3 SQUARE AND SQUARE ROOTS

THEORY

3.1 INTRODUCTION

Numbers which can be expressed as the product of two identical numbers are known as **square numbers**. These numbers are also known as **perfect squares**.

Let p and q are natural numbers such that $p = q^2$ then we say 'p' is the square of number 'q' e.g. $9 = 3^2$, so 9 is square of 3 and we call 9 as a perfect square number. Table below shows number and their squares from 1 to 10.

Number	1	2	3	4	5	6	7	8	9	10
Square	1	4	9	16	25	36	49	64	81	100

Properties of square numbers :

If you examine the table of square numbers, you will observe the following :

(1) If a number ends with 1 or 9, its square ends with the digit 1.
(2) If a number ends with 2 or 8, its square ends with the digit 4.
(3) If a number ends with 3 or 7, its square ends with the digit 9.
(4) If a number ends with 4 or 6, its square ends with the digit 6.
(5) If a number ends with 5, its square also ends with the digit 5.
(6) If a number ends with 0, its square also ends with 0.
(7) No perfect square number can end with 2, 3, 7 or 8.
(8) If a number is even, then its square is also even.
(9) If a number is odd, then its square is also odd.
(10) From above we known perfect square numbers ends with either 0 or 1 or 4 or 5 or 6 or 9.

3.2 SQUARE ROOTS

If $p = q^2$ where p and q are integers, then we say that q is the square root of p. For example $9 = 3^2$, therefore 3 is the square root of 9, similarly 7 is the square root of 49 and 12 is the square root of 144. We can say that if p is a perfect square then its square root is an integer and if p is not a perfect square then it does not have an integral square root.

Symbolically, square roots of a positive number 'n' is written as $\sqrt{n}$ or $\sqrt[2]{n}$ or $(n)^{1/2}$.

Therefore,

$\sqrt{16} = 4$ or $\sqrt[2]{16} = 4$ or $(16)^{1/2} = 4$.

3.2.1 Properties of Square Roots

Based upon the properties of square number discussed, we have the following properties of square roots

Property 1

If the units digit of a number is 2, 3, 7 or 8, then it does not have a square root in N (the set of natural numbers).

Explanation: By property 1, a number having 2, 3, 7 or 8 at unit's place cannot be a perfect square. Hence, a number having 2, 3, 7 or 8 at units place does not have a square root in N.

Property 2

If a number ends in an odd number of zeros, then it does not have a square root. If a square number is followed by an even number of zeros, it has a square root in which the number of zeros in the end is half the number of zeros in the number.

Explanation: By property 2, the number of zeros at the end of a perfect square is always even and is twice the number of zeros at the end of the number.

Property 3

The square root of an even square number is even and that square root of an odd square number is odd.

Explanation: By property 3, the squares of even numbers are even numbers and that of odd numbers are odd numbers.

Property 4

If a number has a square root in N, then its units digit must be 0, 1, 4, 5, 6 or 9.

Explanation : By property 6, the units digits of the square and square root are related as below:

Units digit of square	0	1	4	5	6	9
Units digit of square root	0	1 or 9	2 or 8	5	4 or 6	3 or 7

Property 5

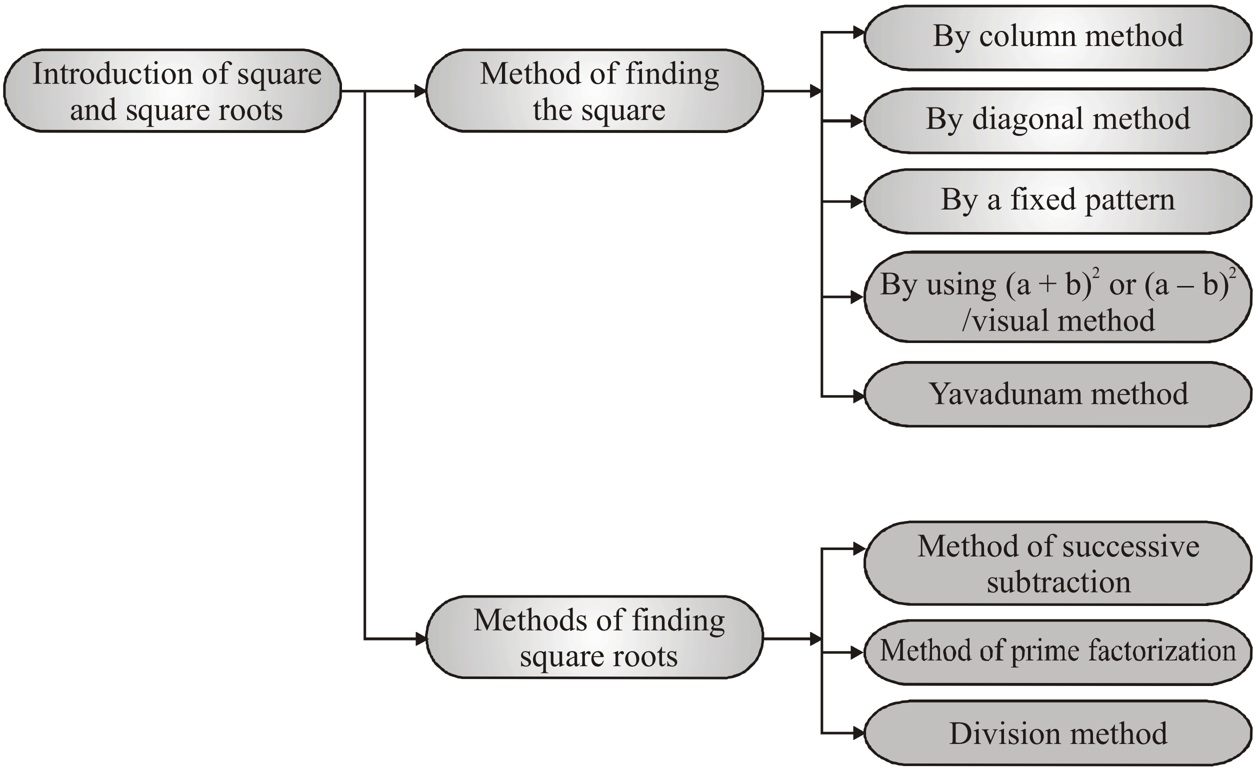
Negative numbers have no square root in the system of rational numbers.

Explanation: We have, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$ and so on. Also, $(-2)^2 = (-2) \times (-2) = 4$, $(-3)^2 = (-3) \times (-3) = 9$, $(-4)^2 = (-4) \times (-4) = 16$ and so on. This means that the square of a number whether positive or negative is always positive. Consequently, negative numbers are not perfect squares. Hence, negative numbers have no square roots.

Property 6

The sum of first n odd natural numbers is n^2 i.e.

$$1 + 3 + 5 + 7 + ... + (2n - 1) = n^2$$



3.3 TO FIND THE SQUARE OF A NUMBER BY THE COLUMN METHOD

The procedure given below explains the application of the column method to find the square of a two digit number.

3.3.1 Procedure

If the given two digit number is of the form ab, where 'a' is the digit in ten place and 'b' is the digit in the units place, then calculate.

a^2	$2ab$	b^2
Col.I	Col.II	Col.III

Now, we have three columns. Consider b^2 in Column III. Underline the units digit of b^2.

Add the tens digit of b^2, if any, to 2ab in Column II and then underline the units digit in Column II.

After underlining the units digit in Column II, add the non-underlined part of column II, if any, to a^2 in Column I.

Underline the number thus obtained in Column I.

The underlined digits when written in the same order as a single number gives the required square.

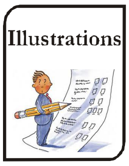


Illustration 1

To find the square of 64.

Solution

Here a = 6 and b = 4

	Column I	Column II	Column III
	a^2	2ab	b^2
Setp (1)	36	4$\underline{8}$	1$\underline{6}$
Setp (2)	36	4$\underline{2}$	$\underline{6}$
Setp (3)	$\underline{40}$	$\underline{9}$	$\underline{6}$

Units digit in Column III is 6.

On adding the tens digit in column III to the number in Column II, we get 48 + 1 = 49

Now the units digit in Column II is 9.

On adding the tens digit in Column II, to the number in the Column I, we get 36 + 4 = 40

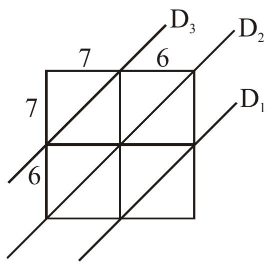
$\therefore$ The square of 64 in 4096.

Note : If the number of digits in the number to be squared is more than two, the use of column method should be avoided, as the method then becomes very difficult of apply.

3.4 TO FIND THE SQUARE OF A NUMBER BY THE DIAGONAL METHOD

(i) Initially, we draw a square. If the number of digits in the given number is 2, then we divide the square into 4 sub-squares and in case the number of digits in the given number is 3, we divide the square into 9 sub-squares and so on.

(ii) Say, the given number is 76 (a two digit number). Construct the diagonals and write the digits of the given number as shown in the figure given below.



(iii) Now multiply each digit on the left of the square with each digit on the top of the column one by one. Write the product in the corresponding sub-square.

(iv) If the number obtained is a single digit number, then write it below the diagonal.

(v) If the number obtained is a two digit number, then write the tens digit above the diagonal and the units digit below the diagonal.

(vi) The numbers in empty places are taken as zero.

(vii) Starting below the lowest diagonal add the digits along the diagonals so obtained. Underline the units digit of the sum and carry over the tens digit, if any, to the diagonal above.

(viii) The underlined unit digits together with, all the digits in the sum obtained above the top most diagonal, give the square of the number.

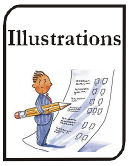
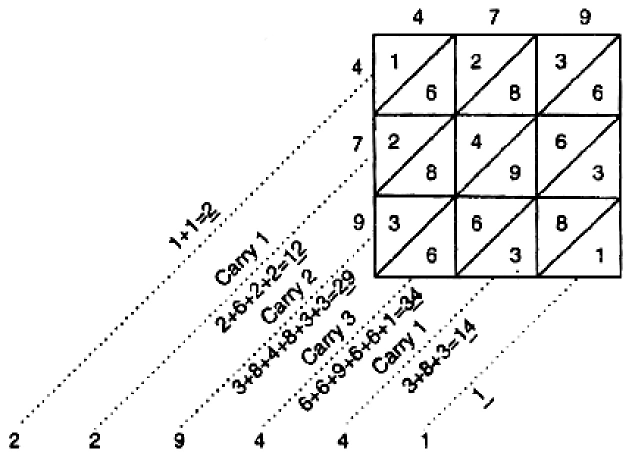


Illustration 2

To find the suqare of 479 by digonal method.

Solution

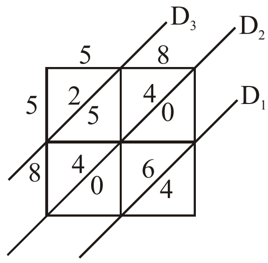


Thus, square of 479 is 229441.

Illustration 3

To find the square of 58.

Solution



In the figure, the number below that lowest diagonal is 4.
Sum of the numbers in between D_1 and D_2 is $0 + 6 + 0 = 6$.
Sum of the numbers in between D_2 and D_3 is $4 + 5 + 4 = 13$.
The units digit of the sum obtained between D_2 and D_3 is 3.
Add the tens digit number of the number 13 to numbers above D_3.
So the sum above D_3 is $2 + 1 = 3$.
$\therefore$ Required square is the combination of all the unit digits in all diagonals = 3364.

3.5 FINDING SQUARES OF THE NUMBERS THAT FOLLOW A FIXED PATTERN

Observe the following pattern.

$11^2 = 121$

$1\underline{0}1^2 = 1\ \underline{0}\ 2\ \underline{0}\ 1$

$1\underline{00}1^2 = 1\ \underline{0\ 0}\ 2\ \underline{0\ 0}\ 1$

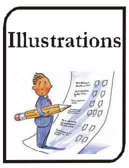


Illustration 4

Find the value of 10001^2.

Solution

From the above patter, we have $10001^2 = 100020001$

observe the following patter.

$9^2 = 81$

$\underline{9}9^2 = \underline{9}801$

$\underline{99}9^2 = \underline{99}8001$

Illustration 5

Find the value of 9999^2.

Solution

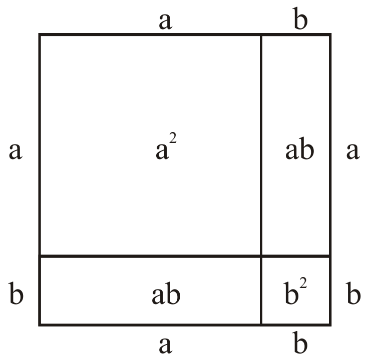
From the above pattern $9999^2 = 99980001$

3.6 TO FIND THE SQUARE OF NUMBER BY USING $(a + b)^2$ OR $(a - b)^2$

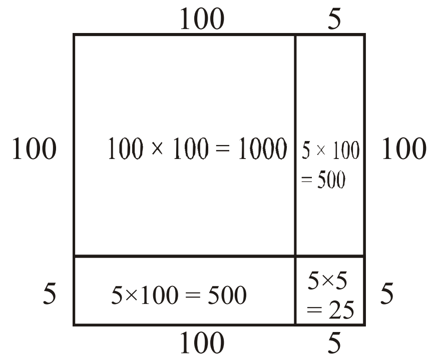
3.6.1 Visual Method

In the column method we have used the algebraic identity $(a + b)^2 = a^2 + 2ab + b^2$ to compute the square of a two digit number. The square of a positive integer can also be computed by closely following the visual representation of $(a + b)^2$. In order to represent $(a + b)^2$, we draw a square of side a + b and divide it into two rectangles of size $a \times (a + b)$ and $b \times (a + b)$ by drawing a vertical line as shown in Fig. We also draw a horizontal line divide the square into two rectangles of size $(a + b) \times b$ and $(a + b) \times a$ as shown in Fig (A). These two lines divide the square into four parts, namely, two squares of size $a \times a$ and $b \times b$ and two rectangles of size $a \times b$ and $b \times a$. The sum of the areas of these four parts is

$$a \times a + a \times b + b \times a + b \times b = a^2 + 2ab + b^2 = (a + b)^2$$



(A)



(B)

We use this visual representation of $(a + b)^2$ to find the square of a number.

Suppose we wish to find the square of 105.

We have, 105 = 100 + 5

So, we draw a square of side 105 units and divide it into four parts as shown in Fig.(B).

The sum of the areas of these four parts is the square of 105.

$\therefore \quad 105^2 = 10000 + 500 + 500 + 25 = 11025$

Note : this method is limited to very few numbers.

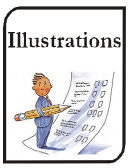


Illustration 6

Find $(102)^2$.

Solution

$(102)^2 = (100 + 2)^2 = (100)^2 + 2(100)(2) + (2)^2 = 10000 + 400 + 4 = 10404$

Illustration 7

Find $\left(49\frac{1}{2}\right)^2$.

Solution

$$\left(49\frac{1}{2}\right)^2 = \left(50-\frac{1}{2}\right)^2 = (50)^2 - 2(50)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 = 2500 - 50 + \frac{1}{4} = 2450\frac{1}{4}$$

Illustration 8

Find the square of the following numbers by Visual method:

(i) 54 (ii) 97

Solution

(i) We have, 54 =50 + 4

So, we draw a square of side 54 units and divide it into parts as shown in Fig. (A)

The sum of the areas of these four parts is the square of 54.

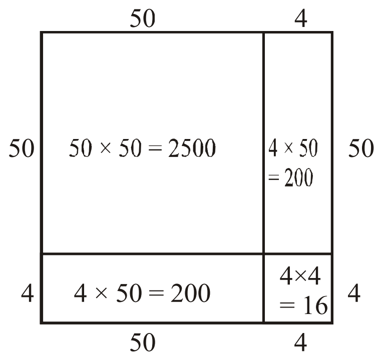
$\therefore$ $54^2 = 2500 + 200 + 200 + 16 = 2916$

(ii) We have, 97 = 90 + 7

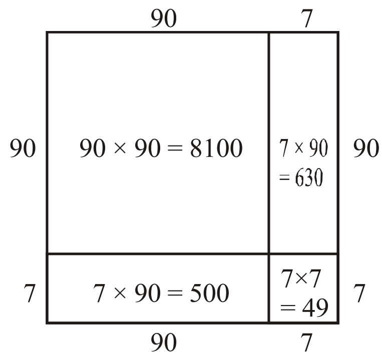
So, we draw a square of side 97 units and divide it into parts as shown in Fig. (B).+

The sum of the areas of these four parts is the square of 97.

$\therefore$ 972 = 8100 + 630 + 630 + 49 = 9409



(A)



(B)

3.6.2 Squaring a number by Yavadunam Method

This method is used for numbers which are near to a base (some power of 10 e.g 10, 100, 1000 etc.). This method is based on one of the vedic mathematics formula "Yavadunam Tavdunikritya Vargamcha Yojayet" which means -whatever the extent of deficiency of a number form base, lesson it to the same extent and set up the square of the deficiency.

Example: If we need to find square of 98 and 104 using 'Yavadunam method', first we observe that both the numbers are near 100, so base in this case is 100. Now 98 is less than 100 by 2, so the deficiency in this case is 2. 104 is more than 100 by 4 so excess in this case is 4.

Now the square can be calculated in two steps.

Example : $(98)^2$ = LHS / RHS

Base = 100 (No of zeros = 2)

LHS = 98 – 2 = 96

RHS = $(2)^2 = 4 = 04$ [digits to be equal to no. of zeros of base]

$\therefore \quad (98)^2 = 9604$

Example : $(104)^2 = (104 + 4) / (4)^2$

$(104)^2 = 10816$

Example : $(1002)^2 = (100^2 + 2) / (2)^2$ [Base = 1000]

=1004/4= 1004/004 = 1004004

Example : $(9999)^2 = (9999 – 1) / (1)^2$ [Base = 10000]

= 9998/1 = 9998/0001

$(9999)^2 = 99980001$

3.7 METHODS FOR FINDING SQUARE ROOTS

3.7.1 Method of Successive Subtraction for Finding the Square Root

We subtract the numbers, 1, 3, 5, 7, 9, 11, ... successively till we get zero. The number of subtractions will give the square root of the number.

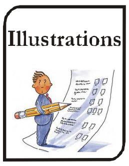


Illustration 9

Find the square root of 64 using the method of successive subtraction.

Solution

64 – 1 = 63; 63 – 3 = 60; 60 – 5 = 55; 55 – 7 = 48; 48 – 9 = 39;

39 – 11 = 28; 28 – 13 = 15; 15 – 15 = 0

$\therefore$ The number of subtractions to yield zero is 8.

$\therefore \quad \sqrt{64} = 8$

3.7.2 Prime Factorization Method for Finding the Square Root

Take the number (n) whose square root is required.

(i) Write all the prime factors of n.

(ii) Pair the factors such that primes in each pair thus formed are equal.

(iii) Choose one prime from each pair and multiply all such primes.

(iv) The product of these primes is the square root of n.

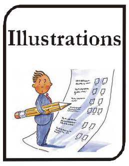


Illustration 10

Find square root of 7225.

Solution

5	7225
5	1445
17	289
17	17
	1

$\therefore \quad 7225 = (5 \times 5) \times (17 \times 17)$

$\therefore \quad \sqrt{7225} = 5 \times 17 = 85$

Illustration 11

Find square root of 4096.

Solution

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

$\therefore \quad 4096 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2)$

$\therefore \quad \sqrt{4096} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

This method of calculation of square root is efficient only if the given number has small prime factors.

3.7.3 Division Method

The number of digits can be determined by placing bars on every pair of digits starting from units digit. If the number of digits are odd then the left most single digit will have a bar on it. The number of bars give the number of digits in the square root of the number. For example : square root of $\overline{20}\,\overline{25}$ will have 2 digits whereas $\sqrt{\overline{2}\,\overline{72}\,\overline{25}}$ has 3 digits.

Steps of Division Method:

(i) Place a bar over every pair of digits starting from the units digit.

(ii) Find the largest number whose square is less than or equal to the number under the left most bar.

(iii) Take this number as the divisor and number under left most bar as dividend. Divide them to get the remainder. You will see that in this step the divisor and quotient are same.

(iv) While down the number under next bar at the right side of the remainder. This is our new dividend.

(v) New divisor is obtained by adding the quotient in the divisor obtained in step (iii) and putting a suitable digit at the right of it . The digit is chosen in such a way that its product with new divisor is equal or just less than new dividend.

Repeat steps (iv) and (v) till all bars have been considered. The final quotient is the square root of the given number.

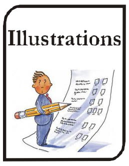


Illustration 12

Find square root of 106929.

Solution

Square root of $\overline{10}\ \overline{69}\ \overline{29}$ will have 3 digits. As 3 is the largest digit whose square is less than 10 (number under left most bar). Here 10 is our dividend and 3 is our divisor and quotient.

$$
\begin{array}{r|l}
 & 327 \\
\hline
3 & \overline{10}\ \overline{69}\ \overline{29} \\
 & 9 \\
\hline
6\underline{2} & 169 \\
 & 124 \\
\hline
64\underline{7} & \times 4529 \\
 & 4529 \\
\hline
 & 0
\end{array}
$$

$\therefore \qquad \sqrt{106929} = 327$

Illustration 13

Find square root of 11664.

Solution

$$
\begin{array}{r|l}
 & 108 \\
\hline
1 & \overline{1}\ \overline{16}\ \overline{64} \\
 & 1 \\
\hline
2\underline{0} & 16 \\
 & 0 \\
\hline
20\underline{8} & 1664 \\
 & 1664 \\
\hline
 & 0
\end{array}
$$

$\therefore \qquad \sqrt{11664} = 108$

Square Root of Rational Numbers whose Numerators and Denominators are Perfect Squares.
We will use the following rules to calculate square root.

(i) $\sqrt{\dfrac{p}{q}} = \dfrac{\sqrt{p}}{\sqrt{q}}$, where $q \neq 0$

(ii) If p and q are positive numbers, then $\sqrt{pq} = \sqrt{p} \times \sqrt{q}$

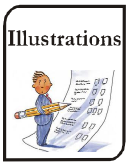


Illustration 14

Find square root of $\dfrac{144}{625}$.

Solution

$$\sqrt{\frac{144}{625}} = \frac{\sqrt{144}}{\sqrt{625}}$$

$$\sqrt{144} = \sqrt{2\times2\times2\times2\times3\times3} = \sqrt{2^2\times2^2\times3^2} = 2\times2\times3 = 12$$

$$\sqrt{625} = \sqrt{5\times5\times5\times5} = 5\times5 = 25$$

$$\therefore \quad \sqrt{\frac{144}{625}} = \frac{12}{25}$$

Square Root of Perfect Square Decimal Number by Division Method :
As we have seen that the square root of these kinds of numbers can be found by first converting them into rational number. However by using division method we can find the square root directly. Follow the steps explained below:

(1) Place the bar on integral part (from left side of decimal) of the number in usual manner.
(2) Place bar on decimal part (from right side of decimal) on every pair of digits.
(3) Apply division method and find square root.
(4) Place the decimal point ill the quotient as soon as the integral part is exhausted.

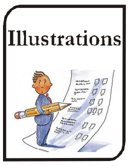


Illustration 15

Find square root of 52.8529

Solution

$$\begin{array}{r|l}
 & 7.27 \\
\hline
7 & \overline{52}\,.\,\overline{85}\,\overline{29} \\
 & 49 \\
\hline
14\underline{2} & 385 \\
 & 284 \\
\hline
144\underline{7} & 101\,29 \\
 & 101\,29 \\
\hline
 & 0
\end{array}$$

$$\therefore \quad \sqrt{52.8529} = 7.27$$

Illustration 16

Find square root of 0.000169

Solution

$$
\begin{array}{r|l}
 & \quad 0.013 \\
\hline
7 & \overline{0}\,.\,\overline{00}\;\overline{01}\;\overline{69} \\
 & \qquad\quad 1 \\
\hline
2\underline{3} & \qquad\; 0\,69 \\
 & \qquad\quad 69 \\
\hline
 & \qquad\quad\; 0
\end{array}
$$

$\therefore \quad \sqrt{0.000169} = 0.013$

Square Root of Numbers which are not Perfect Squares.

Division method can also be applied for finding square root of numbers which are not perfect square numbers. Method is explained with the following illustrative examples.

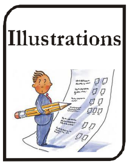


Illustration 17

Find square root of 3 upto 3 decimal places.

Solution

$$
\begin{array}{r|l}
 & \quad 1.732 \\
\hline
1 & \overline{3}\,.\,\overline{00}\;\overline{00}\;\overline{00} \\
 & 1 \\
\hline
2\underline{7} & 2\,00 \\
 & 1\,89 \\
\hline
34\underline{3} & \quad 11\,00 \\
 & \quad 10\,29 \\
\hline
346\underline{2} & \qquad 71\,00 \\
 & \qquad 69\,24 \\
\hline
 & \qquad\;\; 1\,76
\end{array}
$$

Here we have added 3 pairs of zeros after decimal. One pair each for 1 digit after decimal point.

$\therefore \quad \sqrt{3} = 1.732$ upto three decimal places.

Illustration 18

Find square root of $5\frac{2}{15}$ upto 3 decimal places.

Solution

$5\frac{2}{15} = 5.133333$ (approx.)

$$
\therefore\quad
\begin{array}{r|l}
 & 2.265 \\
\hline
2 & \overline{2}\,.\,\overline{13}\,\overline{33}\,\overline{33} \\
 & 4 \\
\hline
42 & 1\,13 \\
 & \;\;84 \\
\hline
446 & \;\;29\,23 \\
 & \;\;26\,76 \\
\hline
4525 & 02\,57\,33 \\
 & \;2\,26\,25 \\
\hline
 & \;0\,31\,08
\end{array}
$$

$$\therefore \qquad \sqrt{5\frac{2}{15}} = \sqrt{5.133333} = 2.265 \text{ (approx.) up to three decimal places.}$$

3.7.4 Relation between the digits of a perfect square and its square root

In order to find the number of digits in the square root of a natural number, we follow the following steps:

Step I Obtain the number.

Step II Place a bar over every pair of digits starting with the units digit.
Each pair and remaining one digit (it any) on the extreme left is called a period.
For example

(i) 2809 will be written as $\overline{28}\,\overline{09}$. In this 28 is called the first period and 09 is called the second period.

(ii) 39204 will be written as $\overline{3}\,\overline{92}\,\overline{02}$. Here, 3 is the first period, 92 is the second period and 04 is the third period.

Step III Count the number of bars. The number of bars is the number of digits in the square root of the given number.
For example, the square root of 2809 has two digits and the square root of 39204 has three digits.

Table : Square Root

x	$\sqrt{x}$	x	$\sqrt{x}$	x	$\sqrt{x}$	x	$\sqrt{x}$
1	1.000	26	5.999	51	7.141	76	8.718
2	1.414	27	5.196	52	7.211	77	8.775
3	1.732	28	5.292	53	7.208	78	8.832
4	2.000	29	5.385	54	7.348	79	8.888
5	2.236	30	5.447	55	7.416	80	8.944
6	2.449	31	5.568	56	7.483	81	9.000
7	2.646	32	5.657	57	7.550	82	9.055
8	2.828	33	5.745	58	7.616	83	9.110
9	3.000	34	5.831	59	5.681	84	9.165
10	3.162	35	5.916	60	7.746	85	9.220
11	3.317	36	6.000	61	7.810	86	9.274
12	3.464	37	6.083	62	7.874	87	9.327
13	3.606	38	6.164	63	7.937	88	9.381
14	3.742	39	6.245	64	8.000	89	9.434
15	3.873	40	6.325	65	8.062	90	9.487
16	4.000	41	6.403	66	8.124	91	9.539
17	4.123	42	6.481	67	8.185	92	9.592
18	4.243	43	6.557	68	8.246	93	9.644
19	4.359	44	6.633	69	8.307	94	9.695
20	4.472	45	6.708	70	8.367	95	9.747
21	4.583	46	6.782	71	8.426	96	9.798
22	4.690	47	6.856	72	8.485	97	9.849
23	4.796	48	6.928	73	8.544	98	9.899
24	4.899	49	7.000	74	8.602	99	9.950
25	5.000	50	7.071	75	8.660		

3.8 METHOD OF FINDING PYTHAGOREAN TRIPLETS

We know that $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

Such a collection of numbers like 5, 12 and 13 is known as Pythagorean triplet

Consider the numbers 9, 40 and 41.

$$9^2 + 40^2 = 81 + 1600 = 1681 = 41^2$$

$\therefore$ 9, 40 and 41 is also called as Pythagorean triplet. We can form Pythagorean triplets using the following method :

Let $k > 1$ be a natural number then we have

$$(k^2 + 1)^2 - (k^2 - 1)^2 = 4k^2 = (2k)^2$$

or $$(k^2 + 1)^2 = (k^2 - 1)^2 + (2k)^2$$

$\therefore$ The set of numbers of the form $k^2 - 1$, $2k$ and $k^2 + 1$ forms a Pythagorean triplet.

$\therefore$ Let us find some Pythagorean triplets using the above form.

Note : All Pythagorean triplets may not be found using the above form.

POINTS TO REMEMBER

- No square number ends in 2, 3, 7 or 8 i.e. unit place digit in a square number can never be 2, 3, 7 or 8.
- A square number end with either 0, 1, 4, 5, 6 or 9. But it does not mean that all numbers that end with 0, 1, 4, 5, 6 or 9 are perfect square number.
- If a number is even then its square is also even. If a number is odd then its square is also odd.
- No. of zeroes at the end of a perfect square number is always even. In other words we can say that numbers ending with odd number of zeroes are never perfect squares.
- A perfect square leaves a remainder 0 or 1 when divided by 3 but all numbers which leave remainder 0 or 1 when divided by 3 need not be a perfect square.
- If p is a square number (perfect square) then 2 p will never be a square number. 4 is a square number, $2 \times 4 = 8$ is not a square number.
- For every natural number 'n' the sum of first 'n' odd natural numbers = n^2
  e.g. $1 + 3 + 5 + 7 + 9 = 25 = 5^2$.
- The set of three number (x, y, z) is called a Pythagorean triplet, if $x^2 + y^2 = z^2$.
- For any natural number m greater than 1, $(2m, m^2 - 1, m^2 + 1)$ is a Pythagorean triplet.
- There are 2n non-perfect square numbers between the squares of the numbers n and (n + 1).
- If a perfect square is of n-digits, then its square root will have $\frac{n}{2}$ digits, if n is even or $\left(\frac{n+1}{2}\right)$ digits, if n is odd.
- The square of any number 'n' can be expressed as the sum of the first 'n' odd natural numbers.

  Eg.
  $1^2 = 1 = 1$
  $2^2 = 4 = 1 + 3$
  $3^2 = 9 = 1 + 3 + 5$
  $4^2 = 16 = 1 + 3 + 5 + 7$

- If x any y are two positive numbers, then
  - $\sqrt{x} \times \sqrt{y} = \sqrt{xy}$
  - $\sqrt{\frac{x}{y}} - \frac{\sqrt{x}}{\sqrt{y}}$ (where $y \neq 0$)
  - $\sqrt{x + y} \neq \sqrt{x} + \sqrt{y}$
  - $\sqrt{x - y} \neq \sqrt{x} - \sqrt{y}$

  Using therse results, we can find the square root of rational numbers.
- Diff. between the square of two consecutive numbers is equal to the sum of the numbers or twice the smaller no +1.
- If (n + 1) and (n – 1) are two consecutive even or odd natural numbers, then $(n + 1)(n - 1) = n^2 - 1$.
  E.g. $10 \times 12 = (11 - 1) \times (11 + 1) = 11^2 - 1$.

SOLVED EXAMPLES

Example 1 :

Is 162 a perfect square?

Solution :

The prime factors of 162 are :

$$162 = 2 \times \underbrace{3\times3} \times \underbrace{3\times3}$$

$$\begin{array}{r|r} 2 & 162 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

If we group the prime factors of 162 into groups of pairs of equal numbers, we find that 2 is left unpaired.

$\therefore$ 162 is not a perfect square.

Example 2 :

Find the smallest number by which we multiply 242 to make it a perfect square.

Solution :

We can write 242 as : $242 = 2 \times \underbrace{11\times11}$

$$\begin{array}{r|r} 2 & 242 \\ \hline 11 & 121 \\ \hline & 11 \end{array}$$

From above we find that the prime factors of 242 do not appear in pairs of equal numbers.

$\therefore$ To make it a perfect square, we must multiply it by 2.

Example 3 :

Without actualy finding the squares of the numbers, find the value of :

(a) $(21)^2 - (20)^2$ **(b)** $(132)^2 - (131)^2$

Solution :

(a) $(21)^2 - (20)^2 = 21 + 20 = 41$

(b) $(132)^2 - (131)^2 = 132 + 131 = 263$

Example 4 :

Write the following numbers as the difference of the squares of two consecutive natural numbers.

(a) **79** **(b)** **131**

Solution :

(a) $79 = 2 \times 39 + 1$

$\therefore$ $79 = (40)^2 - (39)^2$

(b) $131 = 2 \times 65 + 1$

$\therefore$ $131 = (66)^2 - (65)^2$

Example 5 :

Write down the following as sum of odd numbers :

(a) 6^2 **(b)** 7^2

Solution :

(a) 6^2 = sum of first six odd numbers

$= 1 + 3 + 5 + 7 + 9 + 11$

(b) 7^2 = sum of first seven odd numbers

$= 1 + 3 + 5 + 7 + 9 + 11 + 13$

Example 6 :

Show that the following numbers are not perfect squares.

(a) 7927 (b) 1058 (c) 33453 (d) 22222 (e) 360

Solution :

(a) The number 7927 ends in 7, so it is not a perfect square.

(b) The number 1058 ends in 8, so it is not a perfect square.

(c) The number 33453 ends in 3, so it is not a perfect square.

(d) The number 22222 ends in 2, so it is not a perfect square.

(e) The number 360 has odd number of zeros at the end, so it is not a perfect square.

Example 7 :

Find the least square number (perfect square) which is exactly divisible by each one of the numbers 4, 8, 12.

Solution :

The least number divisible by each one of the given numbers 4, 8, 12 is their L.C.M.

L.C.M. of 4, 8, 12 = 2 × 2 × 2 × 3 = 24

2	4 – 8 – 12
2	2 – 4 – 6
	1 – 2 – 3

But $24 = \underbrace{2\times2}\times 2 \times 3$

To make it a perfect square, it must be multiplied by 2 × 3, i.e. 6

$\therefore$ Required number = 24 × 6 = 144

Example 8 :

What least number should be subtracted from 5634 so that the resulting number becomes a perfect square?

Solution :

The remainder 9 shows that if we subtract 9 from 5634,

The square root will be 75 and the resulting number will be a perfect square.

$\therefore$ 9 is to be subtracted.

	75
7	56, 34
	49
145	734
	725
	9

Example 9 :

Find the least number which must be added to 543291 to make it a perfect square.

Solution :

The remainder shows that the given number is greater than $(737)^2$ but will be less than $(738)^2$. If to the given number we add 1468 × 8 – 10391, i.e. 1353, then the sum will be a perfect square.

$\therefore$ 1353 is to be added.

	737
7	54, 32, 91
	49
143	532
	429
1467	10391
	10269
	122

Example 10 :

Without adding, find the sum,

(a) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19

(b) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23

Solution :

We know that sum of first n odd natural numbers is n^2.

(a) Given sum is the sum of first 10 odd natural numbers

$\therefore$ $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = (10)^2 = 100.$

(b) Given sum is the sum of the first 12 odd natural numbers

$\therefore$ $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 = (12)^2 = 144.$

Example 11 :

Write pythagorean triplet whose one number is

(a) 8 (b) 12

Solution :

A triplet of three natural numbers p, q, r is called pythagorean triplet if $p^2 + q^2 = r^2$ and is written as (p, q, r).

For any natural number 'k' > 1, $(2k, k^2 - 1, k^2 + 1)$ is a pythagorean triplet.

(a) Now if $2k = 8$

then $k = 4$

$k^2 - 1 = 16 - 1 = 15$

$k^2 + 1 = 16 + 1 = 17$

$\therefore$ Pythagorean triplet is (8, 15, 17).

(b) $2k = 12 \Rightarrow k = 6$

$k^2 - 1 = 36 - 1 = 35$

$k^2 + 1 = 36 + 1 = 37$

$\therefore$ Pythagorean triplet is (6, 35, 37).

Example 12 :

Find the smallest number by which 252 must be multiplied so that the product becomes a perfect square. Also find the square root of the perfect square so obtained.

Solution :

Writing 252 as its prime factors we get $252 = 2 \times 2 \times 3 \times 3 \times 7$.

We find that prime factors 2 and 3 occur in pairs but prime factor 7 occurs alone.

Therefore 252 must be multiplied with 7 to get a perfect square number

$\therefore$ New number $= 252 \times 7 = 1764$

Now $1764 = (2 \times 2) \times (3 \times 3) \times (7 \times 7)$

$\therefore \quad \sqrt{1764} = 2 \times 3 \times 7 = 42.$

Example 13 :

Find the smallest number by which 15552 must be divided so that it becomes a perfect square. Also find the square root of new perfect square number.

Solution :

By writing 15552 into its prime factors we get

$15552 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3$

$= (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (3 \times 3) \times (3 \times 3) \times 3.$

Here we find that prime factors 2 occur in pair but one of the prime factor 3 occurs alone.

$\therefore$ If we divide 15552 by 3, we get a perfect square.

$\therefore$ New Number $= \dfrac{15552}{3} = 5184$

Now $5184 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (3 \times 3) \times (3 \times 3)$

$\therefore \quad \sqrt{5184} = 2 \times 2 \times 2 \times 3 \times 3 = 8 \times 9 = 72$

Example 14 :

The product of two numbers is 1575 and their quotient is $\frac{9}{7}$ Find the numbers.

Solution :

Let one of the two numbers be K. As the product is 1575, the other number will be $\frac{1575}{K}$.

Quotient of numbers = $\frac{9}{7}$

$\therefore \quad \dfrac{K}{\frac{1575}{K}} = \dfrac{9}{7} \quad \Rightarrow \quad \dfrac{K^2}{1575} = \dfrac{9}{7}$

$\therefore \quad K^2 = \dfrac{9 \times 1575}{7} = 9 \times 225$

$K^2 = 3 \times 3 \times 15 \times 15 = (3 \times 3) \times (3 \times 3) \times 5 \times 5$

$\therefore \quad K = 3 \times 3 \times 5 = 45$

$\therefore \quad$ other number is $\dfrac{1575}{45} = 35$

$\therefore \quad$ The required numbers are 45 and 35.

Example 15 :

Find the greatest number of five digits which is a perfect square.

Solution :

Greatest five digit number is 99999. As 99999 is not a perfect square, we must first find the smallest number to be subtracted from 99999 to make it a perfect square. So we apply method of long division on 99999.

$$
\begin{array}{r|l}
 & 316 \\
\hline
3 & \overline{9}\ \overline{99}\ \overline{99} \\
 & 9 \\
\hline
61 & 0\,99 \\
 & \,61 \\
\hline
626 & 38\,99 \\
 & 37\,56 \\
\hline
 & 1\,43
\end{array}
$$

$\therefore \quad$ We must subtract 143 from 99999 to get largest five digit number which is a perfect square.

$\therefore \quad$ required number = 99999 – 143 = 99856.

Example 16 :

The square of which of the following numbers would be an odd number/an even number? Why?

(i) 727 (ii) 158 (iii) 269 (iv) 1980

Solution :

(i) 727

Since 727 is an odd number.

$\therefore$ It square is also an odd number.

(ii) 158
Since 158 is an even number.
∴ Its square is also an even number.

(iii) 269
Since 269 is an odd number
∴ Its square is also an odd number.

(iv) 1980
Since 1980 is an even number.
∴ Its square is also an even number.

Example 17 :

How many natural numbers lie between 9^2 and 10^2? Between 11^2 and 12^2?

Solution :

(a) Between 9^2 and 10^2
Here, $n = 9$ and $n + 1 = 10$
∴ Natural number between 9^2 and 10^2 are $(2 \times n)$ or 2×9, i.e. 18.

(b) Between 11^2 and 12^2
Here, $n = 11$ and $n + 1 = 12$
∴ Natural numbers between 11^2 and 12^2 are $(2 \times n)$ or (2×11), i.e. 22.

Example 18 :

How many non-square numbers lie between the following pairs of numbers:

(i) 100^2 and 101^2 (ii) 90^2 and 91^2 (iii) 1000^2 and 1001^2

Solution :

(i) Between 100^2 and 101^2
Here, $n = 100$
∴ $n \times 2 = 100 \times 2 = 200$
∴ 200 non square numbers lie between 100^2 and 101^2.

(ii) Between 90^2 and 91^2
Here, $n = 90$
∴ $2 \times n = 2 \times 90$ or 180
∴ 180 non-square numbers lie between 90 and 91.

(iii) Between 1000^2 and 1001^2
Here, $n = 1000$
∴ $2 \times n = 2 \times 1000$ or 2000
∴ 2000 non-square numbers lie between 1000^2 and 1001^2.

Example 19 :

Using the given pattern, find the missing numbers.

$1^2 + 2^2 + 2^2 = 3^2$
$2^2 + 3^2 + 6^2 = 7^2$
$3^2 + 4^2 + 12^2 = 13^2$
$4^2 + 5^2 + \_\_^2 = 21^2$
$5^2 + \_\_^2 + 30^2 = 31^2$
$6^2 + 7^2 + \_\_^2 = \_\_^2$

Note : To find pattern:
Third number is related to first and second number. How?
Fourth number is related to third number. How?

Solution :

The missing numbers are

(i) $4^2 + 5^2 + 20^2 = 21^2$ (ii) $5^2 + 6^2 + 30^2 = 31^2$ (iii) $6^2 + 7^2 + 42^2 = 43^2$

Example 20 :

Find the length of the side of a square whose area is 676 m².

Solution :

$$\begin{array}{r|r} & 26 \\ \hline 2 & 676 \\ & -\ 4 \\ \hline 46 & 276 \\ & -276 \\ \hline & 0 \end{array} \qquad \therefore \quad \sqrt{676} = 26$$

Now, let the side of the square = x m

$\therefore$ Area = x^2

$\Rightarrow$ $x^2 = 676$

$\Rightarrow$ $\sqrt{x^2} = \sqrt{26^2}$

$\Rightarrow$ $x = 26$

$\therefore$ The required side of the square = 26 m

Example 21 :

In a right triangle ABC, ∠B = 90°. If AB = 12 cm, BC = 5 cm, then find AC.

Solution :

We know that, in a right triangle, the side opposite to 90° is hypotenuse.

$\therefore$ AC is the hypotenuse in ΔABC.

According to Phythagoras theorem,

(Hypotenuse)² = [Sum of the square of the other two sides]

$\therefore$ $AC^2 = AB^2 + BC^2$

$\Rightarrow$ $AC^2 = (12)^2 + (5)^2$

$\Rightarrow$ $AC^2 = 144 + 25$

$\Rightarrow$ $AC^2 = 169 = (13)^2$

$\Rightarrow$ $\sqrt{AC^2} = \sqrt{13^2} \Rightarrow AC = 13$ cm

Example 22 :

Which of the following triplets are Pythagorean?

(i) (1, 2, 3) (ii) (3, 4, 5) (iii) (6, 8, 10) (iv) (1, 1, 1)
(v) (2, 2, 3)

Solution :

We know that the three natural numbers m, n and P are called Pythagorean triplets if $m^2 + n^2 = p^2$.

(i) $1^2 + 2^2 = 3^2 \Rightarrow 1 + 4 = 9 \Rightarrow 5 = 9$

But $5 \neq 9$

$\therefore$ (1, 2, 3) are not Pythagorean triplets.

(ii) $3^2 + 4^2 = 5^2 \Rightarrow 9 + 16 = 25 \Rightarrow 25 = 25$

$\therefore$ (3, 4, 5) are Pythagorean triplets.

(iii) $6^2 + 8^2 = 10^2 \Rightarrow 36 + 64 = 100 \Rightarrow 100 = 100$

$\therefore$ (6, 8, 10) are Pythagorean triplets.

(iv) $1^2 + 1^2 = 1^2 \Rightarrow 1 + 1 = 1 \Rightarrow 2 = 1$

But $2 \neq 1$

$\therefore$ (1, 1, 1) are not Pythagorean triplets.

(v) $2^2 + 2^2 = 3^2 \Rightarrow 4 + 4 = 9 \Rightarrow 8 = 9$

But $8 \neq 9$

$\therefore$ (2, 2, 3) are not Pythagorean triplets.

Example 23 :

Find the square root of

(i) $\frac{625}{1296}$ **(ii)** $4\frac{29}{49}$ **(iii)** $23\frac{26}{121}$ **(iv)** **5.774409**

(v) **0.00053361**

Solution :

(i) $\sqrt{\frac{625}{1296}} = \frac{\sqrt{625}}{\sqrt{1296}}$

5	625
5	125
5	25
5	5
	1

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

Now, $\sqrt{625} = \sqrt{\underline{5 \times 5} \times \underline{5 \times 5}}$

$= 5 \times 5 = 25$

$\sqrt{1296} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}}$

$= 2 \times 2 \times 3 \times 3 = 36$

$\therefore \sqrt{\frac{625}{1296}} = \frac{\sqrt{625}}{\sqrt{1296}} = \frac{25}{36}$

(ii) $\sqrt{4\frac{29}{49}} = \sqrt{\frac{225}{49}} = \frac{\sqrt{225}}{\sqrt{49}}$

3	225
3	75
5	25
5	5
	1

7	49
7	7
	1

Now, $\sqrt{225} = \sqrt{\underline{3 \times 3} \times \underline{5 \times 5}}$

$= 3 \times 5 = 15$

$\sqrt{49} = \sqrt{\underline{7 \times 7}} = 7$

$\therefore \sqrt{\frac{225}{49}} = \frac{\sqrt{225}}{\sqrt{49}} = \frac{15}{7} = 2\frac{1}{7}$

(iii) $\sqrt{23\frac{16}{121}} = \sqrt{\frac{2809}{121}} = \frac{\sqrt{2809}}{\sqrt{121}}$

53	2809
53	53
	1

11	121
11	11
	1

Now, $\sqrt{2809} = \sqrt{\underline{53\times53}} = 53$

$\sqrt{121} = \sqrt{\underline{11\times11}} = 11$

$\therefore \quad \sqrt{\frac{2809}{121}} = \frac{\sqrt{2809}}{\sqrt{121}} = \frac{53}{11} = 4\frac{9}{11}$

(iv) $\sqrt{5.774409} = \sqrt{0.000001\times5774409}$

$= \sqrt{(0.1)^6\times\underline{3\times3}\times\underline{3\times3}\times\underline{3\times3}\times\underline{89\times89}}$

$= (0.1)^3 \times 3 \times 3 \times 3 \times 89$

$= 0.001 \times 2403$

$= 2.403$

3	5774409
3	1924803
3	641601
3	213867
3	71289
3	23763
89	7921
89	89
	1

(v) $\sqrt{0.00053361} = \sqrt{.00000001\times53361}$

$= \sqrt{(0.1)^8\times\underline{3\times3}\times\underline{7\times7}\times\underline{11\times11}}$

$= (0.1)^4 \times 3 \times 7 \times 11$

$= 0.0001 \times 231$

$= 0.0231$

3	53361
3	17787
7	5929
7	847
11	121
11	11
	1

Example 24 :

The area of a square field is $101\frac{1}{400}$ square metres. Find the length of one side of the field.

Solution :

Given, Area of the square field $= 101\frac{1}{400}$ Sq. metres $= \frac{40401}{400}$ m^2

Side of the square field $= \sqrt{\frac{40401}{400}} = \frac{\sqrt{40401}}{\sqrt{400}}$ m

$= \frac{\sqrt{\underline{3\times3}\times\underline{67\times67}}}{\sqrt{\underline{2\times2}\times\underline{2\times2}\times\underline{5\times5}}}$ m

$= \frac{3\times67}{2\times2\times5}$ m

$= \frac{201}{20}$ m $= 10\frac{1}{20}$ m

3	40401
3	13467
67	4489
67	67
	1

2	400
2	200
2	100
2	50
5	25
5	5
	1

Example 25 :

Find the least number which must be substracted from 2361 to make it a perfect square.

Solution :

From the process of finding the square root by the division method, we find that if 57 be subtracted from the given number, the square root of the remainder will be 48.

It follows that the given number will be a perfect square. Hence, the required number is 57.

	48
4	$\overline{23}\,\overline{61}$
	16
88	761
	704
	57

Example 26 :

Find the least number of four digits which is a perfect square.

Solution :

The least number of four digits = 1000

	31
3	$\overline{10}\,\overline{00}$
	9
61	100
	61
	39

	32
3	$\overline{10}\,\overline{00}$
	9
62	100
	124

From the above, it is clear that the given number is greater than $(31)^2$, but less than $(32)^2$. If in the given number, we add (124 – 100 = 24), then the sum will be a perfect square.

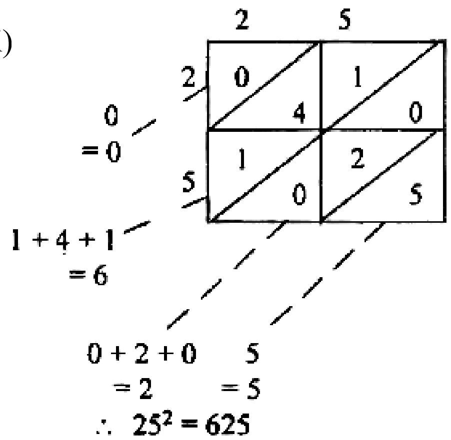
Hence, the required least number of four digits is 1000 + 24 i.e., 1024, which is a perfect square.

Example 27 :

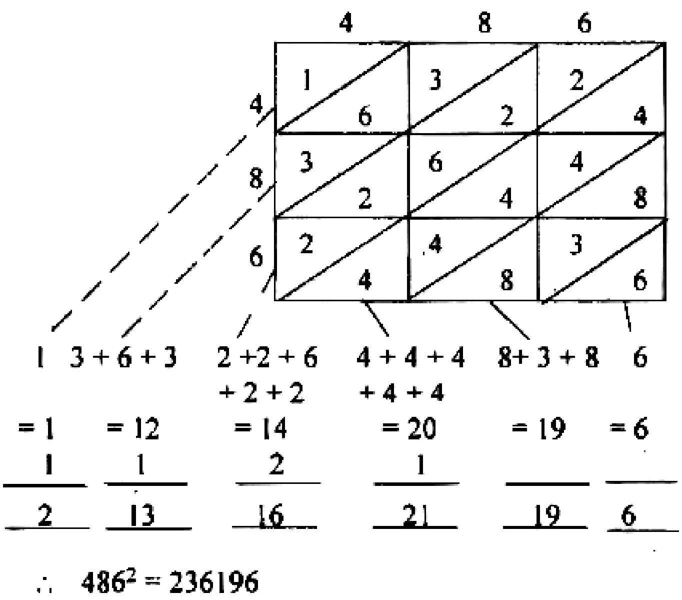
Use diagonal method to find the square of (i) 25 (ii) 486

Solution :

(i)



(ii)



CONCEPT APPLICATION LEVEL - I [NCERT Questions]

EXERCISE - 1

Q.1 What will be the unit digit of the squares of the following numbers?

(i)	**81**	**(ii)**	**272**	**(iii)**	**799**	**(iv)**	**3853**
(v)	**1234**	**(vi)**	**26387**	**(vii)**	**52698**	**(viii)**	**99880**
(ix)	**12796**	**(x)**	**55555**				

Ans. (i) 81

The unit digit of the square of the number 81 will be 1. [$\because$ $1 \times 1 = 1$]

(ii) 272

The unit digit of the square of the number 272 will be 4. [$\because$ $2 \times 2 = 4$]

(iii) 799

The unit digit of the square of the number 799 will be 1. [$\because$ $9 \times 9 = 81$]

(iv) 3853

The unit digit of the square of the number 3853 will be 9. [$\because$ $3 \times 3 = 9$]

(v) 1234

The unit digit of the square of the number 1234 will be 6. [$\because$ $4 \times 4 = 16$]

(vi) 26387

The unit digit of the square of the number 26387 will be 9. [$\because$ $7 \times 7 = 49$]

(vii) 52698

The unit digit of the square of the number 52698 will be 4. [$\because$ $8 \times 8 = 64$]

(viii) 99880

The unit digit of the square of the number 99880 will be 0. [$\because$ $0 \times 0 = 0$]

(ix) 12796

The unit digit of the square of the number 12796 will be 6. [$\because$ $6 \times 6 = 36$]

(x) 55555

The unit digit of the square of the number 55555 will be 5. [$\because$ $5 \times 5 = 25$]

Q.2 The following numbers are obviously not perfect squares. Give reason.

(i)	**1057**	**(ii)**	**23453**	**(iii)**	**7928**	**(iv)**	**222222**
(v)	**64000**	**(vi)**	**89722**	**(vii)**	**222000**	**(viii)**	**505050**

Ans. (i) 1057

The number 1057 is not a perfect square because it ends with 7 whereas the square numbers end with 0, 1, 4, 5, 6 or 9.

(ii) 23453

The number 23453 is not a perfect square it ends with 3 whereas the square numbers end with 0, 1, 4, 5, 6 or 9.

(iii) 7928

The number 7928 is not a perfect square because it ends with 8 whereas the square numbers end with 0, 1, 4, 5, 6 or 9.

(iv) 222222

The number 222222 is not a perfect square because it ends with 2 whereas the square numbers end with 0, 1, 4, 5, 6 or 9.

(v) 64000

The number 64000 is not a square because the number of zeros at the end of a square number ending with zeroes is always even.

(vi) 89722

The number 89722 is not a square number because it ends in 2 whereas the square numbers end with 0, 1, 4, 5, 6 or 9.

(vii) 222000

The number 222000 is not a square number because it has 3 (an odd number of) zeroes at the end whereas the number of zeroes at the end of a square number of zeroes at the end of square number ending with zeros is always even.

(viii) 505050

The number 505050 is not a square number because it has 1 (an odd number of) zeroes at the end whereas the number of zeroes at the end of a square number of zeroes at the end of square number ending with zeros is always even.

Q.3 The square of which of the following would be odd numbers?

(i) 431 (ii) 2826 (iii) 7779 (iv) 82004

Ans. (i) 431

$\because$ 431 is an odd number.

$\therefore$ Its square will also be an odd number.

(ii) 2826

$\because$ 2826 is an even number.

$\therefore$ Its square will not be an odd number.

(iii) 7779

$\because$ 7779 is an odd number.

$\therefore$ Its square will be an odd number.

(iv) 82004

$\because$ 82004 is an even number.

$\therefore$ Its square will not be an odd number.

Q.4 Observe the following pattern and find the missing digits:

$$\mathbf{11^2 = 121}$$
$$\mathbf{101^2 = 10201}$$
$$\mathbf{1001^2 = 1002001}$$
$$\mathbf{100001^2 = 1 \ldots\ldots 2 \ldots\ldots 1}$$
$$\mathbf{10000001^2 = \ldots\ldots\ldots\ldots}$$

Ans. $100001^2 =$ **1000 2 0000 1**

$10000001^2 =$ **1000000 2 0000001**

Q.5 Observe the following pattern and supply the missing numbers:

$$11^2 = 121$$
$$101^2 = 10201$$
$$10101^2 = 102030201$$
$$1010101^2 = \ldots\ldots\ldots$$
$$\ldots\ldots\ldots^2 = 10203040504030201$$

Ans. $1010101^2 = \mathbf{1020304030201}$

$\mathbf{101010101^2} = 10203040504030201$

Q.6 Using the given pattern, find the missing numbers:

$$1^2 + 2^2 + 2^2 = 3^2$$
$$2^2 + 3^2 + 6^2 = 7^2$$
$$3^2 + 4^2 + 12^2 = 13^2$$
$$4^2 + 5^2 + \_\_^2 = 21^2$$
$$5^2 + \_\_^2 + 30^2 = 31^2$$
$$6^2 + 7^2 + \_\_^2 = \_\_^2$$

Ans. $4^2 + 5^2 + \underline{20}^2 = 21^2$

$5^2 + \underline{6}^2 + 30^2 = 31^2$

$6^2 + 7^2 + \underline{42}^2 = \underline{43}^2$

Q.7 Without adding, find the sum

(i) $1 + 3 + 5 + 7 + 9$

(ii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$

(iii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23$

Ans. (i) $1 + 3 + 5 + 7 + 9$ = sum of first five odd natural numbers $= 5^2 = 25$

(ii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$ = sum of first ten odd natural numbers $= 10^2 = 100$

(iii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 =$ sum of first twelve odd natural numbers $= 12^2 = 144$

Q.8 (i) Express 49 as the sum of 7 odd numbers.

(ii) Express 121 as the sum of 11 odd numbers.

Ans. (i) $49\ (= 7^2) = 1 + 3 + 5 + 7 + 9 + 11 + 13.$

(ii) $121\ (= 11^2) = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21.$

Q.9 How many numbers lie between squares of the following numbers?

(i) 12 and 13 (ii) 25 and 26 (iii) 99 and 100

Ans. (i) 12 and 13

Here, $n = 12$

$\therefore \quad 2n = 2 \times 12 = 24$

So, 24 numbers lie between squares of the numbers 12 and 13.

(ii) 25 and 26

Here, $n = 25$

$\therefore \quad 2n = 2 \times 25 = 50$

So, 50 numbers lie between squares of the numbers 25 and 26.

(iii) 99 and 100

Here, $n = 99$

$\therefore \quad 2n = 2 \times 99 = 198$

So, 198 numbers lie between squares of the numbers 99 and 100.

EXERCISE - 2

Q.1 Find the square of the following numbers :

(i) 32 (ii) 35 (iii) 86 (iv) 93

(v) 71 (vi) 46

Ans. (i) 32 $32 = 30 + 2$

Therefore, $32^2 = (30 + 2)^2 = 30\ (30 + 2) + 2\ (30 + 2) = 900 + 60 + 60 + 4 = 1024$

(ii) 35 $35 = 30 + 5$

Therefore, $35^2 = (30 + 5)^2 = 30\ (30 + 5) + 5\ (30 + 5) = 900 + 150 + 150 + 25 = 1225$

(iii) 86 $86 = 80 + 6$

Therefore, $86^2 = (80 + 6)^2 = 80\ (80 + 6) + 6\ (80 + 6) = 6400 + 480 + 480 + 36 = 7396$

(iv) 93 $93 = 90 + 3$

Therefore, $93^2 = (90 + 3)^2 = 90\ (90 + 3) + 3\ (90 + 3) = 8100 + 270 + 270 + 9 = 8649$

(v) 71 $71 = 70 + 1$

Therefore, $71^2 = (70 + 1)^2 = 70\ (70 + 1) + 1\ (70 + 1) = 4900 + 70 + 70 + 1 = 5041$

(vi) 46 $46 = 40 + 6$

Therefore, $46^2 = (40 + 6)^2 = 40\ (40 + 6) + 6\ (40 + 6) = 1600 + 240 + 240 + 36 = 2116$

Q.2 Write a Pythagorean triplet whose one number is

(i) 6 (ii) 14 (iii) 16 (iv) 18

Ans. (i) 6 Here, $2m = 6 \Rightarrow m = \frac{6}{2} = 3$

$m^2 - 1 = 3^2 - 1 = 9 - 1 = 8$

and $m^2 + 1 = 3^2 + 1 = 9 + 1 = 10$

So, a Pythagorean triplet, whose one number is 6, is 9, 8, 10.

(ii) 14 Here, $2m = 14 \Rightarrow m = \frac{14}{2} = 7$

$\therefore$ $m^2 - 1 = 7^2 - 1 = 49 - 1 = 48$

and $m^2 + 1 = 7^2 + 1 = 49 + 1 = 50$

So, a Pythagorean triplet, whose one number is 14, is 14, 48, 50.

(iii) 16 Here, $2m = 16 \Rightarrow m = \frac{16}{2} = 8$

$\therefore$ $m^2 - 1 = 8^2 - 1 = 64 - 1 = 63$

and $m^2 + 1 = 8^2 + 1 = 64 + 1 = 65$

So, a Pythagorean triplet, whose one number is 16, is 16, 63, 65.

(iv) 18 Here, $2m = 18 \Rightarrow m = \frac{18}{2} = 9$

$\therefore$ $m^2 - 1 = 9^2 - 1 = 81 - 1 = 80$

and $m^2 + 1 = 9^2 + 1 = 81 + 1 = 82$

So, a Pythagorean triplet, whose one number is 18, is 18, 80, 82.

EXERCISE - 3

Q.1 What could be the possible 'one's' digits of the square root of each of the following numbers?
(i) 9801 (ii) 99856 (iii) 998001 (iv) 657666025

Ans. (i) 9801, The units digit of the square root of the number 9801 could be 1 or 9.
(ii) 99856, The units digit of the square root of the number 99856 could be 4 or 6.
(iii) 998001, The units digit of the square root of the number 998001 could be 1 or 9.
(iv) 657666025, The units digit of the square root of the number 657666025 could be 5.

Q.2 Without any calculation, find the numbers which are surely not perfect squares.
(i) 153 (ii) 257 (iii) 408 (iv) 441

Ans. (i) 153, The number 153 is surely not a perfect square because it ends in 3 whereas the square numbers end with 0, 1, 4, 5, 6 or 9.

(ii) 257, The number 257 is surely not a perfect square because it ends in 7 whereas the square numbers end with 0, 1, 4, 5, 6 or 9.

(iii) 408, The number 408 is surely not a perfect square because it ends in 8 whereas the square numbers end with 0, 1, 4, 5, 6 or 9.

(iv) 441, The number may be a perfect square surely as the square numbers end with 0, 1, 4, 5, 6 or 9.

Q.3 Find the square roots of 100 and 169 by the method of repeated subtraction.

Ans. (A) 100

(i) $100 - 1 = 99$
(ii) $99 - 3 = 96$
(iii) $96 - 5 = 91$
(iv) $91 - 7 = 84$
(v) $84 - 9 = 75$
(vi) $75 - 11 = 64$
(vii) $64 - 13 = 51$
(viii) $51 - 15 = 36$
(ix) $36 - 17 = 19$
(x) $19 - 19 = 0$

Since from 100 we subtracted successive odd numbers starting from 1 and obtained 0 at the 10th step. Therefore, $\sqrt{100} = 10$

(B) 169

(i) $169 - 1 = 168$
(ii) $168 - 3 = 165$
(iii) $165 - 5 = 160$
(iv) $160 - 7 = 153$
(v) $153 - 9 = 144$
(vi) $144 - 11 = 133$
(vii) $133 - 13 = 120$
(viii) $120 - 15 = 105$
(ix) $105 - 17 = 88$
(x) $88 - 19 = 69$

(xi) $69 - 21 = 48$
(xii) $48 - 23 = 25$
(xiii) $25 - 25 = 0$

Since from 169 we subtracted successive odd numbers starting from 1 and obtained 0 the 13th step, therefore, $\sqrt{169} = 13$.

Q.4 Find the square roots of the following numbers by the Prime factorisation Method.

(i) 729 (ii) 400 (iii) 1764 (iv) 4096
(v) 7744 (vi) 9604 (vii) 5929 (viii) 9216
(ix) 529 (x) 8100

Ans. (i) **729,** The prime factorisation of 729 is

$$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3.$$

By pairing the prime factors, we get

$$729 = \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}$$

So, $\sqrt{729} = 3 \times 3 \times 3 = 27$

$$\begin{array}{r|l} 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 84 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

(ii) **400,** The prime factorisation of 400 is

$$400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5.$$

By pairing the prime factors, we get

$$400 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}$$

Therefore, $\sqrt{400} = 2 \times 2 \times 5 = 20$

$$\begin{array}{r|l} 2 & 400 \\ \hline 2 & 200 \\ \hline 2 & 100 \\ \hline 2 & 50 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

(iii) **1764,** The prime factorisation of 1764 is

$$1764 = 2 \times 2 \times 3 \times 3 \times 7 \times 7.$$

By pairing the prime factors, we get

$$1764 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$$

So, $\sqrt{1764} = 2 \times 3 \times 7 = 42$

$$\begin{array}{r|l} 2 & 1764 \\ \hline 2 & 882 \\ \hline 3 & 441 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

(iv) **4096,** The prime factorisation of 4096 is

$$4096 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2.$$

By pairing the prime factors, we get

$$\begin{array}{r|l} 2 & 4096 \\ \hline 2 & 2048 \\ \hline 2 & 1024 \\ \hline 2 & 512 \\ \hline 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

$$4096 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2}$$

So, $\sqrt{4096} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

(v) **7744,** The prime factorisation of 7744 is

$$7744 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11 \times 11$$

By pairing the prime factors, we get

$$7744 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{11 \times 11}$$

So, $\sqrt{7744} = 2 \times 2 \times 2 \times 11 = 88$

$$\begin{array}{r|l} 2 & 7744 \\ \hline 2 & 3872 \\ \hline 2 & 1936 \\ \hline 2 & 968 \\ \hline 2 & 484 \\ \hline 2 & 242 \\ \hline 11 & 121 \\ \hline & 11 \end{array}$$

(vi) **9604,** The prime factorisation of 9604 is

$$9604 = 2 \times 2 \times 7 \times 7 \times 7 \times 7$$

By pairing the prime factors, we get

$$9604 = \underline{2 \times 2} \times \underline{7 \times 7} \times \underline{7 \times 7}$$

So, $\sqrt{9604} = 2 \times 7 \times 7 = 98$

$$\begin{array}{r|l} 2 & 9604 \\ \hline 2 & 4802 \\ \hline 7 & 2401 \\ \hline 7 & 343 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

(vii) **5929,** The prime factorisation of 9604 is

$$5929 = 7 \times 7 \times 11 \times 11$$

By pairing the prime factors, we get

$$5929 = \underline{7 \times 7} \times \underline{11 \times 11}$$

So, $\sqrt{5929} = 7 \times 11 = 77$

$$\begin{array}{r|l} 7 & 5929 \\ \hline 7 & 847 \\ \hline 11 & 121 \\ \hline & 11 \end{array}$$

(viii) **9216,** The prime factorisation of 9216 is

$$9216 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3.$$

By pairing the prime factors, we get

$$\begin{array}{r|l} 2 & 9216 \\ \hline 2 & 4608 \\ \hline 2 & 2304 \\ \hline 2 & 1152 \\ \hline 2 & 576 \\ \hline 2 & 288 \\ \hline 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$$9216 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$$

So, $\sqrt{9216} = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 96$

(ix) **529,** The prime factorisation of 529 is

$$529 = 23 \times 23$$

By pairing the prime factors, we get

$$529 = \underline{23 \times 23}$$

So, $\sqrt{529} = 23$

$$\begin{array}{r|l} 23 & 529 \\ \hline & 23 \end{array}$$

(x) **8100,** The prime factorisation of 8100 is

$$8100 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$$

By pairing the prime factors, we get

$$8100 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{5 \times 5}$$

So, $\sqrt{8100} = 2 \times 3 \times 3 \times 5 = 90$

2	8100
2	4050
3	2025
3	675
3	225
3	75
5	25
	5

Q.5 For each of the following numbers, find the smallest whole number by which it should be multiplied so as to get a perfect square number. Also find the square root of the square number so obtained.

(i) 252 (ii) 180 (iii) 1008 (iv) 2028

(v) 1458 (vi) 768

Ans. (i) **252,** The prime factorisation of 252 is $252 = 2 \times 2 \times 3 \times 3 \times 7$.

As the prime factor 7 has no pair, 252 is not a perfect square.

If 7 gets a pair, then the number will be a perfect square.

So, we multiply 252 by 7 to get

$252 \times 7 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$

Now each prime factor has a pair.

Therefore, $252 \times 7 = 1764$ is a perfect square.

Thus the required smallest number is 7.

Thus, $\sqrt{1764} = 2 \times 3 \times 7 = 42$

2	252
2	126
3	63
3	21
	7

(ii) **180,** The prime factorisation of 180 is $180 = 2 \times 2 \times 3 \times 3 \times 5$.

As the prime factor 5 has no pair, 180 is not a perfect square.

If 5 gets a pair, then the number will be a perfect square.

So, we multiply 180 by 5 to get

$180 \times 5 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{5 \times 5}$

Now each prime factor has a pair.

Therefore, $180 \times 5 = 900$ is a perfect square.

Thus the required smallest number is 5.

Thus, $\sqrt{900} = 2 \times 3 \times 5 = 30$

2	180
2	90
3	45
3	15
	5

(iii) **1008,** The prime factorisation of 1008 is $1008 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7$

As the prime factor 7 has no pair, 1008 is not a perfect square.

If 7 gets a pair, then the number will be a perfect square.

So, we multiply 1008 by 7 to get

$1008 \times 7 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$

Now each prime factor has a pair.

Therefore, $1008 \times 7 = 7056$ is a perfect square.

Thus the required smallest number is 7.

Thus, $\sqrt{7056} = 2 \times 2 \times 3 \times 7 = 84$

2	1008
2	504
2	252
2	126
3	63
3	21
	7

(iv) **2028,** The prime factorisation of 2028 is $2028 = 2 \times 2 \times 3 \times 13 \times 13$

As the prime factor 3 has no pair, 2028 is not a perfect square.

If 3 gets a pair, then the number will be a perfect square.

So, we multiply 2028 by 3 to get

$2028 \times 3 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{13 \times 13}$

Now each prime factor has a pair.

Therefore, $2028 \times 3 = 6084$ is a perfect square.

Thus the required smallest number is 3.

Thus, $\sqrt{6084} = 2 \times 3 \times 13 = 78$

2	2028
2	1014
3	507
13	169
	13

(v) **1458,** The prime factorisation of 1458 is $1458 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

As the prime factor 2 has no pair, 1458 is not a perfect square.

If 2 gets a pair, then the number will be a perfect square.

So, we multiply 1458 by 2 to get

$1458 \times 2 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}$

Now each prime factor has a pair.

Therefore, $1458 \times 2 = 2916$ is a perfect square.

Thus the required smallest number is 2.

Thus, $\sqrt{2916} = 2 \times 3 \times 3 \times 3 = 54$

2	1458
3	729
3	243
3	81
3	27
3	9
	3

(vi) **768,** The prime factorisation of 768 is $768 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

As the prime factor 3 has no pair, 768 is not a perfect square.

If 3 gets a pair, then the number will be a perfect square.

So, we multiply 768 by 3 to get

$768 \times 3 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$

Now each prime factor has a pair.

Therefore, $768 \times 32 = 2304$ is a perfect square.

Thus the required smallest number is 3.

Thus, $\sqrt{2304} = 2 \times 3 \times 3 \times 3 = 54$

2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
	3

Q.6 **For each of the following numbers, find the smallest whole number by which it should be divided so as to get a perfect square. Also find the square root of the square number so obtained.**

(i) 252 (ii) 2925 (iii) 396 (iv) 2645

(v) 2800 (vi) 1620

Ans. **(i)** **252,** The prime factorisation of 252 is $252 = 2 \times 2 \times 3 \times 7$.

We see that the prime factor 7 has no pair.

So, if we divide 252 by 7, then we get

$$252 \div 7 = \underline{2 \times 2} \times \underline{3 \times 3}$$

Now each prime factor has a pair.

Therefore, $252 \div 7 = 36$ is a perfect square.

Thus, the required smallest number is 7.

Hence, $\sqrt{36} = 2 \times 3 = 6$

2	252
2	126
3	63
3	21
	7

(ii) **2925,** The prime factorisation of 2925 is $2925 = 3 \times 3 \times 5 \times 5 \times 13$.
We see that the prime factor 13 has no pair.
So, if we divide 2925 by 13, then we get

$$2925 \div 13 = \underline{3 \times 3} \times \underline{5 \times 5}$$

Now each prime factor has a pair.
Therefore, $2925 \div 13 = 225$ is a perfect square.
Thus, the required smallest number is 13.

Hence, $\sqrt{225} = 3 \times 5 = 15$

3	2925
3	975
5	325
5	65
	13

(iii) **396,** The prime factorisation of 396 is $396 = 2 \times 2 \times 3 \times 3 \times 11$.
We see that the prime factor 11 has no pair.
So, if we divide 396 by 11, then we get

$$396 \div 11 = \underline{2 \times 2} \times \underline{3 \times 3}$$

Now each prime factor has a pair.
Therefore, $396 \div 11 = 36$ is a perfect square.
Thus, the required smallest number is 11.

Hence, $\sqrt{36} = 2 \times 3 = 6$

2	396
2	198
3	99
3	33
	11

(iv) **2645,** The prime factorisation of 2645 is $2645 = 5 \times 23 \times 23$
We see that the prime factor 5 has no pair.
So, if we divide 2645 by 5, then we get

$$2645 \div 5 = \underline{23 \times 23}$$

Now each prime factor 23 has a pair.
Therefore, $2645 \div 5 = 529$ is a perfect square.
Thus, the required smallest number is 5.

Hence, $\sqrt{529} = 23$

5	2645
23	529
	23

(v) **2800,** The prime factorisation of 2800 is $2800 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 7$
We see that the prime factor 7 has no pair.
So, if we divide 2800 by 7, then we get

$$2800 \div 7 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}$$

Now each prime factor has a pair.
Therefore, $2800 \div 7 = 400$ is a perfect square.
Thus, the required smallest number is 7.

Hence, $\sqrt{400} = 2 \times 2 \times 5 = 20$

2	2800
2	1400
2	700
2	350
5	175
5	35
	7

(vi) **1620,** The prime factorisation of 1620 is $1620 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5$
We see that the prime factor 5 has no pair.
So, if we divide 1620 by 5, then we get

$$1620 \div 5 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}$$

Now each prime factor has a pair.
Therefore, $1620 \div 5 = 324$ is a perfect square.
Thus, the required smallest number is 5.

Hence, $\sqrt{324} = 2 \times 3 \times 3 = 18$

2	1620
2	810
3	405
3	135
3	45
3	15
	5

Q.7 The students of class VIII of a school donated Rs.2401 in all, for Prime Minister's National Relief Fund. Each student donated as many rupees as the number of students in the class. Find the number of students in the class.

Ans. Let the number of students in the class be x.

Then rupees donated by each student = Rs. x.

$\therefore$ Rupees donated by x students = Rs.$x \times x = x^2$

$\because$ The students of class VIII of a school donated Rs.2401 for Prime Minister's National Relief Fund.

$\therefore$ $x^2 = 2401 \Rightarrow x = \sqrt{2401}$

7	2401
7	343
7	49
	7

The prime factorisation of 2401 is

$2401 = \underline{7 \times 7} \times \underline{7 \times 7}$

$\therefore$ $x = \sqrt{2401} = \sqrt{\underline{7 \times 7} \times \underline{7 \times 7}}$

$\Rightarrow$ $x = 7 \times 7 = 49$

Hence, the number of students in the class is 49.

Q.8 2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.

Ans. Let the number of row be x.

Then number of plants in each row = x.

$\therefore$ Number of plants in x rows = $x \times x = x^2$

But 2025 plants are to be planted in a garden.

$\therefore$ $x^2 = 2025 \Rightarrow x = \sqrt{2025}$

3	2025
3	675
3	225
3	75
5	25
	5

The prime factorisation of 2025 is

$2025 = \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{5 \times 5}$

$\therefore$ $x = \sqrt{2025} = \sqrt{\underline{3 \times 3} \times \underline{3 \times 3} \times \underline{5 \times 5}}$

$\Rightarrow$ $x = 3 \times 3 \times 5 \Rightarrow x = 45$

Hence, the number of row is 45 and the number of plants in each row is 45.

Q.9 Find the smallest square number that is divisible by each of the numbers 4, 9 and 10.

Ans. The least number divisible by each one of 4, 9, and 10 is their L.C.M.

The LCM of 4, 9 and 10 is $2 \times 2 \times 3 \times 3 \times 5 = 180$.

Now prime factorisation of 180 is $180 = \underline{2 \times 2} \times \underline{3 \times 3} \times 5$

2	4, 9, 10
2	2, 9, 5
3	1, 9, 5
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

The prime factor 5 is not is pair. Therefore 180 is not a perfect square.

In order to get a perfect square, each factor of 180 must be paired.

So we need to make pair of 5.

Therefore 180 should be multiplied by 5.

Hence, the required smallest square number is $180 \times 5 = 900$.

Q.10 Find the smallest square number that is divisible by each of the numbers 8, 15 and 20.

Ans. The least number divisible by each one of 8, 15 and 20 is their LCM

The LCM of 8, 15 and 20 is $2 \times 2 \times 2 \times 3 \times 5 = 120$.

$$\begin{array}{r|l} 2 & 8, 15, 20 \\ \hline 2 & 4, 15, 20 \\ \hline 2 & 2, 15, 5 \\ \hline 3 & 1, 15, 5 \\ \hline 5 & 1, 5, 5 \\ \hline & 1, 1, 1 \end{array}$$

Now prime factorisation of 120 is

$$120 = \underline{2 \times 2} \times 2 \times 3 \times 5$$

The prime factors 2, 3 and 5 are not in pairs.

Therefore, 120 is not a perfect square.

In order to get a perfect square, each factor of 120 must be paired.

So, we need to make pairs of 2, 3 and 5.

Therefore 120 should be multiplied by $2 \times 3 \times 5$; i.e. 30.

Hence, the required smallest square number is $120 \times 30 = 3600$.

EXERCISE - 4

Q.1 Find the square root of each of the following numbers by Division method.

(i)	**2304**	**(ii)**	**4489**	**(iii)**	**3481**	**(iv)**	**529**
(v)	**3249**	**(vi)**	**1369**	**(vii)**	**5776**	**(viii)**	**7921**
(ix)	**576**	**(x)**	**1024**	**(xi)**	**3136**	**(xii)**	**900**

Ans. **(i)** **2304,**

$$\begin{array}{r|ll} & \multicolumn{2}{l}{48} \\ \hline 4 & \overline{23} & \overline{04} \\ & -16 & \\ \hline 88 & 7 & 04 \\ & -7 & 04 \\ \hline & 0 & \end{array}$$

Therefore, $\sqrt{2304} = 48$

(ii) **4489,**

$$\begin{array}{r|ll} & \multicolumn{2}{l}{67} \\ \hline 6 & \overline{44} & \overline{89} \\ & -36 & \\ \hline 127 & 8 & 89 \\ & -8 & 89 \\ \hline & 0 & \end{array}$$

Therefore, $\sqrt{4489} = 67$

(iii) **3481,**

$$\begin{array}{r|ll} & \multicolumn{2}{l}{59} \\ \hline 5 & \overline{34} & \overline{81} \\ & -25 & \\ \hline 109 & 9 & 81 \\ & -9 & 81 \\ \hline & 0 & \end{array}$$

Therefore, $\sqrt{3481} = 59$

(iv) **529,**

$$\begin{array}{r|ll} & \multicolumn{2}{l}{23} \\ \hline 2 & \overline{5} & \overline{29} \\ & -4 & \\ \hline 43 & 1 & 29 \\ & -1 & 29 \\ \hline & 0 & \end{array}$$

Therefore, $\sqrt{529} = 23$

(v) **3249,**

$$\begin{array}{r|ll} & \multicolumn{2}{c}{57} \\ \hline 5 & \overline{32} & \overline{49} \\ & -25 & \\ \hline 107 & 7 & 49 \\ & -7 & 49 \\ \hline & 0 & \end{array}$$

Therefore, $\sqrt{3249} = 57$

(vi) **1369,**

$$\begin{array}{r|ll} & \multicolumn{2}{c}{37} \\ \hline 3 & \overline{13} & \overline{69} \\ & -9 & \\ \hline 67 & 4 & 69 \\ & -4 & 69 \\ \hline & 0 & \end{array}$$

Therefore, $\sqrt{1369} = 37$

(vii) **5776,**

$$\begin{array}{r|ll} & \multicolumn{2}{c}{76} \\ \hline 7 & \overline{57} & \overline{76} \\ & -49 & \\ \hline 146 & 8 & 76 \\ & -8 & 76 \\ \hline & 0 & \end{array}$$

Therefore, $\sqrt{5776} = 76$

(viii) **7921,**

$$\begin{array}{r|ll} & \multicolumn{2}{c}{89} \\ \hline 8 & \overline{79} & \overline{21} \\ & -64 & \\ \hline 169 & 15 & 21 \\ & -15 & 21 \\ \hline & 0 & \end{array}$$

Therefore, $\sqrt{7921} = 89$

(ix) **576,**

$$\begin{array}{r|ll} & \multicolumn{2}{c}{24} \\ \hline 2 & \overline{5} & \overline{76} \\ & -4 & \\ \hline 44 & 1 & 76 \\ & -1 & 76 \\ \hline & 0 & \end{array}$$

Therefore, $\sqrt{576} = 24$

(x) **1024,**

$$\begin{array}{r|ll} & \multicolumn{2}{c}{32} \\ \hline 3 & \overline{10} & \overline{24} \\ & -9 & \\ \hline 62 & 1 & 24 \\ & -1 & 24 \\ \hline & 0 & \end{array}$$

Therefore, $\sqrt{1024} = 32$

(xi) **3136,**

$$\begin{array}{r|l} & \;\;5\quad 6 \\ \hline 5 & \overline{31}\;\overline{36} \\ & -25 \\ \hline 106 & \;\;6\;\;36 \\ & -6\;\;36 \\ \hline & \quad 0 \end{array}$$

Therefore, $\sqrt{3136} = 56$

(xii) **900,**

$$\begin{array}{r|l} & 30 \\ \hline 3 & \bar{9}\;\overline{00} \\ & -9 \\ \hline 60 & \;00 \\ & -00 \\ \hline & \;\;0 \end{array}$$

Therefore, $\sqrt{900} = 30$

Q.2 Find the number of digits in the square root of each of the following numbers (without any calculation)

(i) 64 **(ii) 144** **(iii) 4489** **(iv) 27225**

(v) 390625

Ans. **(i)** **64,** Number (n) of digits in 64 = 2 which is even.

$\therefore$ Number of digits in the square root of 64 = $\frac{n}{2} = \frac{2}{2} = 1$

(ii) **144,** Number (n) of digits in 144 = 3 which is odd.

$\therefore$ Number of digits in the square root of 144 = $\frac{n+1}{2} = \frac{3+1}{2} = \frac{4}{2} = 2$

(iii) **4489,** Number (n) of digits in 4489 = 4 which is even.

$\therefore$ Number of digits in the square root of 4489 = $\frac{n}{2} = \frac{4}{2} = 2$

(iv) **27225,** Number (n) of digits in 27225 = 5 which is odd.

$\therefore$ Number of digits in the square root of 27225 = $\frac{n+1}{2} = \frac{5+1}{2} = \frac{6}{2} = 3$

(v) **390625,** Number (n) of digits in 390625 = 6 which is even.

$\therefore$ Number of digits in the square root of 390625 = $\frac{n}{2} = \frac{6}{2} = 3$

Q.3 Find the square root of the following decimal numbers.

(i) 2.56 (ii) 7.29 (iii) 51.84 (iv) 42.25
(v) 31.36

Ans. (i) 2.56,

$$\begin{array}{r|ll} & 1. & 6 \\ \hline 1 & \overline{2}. & \overline{56} \\ & -1 & \\ \hline 26 & 1 & 56 \\ & -1 & 56 \\ \hline & & 0 \end{array}$$

Hence, $\sqrt{2.56} = 1.6$

(ii) 7.29,

$$\begin{array}{r|ll} & 2. & 7 \\ \hline 2 & 7. & \overline{29} \\ & -4 & \\ \hline 47 & 3 & 29 \\ & -3 & 29 \\ \hline & & 0 \end{array}$$

Hence, $\sqrt{7.29} = 2.7$

(iii) 51.84,

$$\begin{array}{r|ll} & 7. & 2 \\ \hline 7 & 51. & 84 \\ & -49 & \\ \hline 142 & 2 & 84 \\ & -2 & 84 \\ \hline & & 0 \end{array}$$

Hence, $\sqrt{51.84} = 7.2$

(iv) 42.25,

$$\begin{array}{r|ll} & 6. & 5 \\ \hline 6 & \overline{42}. & \overline{25} \\ & -36 & \\ \hline 125 & 6 & 25 \\ & -6 & 25 \\ \hline & & 0 \end{array}$$

Hence, $\sqrt{2.56} = 1.6$

(v) 31.36,

$$\begin{array}{r|ll} & 5. & 6 \\ \hline 5 & \overline{31}. & \overline{36} \\ & -25 & \\ \hline 106 & 6 & 36 \\ & -6 & 36 \\ \hline & & 0 \end{array}$$

Hence, $\sqrt{31.36} = 5.6$

Q.4 Find the least number which must be subtracted from each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.

(i) 402 (ii) 1989 (iii) 3250 (iv) 825

(v) 4000

Ans. (i) 402, We have

$$\begin{array}{r|r} & 2\quad 0 \\ \hline 2 & \overline{4}\quad \overline{02} \\ & -4\quad\;\; \\ \hline 40 & 02 \\ & -00 \\ \hline & 2 \end{array}$$

This shows that 20^2 is less than 402 by 2. This is means that if we subtract the remainder from the number, we get a perfect square. So, the required least number is 2.

Therefore, the required perfect square is $402 - 2 = 400$

Hence, $\sqrt{400} = 20$

(ii) 1989, We have

$$\begin{array}{r|r} & 4\quad 4 \\ \hline 4 & \overline{19}\quad \overline{89} \\ & -16\quad\;\;\; \\ \hline 84 & 3\quad 89 \\ & -3\quad 36 \\ \hline & 53 \end{array}$$

This shows that 44^2 is less than 1989 by 53. This is means that if we subtract the remainder from the number, we get a perfect square. So, the required least number is 53.

Therefore, the required perfect square is $1989 - 53 = 1936$

Hence, $\sqrt{1936} = 44$

(iii) 3250, We have

$$\begin{array}{r|r} & 5\quad 7 \\ \hline 5 & \overline{32}\quad \overline{50} \\ & -25\quad\;\;\; \\ \hline 107 & 7\quad 50 \\ & -7\quad 49 \\ \hline & 1 \end{array}$$

This shows that 57^2 is less than 3250 by 1. This means that if we subtract the remainder from the number, we get a perfect square. So, the required least number is 1.

Therefore, the required perfect square is $3250 - 1 = 3249$

Hence, $\sqrt{3249} = 57$

(iv) **825,** We have

$$\begin{array}{r|rr} & 2 & 8 \\ \hline 2 & \overline{8} & \overline{25} \\ & -4 & \\ \hline 48 & 4 & 25 \\ & -3 & 84 \\ \hline & & 41 \end{array}$$

This shows that 28^2 is less than 825 by 41. This means that if we subtract the remainder from the number, we get a perfect square. So, the required least number is 41.
Therefore, the required perfect square is $825 - 41 = 784$

Hence, $\sqrt{784} = 28$

(v) **4000,** We have

$$\begin{array}{r|rr} & 63 & \\ \hline 6 & \overline{40} & \overline{00} \\ & -36 & \\ \hline 123 & 4 & 00 \\ & -3 & 69 \\ \hline & & 31 \end{array}$$

This shows that 63^2 is less than 4000 by 31. This means that if we subtract the remainder from the number, we get a perfect square. So, the required least number is 31.
Therefore, the required perfect square is $4000 - 31 = 3969$

Hence, $\sqrt{3969} = 63$

Q.5 Find the least number which must be added to each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.

(i) **525** **(ii)** **1750** **(iii)** **252** **(iv)** **1825**

(v) **6412**

Ans. **(i)** **525,** We have

$$\begin{array}{r|rr} & 22 & \\ \hline 2 & \overline{5} & \overline{25} \\ & -4 & \\ \hline 42 & 1 & 25 \\ & & -84 \\ \hline & & 41 \end{array}$$

This shows that $22^2 < 525$.
Next perfect square is $23^2 = 529$.
Hence, the number to be added is $23^2 - 525 = 529 - 525 = 4$
Therefore, the perfect square so obtained is $525 + 4 = 529$

Hence, $\sqrt{529} = 23$

(ii) **1750,** We have

$$\begin{array}{r|rr} & 41 & \\ \hline 4 & \overline{17} & \overline{50} \\ & -16 & \\ \hline 81 & 1 & 50 \\ & & -81 \\ \hline & & 69 \end{array}$$

This shows that $41^2 < 1750$.
Next perfect square is $42^2 = 1764$.
Hence, the number to be added is $42^2 - 1750 = 1764 - 1750 = 14$
Therefore, the perfect square so obtained is $1750 + 14 = 1764$

Hence, $\sqrt{1764} = 42$

(iii) **252,** We have

This shows that $15^2 < 252$.

Next perfect square is $16^2 = 256$.

Hence, the number to be added is $16^2 - 252 = 256 - 252 = 4$

Therefore, the perfect square so obtained is $252 + 4 = 256$

Hence, $\sqrt{256} = 16$

$$\begin{array}{r|rr} & 1 & 5 \\ \hline 1 & \overline{2} & \overline{52} \\ & -1 & \\ \hline 25 & 1 & 52 \\ & -1 & 25 \\ \hline & & 27 \end{array}$$

(iv) **1825,** We have

This shows that $42^2 < 1825$.

Next perfect square is $43^2 = 1849$.

Hence, the number to be added is $43^2 - 1825 = 1849 - 1825 = 24$

Therefore, the perfect square so obtained is $1825 + 24 = 1849$

Hence, $\sqrt{1849} = 43$

$$\begin{array}{r|rr} & 4 & 2 \\ \hline 4 & \overline{18} & \overline{25} \\ & -16 & \\ \hline 82 & 2 & 25 \\ & -1 & 64 \\ \hline & & 61 \end{array}$$

(v) **6412,** We have

This shows that $80^2 < 6412$.

Next perfect square is $81^2 = 6412$.

Hence, the number to be added is $81^2 - 6412 = 6561 - 6412 = 149$

Therefore, the perfect square so obtained is $6412 + 149 = 6561$

Hence, $\sqrt{6561} = 81$

$$\begin{array}{r|rr} & 8 & 0 \\ \hline 8 & \overline{64} & \overline{12} \\ & -64 & \\ \hline 160 & & 12 \\ & & -0 \\ \hline & & 12 \end{array}$$

Q.6 Find the length of the side of a square whose area is 441 m^2.

Ans. Area of the square = 441 m^2

$\therefore$ Length of the side of the square = $\sqrt{441}$ m

Therefore, $\sqrt{441} = 21$ m

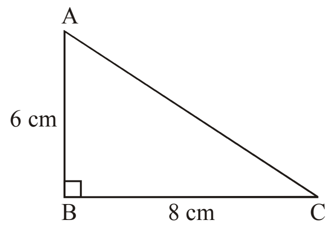
Hence, the length of the side of the square is 21 m.

$$\begin{array}{r|rr} & 2 & 1 \\ \hline 2 & \overline{4} & \overline{41} \\ & -4 & \\ \hline 41 & & 41 \\ & & -41 \\ \hline & & 0 \end{array}$$

Q.7 In a right triangle ABC $\angle B = 90°$.

(a) If AB = 6 cm, BC = 8 cm, find AC

(b) If AC = 13 cm, BC = 5 cm, find AB.



Ans. **(a)** In the right triangle ABC,

$\because$ $\angle B = 90°$ [Given]

$\therefore$ By phytagoras theorem

$AC^2 = AB^2 + BC^2$

$\Rightarrow$ $AC^2 = AB^2 + BC^2$

$\Rightarrow$ $AC^2 = 6^2 + 8^2$

$\Rightarrow$ $AC^2 = 36 + 64$

$\Rightarrow$ $AC = \sqrt{100}$

$$\begin{array}{r|rr} & 1 & 0 \\ \hline 1 & \overline{1} & \overline{00} \\ & -1 & \\ \hline 20 & & 00 \\ & & -0 \\ \hline & & 0 \end{array}$$

(b) In the right triangle ABC,

$\because$ $\angle B = 90°$ [Given]

$\therefore$ By phytagoras theorem

$AC^2 = AB^2 + BC^2$

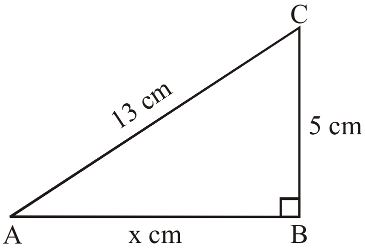
$\Rightarrow$ $13^2 = AB^2 + 5^2$

$\Rightarrow$ $169 = AB^2 + 25$

$\Rightarrow$ $AB^2 = 169 - 25$

$\Rightarrow$ $AB^2 = 144$

$\Rightarrow$ $AB = \sqrt{144}$



$$\begin{array}{r|l} & \ \ 12 \\ \hline 1 & \overline{1}\ \overline{44} \\ & -1 \\ \hline 22 & \ \ \ 44 \\ & \ -44 \\ \hline & \ \ \ \ 0 \end{array}$$

Therefore, $\sqrt{144} = 12$

Hence, AB is equal to 12 cm.

Q.8 A gardener has 1000 plants. He wants to plant these in such a way that the number of rows and the number of columns remain same. Find the minimum number of plants he needs more for this.

Ans. Let the number of rows be x.

$$\begin{array}{r|l} & \ \ 31 \\ \hline 3 & \overline{10}\ \overline{00} \\ & -9 \\ \hline 61 & \ 100 \\ & -61 \\ \hline & \ \ 39 \end{array}$$

Then the number of columns is x.

So, the number of plants is $x \times x = x^2$ which is a perfect square.

Let us find out the square root of 1000 by division method.

This shows that $31^2 < 1000$.

Next perfect square number is $32^2 = 1024$.

Hence, the minimum number of plants he needs more for this = 1024 – 1000 = 24.

Q.9 There are 500 children in a school. For a P.T. drill they have to stand in such a manner that the number of rows is equal to number of column. How many children would be left out in this arrangement?

Ans. Let the number of rows be x.

$$\begin{array}{r|l} & \ \ 22 \\ \hline 2 & \overline{5}\ \overline{00} \\ & -4 \\ \hline 42 & \ 100 \\ & -84 \\ \hline & \ \ 16 \end{array}$$

Then the number of columns is x.

So, The number of children is $x \times x = x^2$ which is a perfect square.

Let us find out the square.

Let us find out the square root of 500 by division method.

We get the remainder 16. It shows that 22^2 is less than 500 by 16.

This means that 16 children would be leftout in this arrangement.

CONCEPT APPLICATION LEVEL - II

SECTION - A

- **FILL IN THE BLANKS**

Q.1 Square numbers can only have ___________ number of zeros at the end.

Q.2 Numbers obtained when a number is multiplied by itself three times are known as ___________.

Q.3 The number of zeroes at the end of the square of a number is ___________ the number of zeroes at the end of the number.

Q.4 The smallest number by which 81 should be divided to make it a perfect cube is ___________.

Q.5 If a number ends in two 9's then its cube ends in ___________ number of 9's.

Q.6 The square root of a 4-digit or a 3 digit number is a ___________ digit number.

Q.7 A number n is a perfect cube only if there is an integer m such that ___________.

Q.8 Square of a ___________ number between 0 and 1 is __________ than the number itself.

Q.9 If 'a' is a square root of 'b' then 'b' is ___________ of 'a'.

Q.10 A number whose square root it exact is called a ___________.

Q.11 Square root of 0.01 is _________

Q.12 When a 'n' digit number is squared, then the number of digits in the square thus obtained is _________.

Q.13 If $7^2 = 49$ and $0.7^2 = 0.49$, then $0.007^2 =$ _________.

Q.14 The square of a proper fraction is always _________ than itself.

SECTION - B

- **MULTIPLE CHOICE QUESTIONS**

Q.1 The smallest number by which 136 must be multiplied so that it becomes a perfect square is
(A) 2 (B) 17 (C) 34 (D) None of these

Q.2 The product of two numbers is 1936. If one number is 4 times the other, the numbers are
(A) 16, 121 (B) 22, 88 (C) 44, 44 (D) None of these

Q.3 The least square number exactly divisible by 4, 6, 10, 15 is
(A) 400 (B) 100 (C) 25 (D) 900

Q.4 The value of $\sqrt{388+\sqrt{127+\sqrt{289}}}$ is
(A) 17 (B) 12 (C) 20 (D) None of these

Q.5 A gardener arranges plants in rows to form a square. He finds that in doing so 15 plants are left out. If the total number of plants are 3984, the number of plants in each row are
(A) 62 (B) 63 (C) 64 (D) None of these

Q.6 If a is a natural number then $a^2 + \frac{1}{a^2}$ is always greater than or equal to

(A) 6 (B) 4 (C) 3 (D) 2

Q.7 If $\sqrt{0.04 \times 0.4 \times a} = 0.4 \times 0.04 \times \sqrt{b}$, then value of $\frac{b}{a}$ is

(A) 0.016 (B) $\frac{125}{2}$ (C) 0.16 (D) None of these

Q.8 The hypotenuse of an isosceles right angled triangular field has a length of $30\sqrt{2}$ m, then length of other side is

(A) $30\sqrt{2}$ m (B) 30 m (C) 25 m (D) None of these

Q.9 The sides of a triangle are denoted by x, y and z. Area of the triangle and semi perimeter of the triangle are denoted by P and q respectively. If P = $\sqrt{q(q-x)(q-y)(q-z)}$ and $x + y - z = y + z - x = z + x - y = 4$. Find P (in square units).

(A) $2\sqrt{3}$ (B) $3\sqrt{3}$ (C) $4\sqrt{3}$ (D) $6\sqrt{3}$

Q.10 Find the square root of $\frac{81b^2a^4}{36x^2y^6}$.

(A) $\frac{3ba^2}{2xy^3}$ (B) $\frac{3b^2a}{2x^2y}$ (C) $\frac{3ba}{2x^3y}$ (D) $\frac{3ba}{2xy^2}$

Q.11 Which of the following can be a perfect square?

(A) A number ending in 3 or 7 (B) A number ending with odd number of zeros
(C) A number ending with even number of zeros (D) A number ending in 2.

Q.12 Which of the following can be the square of a natural number 'n'?

(A) sum of the squares of first n natural numbers (B) sum of the first n natural numbers
(C) sum of first (n – 1) natural numbers (D) sum of first 'n' odd natural numbers

Q.13 Which of the following is the number of non-perfect square number' between the squares of the numbers n and n + 1?

(A) n + 1 (B) n (C) 2n (D) 2n + 1

Q.14 Which of the following is the difference between the squares of two consecutive natural numbers is

(A) sum of the two numbers (B) difference of the numbers
(C) twice the sum of the two numbers (D) twice the sum of the two numbers

Q.15 Which of the following is the number of non-perfect square number between 17^2 and 18^2?
(A) 613 (B) 35 (C) 34 (D) 70

Q.16 Which of the following is the difference between the squares of 21 and 22?
(A) 21 (B) 22 (C) 42 (D) 43

Q.17 Which of the following is the number of zeros in the square of 900?
(A) 3 (B) 4 (C) 5 (D) 2

Q.18 If a number of n-digits is a perfect square and 'n' is an even number, then which of the following is the number of digits of its square root?
(A) $\frac{n-1}{2}$ (B) $\frac{n}{2}$ (C) $\frac{n+1}{2}$ (D) 2n

Q.19 If a number of n-digit is perfect square and 'n' is an odd number then which of the following is the number of digits of its square root?
(A) $\frac{n-1}{2}$ (B) $\frac{n}{2}$ (C) $\frac{n+1}{2}$ (D) 2n

Q.20 Which of the following is a pythagorean-triplet?
(A) n, $(n^2 - 1)$ and $(n^2 + 1)$ (B) $(n - 1)$, $(n^2 - 1)$ and $(n^2 + 1)$
(C) $(n + 1)$, $(n^2 - 1)$ and $(n^2 + 1)$ (D) 2n, $(n^2 - 1)$ and $(n^2 + 1)$

Q.21 The greatest four digit number which is also a perfect square is
(A) 9701 (B) 9801 (C) 9901 (D) None of these

Q.22 The greatest perfect square of a natural number smaller than $(51)^2$ is
(A) 50 (B) 2500 (C) 3600 (D) 2551

Q.23 $\sqrt{176+\sqrt{2401}}$ is equal to
(A) 12 (B) 13 (C) 14 (D) 15

Q.24 If $\frac{1872}{\sqrt{x}} = 234$, then x is equal to
(A) 8 (B) 64 (C) 256 (D) 4

Q.25 If $140\sqrt{x} + 315 = 1015$, then x is equal to
(A) 15 (B) 225 (C) 5 (D) 25

Q.26 $\frac{\sqrt{25}+\sqrt{121}}{\sqrt{256}}$ is equal to
(A) 2 (B) 1 (C) 3 (D) 4

Q.27 $\sqrt{110\frac{1}{4}}$ is equal to

(A) 10.25 (B) 10.5 (C) 10.45 (D) 10.75

Q.28 $\dfrac{(0.9)^2+(0.1)^2+2\times(0.9)(0.1)}{(0.8)^3+(0.2)^3+3\times(0.8)^2(0.2)+3\times(0.8)(0.2)^3}$ is equal to

(A) $\frac{9}{8}$ (B) 1 (C) 2 (D) $\frac{91}{82}$

Q.29 If $\sqrt{24}=4.899$, then the value of $\sqrt{\frac{8}{3}}$ is

(A) 2.633 (B) 1.633 (C) 1.666 (D) 2.666

Q.30 The least square number which is exactly divisible by 10, 12, 15 and 18 is

(A) 3600 (B) 900 (C) 1600 (D) 2500

Q.31 If $x * y * z = \sqrt{\frac{(x+2)(y+3)}{(z+1)}}$,then the value of $7 * 6 * 8 *$ is

(A) 2 (B) 9 (C) 3 (D) 4

Q.32 The value of $(0.9)^2 - (0.1)^2$ is

(A) 1 (B) 0.8 (C) 0.64 (D) 10.16

Q.33 A general wishes to arrange his 36581 soldiers in the form of a square. After arranging them he found that some of them are left over. The number of soldiers left over is

(A) 81 (B) 100 (C) 121 (D) 144

Q.34 A man plants 15129 apple trees in his garden and arrange them so that there are as many rows as there are apple trees in each row , then the number of rows is

(A) 124 (B) 125 (C) 122 (D) 123

Q.35 If $\frac{\sqrt{1296}}{x}=\frac{x}{2.25}$, then x is equal to

(A) 7 (B) 8 (C) 9 (D) None of these

Q.36 The product of two numbers is 1575 and their quotient is $\frac{9}{7}$. Find the numbers.

(A) 21, 75 (B) 35, 45 (C) 63, 25 (D) 105, 15

Q.37 Find the smallest square number divisible by each one of the numbers 8, 9 and 10.

(A) 360 (B) 720 (C) 3600 (D) 2500

Q.38 Find the least number which must be subtracted from 182565 to make it a perfect square
(A) 236 (B) 40 (C) 265 (D) 65

Q.39 Find the least number which must be added to 306452 to make it a perfect square.
(A) 460 (B) 462 (C) 464 (D) 468

Q.40 Find the greatest number of six digits which is a perfect square.
(A) 999999 (B) 100000 (C) 998001 (D) 998000

Q.41 Find the value of $\sqrt{99} \times \sqrt{396}$
(A) 196 (B) 197 (C) 198 (D) 199

Q.42 Find the value of $\sqrt{147} \times \sqrt{243}$
(A) 189 (B) 181 (C) 180 (D) 294

Q.43 Find the square root of 0.00008281.
(A) 0.0091 (B) 0.0092 (C) 0.0093 (D) 0.0094

Q.44 Find the value of $\sqrt{15625}$ and the use it to find the value of $\sqrt{156.25} + \sqrt{1.5625}$.
(A) 13.25 (B) 13.35 (C) 13.65 (D) 13.75

Q.45 Find the square root of 2 correct to three places of decimal.
(A) 1.401 (B) 1.141 (C) 1.414 (D) 1.410

Q.46 7396 students are sitting in an auditorium in such a manner that there are as many students in a row as there are rows in the auditorium. How many rows are there in the auditorium?
(A) 96 (B) 86 (C) 87 (D) 98

SECTION - C

- **MATCH THE COLUMN**

Q.1 Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.

	Column I		Column II
(A)	Value of $1-(0.5)^2$ is	(p)	33.30
(B)	If $\sqrt{6.4} = 2.53$ then value of $\sqrt{640} + \sqrt{64}$	(q)	0.029
(C)	Value of $\sqrt{2980}$ correct to two decimal place is	(r)	54.59
(D)	Given $\sqrt{8.5} = 2.915$ and $\sqrt{85} = 9.320$. Value of $\sqrt{0.00085}$ is	(s)	0.75

Q.2 Statements (A, B, C, D) in column I have to be matched with statements (p, q, r, s) in column II.

	Column I		**Column II**
(A)	If $\sqrt{(75.24+x)}=8.71$ then the value of x is	(p)	324
(B)	If $\sqrt{0.04\times0.4\times a}=0.4\times0.04\times\sqrt{b}$ then the value of $\frac{a}{b}$ is	(q)	64
(C)	If $\sqrt{256}\div\sqrt{x}=2$ then the value of x is	(r)	0.016
(D)	If $\sqrt{\frac{x}{169}}=\frac{54}{39}$ then the value of x is	(s)	0.6241

Q.3 Match the column

	Column I		**Column II**
(A)	There are '2n' non-perfect square numbers between the square of the number _____	(p)	$\frac{n+1}{2}$
(B)	For any natural number greater than 1, _____ are called a Pythagoren Triplet	(q)	$\frac{n}{2}$
(C)	If n is an even number of digits of a square number then the number of digits in its square root are ______	(r)	n and n + 1
(D)	If n is an odd number of digits of a square-number then the number of digits in its square-root are _____	(s)	$2n, (n^2-1)$ and (n^2+1)

ANSWER KEY

CONCEPT APPLICATION LEVEL - II

SECTION - A

Q.1	even	Q.2	cube numbers	Q.3	twice	Q.4	3
Q.5	two	Q.6	two	Q.7	$n = m^3$	Q.8	decimal, less
Q.9	square	Q.10	perfect square	Q.11	0.1	Q.12	2n or 2n – 1
Q.13	0.000049	Q.14	Less				

SECTION - B

Q.1	C	Q.2	B	Q.3	D	Q.4	C	Q.5	B	Q.6	D	Q.7	B
Q.8	B	Q.9	C	Q.10	A	Q.11	C	Q.12	D	Q.13	C	Q.14	A
Q.15	C	Q.16	C	Q.17	B	Q.18	B	Q.19	C	Q.20	D	Q.21	B
Q.22	B	Q.23	D	Q.24	B	Q.25	D	Q.26	B	Q.27	B	Q.28	B
Q.29	B	Q.30	B	Q.31	C	Q.32	B	Q.33	B	Q.34	D	Q.35	C
Q.36	B	Q.37	C	Q.38	A	Q.39	C	Q.40	C	Q.41	C	Q.42	A
Q.43	A	Q.44	D	Q.45	C	Q.46	B						

SECTION - C

Q.1 (A)- s; (B)- p; (C)- r; (D)-q

Q.2 (A)- s; (B)- r; (C)- q; (D)- p

Q.3 (A)-(r); (B)-(s); (C)-(q); (D)-(p)