11 PLAYING WITH NUMBERS

11.1 WRITING A NUMBER IN GENERAL FORM

In the previous classes, we have read about place value of a digit in a number. We have also learnt how we express a number in expanded form :

Observe the following :

In 12, 1 is at tens place, so place value of

$1 = 10 \times 1 = 10.$

and 2 is at ones place, so place value of

$2 = 1 \times 2 = 2$

Thus, 12 can be expressed as $12 = 10 \times 1 + 2$

Other 2-digit numbers can also be expressed in the similar way :

eg. $85 = 10 \times 8 + 5$

$99 = 10 \times 9 + 9$

In general, any 2-digit number ab made up of digits a and b can be written as

$ab = 10 \times a + b = 10a + b$

On reversing the order of the digits of ab, we get the new 2-digit number as

$ba = 10 \times b + a = 10\,b + a$

Note : Here ab does not mean $a \times b$.

Again, observe the following :

In 123,

1 is at hundreds place, so place value of

$1 = 100 \times 1 = 100$

2 is at tens place, so place value of

$2 = 10 \times 2 = 20$

3 is at units place, so place value of

$3 = 1 \times 3 = 3$

Thus, 123 can be expressed as

$123 = 100 \times 1 + 10 \times 2 + 3$

Similarly, we have

$157 = 100 \times 1 + 10 \times 5 + 7$

In general, for a 3-digit number abc, add up of digits a, b and c can be written as

$$\begin{aligned} abc &= 100 \times a + 10 \times b + c \\ &= 100a + 10b + c \end{aligned}$$

On reversing the order of the digit of abc, we get the new 3-digit number as

$cba = 100 \times c + 10 \times b + a = 100\,c + 10b + a$

11.2 GAMES WITH NUMBERS

Reversing the Digits of a 2-digit Number

Consider a 2-digit number ab whose expanded form is 10a + b. On reversing its digits, we get another number ba = 10b + a.

On adding the two numbers, we get

$$ab + ba = 10a + b + 10b + a$$
$$= 11a + 11b = 11(a + b)$$

$$\Rightarrow \frac{ab+ba}{11} = a + b \quad \text{or} \quad \frac{ab+ba}{(a+b)} = 11$$

Thus, the sum of any 2 digit number ab and the number ba obtained by reversing its digits, is completely divisible by

(i) 11 and the quotient is a + b

(ii) a + b and the quotient is 11.

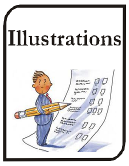


Illustration 1

Without performing the actual division and addition, find the quotient when the sum of 87 and 78 is divided by (i) 11 (ii) 15.

Solution

87 and 78 are the two numbers such that one can be obtained by interchanging the digits of the other.

(i) If we divide their sum by 11, the quotient is the sum of the digits i.e.,

$$\frac{87+78}{11} = 7 + 8 = 15$$

(ii) If we divide their sum by the sum of the digits, we get 11 as the quotient.

$$\text{or} \quad \frac{87+78}{7+8} = 11 \text{ or } \frac{87+78}{15} = 11$$

Again consider

ab = 10a + b and ba = 10b + a

(i) If a > b, then the difference between the numbers is

$$ab - ba = 10a + b - 10b - a$$
$$= 9a - 9b = 9(a - b)$$

$$\Rightarrow \frac{ab-ba}{9} = a - b \quad \text{or} \quad \frac{ab-ba}{a-b} = 9$$

(ii) If b > a, then difference between the numbers is

$$ba - ab = 10b + a - 10a - b$$
$$= 9b - 9a = 9(b - a)$$

$$\Rightarrow \frac{ba-ab}{9} = b - a \text{ or } \frac{ba-ab}{b-a} = 9$$

(iii) If a = b, then ab – ba = 0

$\Rightarrow \quad \frac{ab - ba}{9} = 0$

Thus, the difference of any 2-digit number ab and the number obtained ba by reversing its digits, is completely divisible by
(i) 9 and the quotient is the difference of the digits
(ii) the difference of the digits, ($a \neq b$) and the quotient is 9.

Illustration 2

Without performing actual subtraction and division, find the quotient which the difference of 94 and 49 is divided by :
(i) 9 (ii) 5

Solution

94 and 49 are the two numbers, such that one can be obtained by interchanging the digits of the other.
Thus,
(i) If we divided their difference by 9, the quotient is the difference of the digits

or $\frac{94-49}{9} = 9 - 4 = 5$

(ii) If we divide their difference by the difference of the digits, we get 9 as the quotient.

or $\frac{94-49}{9-4} = 9$ or $\frac{94-49}{5} = 9$

11.3 REVERSING THE DIGITS OF A 3-DIGIT NUMBER

Consider a 3-digit number abc whose expanded form is 100a + 10b + c. On reversing its digit, we get another number,

cba = 100c + 10b + a

(i) If a > c, then the difference of the number is
abc – cba = 100a + 10b + c – 100c – 10b – a
= 99a – 99c = 99(a – c)

$\Rightarrow \quad \frac{abc - cba}{99} = a - c$ or $\frac{abc - cba}{a - c} = 99$

(ii) If c > a, then the difference of the numbers is
cba – abc = 100c + 10b + a – 100a – 10b – c
= 99c – 99a = 99 (c – a)

$\Rightarrow \quad \frac{cba - abc}{99} = c - a$ or $\frac{cba - abc}{c - a} = 99$

(iii) If a = c, then the difference of the numbers is

abc – cba

= 100a + 10b + c – 100c – 10b – a = 0

$\Rightarrow \quad \dfrac{abc - cba}{99} = 0$

Thus, the difference of any 3-digit number abc and the number cba by reversing its digits, is exactly divisible by

(i) 99 and the quotient is the difference of hundreds and ones digit of the number.

(ii) a – c or c – a, $(a \neq c)$ and the quotient is 99.

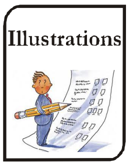


Illustration 3

Without performing actual subtraction and division, find the quotient when the difference of 589 and 985 is divided by :

(i) 99 (ii) 4

Solution

(i) 589 and 985 are the two 3-digit numbers such that one can be obtained by reversing the digits of the correct.

Thus, (i) If we divide their difference by 99, the quotient is the difference of hundreds and ones digits.

or $\dfrac{985 - 589}{99} = 9 - 5 = 4$

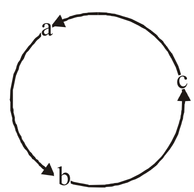
(ii) If we divide their difference by the difference of hundreds difference by the difference of hundreds and ones digits, the quotient is 99.

or $\dfrac{985 - 589}{9 - 5} = 99$ or $\dfrac{985 - 589}{4} = 99$

11.4 FORMING 3-DIGIT NUMBERS WITH GIVEN THREE DIGITS

Consider a 3 - digit number abc = 100a + 10b + c.

By changing the order of its digits in cyclic order, we get two more 3-digit number.



bca = 100b + 10c + a

and cab = 100c + 10a + b

On adding these three numbers, we get

abc + bca + cab = 100a + 10b + c + 100b + 10c + a + 100c + 10a + b

= 111a + 111b + 111c

$\Rightarrow$ abc + bca + cab = 111 (a + b +c) = 3 × 37 (a + b + c)

Sol. (i) $\dfrac{abc + bca + cab}{111} = a + b + c$ (ii) $\dfrac{abc + bca + cab}{a + b + c} = 111$

(iii) $\dfrac{abc + bca + cab}{37} = 3(a + b + c)$ (iv) $\dfrac{abc + bca + cab}{3} = 37(a + b + c)$

(v) $\dfrac{abc + bca + cab}{3(a + b + c)} = 37$ (vi) $\dfrac{abc + bca + cab}{37(a + b + c)} = 3$

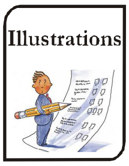


Illustration 4

Without performing actual addition and division, find the quotient when the sum of 584, 845 and 458 is divisible by :

(i) 17 (ii) 37 (iii) 111 (iv) 3 (v) 51 (vi) 629

Solution

584, 845, 458 are three numbers obtained when the digits 5, 8 and 4 are arranged in the cyclic order.

So, the quotient when the sum of these numbers is divided by :

(i) $5 + 8 + 4 = 17$ is 111

(ii) 37 is $3 \times (5 + 8 + 4)$ i.e., 51

(iii) 111 is $(5 + 8 + 4)$ i.e., 17

(iv) 3 is $37 \times (5 + 8 + 4)$ i.e., 629

(v) $3 \times (5 + 8 + 4) = 51$ is 37

(vi) $37 \times (5 + 8 + 4) = 629$ is 3.

11.5 LETTERS FOR DIGITS (CRYPTARITHMS)

A mathematical puzzle written by using letters in place of digits is called a cryptogram. Here we shall discuss problems on addition and multiplication only. While dealing with such puzzles, we keep in mind the following rules :

1. Each letter in the puzzle must stand for just one digit, and each digit must be represented by only one letter.
2. The first digit of a number cannot be zero. For example, twenty one is written as 21 not as 0.21 or 0021 etc.
3. A puzzle must have only one answer

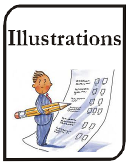


Illustration 5

Find the values of letters in each of the following, giving reasons for each step.

(i) $\begin{array}{rrr} & A & B \\ + & 3 & 7 \\ \hline & 6 & A \\ \hline \end{array}$ (ii) $\begin{array}{rrrr} & 1 & 2 & A \\ + & 6 & A & B \\ \hline & A & 0 & 9 \\ \hline \end{array}$ (iii) $\begin{array}{rrrr} & 2 & A & B \\ + & A & B & 1 \\ \hline & B & 1 & 8 \\ \hline \end{array}$

(iv) $\begin{array}{rrr} & A & B \\ \times & & 3 \\ \hline C & A & B \\ \hline \end{array}$ (v) $\begin{array}{rrr} & A & B \\ \times & & 6 \\ \hline B & B & B \\ \hline \end{array}$ (vi) $\begin{array}{rrr} & A & A \\ \times & A & A \\ \hline A & B & A \\ \hline \end{array}$

Solution

(i) We have

$$\begin{array}{rrr} & A & B \\ + & 3 & 7 \\ \hline & 6 & A \\ \hline \end{array}$$

Here there are two letters A and B.

Look at the ones column.

B + 7 = A, so B can be 0, 1, 2, 3, ...9

For B = 0, A = 7, but A = 7 does not fit in tens column
For B = 1, A = 8, but A = 8, does not fit in tens column.
For B = 2, A = 9, but A = 9, does not fit in tens column.
For B = 3, A = 0, but A = 0, does not fit in tens column.
For B = 4, A = 1 but A = 1, does not fit in tens column.
For B = 5, A = 2, the sum becomes

$$\begin{array}{r} 2\ 5 \\ +\ 3\ 7 \\ \hline 6\ 2 \\ \hline \end{array}$$

Hence, A = 2 and B = 5.

(ii) We have

$$\begin{array}{r} 1\ 2\ A \\ +\ 6\ A\ B \\ \hline A\ 0\ 9 \\ \hline \end{array}$$

Here, we have two letters A and B
Look at the tens column 2 + A = 0
$\Rightarrow$ A = 8
In ones column, for A = 8, B = 1
So, for A = 8, B = 1, the sum becomes

$$\begin{array}{r} 1\ 2\ 8 \\ +\ 6\ 8\ 1 \\ \hline 8\ 0\ 9 \\ \hline \end{array}$$

Hence, A = 8, B = 1.

(iii) We have

$$\begin{array}{r} 2\ A\ B \\ +\ A\ B\ 1 \\ \hline B\ 1\ 8 \\ \hline \end{array}$$

Look at the ones column
B + 1 = 8 $\Rightarrow$ B = 7
In tens column for B = 7, A = 4
So, for A = 4 and B = 7, the sum becomes

$$\begin{array}{r} 2\ 4\ 7 \\ +\ 4\ 7\ 1 \\ \hline 7\ 1\ 8 \\ \hline \end{array}$$

(iv) We have

$$\begin{array}{r} A\ B \\ \times\ \ \ 3 \\ \hline C\ A\ B \\ \hline \end{array}$$

Here, there are three letters.
Since the ones digit of B × 3 = B, so B = 0 or B = 5
Now, A × 3 = A $\Rightarrow$ A = 5 or A = 0

But A = 0 will make AB a one-digit number, so we reject A = 0.

So, we have either
$$\begin{array}{r} 5\ 5 \\ \times\ 3 \\ \hline 1\ 6\ 5 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 5\ 0 \\ \times\ 3 \\ \hline 1\ 5\ 0 \\ \hline \end{array}$$

But, $\begin{array}{r} 5\ 0 \\ \times\ 3 \\ \hline 1\ 5\ 0 \\ \hline \end{array}$ is correct.

Hence, A = 5 and B = 0

(v) We have
$$\begin{array}{r} A\ B \\ \times\ \ 6 \\ \hline B\ B\ B \\ \hline \end{array}$$

Ones digit of B × 6 is again B, so possible values of B are 2, 4, 6 or 8.
For B = 2, AB × 6 = BBB

$\Rightarrow$ (A2) × 6 = 222 $\Rightarrow$ $(10A + 2) = \frac{222}{6} = 37$

$\Rightarrow$ 10A = 37 – 2 = 35

$\Rightarrow$ $A = \frac{35}{10}$,which is not possible

For B = 4, AB × 6 = BBB

$\Rightarrow$ (A4) × 6 = 444 $\Rightarrow$ $(10A + 4) = \frac{444}{6} = 74$

$\Rightarrow$ 10A = 74 – 4 = 70 $\Rightarrow$ A = 7
Hence, A = 7, B = 4

(vi) We have
$$\begin{array}{r} A\ A \\ \times\ A\ A \\ \hline A\ B\ A \\ \hline \end{array}$$

Here, A × A = A, so A = 1 or A = 5 or A = 6
For A = 1, 11 × 11 = 121 $\Rightarrow$ B = 2
No other values of A satisfy the given condition.
Hence, A = 1 and B = 2.

11.6 DIVISIBILITY TEST

(a) **Divisibilit by 2 :** A given number is divisible by 2, if its unit digit is any of 0, 2, 4, 6, 8 e.g. 4268134 is divisible by 2 while 311267429 is not divisible by 2.

(b) **Divisibility by 3 :** A number is divisible by 3, if sum of its digits is divisible by 3 e.g. 252771 is divisible by 3 as sum of its digits $(2 + 5 + 2 + 7 + 7 + 1 = 24)$ is divisible by 3.

(c) **Divisibility by 4 :** A number is divisible by 4 if the last two digits of the number is divisible by 4 or the number ends with '00'. e.g. 213428 is divisible by 4 as last two digits is 28 which is divisible by 4. 1246800 is also divisible by 4 as the number ends with 00.

(d) **Divisibility by 5 :** A number is divisible by 5 if its unit place digit is either 0 or 5.

(e) **Divisibility by 6 :** A number is divisible by 6 if it is divisible by 2 and 3 both. e.g., 254784 is divisible by 6 because it is an even number and hence divisible by 2, also the sum of digits i.e., $2 + 5 + 4 + 7 + 8 + 4 = 30$ is divisible by 3. Hence the number is divisible by 6.

(f) **Divisibility by 8 :** A given number is divisible by 8 if the number formed by last three digits of the number is divisible by 8 or the number ends with '000' e.g., 342840 is divisible by 8 because 840 is divisible by 8. 29342000 is also divisible by 8.

(g) **Divisibility by 9 :** A number is divisible by 9 if the sum of digits of the number is divisible by 9. e.g. 284796 is divisible by 9 because sum of digits $2 + 8 + 4 + 7 + 9 + 6 = 36$ is completely divisible by 9.

(h) **Divisibility by 10 :** A number is divisible by 10 if the unit place digit of given number is '0' e.g., 21380, 3142900 are divisible by 10, whereas 214385, 329212, 46843 are not divisible by 10.

(i) **Divisibility by 11 :** A number is divisible by 11 if the difference between the sum of the digits in the odd places and the sum of the digits in the even places is either O or completely divisible by 11. e.g., 6584919 is divisible by 11 because. (Sum of digits at odd places) – (Sum of digits at even places)

$\Rightarrow \quad (6 + 8 + 9 + 9) - (5 + 4 + 1)$

$\Rightarrow \quad 32 - 10 = 22$, which is divisible by 11.

(j) **Divisibility by 12 :** If a given number is divisible by both 3 and 4 then it is also divisible by 12 e.g., 16128 is divisible by both 3 and 4 and hence it is divisible by 12 also.

11.7 PRIME FACTORISATION

If a natural number is expressed as the product of prime numbers, then the factorisation of the number is called its prime factorisation.

For example : $84 = 2 \times 2 \times 3 \times 7$.

11.8 HIGHEST COMMON FACTOR (HCF)

HCF of two or more numbers is the largest number that divides all the given number exactly. It is also known as Greatest Common Divisor (G.C.D.) or Greatest Common Measure (GCM).

11.8.1 Method of Finding HCF

HCF can be found by Prime Factorisation Method and Division Method.

(i) **Prime Factorisation Method :** When the numbers whose HCF has to be found are relatively small, we use prime factorisation method. First we resolve the given numbers into their prime factors and then find out the product of common factors of given number. The product thus obtained is the H.C.F.

(ii) Division Method : When the numbers whose HCF is required are large, prime factorisation method is time consuming. Division method is more suitable in those cases. Following are the steps for division method.

(a) Divide the larger number by smaller number. If the remainder is zero, the divisor is HCF.

(b) If remainder in step-a is not zero, then remainder becomes divisor and divisor of step-a becomes dividend. If the remainder is zero the divisor is HCF, otherwise step-b continues.

11.9 LEAST COMMON MULTIPLE (LCM)

The LCM of two or more numbers is the smallest number which is exactly divisible by them. In other words, LCM of two or more numbers is the smallest number which is a multiple of each of the numbers e.g. LCM of 4 and 6 is 12. LCM is also known as Lowest Common Dividend (LCD).

11.9.1 Methods of finding LCM

(i) Prime Factorisation Method :

(a) Write the given numbers as product of its prime factors.

(b) LCM of given number is the product of the factors with highest power.

(ii) Division Method : In this method we arrange the given numbers in any order and start dividing these numbers by prime numbers starting from 2 (smallest prime number) till the at least two number are divisible. The product of the divisor (s) and the undivided number is the LCM of the given numbers.

11.9.2 Relation between LCM and HCF

The product of HCF and LCM of two numbers is equal to the product of the given numbers.

$$\text{HCF} \times \text{LCM} = \text{Product of two numbers}$$

or $$\text{LCM} = \frac{\text{Product of two numbers}}{\text{HCF}}$$

or $$\text{HCF} = \frac{\text{Product of two numbers}}{\text{LCM}}$$

Note : The above relations is not true for three or more numbers.

11.10 FRACTIONS

A fraction is a number that represent part of a whole. It is written in the form of x/y, here y tells about how many equal parts are taken.

In fraction = $\frac{x}{y}$, x is called numberator and y is called denominator.

$\frac{3}{4}, \frac{9}{17}, \frac{7}{9}$ are some examples of fractions.

11.10.1 Types of Fractions

(i) **Proper Fractions :** A fraction is called proper fraction if its numerator is less than denominator,

e.g. : $\frac{2}{3}, \frac{6}{13}, \frac{3}{7}$, etc. are all proper fractions.

(ii) **Improper Fractions :** A fraction is called improper fraction if its numerator is greater than denominator e.g. : $\frac{3}{2}, \frac{7}{3}, \frac{15}{7}$, etc, are all improper fractions.

Improper fractions can be converted to mixed fraction e.g. : $\frac{3}{2}$ can be written as $1\frac{1}{2}$, etc.

(iii) **Like Fraction :** Fraction with same denominators are called like fractions.

e.g..., $\frac{1}{2}, \frac{3}{2}, \frac{7}{2}, \frac{9}{2}$, etc are all like fractions.

(iv) **Unlike Fractions :** Fraction with different denominators are called unlike fractions

e.g.: $\frac{3}{4}, \frac{7}{9}, \frac{10}{17}$ etc. unlike fractions.

(v) **Equivalent Fraction :** Two fractions are equivalent if they represent the same ratio or number

e.g. $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{10}{15}$ etc. are all equivalent fraction.

11.10.2 Simplest form of a Fraction

A fraction is said to be in the simplest or lowest form if its numerator and denominator have no common factor except 1.

e.g. lowest form of $\frac{36}{24}$ is $\frac{3}{2}$. We can arrive at the lowest form by dividing numerator and denominator by their HCF.

11.11 EUCLID'S DIVISION ALGORITHM

It is a technique to determine the highest common factor (HCF) of two given positive integers. It is based on Euclid Dvision Lemma.

HCF : We know that HCF of two positive integers 'a' and 'b' is the largest positive integer 'c' that divides both 'a' and 'b'

11.11.1 Steps of Euclid Division Algorithm

According to this algorithm HCF of two positive integers a and b ($a > b$) can be found by following steps :

(I) Apply the Euclid'sDivision Lemma to 'a' and 'b' to find the whole numbers q and r such that $a = bq + r, \quad 0 \le r < b$

(II) If $r = 0$ then HCF of a and b is 'b'. If $r \ne 0$, then apply Euclid's Divison Lemma again to b and r

(III) Continue the process till the remainder becomes zero At this stage the divisor is the HCF of 'a' and 'b' .

SOLVED EXAMPLES

Example 1

If $\overline{24y5}$ is a multiple of 3, where y is a digit, what might be the value of y?

Solution

Since $\overline{24y5}$ is a multiple of 3.

$\therefore$ $2 + 4 + y + 5$ is a multiple of 3.

$\Rightarrow$ $11 + y$ is a multiple of 3.

$\Rightarrow$ $11 + y = 0, 3, 6, 9, 12, 15, 18, 21, ...,$ ...(i)

But, y is a digit of the number $\overline{24y5}$. So y can take values 0, 1, 2, ..., 9.

$\therefore$ $11 + y$ can take values 11, 12, 13, ...22. ...(ii)

From (i) and (ii), we get

$11 + y = 12$ or 15 or 18

$\Rightarrow$ $11 + y = 15$ or $11 + y = 18$

$\Rightarrow$ $y = 4$ or, $y = 7$

Hence, y = 4 or, 7 or 1.

Example 2

If $\overline{31z5}$ is a multiple of 3, where z is a digit, what might be the value of z?

Solution

Since $\overline{31z5}$ is a multiple of 3.

$\therefore$ $3 + 1 + z + 5$ is a multiple of 3.

$\Rightarrow$ $z + 9$ is a multiple of 3.

$\Rightarrow$ $z + 9 = 0, 3, 6, 9, 12, 15, 18,....$

But, z is a digit of the number $\overline{31z5}$. So z can take values 0, 1, 3, ..., 9

$\Rightarrow$ $z + 9$ can take values 9, 10, 11, 12, ..., 18

From (i) and (ii), we get

$z + 9$ can take values 9 or , 12 or, 15 or, 18

$\Rightarrow$ $z + 9 = 9$ or, $z + 9 = 12$ or, $z + 9 = 15$ or, $z + 9 = 18$

$\Rightarrow$ $z = 0, 3, 6, 9$

Hence, z can take values 0, 3, 6, 9

Example 3

Without actual division find the remainder when 379843 is divided by 3.

Solution

The remainder obtained by dividing 379843 by 3 is same as the remainder obtained by dividing the sum of its digits by 3.

We have,

Sum of the digits $= 3 + 7 + 9 + 8 + 4 + 3 = 34$

When 34 is divided by 3 we get 1 as the remainder.

Hence, division of 379843 by 3 leaves remainder of 1.

Example 4

If $\overline{24x}$ is a multiple of 6, where x is a digit, what is the value of x ?

(A) 0, 8 (B) 0, 6 (C) 8, 6 (D) 2, 8

Solution

It is given that the number $\overline{24x}$ is a multiple of 6. Therefore, it is a multiple of both 2 and 3.
Now,

$\overline{24x}$ is a multiple of 3
$\Rightarrow$ 2 + 4 + x is a multiple of 3
$\Rightarrow$ 6 + x is a multiple of 3
$\Rightarrow$ $6 + x = 0, 3, 6, 9, 12, 15, 18, ...$
and,
$\overline{24x}$ is a multiple of 2
$\Rightarrow$ x is an even digit
$\Rightarrow$ $x = 0, 2, 4, 6, 8$
$\Rightarrow$ $6 + x = 6, 8, 10, 12, 14$
From (i) and (ii), we have
$6 + x = 6$ or, $6 + x = 12 \Rightarrow x = 0, x = 6$
Hence, $x = 0$ or, $x = 6$

Example 5

If $\overline{2y5}$ is divisible by 11, where y is a digit, what is the value of y ?

Solution

We have,
Sum of the digits in odd places = 2 + 5 = 7, Sum of the digits in even places = y.
$\therefore$ Sum of the digits in even places – Sum of the digits in odd places = y – 7.
If $\overline{2y5}$ is divisible by 11, then
y – 7 must be a multiple of 11
$\Rightarrow$ $y - 7 = 0$ or, 11 or, 22 or 33, ...
$\Rightarrow$ $y = 7$ or, 18 or, 29, ...(i)
But, y is a digit. So, y can take values 0, 1, 2, 3, ..., 9 .....(ii)
From (i) and (ii), we get y = 7.

Example 6

Given that the number $\overline{148101a095}$ is divisible by 11, where a is some digit, what are the possible values of a ?

Solution

If $\overline{148101a095}$ is divisible by 11, then
$(1 + 8 + 0 + a + 9) - (4 + 1 + 1 + 0 + 5)$ is a multiple of 11.
$\Rightarrow$ $(a + 18) - 11$ is a multiple of 11
$\Rightarrow$ $a + 7$ is a multiple of 11
$\Rightarrow$ $a + 7 = 0$ or, 11 or, 22 or, 33, ...(i)
But, a is a digit of some number. So, a can take one of the values from 0 to 9.
Therefore, a + 7 can take values 7, 8, 9, ..., 16 ...(ii)
From (i) and (ii) $\Rightarrow$ a = 4

Example 7

Given that the number $\overline{1735538a05}$ is divisible by 9, where 'a' is a digit, what are the possible values of a ?

Solution

If the number $\overline{1735538a05}$ is divisible by 9, then 1 + 7 + 3 + 5 + 5 + 3 + 8 + a + 0 + 5 is a multiple of 9.

$\Rightarrow$ a + 37 is a multiple of 9 ...(i)

But, a is some digit of a number. So, a can take values 0, 1, 2, 4, ..., 9.

$\therefore$ a + 37 = 37, 38, 39, 40, ..., 46

From (i) and (ii), we have ...(ii)

a + 37 = 45 $\Rightarrow$ a = 8

Hence, a can take only one value equal to 8.

Example 8

Given that the number $\overline{60ab57377}$ is divisible by 99, where a and b are digits, what are the values of a and b ?

Solution

Since a and b are digits of a number. So, a and b can take values from 0 to 9.

It is given that $\overline{60ab57377}$ is divisible by 99 which is divisible by both 9 and 11.

$\therefore$ $\overline{60ab57377}$ is divisible by both 9 and 11

$\Rightarrow$ 6 + 0 + a + b + 5 + 7 + 3 + 7 + 7 is a multiple of 9

and (6 + a + 5 + 3 + 7) – (0 + b + 7 + 7) is a multiple of 11.

$\Rightarrow$ a + b + 35 is a multiple of 9

and, (a + 21) – (b + 14) is a multiple of 11

and, a – b + 7 is a multiple of 11

Since a and b can take values from 0 to 9. Therefore, a + b can take values from 0 to 18 and hence, a + b + 35 can take values from 35 to 53.

Also, a + b + 35 is a multiple of 9.

$\therefore$ either a + b + 35 = 36 or, a + b + 35 = 45

$\Rightarrow$ a + b = 1 or, a + b = 10

Now, a – b + 7 is a multiple of 11

$\Rightarrow$ a – b + 7 = 0 or, a – b + 7 = 11 $\left[\begin{array}{l}\because 0 \le a, b \le 9 \ \therefore -9 \le a - b \le 9 \\ \Rightarrow -2 \le a - b + 7 \le 16\end{array}\right]$

$\Rightarrow$ a – b = – 7 or, a – b = 4

Thus, we have following equations giving values of a and b.

a + b = 1 or, a + b = 10

a – b = – 7 or, a – b = 4

These equations gives the follwing pairs of equations in a and b.

a + b = 1 and a – b = 7 ...(i)

a + b = 1 and a – b = 4 ...(ii)

a + b = 10 and a – b –7 ...(iii)

a + b = 10 and a – b = 4 ...(iv)

Adding and subtracting equations in (iv), we get

2a = 14 and 2b = 6 $\Rightarrow$ a = 7 and b = 3

other pairs of equations do not give non-negative integral value of a and b.

Hence, a = 7 and b = 3.

Example 9

Without performing actual division, find the remainders left when 192837465 is divided by
(i) 9 (ii) 11

Solution

(i) In sub-section 5.4.4, we have seen that any natural number n, can be written as n = a multiple of 9 + Sum of the digits of n

$\therefore$ 192837465 = a multiple of 9 + (1 + 9 + 2 + 8 + 3 + 7 + 4 + 6 + 5)

$\Rightarrow$ 192837465 = a multiple of 9 + 45

$\Rightarrow$ 192837465 = a multiple of 9 + 9 × 5

$\Rightarrow$ 192837465 = multiple of 9

So, the remainder left when 192837465 is divided by 9 is zero.

(ii) In sub-section 5.4.7, we have seen that any odd digit natural number n can be written as n = a multiple of 11 + Sum of its digits in odd places – Sum of its digit in even places

$\Rightarrow$ 192837465 = a multiple of 11 + (1 + 2 + 3 + 4 + 5) – (9 + 8 + 7 + 6)

$\Rightarrow$ 192837465 = a multiple of 11 + 15 – 30

$\Rightarrow$ 192837465 = a multiple of 11 – 15

$\Rightarrow$ 192837465 = a multiple of 11 – 22 + 7

$\Rightarrow$ 192837465 = a multiple of 11 – 2 × 11 + 7

$\Rightarrow$ 192837465 = (a multiple of 11) + 7

So, the remainder left when 192837465 is divided by 11 is 7.

Example 10

Solve the cryptarithm : $\overline{ON} + \overline{ON} = \overline{GO}$ or $2 \times \overline{ON} = \overline{GO}$.

Solution

We have, $\overline{ON} + \overline{ON} =$ or, $2\times\overline{ON}=\overline{GO}$

Clearly, $\overline{GO}$ is a two digit number whose maximum value can be 99.

Therefore, maximum value of $\overline{ON}$ is also an even number.

Consequently, O can take even values only.

$\therefore$ Digit O can take values 2 or 4.

CASE I

When digit O takes value 2.

Substituting 2 in place of digit O in equation (i), we get

$2\times\overline{2N}=\overline{G2}$

$\Rightarrow$ Multiplication of 2 and N must be either 2 or a two digit number between 10 and 19 having 2 at ones place.

$\Rightarrow$ N = 1 or, N = 6

When N = 1, equation (ii) gives

$2 \times 21 = \overline{G2}$ $\Rightarrow 42 = \overline{G2}$ $\Rightarrow G = 5$

Thus, we have

O = 2, G = 4 and N = 1 or, O = 2, G = 5 and N = 6.

Example 11 **Solve the following Cryptarithms:**

(i)
$$\begin{array}{r} 1\ A \\ \times\ \ A \\ \hline 9\ A \end{array}$$

(ii)
$$\begin{array}{r} A\ B \\ \times\ \ 6 \\ \hline B\ B\ B \end{array}$$

(iii)
$$\begin{array}{r} A\ B \\ \times\ \ 5 \\ \hline C\ A\ B \end{array}$$

Solution

(i) We have

$$\begin{array}{r} 1\ A \\ \times\ \ A \\ \hline 9\ A \end{array}$$

This means that the product of A with itself is either A or it has units digit as A. Since A = 1 satisfies A × A = 1 but it is not possible as the product is 9 A. The other value of A is 6 whose product with itself is a number having 6 at units place.

Taking A = 6, we have

$$\begin{array}{r} 1\ 6 \\ \times\ \ 6 \\ \hline 9\ 6 \end{array}$$

Clearly, it satisfies the given product.

Hence, A = 6.

(ii)
$$\begin{array}{r} A\ B \\ \times\ \ 6 \\ \hline B\ B\ B \end{array}$$

This means that 6 × B is a number having its ones digit as B. Such values of B are 2, 4, 6 and 8, because × 2 = 24, 6 × 6 = 36 and 6 × 8 = 48. So, we have following cases :

CASE I

When B = 2

In this case, we have

$$\overline{AB} \times 6 = \overline{BBB}$$

$\Rightarrow \quad \overline{A2} \times 6 = 222 \quad \Rightarrow \quad (10A + 2) \times 6 = 222$

$\Rightarrow \quad 60A + 12 = 222 \quad \Rightarrow \quad 60A = 210$

$\Rightarrow \quad 2A = 7 \quad \Rightarrow \quad A = \frac{7}{2}$, which is not possible.

CASE II

When B = 4

In this case, we have

$$\overline{AB} \times 6 = \overline{BBB}$$

$\Rightarrow \quad \overline{A4} \times 6 = 444 \quad \Rightarrow \quad (10A + 4) \times 6 = 444$

$\Rightarrow \quad 10A + A = \frac{444}{6} \quad \Rightarrow \quad 10A + 4 = 74$

$\Rightarrow \quad 10A = 70 \quad \Rightarrow \quad A = 7$

$\therefore \quad$ A = 7 and B = 4 is the required solution.

CASE III

When B = 6

In this case, we have

$\overline{A6} \times 6 = 666$

$\Rightarrow \quad \overline{A6} = 111$

This is not possible as LHS is a two digit number and RHS is a three digit number.

CASE IV

When B = 8

In this case, we have

$\overline{AB} \times 6 = \overline{BBB}$

$\Rightarrow \quad \overline{A8} \times 6 = 888 \qquad \Rightarrow \quad \overline{A8} = 148$

This is not possible as LHS is a two digit number and RHS is a three digit number.

(iii) We have,

$$\begin{array}{r} A\ B \\ \times\ \ 5 \\ \hline C\ A\ B \\ \hline \end{array}$$

This means that 5 × B is a number whose units digit is B. Clearly, B can take value 5.

Taking B = 5, we have

$\overline{A5} \times 5 = \overline{CA5}$

$\Rightarrow \quad (10A + 5) \times 5 = 100C + 10A + 5$

$\Rightarrow \quad 10A + 5 = 20C + 2A + 1$

$\Rightarrow \quad 8A + 4 = 20C$

$\Rightarrow \quad 2A + 1 = 5C$ ...(i)

$\Rightarrow \quad 2A + 1$ is an odd multiple of 5 $[\because\ O < A \le 9]$

$\Rightarrow \quad 2A + 1 = 5,\ 2A + 1 = 15$ $[\because\ O < A \le 9]$

$\Rightarrow$ Putting A = 2 in (in), we get C = 1

$\therefore \quad A = 2,\ B = 5$ and $C = 1$

Putting A = 7 in (i), we get C = 3

$\therefore \quad A = 7,\ B = 5$ and $C = 3$

Example 12

Solve the Cryptarithm :

$$\begin{array}{r} \mathbf{B\ A} \\ \times\ \mathbf{B\ 3} \\ \hline \mathbf{5\ 7\ A} \\ \hline \end{array}$$

(A) A = 2, B = 5 (B) A = 5, B = 2 (C) A = 1, B = 7 (D) A = 7, B = 1

Solution

Here, we have to find the values of A and B.

Since ones digit of 3 × A is A. Therefore, A = 0 or A = 5.

Now, $\overline{BA} \times \overline{B3} = \overline{57A}$ ...(i)

$\Rightarrow \quad \overline{BA} \times \overline{B3}$ is a three digit number between 500 and 600.

If B = 1, then $\overline{BA}\times\overline{B3}$ can have maximum value 19 × 13 = 247. Therefore,

B ≠ 1. If B = 3, then $\overline{BA}\times\overline{B3}$ can have minimum value 30 × 33 = 990.
Therefore, B ≠ 3. Thus, we have B = 2.
Putting B = 2 in (i), we get

$$\overline{2A}\times 23 = \overline{57A}$$

$\Rightarrow$ (20 + A) × 23 = 500 + 70 + A

$\Rightarrow$ 460 + 23A = 570 + A

$\Rightarrow$ 22A = 110

$\Rightarrow$ A = 5

Hence, A = 5 and B = 2 and,
$$\begin{array}{r} 25 \\ \times 23 \\ \hline 575 \end{array}$$

Example 13

Solve the Cryptarithm : $\overline{AB}\times\overline{AB}=\overline{ACB}$

Solution

We have, $\overline{AB}\times\overline{AB}=\overline{ACB}$

This means that the units digit of B × B is B. Therefore, B = 1 or B = 6.

Again, $\overline{AB}\times\overline{AB}=\overline{ACB}$

$\Rightarrow$ The square of a two digit number is a 3 number.

So, A can take values 1, 2 and 3.

We find that A = 1, B = 1 satisfies equation (i). For these values of A and B, we have

11 × 11 = 121

$\therefore$ C = 2

No other pairs of values of A and B satisfy equation (i)

Hence, A = 1, B = 1 and C = 2

Example 14

The sum of the digits of a 2-digit number is 8. If the digits are reversed, the new number increases by 18. Find the number.

Solution

Suppose the units digit = x

Tens digit = y

$\therefore$ The number = 10y + x

The number obtained on reversing the digits = 10x + y

By the given condition,

New number = Original number + 18

10x + y = 10y + x + 18

or 10x + y – 10y – x = 18

or 9x – 9y = 18

or x – y = 2 ...(i)

As the sum of the digits is 8

$\therefore$ x + y = 8 ...(ii)

$$\begin{array}{c} 8 \\ x \rightarrow \left\{ \begin{array}{cc} 1 & 7 \\ 2 & 6 \\ 3 & 5 \\ 4 & 4 \\ 5 & 3 \\ 6 & 2 \\ 7 & 1 \end{array} \right\} \leftarrow Y \end{array}$$

Now we find all pairs of digits from 1 to 9 whose difference is 2 and sum is 8.

We find there are only two pairs which satisfy both the conditions. Thus either
$x = 3$ and $y = 5$ or $x = 5$ and $y = 3$
But $x \neq 3$ because it does not satisfy (i)
$\therefore$ $x = 5$ and $y = 3$
The required number = 35 ....$(10 \times 3 + 5)$

Another method

Suppose units = x
$\therefore$ Tens digit = $8 - x$ ...($\because$ Sum of the digits is 8)
The number = $10(8 - x) + x$
$= 80 - 10x + x = 80 - 9x$ ...(i)
On reversing the digits, the number $= 10x + (8 - x)$
$= 10x + 8 - x = 9x + 8$ ...(ii)
By the given condition,
New number = Original number + 18
$\therefore$ $9x + 8 = 80 - 9x + 18$ [Using (i) and (ii)]
or $9x + 9x = 80 + 18 - 8$
or $18x = 90$
$\therefore$ $x = \frac{90}{18} = 5$
Putting $x = 5$ in $8 - x$, we get
Tens digit $= 8 - 5 = 3$
$\therefore$ The number = 35 ....$(10 \times 3 + 5)$
Check, On reversing the digit, we get the number 53 which is 18 more than 35. So the answer is correct.

Example 15

The product of two, 2-digit numbers is 2117. The product of their units digits is 27 and that of tens digits is 14. Find the numbers.

Solution

Let the 2-digit numbers be $10a + b$ and $10c + d$
By the given conditions $bd = 27$ ...(i)
and $ac = 14$ ...(ii)
Now find all possible pairs of digits 1 to 9 whose product is 27.
We get $b = 3$, $d = 9$ or $b = 9$, $d = 3$.
Similarly, possible pairs of digits whose product is 14
are $c = 2$, $a = 7$ or $c = 7$, $a = 2$
The possible numbers are

(c) 2 (a) 7
(d) 3 9 (b) 3 9
(10c + d) 23 29 73 79 (10a + b)

Try and see if you get the same answer 29 and 73 when c = 7 and z = 2

Units digit in the given product is 7

Now $23 \times 79 = 1817$ and 29×73
$\therefore$ $23 \times 79 = 1817$ and $29 \times 73 = 2117$
$\therefore$ The required numbers are 29 and 73

Example 16

Use digits from 0 to 9 to solve the following, where each letter stands for a different digit

$$\begin{array}{r} S\ E\ N\ D \\ +\ M\ O\ R\ E \\ \hline M\ O\ N\ E\ Y \end{array}$$

Solution

Letter M cannot be 0 as it is the left most digit of the sum. Also it cannot be more than 1 as it is the carryover number. So M is 1. The eqaution becomes

$$\begin{array}{r} S\ E\ N\ D \\ +\ 1\ O\ R\ E \\ \hline 1\ O\ N\ E\ Y \end{array}$$

Carried over here cannot be more than 1.

Letter S cannot be less than 9, because sum of S and 1 carries over 1. So S is 9. The question becomes

$$\begin{array}{r} 9\ E\ N\ D \\ +\ 1\ O\ R\ E \\ \hline 1\ O\ N\ E\ Y \end{array}$$

O cannot be 1 as $O \neq S$

Clearly, the letter O is 0 because 9 + 1 = 10.

Now the question reduces to

$$\begin{array}{r} 9\ E\ N\ D \\ +\ 1\ 0\ R\ E \\ \hline 1\ 0\ N\ E\ Y \end{array}$$

Try some value of E other then 0, 1, 9

Let us take E as 5. The question becomes

$$\begin{array}{r} 9\ 5\ N\ D \\ +\ 1\ 0\ R\ 5 \\ \hline 1\ 0\ N\ 5\ Y \end{array}$$

Because $5 + 0 \neq N$, so N = 5 + 1 (carried over from the tens digit), i.e., 6. The question reduces to

$$\begin{array}{r} 9\ 5\ 6\ D \\ +\ 1\ 0\ R\ 5 \\ \hline 1\ 0\ 6\ 5\ Y \end{array}$$

Now 6 + R = 15 gives R = 9 which is not possible because we have taken S = 9, so R = 8 and 1 is to be carried over from units digit. Now the question reduces to

$$\begin{array}{r} 9\ 5\ 6\ D \\ +\ 1\ 0\ 8\ 5 \\ \hline 1\ 0\ 6\ 5\ Y \end{array}$$

D is one of the digits 2, 3, 4, 7 since other digits have aleardy been used. D cannot be 2, 3, or 4 since we need 1 carried over, so D = 7.

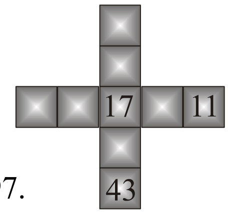
Then y will be 2. Hence the solution is

$$\begin{array}{r} 9\ 5\ 6\ 7 \\ +\ 1\ 0\ 8\ 5 \\ \hline 1\ 0\ 6\ 5\ 2 \end{array}$$

NOTE : If you take E other than 5, it will not work if you proceed further from that step.

Example 17

Fill in the boxes by 2-digit prime numbers, so that the sum of the numbers horizontally and vertically is 161.



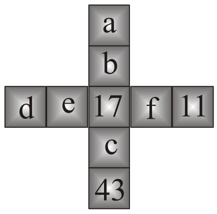
Solution

The 2-digit prime numbers are :

11, 13, 17, 19, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Let the numbers be a, b, c, d, e, f such that

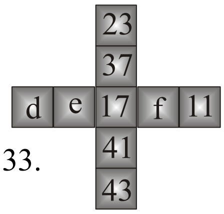
$a + b + 17 + c + 43 = 161$ or $a + b + c = 161 - 60$
$= 101$



And $d + e + 17 + f + 11 = 161$ or $d + e + f = 161 - 28 = 133$

Now to get $a + b + c = 101$, we need to select three numbers from the above list of 2-digit prime numbers whose sum is 101.

One such possible group of numbers is 23, 37, 41.



Again we need to select three numbers from the list such that their sum is 133.

One such possible group of numbers is 13, 47, 73.

Note : 1. Each group of numbers selected above can be written in an order.

2. There may be more than one solution to such type of problems.

Example 18

Veena told Rita that "every number can be written as a difference of two squares", was discovered during Vedic times. For example, $24 = 7^2 - 5^2$. Do you know the reason ? Write down 25, 31 and 6 as difference of two square.

Solution

We know that $4ab = (a + b)^2 - (a - b)^2$

or $$ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$$

So every number can be written as the difference of two squares by using the above formula.

Now $25 = 25 \times 1$

$$\therefore \quad 25 = \left(\frac{25+1}{2}\right)^2 - \left(\frac{25-1}{2}\right)^2 = 13^2 - 12^2$$

$31 = 31 \times 1$

$$\therefore \quad 31 = \left(\frac{31+1}{2}\right)^2 - \left(\frac{31-1}{2}\right)^2 = 16^2 - 15^2$$

$6 = 3 \times 2$

$$\therefore \quad 6 = \left(\frac{3+2}{2}\right)^2 - \left(\frac{3-2}{2}\right)^2 = \left(\frac{5}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

Example 19

Think of a number. Double it. Add 18. Take away 10. Halve it. Take away 4. Now what do you get and why ?

Solution

Let us take the number 13.

$\therefore$ Double of 13 = 13 + 13

After adding 18, we get = 13 + 13 + 18

After taking away 10, we get = 13 + 13 + 18 – 10

After dividing by 2, we get = $\frac{13+13+18-10}{2} = 17$

After taking away 4, we get = 17 – 4 = 13

We get the original number.

We can write it as

13 = 13 + 13 + 18 – 10 – 17 – 4 = 13 + (31) – (31)

Adding and subtracting the same number to a given number does not change the given number.

Example 20

Replace* by the smallest digit, so that 1 * 4 is divisible by (i) 3, (ii) 9. Find the numbers also.

Solution

(i) Sum of the known digits of 1 * 4 = 1 + 4 = 5

The number next of 5 which is divisible by 3 is 6, so * is to be replaced by 1. 1 + 1 + 4 = 6

$\therefore$ The number becomes 114.

(ii) Sum of the known digits of 1 * 4 = 1 + 4 = 5

The number next to 5 which is divisible by 9 is 9, so * is to be replaced by 4. 1 + 4 + 4 = 9

$\therefore$ The number becomes 144.

Example 21

Given two numbers which are divisible by 3 but not by 9. Can you find a number which is divisible by 9 but not by 3 ?

Solution

The numbers 15 an 174 are both divisible by 3 but not by 9.

There does not exist a number which is divisible by 9 but not by 3.

Example 22

Form as many as possible 3-digit numbers by using the digits 1, 0, 5 which are divisible by (i) 2, (ii) 3.

Solution

(i) We know that a number is divisible by 2, if its unit place is divisible by 2.

$\therefore$ The possible 3-digit numbers are 150, 510.

(ii) We know that a number is divisible by 3, if the sum of its digits is divisible by 3.

0 is not used on the extreme left of a number

$\therefore$ The possible 3-digit numbers are 105, 150 501, 510.

Example 23

Answer the following questions :

(i) Using mathematical operation + and – only make a 3, using the digit 3 exactly three times.

(ii) Using various mathematical operation make a 1000 using the digit 8 exactly 8 times.

(iii) Using various mathematical operation make a 5, using the digit 5 exactly 5 time.

(iv) Using the mathematical operations + and – only and all digits 1 to 9 each exactly once, make digit 1.

Solution

(i) $3 + 3 - 3 = 3$

(ii) $[888 + 88] + [8 + 8 + 8] = 1000$

(iii) $5 + 5 - 5 + 5 - 5 = 5$

(iv) $9 - 8 + 7 - 6 + 5 - 4 - 3 + 2 - 1 = 1$

CONCEPT APPLICATION LEVEL - I [NCERT Questions]

EXERCISE -1

Q.1 Find the values of the letters in the following and give reasons for the steps involved.

1.
$$\begin{array}{cccc} & & 3 & A \\ + & & 2 & 5 \\ \hline & & B & 2 \\ \hline \end{array}$$

2.
$$\begin{array}{cccc} & & 4 & A \\ + & & 9 & 8 \\ \hline & C & B & 3 \\ \hline \end{array}$$

3.
$$\begin{array}{cc} 1 & A \\ \times & A \\ \hline 9 & A \\ \hline \end{array}$$

4.
$$\begin{array}{cccc} & & A & B \\ + & & 3 & 7 \\ \hline & & 6 & A \\ \hline \end{array}$$

5.
$$\begin{array}{ccc} & A & B \\ \times & 3 & \\ \hline C & A & B \\ \hline \end{array}$$

6.
$$\begin{array}{ccc} & A & B \\ \times & 5 & \\ \hline C & A & B \\ \hline \end{array}$$

7.
$$\begin{array}{ccc} & A & B \\ & \times & 6 \\ \hline B & B & B \\ \hline \end{array}$$

8.
$$\begin{array}{cccc} & & A & 1 \\ + & & 1 & B \\ \hline & & B & 0 \\ \hline \end{array}$$

9.
$$\begin{array}{cccc} & 2 & A & B \\ + & A & B & 1 \\ \hline & B & 1 & 8 \\ \hline \end{array}$$

10.
$$\begin{array}{cccc} & 1 & 2 & A \\ + & 6 & A & B \\ \hline & A & 0 & 9 \\ \hline \end{array}$$

Sol. 1. Here, there are two letters A and B whose values are to be found out.
Let us see the sum in unit's column. It is $A + 5$ and we get 2 from this.
So A has to be 7. $(\because A + 5 = 7 + 5 = 12)$
Now, for sum in ten's column, we have
$1 + 3 + 2 = B \quad \Rightarrow \quad B = 6$
Therefore, the puzzle is solved as shown below :

$$\begin{array}{ccc} & 3 & 7 \\ + & 2 & 5 \\ \hline & 6 & 2 \\ \hline \end{array}$$

That is, $A = 7$ and $B = 6$

2. Here, there are three letters A, B and C whose values are to be found out.
Let us see the sum in units column. It is $A + 8$ and we get 3 from this.
So a has to be 5 $(\because A + 8 = 5 + 8 = 13)$
Now, for the sum in ten's column, we have
$1 + 4 + 9 = 14 \quad \Rightarrow \quad B = 4, C = 1$
Therefore, the puzzle is solved as shown below :

$$\begin{array}{ccc} & 4 & 5 \\ + & 9 & 8 \\ \hline 1 & 4 & 3 \\ \hline \end{array}$$

That is, $A = 5$, $B = 4$ and $C = 1$

3. $\because$ Unit's digit of $A \times A$ is A.

$\therefore$ A = 1 and A = 5 or A = 6

When A = 1, Then

$$\begin{array}{r} 1\ \ 1 \\ \times\ \ 1 \\ \hline 1\ \ 1 \\ \hline \end{array}$$

$\therefore$ A = 1 is not possible. When A = 5, then

$$\begin{array}{r} 1\ \ 5 \\ \times\ \ 5 \\ \hline 7\ \ 5 \\ \hline \end{array}$$

$\therefore$ A = 1 is not possible. When A = 5, then

$$\begin{array}{r} 1\ \ 5 \\ \times\ \ 5 \\ \hline 7\ \ 5 \\ \hline \end{array}$$

$\therefore$ A = 5 is not possible. When A = 6, Then

$$\begin{array}{r} 1\ \ 6 \\ \times\ \ 6 \\ \hline 9\ \ 6 \\ \hline \end{array}$$

which is admissible.

$\therefore$ A = 6

4. Here, there are two letters A and B whose values are to be found out.

B + 7 gives A and A + 3 gives 6. The possible values are :

$0 + 7 = 7$

$\Rightarrow$ A = 7 but $7 + 3 \neq 6$. So, not acceptable.

$1 + 7 = 8$

$\Rightarrow$ A = 8 but $8 + 3 \neq 6$. So, not acceptable.

$2 + 7 = 9$

$\Rightarrow$ A = 9 but $9 + 3 \neq 6$. So, not acceptable.

$3 + 7 = 10$

$\Rightarrow$ A = 10 but $1 + 0 + 3 \neq 6$. So, not acceptable.

$4 + 7 = 11$

$\Rightarrow$ A = 11 but $1 + 1 + 3 \neq 6$. So, not acceptable.

$5 + 7 = 12$

$\Rightarrow$ A = 2. Also $1 + 2 + 3 = 6$.

So, B = 5 works and then we get A as 2.

Therefore, the puzzle is solved as shown below :

$$\begin{array}{r} 2\ \ 5 \\ +\ \ 3\ \ 7 \\ \hline 6\ \ 2 \\ \hline \end{array}$$

That is, A = 2 and B = 5

5. Here, there are three letters A, B and C whose values are to be found out.

$\because$ Unit's digit of $3 \times B$ is B. so, it is essential that $B = 0$.

Hence, we get

$$\begin{array}{r} A\ 0 \\ \times\ 3 \\ \hline C\ A\ 0 \\ \hline \end{array}$$

$\therefore$ Unit's digit of $3 \times A$ is a. So, it is essential that $A = 5$.

Hence, we get

$$\begin{array}{r} 5\ 0 \\ \times\ 3 \\ \hline 1\ 5\ 0 \\ \hline \end{array}$$

$\therefore$ $A = 5$, $B = 0$ and $C = 1$

6. Here, there are three letters A, B and C, whose values are to be found out.

$\because$ Units's digit of $5 \times B$ is B

$\therefore$ $B = 0$ or $B = 5$

If $b = 0$, then we have

$$\begin{array}{r} A\ 0 \\ \times\ 5 \\ \hline C\ A\ 0 \\ \hline \end{array}$$

Now, $5 \times A = A \Rightarrow A = 0$ or 5

But a 0 as in the answer there is a third letter too.

If $A = 5$, then we have

$$\begin{array}{r} 5\ 0 \\ \times\ 5 \\ \hline 2\ 5\ 0 \\ \hline \end{array}$$

$\therefore$ $A = 5$, $B = 0$ and $C = 2$. If $B = 5$, then we have

$$\begin{array}{r} A\ 5 \\ \times\ 5 \\ \hline C\ A\ 5 \\ \hline \end{array}$$

Now, $5 \times A + 2 = A$

$\Rightarrow$ $A = 2 \mid$ in the form of $5 \times 2 + 2 = 12$

$\therefore$ Unit's digit is 2 which is equal to a.

Then, we have

$$\begin{array}{r} 2\ 5 \\ \times\ 5 \\ \hline 1\ 2\ 5 \\ \hline \end{array}$$

$\therefore$ $A = 2$, $B = 5$ and $C = 1$.

7. Here, there are two letters A and B whose values are to be found out. We have

$$\begin{array}{rrr} & A & B \\ & \times & 6 \\ \hline B & B & B \\ \hline \end{array}$$

The possible values of BBB are

111, 222, 333 etc.

Let us divide these numbers by 6, we get

$111 \div 6 = 18$, remainder 3

$\therefore$ 111 is not acceptable.

$222 \div 6 = 37$, remainder 0.

Hence, the quotient is not of the form A_2.

$\therefore$ 222 is not acceptable.

$333 \div 6 = 66$, remainder 3.

$\therefore$ 33 is inadmissible.

$444 \div 6 = 74$, remainder 0.

Here the quotient 74 is of the form A_4 which clearly works well.

hence, the puzzle is solved as shown below :

$$\begin{array}{rrr} & 7 & 4 \\ & \times & 6 \\ \hline 4 & 4 & 4 \\ \hline \end{array}$$

$\therefore$ A = 7 and B = 4

8. Here, there are two letters A and B whose values are to be found out.

Let us see the sum in one's column, we have

1 + B gives 0, i.e., a number whose unit's digit is 0.

$\therefore$ B = 9

$\because$ B is itself a one digit number.

Then, we have the puzzle as

$$\begin{array}{rrr} & A & 1 \\ + & 1 & 9 \\ \hline & 9 & 0 \\ \hline \end{array}$$

$90 - 19 = 71 \quad \Rightarrow \quad A1 = 71 \quad \Rightarrow \quad A = 7$

Therefore, A = 7 and B = 9.

9. We are to find out a and b.

Let us see the sum in unit's column

$B + 1 = 8 \quad \Rightarrow \quad B = 7$

$\because$ B is itself a one digit number.

Then, the puzzle becomes

$$\begin{array}{rrrr} & 2 & A & 7 \\ + & A & 7 & 1 \\ \hline & 7 & 1 & 8 \\ \hline \end{array}$$

Now, see the sum in ten's column.

A + 7 gives 1, i.e,. a number whose unit's digit is 1.

$\therefore$ A = 4

$\because$ A itself is a one digit number.

Hence, the puzzle is solved as shown below.

$$\begin{array}{r} 2\ 4\ 7 \\ +\ 4\ 7\ 1 \\ \hline 7\ 1\ 8 \\ \hline \end{array}$$

Here, A = 4, B = 7

10. We are to find out the values of A and B.

Let us see the sum in ten's column. It is 2 + A and we get 0 from this.

$\because$ A itself is a one digit number.

The puzzle then becomes

$$\begin{array}{r} 1\ 2\ 8 \\ +\ 6\ 8\ B \\ \hline 8\ 0\ 9 \\ \hline \end{array}$$

Now, see the sum in one's column. It is 8 + B and we get 9 from this.

$\therefore$ B = 1

Hence, the puzzle is finally solved as shown below

$$\begin{array}{r} 1\ 2\ 8 \\ +\ 6\ 8\ 1 \\ \hline 8\ 0\ 9 \\ \hline \end{array}$$

Therefore, A = 8 and B = 1.

TRY YOUR SELF

Q.1 Write the following numbers in generalised form.

(i) 25 (ii) 73 (iii) 129 (iv) 302

Sol. (i) $25 = 20 + 5 = 10 \times 2 + 1 \times 5$

(ii) $73 = 70 + 3 = 10 \times 7 + 1 \times 3$

(iii) $129 = 100 + 20 + 9 = 100 \times 1 + 10 \times 2 + 1 \times 9$

(iv) $302 = 300 + 00 + 2 = 100 \times 3 + 10 \times 0 + 1 \times 2$

Q.2 Write the following in the usual form

(i) $10 \times 5 + 6$ (ii) $100 \times 7 + 10 \times 1 + 8$ (iii) $100 \times a + 10 \times c + b$

Sol. (i) $10 \times 5 + 6 = 50 + 6 = 56$

(ii) $100 \times 7 + 10 \times 1 + 8 = 700 + 10 + 8 = 718$

(iii) $100 \times a + 10 \times c + b = acb$

Q.3 Check what the result would have been if Sundaram has chosen the numbers shown below

(1) 27 (2) 39 (3) 64 (4) 17

Sol. (1) Chosen number = 27

Reversed number = 72

Sum of these two numbers = 27 + 72 = 99

finally, 99 ÷ 11 = 9 with no remainder

also, 9 = 2 + 7,

i.e., the sum of the digits of the chosen number.

(2) Chosen number = 39
Reversed number = 93
Sum of these two numbers = 39 + 93 = 132
Finally, $132 \div 11 = 12$ with no remainder.
Also, $12 = 3 + 9$

(3) Chosen number = 64
Reversed number = 46
Sum of these two numbers = 64 + 46 = 110
Finally, $110 \div 11 = 10$ with no remainder.
also, $10 = 6 + 4$
i.e., the sum of the digits of the chosen number.

(4) Chosen number = 17
Reversed number = 71
Sum of these two numbers = 17 + 71 = 88
Finally, $88 \div 11 = 8$ with no remainder.
Also, $8 = 1 + 7$
i.e., the sum of the digits of the chosen number.

Q.4 Check what the results would have been if Sundaram had chosen the numbers shown below
(1) 17 (2) 21 (3) 96 (4) 37

Sol. (1) Chosen number = 17
Reversed number = 71
On subtracting the smallest number from the larger one, we get
$= 71 - 17 = 54$
Finally, $54 \div 9 = 6$, with no remainder.
also, $6 = 7 - 1$,
i.e., the difference of the digits of the chosen number.

(2) Chosen number = 21
Reversed number = 12
On subtracting the smallest number from the larger one, we get
$= 21 - 12 = 9$
Finally, $9 \div 9 = 1$, with no remainder.
also, $1 = 2 - 1$,
i.e., the difference of the digits of the chosen number.

(3) Chosen number = 96
Reversed number = 69
On subtracting the smallest number from the larger one, we get
$= 96 - 69 = 27$
Finally, $27 \div 9 = 3$, with no remainder.
also, $3 = 9 - 6$,
i.e., the difference of the digits of the chosen number.

(4) Chosen number = 37
Reversed number = 73
On subtracting the smallest number from the larger one, we get
$= 73 - 37 = 36$
Finally, $36 \div 9 = 4$, with no remainder.
also, $4 = 7 - 3$,
i.e., the difference of the digits of the chosen number.

Q.5 Check what the result would have been if Minakshi had chosen the numbers shown below. In each case keep a record of the quotient obtained at the end.

(1) 132 (2) 469 (3) 737 (4) 901

Sol. (1) Chosen number = 132
Reversed number = 231
On subtracting the smallest number from the larger one, we get
$= 231 - 132 = 99$
Finally, $99 \div 99 = 1$, with no remainder.
also, $1 = 2 - 1$,
i.e., $1 = 2 - 1$, i.e., the difference of the digits at the hundred's place and the unit's place.

(2) Chosen number = 469
Reversed number = 964
On subtracting the smallest number from the larger one, we get
$= 964 - 469 = 495$
Finally, $495 \div 99 = 5$, with no remainder.
also, $5 = 9 - 4$,
i.e., the difference of the digits at the hundred's place and the unit's place.

(3) Chosen number = 737
Reversed number = 737
On subtracting the smallest number from the larger one, we get
$= 737 - 737 = 0$
Finally, $0 \div 99 = 0$, with no remainder.
also, $0 = 7 - 7$,
i.e., the difference of the digits at the hundred's place and the unit's place.

(4) Chosen number = 901
Reversed number = 109
On subtracting the smallest number from the larger one, we get
$= 901 - 109 = 792$
Finally, $792 \div 99 = 8$, with no remainder.
Also, $8 = 9 - 1$, i.e., the difference of the digits at the hundred's place and the unit's place.

Q.6 Check what the result would have been if sundaram has chosen the numbers shown below.

(1) 417 (2) 632 (3) 117 (4) 937

Sol. (1) Chosen number = 417

Two more 3-digit numbers = 741 and 174

Sum of the three numbers =

$$\begin{array}{rrrr} = & 4 & 1 & 7 \\ + & 7 & 4 & 1 \\ + & 1 & 7 & 4 \\ \hline 1 & 3 & 3 & 2 \\ \hline \end{array}$$

Finally, 1332 ÷ 37 = 36, no remainder.

(2) Chosen number = 632

Two more 3-digit numbers = 263 and 326

Sum of the three numbers =

$$\begin{array}{rrrr} = & 6 & 3 & 2 \\ + & 2 & 6 & 3 \\ + & 3 & 2 & 6 \\ \hline 1 & 2 & 2 & 1 \\ \hline \end{array}$$

Finally, 1221 ÷ 37 = 33, no remainder.

(3) Chosen number = 117

Two more 3-digit numbers = 711 and 171

Sum of the three numbers =

$$\begin{array}{rrrr} = & 1 & 1 & 7 \\ + & 7 & 1 & 1 \\ + & 1 & 7 & 1 \\ \hline & 9 & 9 & 9 \\ \hline \end{array}$$

Finally, 999 ÷ 37 = 27, no remainder.

(4) Chosen number = 937

Two more 3-digit numbers = 793 and 379

Sum of the three numbers =

$$\begin{array}{rrrr} = & 9 & 3 & 7 \\ + & 7 & 9 & 3 \\ + & 3 & 7 & 9 \\ \hline 2 & 1 & 0 & 9 \\ \hline \end{array}$$

Finally, 2109 ÷ 37 = 57, no remainder.

Q.7 Write a 2-digit number ab and the number obtained by reversing its digits i.e., ba. Find their sum. Let the sum be a 3-digit number dad.

i.e., ab + ba = dad

(10a + b) + (10b + a) = dad

11(a + b) = dad

The sum a + b cannot exceed 18 (why) ?

Is dad a multiple of 11 ?

Is dad less than 198?

Write all the 3-digit numbers which are multiples of 11 upto 198.

Find the value of a and b.

Sol. $\because$ The value of a or b cannot exceed 9

$\therefore$ The sum a + b cannot exceed 18.

Yes ! dad is a multiple of 11.

dad is less than or equal to 198.

All the 3-digit numbers which one multiples of 11 upto 198 are

110, 121, 132, 143, 154, 165, 176, 187 and 198

Clear

dad = 121

$\Rightarrow$ d = 1, a = 2

Q.8 If the division N / 5 leaves a remainder of 3, what might be the ones digit of N?

Sol. The one's digit, when divided by 5, must leave a remainder of 3. so the one's digit must be either 3 or 8

Q.9 If the division N / 5 leaves a remainder of 1, what might be one's digit of N?

Sol. The one's digit, when divided by 5, must leave a remainder of 1. So the one's digit must be either 1 or 6.

Q.10 If the division N / 5 leaves a remainder of 4, what might be one's digit of N?

Sol. The one's digit, when divided by 5, must leave a remainder of 4. So the one's digit must be either 4 or 9.

Q.11 If the division N / 2 leaves a remainder of 1, what might be the one's digit of N?

Sol. N is odd; so its one' digit is odd. Therefore, the one's digit must be 1, 3, 5, 7 or 9)

Q.12 If the division N / 2 leaves no remainder (i.e., zero remainder), what might be the one's digit of N?

Sol. N is even; so its units digit is even. Therefore, the one's digit must be 0, 2, 4, 6 or 8.

Q.13 Suppose that the division N / 5 leaves a remainder of 4, and the division N / 2 leaves a remainder of 1.what must be one's digit of N?

Sol. $\because$ The division N / 5 leaves a remainder of 4.

$\therefore$ The one's digit, when divided by 5, must leave a remainder of 4.

Again, Q The division N / 2 leaves a remainder of 1.

$\therefore$ N is odd.

$\therefore$ Its unit's digit is odd.

$\therefore$ The one's digit must be 1, 3, 5, 7 or 9 .....(2)

In view of (1) and (2),

the one's digit must be 9.

Q.14 Check the divisibility of the following numbers by 9.
(1) 108 (2) 616 (3) 294 (4) 432 (5) 927

Sol. (1) The sum of the digits of 108 is 1 + 0 + 8 = 9. This number is divisible by 9 (for 9 / 9 = 1). We conclude that 108 is divisible by 9.

Double-check $\frac{108}{9} = 12$(the division is exact)

(2) 616

The sum of the degree digit of 616 is 6 + 1 + 6 = 13. This number is not divisible by 9.

$\left(\text{for } 13 \div 9 = 1\frac{4}{9}\right)$

We conclude that 616 is not divisible by 9.

Double-check $\frac{616}{9} = 68\frac{4}{9}$

(3) 294

The sum of the digits of 294 is 2 + 9 + 4 = 15. This number is not divisible by 9.

$\left(\text{for } 15 \div 9 = 1\frac{6}{9}\right)$

We conclude that 294 too is not divisible by 9.

Double-check $\frac{294}{9} = 32\frac{6}{9}$

(4) 432

The sum of the digits of 4 + 3 + 2 = 9. This number is divisible by 9 (for 9 / 9 = 1). We conclude that 432 is divisible by 9.

Double-check $\frac{432}{9} = 48$ (the division is exact)

(5) 927

The sum of the digits of 927 is 9 + 2 + 7 = 18. This number is divisible by 9 (for 18 / 9 = 2). We conclude that 927 is divisible by 9.

Double-check $\frac{297}{9} = 103$ (the division is exact)

Q.15 You have seen that a number 450 is divisible by 10. It is also divisible by 2 and 5 which are factor of 10. Similarly, a number 135 is divisible 9. It is also divisible by 3 which is a factor of 9. Can you say that if a number is divisible by any number m, then it will also be divisible by each of the factor of m ?

Sol. Yes! we can say that if a number is divisible by any number m, then it will also be divisible by each of the factor of m.

Q.16 (i) Write a 3-digit number abc as 100a + 10b + c
= 99a + 11b + (a – b + c)
= 11(9a + b) + (a – b + c)
If the number abc is divisible by 11, then what can you say about (a – b + c) should it be divisible by 11?

(ii) Write a 4-digit number abcd as 1000a + 100b + 10c + d.
= (1001a + 99b + 11c) – (a – b + c – d)
= 11(91a + 9b + c) + [(b + d) – (a + c)]
if the number acd is divisible by 11, then what can you say about [(b + d) – (a + c)]?

(iii) From (i) and (ii) above, can you say that a number will be divisible by 11 if the difference between the sum of digits at its odd places and that of digits at the even places is divisible by 11?

Sol. (i) (a – b + c) is divisible by 11.

(ii) [(b + d) – (a + c)] is divisible by 11.

(iii) Yes! we can say that a number will be divisible by 11 if the difference the sum of digits at its odd places and that of digits at the even places is divisible by 11.

Q.17 Check the divisibility of the following numbers by 3.
(1) 108 (2) 616 (3) 294 (4) 432 (5) 927

Sol. (1) The sum of the digits of 108 is 1 + 0 + 8 = 9. 9 is divisible by 3. (for 9 ÷ 3 = 3). We conclude that 108 is divisible by 3.

Double-check $\frac{108}{3}$ = 36 (the division is exact)

(2) The sum of the digits of 616 is 6 + 1 + 6 = 13. 13 is not divisible by 3 $\left(\text{for } 13 \div 3 = 4\frac{1}{3}\right)$. We conclude that 616 is not divisible by 3.

Double-check $\frac{616}{3} = 205\ \frac{1}{3}$

(3) The sum of the digits of 294 is 2 + 9 + 4 = 15. 15 is divisible by 3. (for 15 ÷ 3 = 5). We conclude that 294 is divisible by 3.

Double check $\frac{294}{3}$ = 98 (the division is exact)

(4) The sum of the digits of 432 is 4 + 3 + 2 = 9. 9 is divisible by 3 (for 9 ÷ 3 = 3). We conclude that 432 is divisible by 3.

Double check $\frac{432}{3}$ = 144 (the division is exact)

(5) The sum of the digits of 927 is 9 + 2 + 7 = 18. This number is divisible by 3 (for 18 ÷ 3 = 6). We conclude that 927 is divisible by 3.

Q.18 If 21y5 is a multiple of 9, where y is a digit, what is the value of y ?

Sol. Since 21y5 is a multiple of 9, its sum of digits 2 + 1 + y + 5 = 8 + y is a multiple of 9; so 8 + y is one of these numbers 0, 9, 18, 27, 36, 45, ... But since y is a digit, it can only be that 8 + y = 9. Therefore, y = 1

Q.19 If 31z5 is a multiple of 9, where z is a digit, what is the value of z ?
You will find that there are two answers for the last problem. Why is this so ?

Sol. Since 31z5 is a multiple of 9, its sum of digits 3 + 1 + z + 5 = 9 + z is a multiple of 9; so 9 + z = 9 or 18. Therefore, z = 0 or 9.

Q.20 If 24x is a multiple of 3, where x is a digit, what is the value of x ?
(Since 24x is a multiple of 3, its sum of digits 6 + x is a multiple of 3; so 6 + x is one of these numbers; 0, 3, 6, 9, 12, 15, 18, ... but these numbers : 0, 3, 6, 9, 12, 15, 18, ... But since x is a digit, it can only be that 6 + x = 6 or 9 or 12 or 16. Therefore, x = 9, or 3 or 6 or 9. Thus, x can have any of four different values.

Ans. Solution given with question.

Q.21 If 31z5 is a multiple of 3, where z is a digit, what might be the values of z ?

Ans. Since 31z5 is a multiple of 3, its sum of digits 3 + 1 + z + 5 = 9 + z is a multiple of 3; so 9 + z is one of these numbers : 0, 3, 6, 9, 12, 15, 18, ... But since z is a digit, it can only be that 9 + z = 9 or 12 or 15 or 18. Therefore, z = 0 or 3 or 6 or 9.

CONCEPT APPLICATION LEVEL - II

SECTION - A

- **FILL IN THE BLANKS**

Q.1 If we divide the sum of any 2-digit numbers ab and ba by (a + b), then the quotient is __________.

Q.2 The difference between two 2-digit numbers ab and ba, where a > b is divided by 3. The quotient is __________.

Q.3 If the difference of 582 and 285 is divided by 11, the quotient is __________.

Q.4 The sum of three 3-digit numbers zyz, yzx and zxy is divided by (x + y + z), the quotient is __________.

Q.5 If $1x \times x = 9x$, then find the value of x. __________.

Q.6 If $N \div 2$ leaves a remainder of 0, then what might be the ones digit of N ? __________.

Q.7 Write a 3-digit number which is divisible by 4 and 8 but not by 32. __________.

Q.8 All even natural numbers which are divisible by 3 are also divisible by 6. Is true ?__________.

Q.9 Standard form of 0.000000000839 is __________.

Q.10 Cube of 14 is = __________.

SECTION - B

➢ **MULTIPLE CHOICE QUESTIONS**

Q.1 Which occupies more space : 1 kg gold or 1 kg cotton?
(A) Gold (B) Cotton (C) Both (D) None

Q.2 Which encloses more area if their perimeters are same - an equilateral triangle or a square?
(A) Equilaterals (B) Square (C) Both equal (D) None

Q.3 I am as much older than my brother who is 10 years as I am younger than my father who is 70 years. How old I am?
(A) 40 years (B) 30 years (C) 20 years (D) 10 years

Q.4 The sum of the digits of a 2-digits number is 12. If the digits are reversed, the new number decreases by 36. Find the number.
(A) 48 (B) 84 (C) 75 (D) 57

Q.5 The product of two 2-digit numbers is 1665. The product of their units digits is 35 and that of tens digits is 12. Find the numbers.
(A) 37, 45 (B) 47, 35 (C) 67, 25 (D) 65, 27

Q.6 If * and ⊙ are two operations such that $a * b = a + b + 2$ and $a \odot b = a \times b - 4$, find
(i) $(3 * 4) \odot 5$ and (ii) $3 * 4\ (4 \odot 5)$
Are they equal?
(A) Yes (B) No (C) Can't say (D) Can't be determined

Direction (Q.7 to 9) : Choose digits (0 – 9) for each letter which satisfy the following. Each letter has different value.

Q.7 $x^y = y^x$

(A) x = 2, y = 4 (B) x = 4, y = 2 (C) x = 3, y = 2 (D) x = 2, y = 3

Q.8 $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$

(A) 2, 3, 6 (B) 1, 1, 1 (C) 3, 3, 3 (D) 3, 2, 6

Q.9 X × Y × YZ = XXX

(A) 2, 3, 7 (B) 4, 3, 2 (C) 1, 2, 3 (D) 2, 1, 7

Q.10 Use the symbols +, –, ÷, × and $\sqrt{\ }$ to write the numberal 9 using 4 fours. For example, 44 ÷ 44 = 1 or $4 + 4 + 4 - \sqrt{4} = 10$.

(A) 4 + 4 + 4 ÷ 4 (B) 4 + 4 – 4 ÷ 4 (C) 4 + 4 – (4 ÷ 4) (D) 4 + 4 + (4 ÷ 4)

Q.11 Select a number, add 2, now multiple by 3 then subtract 6 and lastly divide by 3. What do you get?

(A) always 3 (B) Never 3

(C) Always number itself (D) sometimes number itself

Q.12 Select a number between 1 and 100. Add 28, then multiply by 6 and subtract 3. Now divide by 3 and then subtract 3 more than the original number. Now add 8, subtract 1 less than the original number. Lastly multiply by 7. Find the answer.

(A) 427 (B) 320 (C) Number itself (D) 420

Q.13 The sum of the digits of a 2-digit number is 7. If the number obtained by interchanging the digits is 27 more than the original number, find the original number.

(A) 25 (B) 52 (C) 43 (D) 34

Q.14 The sum of the digits of a 2-digit number is equal to 12. The digit in one's place is 3 times the digit in tens place. Find the number.

(A) 39 (B) 93 (C) 57 (D) 75

Q.15 The sum of the digits of a 2-digit number is 10. The digit in one's place is nine times the digit at ten's place. Find the number.

(A) 91 (B) 19 (C) 90 (D) None

Q.16 The sum of digits of a 2-digit number is 14. If the digits are reversed, the new number decreases by 36. Find the number.

(A) 95 (B) 96 (C) 69 (D) 59

Q.17 The smallest three digit number divisible by 3 is :

(A) 100 (B) 101 (C) 102 (D) 103

Q.18 If a number 93*5 is divisible by 9, the possible digit which can replace * is :

(A) 1 (B) 2 (C) 3 (D) 4

Q.19 If 23* is divisible by 11, the possible digit which can replace * is :
(A) 1 (B) 2 (C) 3 (D) 4

Q.20 If the sum of the digits of the number is 33, then the number is divisible by :
(A) 3 only (B) 9 only (C) neither 3 nor 9 (D) 3 and 9 both

Q.21 If a number is divisible by 10, then the number is also divisible by :
(A) 2 only (B) 5 only (C) neither 2 nor 5 (D) 2 and 5 both

Q.22 The number 6912 is divisible by :
(A) 2 and 3 (B) 2 and 9 only (C) 3 and 9 only (D) 2; 3 and 9

Q.23 A number having 0 at unit's digit is divisible by :
(A) 2 only (B) 5 only (C) 10 only (D) 2, 5 and 10

Q.24 If a number 87*2 is divisible by 9, the possible digit which can replace * is :
(A) 1 (B) 2 (C) 3 (D) 4

Q.25 Largest three digit number divisible by 5 and 10 both is :
(A) 999 (B) 995 (C) 990 (D) 900

Q.26 If 56*72 is divisible by 11, the possible digit which can replace * is :
(A) 6 (B) 2 (C) 3 (D) 4

Q.27 If [1 X 2 Y 6 Z] is a number divisible by 9, then the least value of X is :
(A) 0 (B) 1 (C) 9 (D) 5

Q.28 The number 2 8 2 2 1 is divisible by which of the following :
(A) 2 (B) 3 (C) 6 (D) 9

Q.29 Which of the following is one's digit of a number, when divided by 5 gives a remainder of 3 ?
(A) 8 (B) 3 (C) 3 or 8 (D) none of these

Q.30 If the 4-digit number 2 X Y 7 is exactly divisible by 3, then which of the following is the least value of (X + Y) ?
(A) 3 (B) 4 (C) 6 (D) 9

Q.31 If a number is divisible by 2, then which of the following cannot be a one's digit in it ?
(A) 0 (B) 1 (C) 2 (D) 4

Q.32 A is a digit and 3A15 is a multiple of 9. Which of the following can be the value of A ?
(A) 1 or 9 (B) 0 or 8 (C) 0 or 7 (D) 0 or 9

Q.33 The value of A and B in
$$\begin{array}{r} A\ \ 1 \\ +\ 1\ \ B \\ \hline B\ \ 0 \\ \hline \end{array}$$
is :

(A) A = 9, B = 9 (B) A = 7, B = 9 (C) A = 7, B = 7 (D) A = 9, B = 7

Q.34 Sum of an even number and an odd number is :
(A) an even number (B) an odd number (C) a multiple of 3 (D) a multiple of 5

Q.35 Sum of (10a + b) and (10b + a) is always divisible by :
(A) 11 (B) 9 (C) 7 (D) 3

Q.36 Expanded form of 729 is :
(A) 7 2 9 (B) 7 + 2 + 9
(C) $7 \times 10 + 2 \times 10 + 9$ (D) $7 \times 100 + 2 \times 10 + 9$

Q.37 If a number is divisible by both 4 and 6 then it is always divisible by :
(A) 24 (B) 12 (C) 36 (D) 48

Q.38 The number 1 + 6354 is divisible by :
(A) 5 (B) 9 (C) 7 (D) 6

Q.39 In the following cryptarithms value of Q is

$$\begin{array}{r} PQ \\ \times P3 \\ \hline 57Q \\ \hline \end{array}$$

(A) 2 (B) 5 (C) 4 (D) 3

Q.40 Multiple of 11 closest to 10,00,000 is :
(A) 99991 (B) 99999 (C) 999999 (D) 999899

Q.41 Least value of A such that 36825A6 is divisible by 11 is :
(A) 2 (B) 3 (C) 4 (D) 7

Q.42 The number divisible by 15 is
(A) 9554 (B) 9555 (C) 8555 (D) 7555

Q.43 What should be added to $\frac{-7}{10}$ to get $\frac{19}{30}$?

(A) $\frac{3}{4}$ (B) $\frac{14}{15}$ (C) $\frac{4}{3}$ (D) $\frac{12}{5}$

Q.44 $x^m \times x^n =$
(A) $x^{m \times n}$ (B) $x^{m + n}$ (C) $x^m \times n$ (D) $x^m + x^n$

Q.45 Which of the following number is a perfect square number ?
(A) 840 (B) 841 (C) 1088 (D) 1368

Q.46 Multiplicative inverse of $\frac{-4}{9} \times \frac{12}{7}$ is :

(A) $-\frac{21}{16}$ (B) $\frac{16}{21}$ (C) $\frac{-16}{21}$ (D) $\frac{-13}{16}$

Q.47 Cubes of the numbers ending with 7 ends with :
(A) 3 (B) 2 (C) 9 (D) 1

Q.48 Sanchi is now 12 years old and Sam is two years old. In how many years will Sanchi be three times as old as Sam? **[IMO-2016]**

(A) 3 (B) 4 (C) 5 (D) 6

Q.49 The sum of the digits of a 2-digit number is 7. If the digits are reversed, the number formed is 9 less than the original number. Find the number. **[IMO-2016]**

(A) 40 (B) 43 (C) 49 (D) 53

Q.50 While solving a problem, by mistake, Minakshi squared a number and then subtracted 25 from it rather than first subtracting 25 from the number and then squaring it. But she got the answer right. What was the given number? **[IOM-2016]**

(A) 13 (B) 38 (C) 48 (D) 58

SECTION - C

Q.1 Match the following

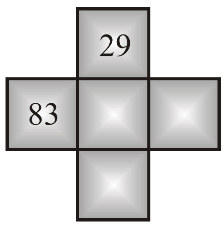
	Column I		Column II
(A)	If N ÷ 2 leaves a remainder 1, then one's digit of N is	(p)	1 or 6
(B)	If 36 D is multiple of 3 and D is a digit then value of D is	(q)	1, 3, 5, 7 or 9
(C)	If N ÷ 5 leaves a remainder 1, then one's digit of 'N' is	(r)	1
(D)	If D is a digit and the number 21D5 is divisible by 9. The value of D is	(s)	0, 3, 6 or 9

SECTION -D

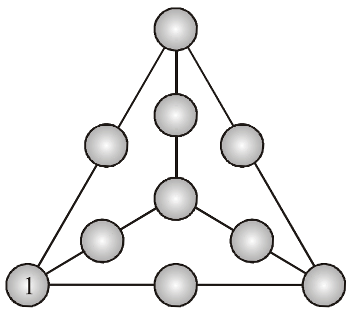
MAGICAL FIGURES

Q.1 Fill in the boxes with 2-digit prime numbers so that the sum of the numbers horizontally and vertically is 123.

(A) 23
(B) 17
(C) 71
(D) 61



Q.2 Place the digits 1 to 9 on the number star so that the three digits on each of the lines add up to :



(A) 12 (B) 13

ANSWER KEY

CONCEPT APPLICATION LEVEL - II

SECTION - A

Q.1	11	Q.2	3(a – b)	Q.3	27	Q.4	111	Q.5	6
Q.6	0, 2, 4, 6, 8	Q.7	104	Q.8	Yes	Q.9	8.39×10^{-10}	Q.10	2744

SECTION - B

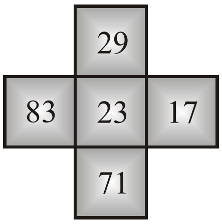
Q.1	B	Q.2	B	Q.3	A	Q.4	B	Q.5	A	Q.6	B	Q.7	A
Q.8	C	Q.9	A	Q.10	D	Q.11	C	Q.12	A	Q.13	A	Q.14	A
Q.15	B	Q.16	A	Q.17	C	Q.18	A	Q.19	A	Q.20	A	Q.21	D
Q.22	D	Q.23	D	Q.24	A	Q.25	C	Q.26	A	Q.27	A	Q.28	B
Q.29	A	Q.30	A	Q.31	B	Q.32	D	Q.33	B	Q.34	B	Q.35	A
Q.36	D	Q.37	B	Q.38	A	Q.39	B	Q.40	C	Q.41	B	Q.42	B
Q.43	C	Q.44	B	Q.45	B	Q.46	A	Q.47	A	Q.48	A	Q.49	B
Q.50	A												

SECTION - C

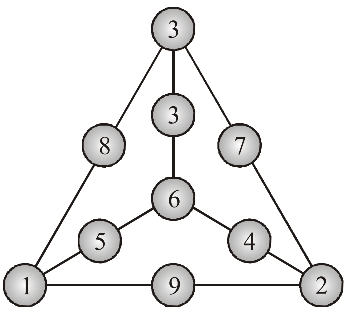
Q.1 (A)-(q); (B)-(s);(C)-(p); (D)-(r)

SECTION - D

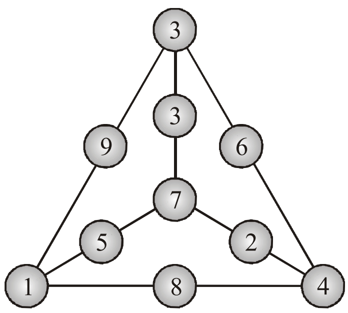
Q.1



Q.2 (A)



(B)



Hint. Make all possible groups of 3 integers with 1 so that the sum is 12, e.g. 1, 8, 3; 1, 9, 2 etc.