frac{360^\circ}{15^\circ} = 24$$

Hence, the number of sides is 24.

Q.5 (a) Is it possible to have a regular polygon with measure of each exterior angle as 22° ?

(b) Can it be an interior angle of a regular polygon? Why?

Sol. (a) No, (since 22° is not a divisor of 360°).

(b) No, (because each exterior angle is $180^\circ - 22^\circ = 158^\circ$, which is not a divisor of 360°).

Q.6 (a) What is the minimum interior angle possible for a regular polygon? Why?

(b) What is the maximum exterior angle possible for a regular polygon?

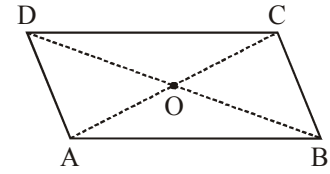
Sol. (a) The equilateral triangle being a regular polygon of 3 sides has the least measure of an interior angle = 60°

(b) By (a) we can see that the greatest exterior angle is $180^\circ - 60^\circ = 120^\circ$.

EXERCISE - 3

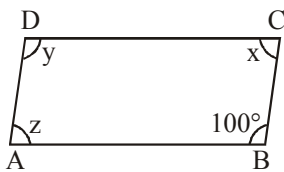
Q.1 Given a parallelogram ABCD. Complete each statement along with the definition or property used.

- (i) $AD =$
- (ii) $\angle DCB =$
- (iii) $OC =$
- (iv) $\angle DAB + \angle CDA =$

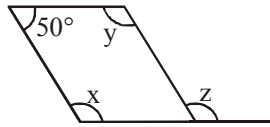


Sol. (i) $AD = BC$ [Opposite sides are equal]
 (ii) $\angle DCB = \angle DAB$ [Opposite angles are equal]
 (iii) $OC = OA$ [\because Diagonals bisect each other]
 (iv) $\angle DAB + \angle CDA = 180^\circ$ [Adjacent angles in a parallelogram are supplementary]

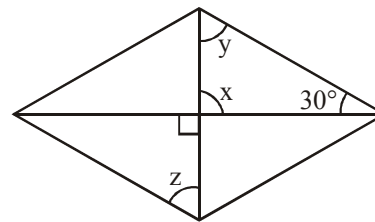
Q.2 Consider the following parallelograms. Find the degree values of the unknowns x, y, z .



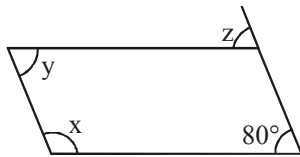
(i)



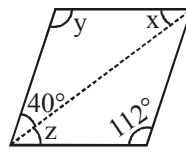
(ii)



(iii)



(iv)



(v)

Sol. (i) $y = 100^\circ$ [Opposite angles of a parallelogram are equal]
 $x + 100^\circ = 180^\circ$ [Adjacent angles in a parallelogram are supplementary]
 $\Rightarrow x = 180^\circ - 100^\circ$
 $\Rightarrow x = 80^\circ$
 $\Rightarrow z = x = 80^\circ$

(ii) $x + 50^\circ = 180^\circ$ [Adjacent angles in a parallelogram are supplementary]
 $\Rightarrow x = 180^\circ - 50^\circ = 130^\circ$
 $\Rightarrow y = x = 130^\circ$ [The Opposite angles of a parallelogram are of equal measure]
 $\Rightarrow 180^\circ - z = 50^\circ$
 $\Rightarrow z = 180^\circ - 50^\circ = 130^\circ$

(iii) $x = 90^\circ$
 $x + y + 30^\circ = 180^\circ$ [By angle sum property of a triangle]
 $\Rightarrow 90^\circ + y + 30^\circ = 180^\circ$
 $\Rightarrow 120^\circ + y = 180^\circ$
 $\Rightarrow y = 180^\circ - 120^\circ = 60^\circ$
 $z + 30^\circ + 90^\circ = 180^\circ$ [By angle sum property of a triangle]
 $\Rightarrow z = 60^\circ$

- (iv) $y = 80^\circ$
 $x + 80^\circ = 180^\circ$ [Adjacent angles in a parallelogram are supplementary]
 $\Rightarrow x = 180^\circ - 80^\circ$
 $\Rightarrow x = 100^\circ$
 $180^\circ - z + 80^\circ = 180^\circ$
 [Linear pair property and adjacent angles in a parallelogram are supplementary]
 $\Rightarrow z = 80^\circ$
- (v) $y = 112^\circ$ [Opposite angles of a parallelogram are equal]
 $x + y + 40^\circ = 180^\circ$ [By angle sum property of a triangle]
 $\Rightarrow x + 112^\circ + 40^\circ = 180^\circ$
 $\Rightarrow x + 152^\circ = 180^\circ$
 $\Rightarrow x = 180^\circ - 152^\circ$
 $\Rightarrow x = 28^\circ$
 $z = x = 28^\circ$ [Alternate angles]

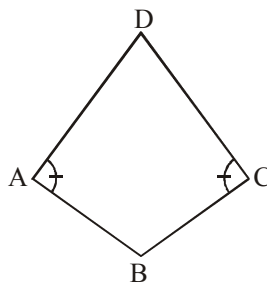
Q.3 Can a quadrilateral ABCD be a parallelogram if

- (i) $\angle D + \angle B = 180^\circ$
 (ii) $AB = DC = 8 \text{ cm}, AD = 4 \text{ cm}$ and $BC = 4.4 \text{ cm}$
 (iii) $\angle A = 70^\circ$ and $\angle C = 65^\circ$

- Sol.** (i) Can be, but need not be.
 (ii) No, in a parallelogram, opposite sides are equal, but here $AD \neq BC$
 (iii) No, in a parallelogram, opposite angles are equal, but here $\angle A \neq \angle C$

Q.4 Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

Sol. A kite, for example



Q.5 The measures of two adjacent angles of a parallelogram are in the ratio 3 : 2. Find the measure of each of the angle of the parallelogram.

Sol. Let the two adjacent angles by $3x^\circ$ and $2x^\circ$.

Then, $3x^\circ + 2x^\circ = 180^\circ$ [\because Sum of the two adjacent angles of a parallelogram is 180°]

$\Rightarrow 5x^\circ = 180^\circ$

$\Rightarrow x^\circ = \frac{180^\circ}{5}$

$\Rightarrow x^\circ = 36^\circ$

$\Rightarrow 3x^\circ = 3 \times 36^\circ = 108^\circ$

and $2x^\circ = 2 \times 36^\circ = 72^\circ$

Since, the opposite angles of parallelogram are of equal measure, therefore the measures of the angles of the parallelogram are $72^\circ, 108^\circ, 72^\circ$ and 108° .

Q.6 Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.

Sol. Let the two adjacent angles of a parallelogram be x° each.

Then, $x^\circ + x^\circ = 180^\circ$ [\because Sum of the two adjacent angles of parallelogram is 180°]

$\Rightarrow 2x^\circ = 180^\circ$

$\Rightarrow x^\circ = \frac{180^\circ}{2}$

$\Rightarrow x^\circ = 90^\circ$

Since, the opposite angles of a parallelogram are of equal measured, therefore the measure of each of the angles of the parallelogram is 90° , i.e. each angle of the parallelogram is a right angle.

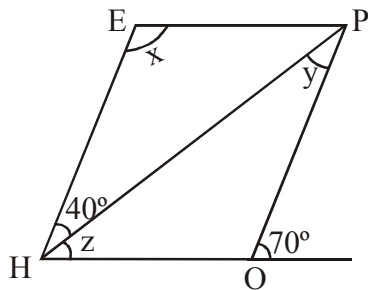
Q.7 The adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the properties you use to find them.

Sol. $x = 180^\circ - 70^\circ = 110^\circ$

[Linear pair property and the opposite angles of a parallelogram are equal measure.]

$y = 40^\circ$

$40^\circ + z + x = 180^\circ$ [The adjacent angles in a parallelogram are supplementary]



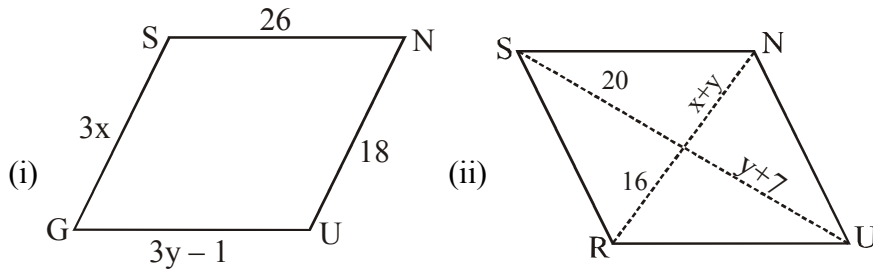
$\Rightarrow 40^\circ + z + 110^\circ = 180^\circ$

$\Rightarrow z + 150^\circ = 180^\circ$

$\Rightarrow z = 180^\circ - 150^\circ$

$\Rightarrow z = 30^\circ$

Q.8 The following figure GUNS and RUNS are parallelograms. Find x and y. (Lengths are in cm)



Sol. (i) For Figure GUNS

Since the opposite sides of a parallelogram are of equal length, therefore,

$$3x = 18$$

$$\Rightarrow x = \frac{18}{3} = 6$$

and $3y - 1 = 26$

$$\Rightarrow 3y = 26 + 1$$

$$\Rightarrow 3y = 27$$

$$\Rightarrow y = \frac{27}{3} = 9$$

(ii) For figure RUNS

Since the diagonals of a parallelogram bisect each other, therefore,

$$x + y = 16 \quad \dots(1)$$

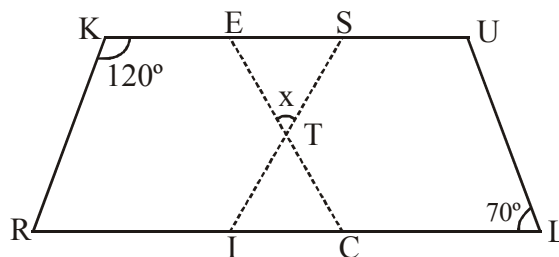
and, $y + 7 = 20 \quad \dots(2)$

From (2), $y = 20 - 7 = 13$

Putting $y = 13$ in (1), we get

$$x + 13 = 16 \Rightarrow x = 16 - 13 = 3$$

Q.9 In the below figure both RISK and CLUE are parallelograms. Find the value of x.



Sol. \therefore RISK is a parallelogram

$$\therefore \angle RIS = \angle RKS = 120^\circ$$

The opposite angles of a parallelogram are of equal measure

Also, $\angle RIS = \angle ISK = 180^\circ$.

The adjacent angles in a parallelogram are supplementary

$$\Rightarrow 120^\circ + \angle ISK = 180^\circ$$

$$\Rightarrow \angle ISK = 180^\circ - 120^\circ$$

$$\angle ISK = 60^\circ \quad \dots(1)$$

\therefore CLUE is a parallelogram

$$\therefore \angle CES = \angle CLU = 70^\circ \quad \dots(2)$$

The opposite angles of a parallelogram are of equal measure

In triangle EST,

$$x^\circ + \angle TSE + \angle TES = 180^\circ$$

By angle sum property of a triangle

$$\Rightarrow x^\circ + \angle ISK + \angle CES = 180^\circ$$

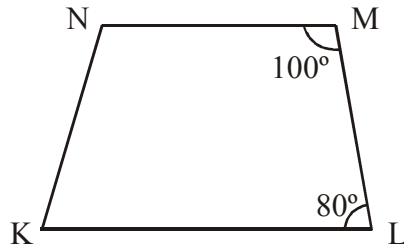
$$\Rightarrow x^\circ + 60^\circ + 70^\circ = 180^\circ$$

[Form (1) and (2)]

$$\Rightarrow x^\circ + 130^\circ = 180^\circ$$

$$\Rightarrow x^\circ = 180^\circ - 130^\circ = 50^\circ \Rightarrow x = 50^\circ.$$

Q.10 Explain how this figure is a trapezium. Which of its two sides parallel ?



Sol.

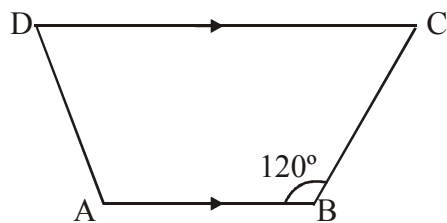
$$\therefore \angle KLM + \angle NML = 80^\circ + 100^\circ = 180^\circ$$

$$\therefore KL \parallel NM$$

i.e., the sum of interior opposite angles is 180°

\therefore Figure KLMN is a trapezium.

Q.11 Find $m \angle C$ in the figure, if $\overline{AB} \parallel \overline{DC}$



Sol.

$$AB \parallel DC$$

$$\therefore m \angle C + m \angle B = 180^\circ$$

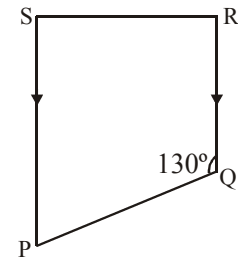
i.e., the sum of interior opposite angles is 180°

$$\Rightarrow m \angle C + 120^\circ = 180^\circ$$

$$\Rightarrow m \angle C = 180^\circ - 120^\circ = 60^\circ$$

Q.12 Find the measure of $\angle P$ and $\angle S$. If $\overline{SP} \parallel \overline{RQ}$ in the figure. (If you find $m \angle R$, is there more than one method to find $m \angle P$?)

Sol. $\because SP \parallel RQ$
 $\therefore \angle P + \angle Q = 180^\circ$
 i.e., the sum of interior opposite angles is 180°
 $\Rightarrow \angle P + 130^\circ = 180^\circ$
 $\Rightarrow \angle P = 180^\circ - 130^\circ$
 $\Rightarrow \angle P = 50^\circ$
 $\angle R = 90^\circ$



EXERCISE - 4

Q.1 State whether True or False :

- All rectangles are squares
- All rhombuses are parallelograms
- All squares are rhombuses and also rectangles
- All squares are not parallelograms
- All kites are rhombuses
- All rhombuses are kites
- All parallelograms are trapeziums
- All squares are trapeziums.

Sol. (b), (c), (f), (g), (h) are true others are false.

Q.2 Identify all the quadrilaterals that have.

- four sides of equal length
- four right angles

Sol. (a) Rhombus : square (b) Square : rectangle

Q.3 Explain how a square is

- a quadrilateral
- a parallelogram
- a rhombus
- a rectangle.

Sol. (i) a quadrilateral
 A square is 4 sided, so it is a quadrilateral.
 (ii) A parallelogram
 A square has its opposite sides parallel; so it is a parallelogram.
 (iii) A rhombus
 A square is a parallelogram with all the 4 side equal, so it is a rhombus.
 (iv) a rectangle
 A square is a parallelogram with each angle a right angle; so it is a rectangle.

Q.4 Name the quadrilaterals whose diagonals.

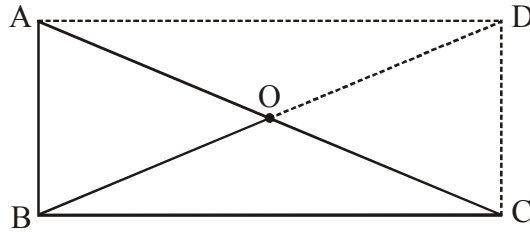
- bisect each other
- are perpendicular bisectors of each other
- are equal.

Sol. (i) bisect each other
 The name of the quadrilaterals whose diagonals bisect each other are parallelogram; rhombus; square; rectangle.
 (ii) are perpendicular bisectors of each other
 The names of the quadrilaterals whose diagonals are perpendicular bisectors of each other are rhombus; square.
 (iii) are equal
 The names of the quadrilaterals whose diagonals are equal are square; rectangle.

Q.5 Explain why a rectangle is a convex quadrilateral.

Sol. A rectangle is a convex quadrilateral because both of its diagonals lie in its interior.

Q.6 ABC is a right-angled triangle and O is the mid-point of the side opposite of the right angle. Explain why O is equidistant from A, B and C. (The dotted lines are drawn additionally to help you).



Sol. Construction : Produce BO to D such that $BO = OD$. Join AD and CD.

Proof. $\overline{AD} \parallel \overline{BC} ; \overline{AB} \parallel \overline{DC}$.

So in parallelogram ABCD, the midpoint of the diagonal \overline{AC} is O. Hence, O is equidistant from A, B and C.

TRY THESE

Q.1 Match the following (Column A figure may match to more than one type)

Figure	Type
(1)	(a) Simple closed curve
(2)	(b) A closed curve that is not simple
(3)	(c) Simple curve that is not closed
(4)	(d) Not a simple curve

Sol. (1)- (c); (2)-(b); (3)-(a); (4)-(d)

Q.2 Does the exterior have a boundary?

Sol. No, the exterior does not have a boundary.

Q.3 A mason has made a concrete slab. He needs it to be rectangular. In what different ways can he make sure that it is rectangular ?

Sol. Different ways

- (i) If opposite sides are of equal length.
- (ii) If each angle at the corner is 90° in measure.
- (iii) If the diagonals are equal in length.

Q.4 Can a trapezium have all angles equal ? Can it have all side equal ? Explain.

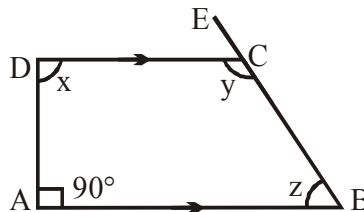
Sol. If a trapezium has all angles equal, then either it becomes a rectangle or a square. If a trapezium has all sides equal, then either it becomes a rhombus or a square.

CONCEPT APPLICATION LEVEL - II

SECTION -A

➤ FILL IN THE BLANKS

- Q.1 The minimum interior angle possible for a regular polygon is _____.
- Q.2 The sum of the measures of interior angle of a polygon of n-sides is 360° . Is it True?
- Q.3 Can we have a regular polygon whose each exterior angle is 120° ? _____.
- Q.4 One angle of a parallelogram is 100° then its opposite angle and adjacent angle are _____, _____ respectively.
- Q.5 If one angle of a rhombus is 60° , then the other angles is _____.
- Q.6 Is every square a rhombus? _____
- Q.7 Is every rhombus a square? _____
- Q.8 Is every parallelogram a rhombus? _____.
- Q.9 If $\angle A = 90^\circ$, $\angle ECD = 60^\circ$, then the measures of x, y and z in the trapezium ABCD is _____, _____, _____.



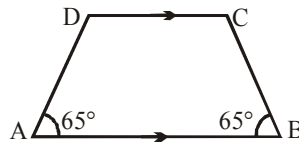
- Q.10 Diagonals of a rhombus are equal and perpendicular to each other. Is it true? _____.

SECTION -B

➤ MULTIPLE CHOICE QUESTIONS

- Q.1 The number of sides of a regular polygon whose each exterior angle has a measure of 45° , is
 (A) 5 (B) 6 (C) 7 (D) 8
- Q.2 If the sides of a quadrilateral are produced in an order, the sum of the four exterior angles so formed is
 (A) 180° (B) 360° (C) 540° (D) 720°
- Q.3 The measure of each angle of a convex quadrilateral is
 (A) less than 180° (B) equal to 180° (C) greater than 180° (D) none of these
- Q.4 The angle of a quadrilateral are in the ratio 1 : 2 : 3 : 4. The largest angle is
 (A) 36° (B) 72° (C) 108° (D) 144°

Q.5 In the figure, the measure of $\angle C$ is



- (A) 65° (B) 115° (C) 135° (D) 125°

Q.6 A quadrilateral has three acute angles each measuring 70° . The measure of fourth angle is

- (A) 140° (B) 150° (C) 105° (D) 120°

Q.7 If the angle of a quadrilateral are x° , $(x - 10)^\circ$, $(x + 30)^\circ$ and $2x^\circ$, then the greatest angle is

- (A) 136° (B) 180° (C) 68° (D) 148°

Q.8 The measures of two angles of a quadrilateral are 115° and 45° , and the other two angles are equal. The measure of each of the equal angles is

- (A) 200° (B) 120° (C) 100° (D) 160°

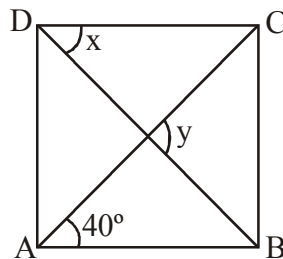
Q.9 In a square PQRS, the diagonals bisect at T. Then ΔPTQ is.

- (A) An equilateral triangle (B) An isosceles but not right angled
(C) A right angled but not isosceles (D) An isosceles right angled

Q.10 A diagonal of a rectangle is inclined to one side of the rectangle at 35° . The acute angle between the diagonals is

- (A) 35° (B) 45° (C) 70° (D) 55°

Q.11 In fig. ABCD is a rhombus. The value of $y - x$ is



- (A) 40° (B) 50° (C) 20° (D) 10°

Q.12 The sum interior angles of a hexagon is

- (A) 180° (B) 360° (C) 540° (D) 720°

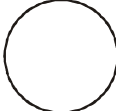
Q.13 The diagonals of a rhombus ABCD intersect at O, $AO = 3$ cm, $BO = 4$ cm then, length of BC is

- (A) 6 cm (B) 8 cm (C) 5 cm (D) none.

Q.14 A quadrilateral whose angles are equal but only adjacent side are equal, then the quadrilateral is a

- (A) square (B) rectangle (C) rhombus (D) parallelogram

- Q.15 The adjacent angles of a prallelogram are in the ratio 4 : 5, then the measure of the adjacent angles is
 (A) $40^\circ, 50^\circ$ (B) $80^\circ, 80^\circ$ (C) $100^\circ, 100^\circ$ (D) $80^\circ, 100^\circ$
- Q.16 One of the diagonals of a rhombus is of same length as the of the side of the rhombus. The angles of the rhombus measure.
 (A) $80^\circ, 100^\circ$ (B) $60^\circ, 80^\circ$ (C) $90^\circ, 90^\circ$ (D) $60^\circ, 120^\circ$
- Q.17 Which of the following is not true ?
 (A) A plane figure formed by joining a number of points without lifting the pencil from the paper and without retracting any portion of the drawing other then single point is called a curve.
 (B) a simple closed curve made up of only line segments is called a polygon.

(C)  is a figure of closed curve.

(D) None of these

- Q.18 Adjacent sides of a polygon are
 (A) any two sides of the polygon
 (B) any two sides connecting two non-consecutive vertices of a polygon
 (C) any two sides with a common vertex
 (D) None of these
- Q.19 Adjacent vertices are
 (A) uncommon vertices of two adjacent sides of a polygon
 (B) end points of the same side of a polygone
 (C) end points of the diagonal of a polygon
 (D) none of these

- Q.20 In the given figure

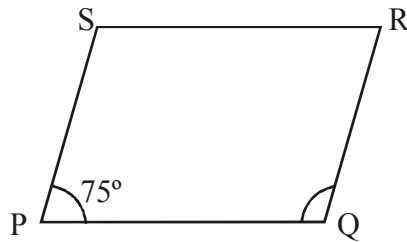


- (A) point A and B are in the interior of the curve
 (B) point B and C are at the exterior of the curve
 (C) point A is at the exterior of the curve and point C is in the interior of the curve
 (D) point A is in the interior of the curve and point C at the exterior of the curve
- Q.21 Which of the following is not true ?
 (A) a polygon is a convex polygon if the line segement joining any two points inside it lies completely inside the polygon
 (B) if a polygon has position of its diagonal in tis exterior then it is known as a concave polygon
 (C) a polygon having all sides and all agnles equal is a regular polygon
 (D) rohombus is a regular polygon

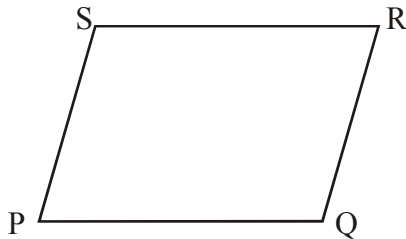
- Q.22 Which of the following is not true?
 (A) equilateral triangle is a regular polygon (B) square is a regular polygon
 (C) rectangle is a regular polygon (D) a regular polygon is both equiangular and equilateral.

- Q.23 Which of the following is not true
 (A) every trapezium is a parallelogram but every parallelogram is not a trapezium
 (B) opposite sides of a parallelogram are not equal
 (C) opposite angles of a parallelogram are equal
 (D) both (A) and (B)

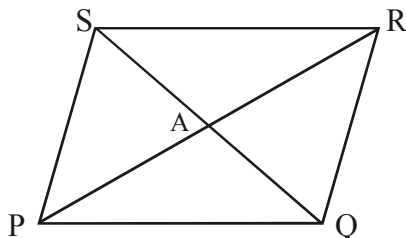
- Q.24 In the given figure, PQRS is a parallelogram. If $\angle P = 75^\circ$, then $\angle Q$ is



- (A) 75° (B) 90° (C) 105° (D) 100°
- Q.25 In the given figure, PQRS is a parallelogram. If perimeter of $\parallel\text{gm}$ PQRS is 40 cm and $PQ = 12$ cm then PS is equal to

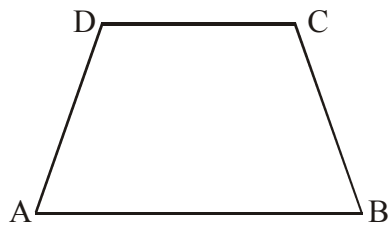


- (A) 12 cm (B) 10 cm (C) 8 cm (D) 9 cm
- Q.26 In the given figure, PQRS is a parallelogram and diagonal PR and QS intersect each other at A. If $QA = 3$ cm, $AR = 5$ cm and $PS = 6$ cm, then perimeter of $\triangle AQR$ is



- (A) 16 cm (B) 14 cm (C) 12 cm (D) 10 cm

Q.27 In the given figure, ABCD is a trapezium in which $AB \parallel CD$. If $\angle A = 50^\circ$ then $\angle D$ is equal to

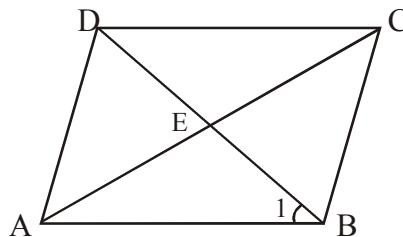


- (A) 50° (B) 100° (C) 130° (D) 120°

Q.28 Which of the following is not the property of a square ?

- (A) each angle of a square is a right angle
 (B) the diagonals of a square are not equal
 (C) the sides of a square are equal
 (D) the diagonals of a square bisect each other at right angle

Q.29 In the given figure, ABCD is a rhombus. Diagonals AC and BD intersect each other at E. If $\angle 1 = 50^\circ$ then $\angle BCD$ is equal to



- (A) 100° (B) 90° (C) 80° (D) none of these

Q.30 How many diagonals does a regular hexagon have ?

- (A) 2 (B) 0 (C) 4 (D) 9

Q.31 The angle sum of a convex polygon with number of sides 7 is

- (A) 900° (B) 1080° (C) 1440° (D) 720°

Q.32 Two adjacent angles of a quadrilateral measure 130° and 40° . The sum of the remaining two angles is

- (A) 190° (B) 180° (C) 360° (D) 90°

Q.33 the measure of each exterior angle of a regular polygon of 15 sides is

- (A) 30° (B) 45° (C) 60° (D) 24°

Q.34 How many sides does a regular polygon have if each of its interior angles is 165° ?

- (A) 12 (B) 24 (C) 9 (D) 6

Q.35 In a regular polygon of n sides, the measure of each internal angle is

- (A) $\frac{360^\circ}{n}$ (B) $\left(\frac{2n-4}{n}\right)90^\circ$ (C) $n \cdot 90^\circ$ (D) $2n$ right angles.

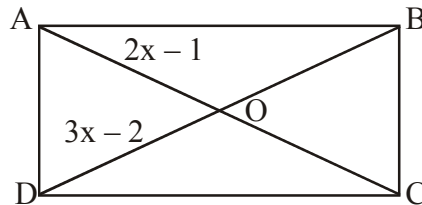
Q.36 If one angle of a parallelogram is of 65° then the measure of the adjacent angle is

- (A) 65° (B) 115° (C) 25° (D) 90°

Q.37 In a kite, what is false ?

- (A) The diagonals are perpendicular to each other
 (B) The diagonals equal to each other
 (C) Only one pair of opposite angles is equal
 (D) All the four sides are equal

Q.38 ABCD is rectangle. Its diagonals meet at O.



OA = $2x - 1$, OD = $3x - 2$. Find x

- (A) 1 (B) 2 (C) 3 (D) - 1

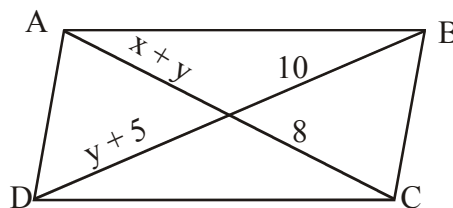
Q.39 In a parallelogram $\angle A : \angle B = 1 : 2$. Then $\angle A =$

- (A) 30° (B) 60° (C) 45° (D) 90°

Q.40 Two adjacent angles of a parallelogram are of equal measure. The measure of each angle of the parallelogram is

- (A) 45° (B) 30° (C) 60° (D) 90°

Q.41 ABCD is a parallelogram as shown. Find x and y .



- (A) 1, 7 (B) 2, 6 (C) 3, 5 (D) 4, 4

ANSWER KEY

CONCEPT APPLICATION LEVEL - II**SECTION -A**

Q.1	60°	Q.2	no	Q.3	yes	Q.4	100°, 80°	Q.5	120°, 60°, 120°
Q.6	yes	Q.7	no	Q.8	no	Q.9	90°, 120°, 60°	Q.10	no

SECTION -B

Q.1	D	Q.2	B	Q.3	A	Q.4	D	Q.5	B	Q.6	B	Q.7	A
Q.8	C	Q.9	D	Q.10	C	Q.11	A	Q.12	D	Q.13	C	Q.14	A
Q.15	D	Q.16	D	Q.17	D	Q.18	C	Q.19	B	Q.20	D	Q.21	D
Q.22	C	Q.23	D	Q.24	C	Q.25	C	Q.26	B	Q.27	C	Q.28	B
Q.29	C	Q.30	D	Q.31	A	Q.32	A	Q.33	D	Q.34	B	Q.35	B
Q.36	B	Q.37	D	Q.38	A	Q.39	B	Q.40	D	Q.41	C		