7 LINEAR EQUATIONS IN ONE VARIABLE

7.1 INTRODUCTION TO LINEAR EQUATION

This chapter is very important because, while solving the problems, in most cases, we need to frame an equation first. In this chapter, we learn how to frame and solve equation. Framing an equation is more difficult than solving an equation. First, we shall have to understand the meaning of certain terms which are associated with equation like number, symbols, knowns, unknowns, constant variables, expressions, sentences, statements etc.

7.2 COMMON TERMS

7.2.1 Numbers and symbols

In lower classes, we worked with numbers like 1, 2, 3, 1.2, – 2.3 as well as letters like a, b, c, or x, y, z, which can be used instead of number, These letters can be used for some known or unknown numbers. Accordingly, they are called knowns or unknowns. We'll also come across situations in which the letters represent some particular numbers or a whole set of numbers. Accordingly, we call them constants or variables.

7.2.2 Numerical expressions

Expressions of the from 3×5, $(2 + 6)\ 5 \div (-4)$, $3^2 + 4^{1/2}$, $\sqrt{2} + 5 \div 3$ are numerical expressions. Numerical expressions are made up of numbers, the basic arithmetical operations (+, –, ×, ÷), involution (raising to a power) and evolution (root extraction).

7.2.3 Algebraic Expressions

Expressions of the form $2x$, $(3x + 5)$, $(4x - 2y)$, $2x^2 + 3\sqrt{y}$, $3x^2/2\sqrt{y}$ are algebraic expressions. $3x$ and 5 are the terms of $(3x + 5)$, and $4x$ and $2y$ are the terms of $4x - 2y$. Algebraic expressions are made up of number, symbols and the basic arithmetical operations.

7.2.4 Equations

An open sentence containing the equality sign is an equation. In order words, an equation is a sentence in which there is an equality sign between two algebraic expressions.

For example, $2x + 5 = x + 3$, $3y - 4 = 20$, $5x + 6 = x + 1$ are equation. Here x and y are unknown quantities and 5, 3, 20, etc are known equations.

7.3 LINEAR EQUATION

An equation in which the highest index of the unknowns present is one is a linear equation.

$2(x + 5) = 18$, $3x - 2 = 5$

$x + y = 20$ and $3x - 2y = 5$ are same linear equations.

7.3.1 A linear equation in one variable

A linear equation which has only one unknown is called a linear equation in one variable.

$3x + 4 = 16$ and $2x - 5 = x + 3$ are examples of linear equation in one variable. The part of an equation which is to the left side of the equality sign is known as the left hand side, abbreviated as LHS. The part of an equation which is to the right side of the equality sign is known as the right hand side, abbreviated as RHS. The process of finding the value of an unknown in an equation is called the solution (s) or the root (s) of the equation.

Before we learn how to solve an equation, let us review the basic properties of equality.

7.3.2 Properties of equation

(A) REFLEXIVE PROPERTY

Every number is equal to itself.

Example : $5 = 5$, $2 = 2$ and so on.

(B) SYMMETRIC PROPERTY

For any two numbers, if the first number is equal to the second, then the second number is equal to the first.

If x and y are two numbers and $x = y$, then $y = x$

Example : $3 + 4 = 5 + 2$

$\Rightarrow$ $5 + 2 = 3 + 4$

(C) TRANSITIVE PROPERTY

If x, y and z are three number such that $x = y$ and $y = z$ then $x = z$.

Example : $9 + 3 = 12$, $12 = 3 \times 4$

$\therefore$ $9 + 3 = 3 \times 4$

(D) ADDITION PROPERTY

If equal numbers are added to both side of an equality, the equality remains the same.

If $x = y$, then $x + z = y + z$.

(E) SUBTRACTION PROPERTY

If equal number are subtacted from both side of an equality, the equality remains the same.

If $x = y$, then $x - z = y - z$.

(F) MULTIPLICATION PROPERTY

If both sides of an equality are multiplied by the same number, the equality remains the same.

If $x = y$, then $(x)(z) = (y)(z)$

(G) DIVISION PROPERTY

If both sides of an equality are divided by a non-zero number, the equality remains the same.

If $x = y$, then $x/z = y/z$, where $z \neq 0$.

If x, y and z are three numbers such that $x = y$ and $x = z$, then $y = z$.

Example : $24 = 8 \times 3$, $24 = 14 + 10$

$\Rightarrow$ $8 \times 3 = 14 + 10$

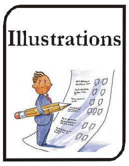

Illustration 1

Solve $\mathbf{\frac{2x-3}{4}+2=\frac{3x-2}{3}}$

Solution

Given, $\frac{2x-3}{4}+2=\frac{3x-2}{3}$

Multiplying both sides of the equation by the LCM of 4 and 3 i.e., 12.

$\Rightarrow$ $\left(\frac{2x-3}{4}\right)12+(2)12=\left(\frac{3x-2}{3}\right)12$

$\Rightarrow$ $(2x-3)\,3+24=(3x-2)\,4$

$\Rightarrow$ $6x-9+24=12x-8$ $\Rightarrow$ $6x-12x=-8+9-24$

$\Rightarrow$ $-6x=-23$ $\Rightarrow$ $x=\frac{23}{6}$

Illustration 2

The present ages of Tom and Jerry are in the ratio 3 : 2. One year from now, the sum of their ages would be 7 year. Find their present ages.

Solution

Let the present ages of Tom and Jerry be 3x year and 2x years respectively.

After one year, age of Tom (3x + 1) years.

After one year, age of Jerry = (2x + 1) years.

Given, (3x + 1) + (2x + 1) = 7

$\Rightarrow$ $5x+2=7$ $\Rightarrow$ $5x=7-2$

$\Rightarrow$ $5x=5$ $\Rightarrow$ $x=1$

Present age of Tom = 3x years = 3 years

present age of Jerry = 2 y years = 2 years

Illustration 3

The sum of the numerator and the denominator of a fraction is 8. If the numerator and the denominator each increased by 1, then the fraction is 2/3. Find the fraction.

Solution

Let the numerator be x

$\therefore$ The denominator 8 – x

Given, $\frac{x+1}{8-x+1}=\frac{2}{3}$

$\Rightarrow$ $3\,(x+1)=2\,(9-x)\Rightarrow 3x+3=18-2x\Rightarrow 5x=15\Rightarrow x=15/5\Rightarrow x=3$

$\therefore$ The numerator (x) = 3

The denominator (8 – x) = 8 – 3 = 5

$\therefore$ The Fraction = 3/5

POINTS TO REMEMBER

1. An equation in which the highest index of the unknowns present is one, is called a linear equation.
2. A linear equation which has only one unknown is called a simple equation or a linear equation in one variable.
3. The value of the unknown in a linear equation which satisfies the equation, is called the solution or the root of the equation.
4. If a = b, then ac = bc.
5. If a = b and c ≠ 0, then $\frac{a}{c} = \frac{b}{c}$
6. If a = b, then a + c = b + c.
7. If a = b, then a – c = b – c.
8. Changing a term from one side to another side is called transposition.

SOLVED EXAMPLE

Example 1 :

Solve the following equations :

(a) $3x + \frac{1}{2} = \frac{3}{8} + \frac{x}{4}$ **(b)** $2x + 3(x - 1) = \frac{7}{2}$

Solution :

(a) $3x + \frac{1}{2} = \frac{3}{8} + \frac{x}{4}$

Multiplying both sides by 8, we get ...(L.C.M. of 2, 8, 4)

$24x + 4 = 3 + 2x$

$\Rightarrow$ $24x - 2x = 3 - 4$ ... (Transposing)

or $22x = -1$ $\therefore$ $x = -\frac{1}{22}$

(b) $2x + 3(x - 1) = \frac{7}{2}$ $\Rightarrow$ $2x + 3(x - 1) = \frac{7}{2}$

$\Rightarrow$ $2x + 3x - 3 = \frac{7}{2}$ $\Rightarrow$ $2x + 3x = \frac{7}{2} + 3$ ... (Transposing 3)

$\Rightarrow$ $5x = \frac{7+6}{2}$ $\Rightarrow$ $\frac{5x}{5} = \frac{13}{2 \times 5}$ ... (Dividing by 5)

$\therefore$ $x = \frac{13}{10}$

Example 2 :

Solve : $\frac{2}{3}(4x - 1) - \left(4x - \frac{1-3x}{2}\right) = \frac{x-7}{2}$

Solution :

The given equation is $\frac{2}{3}(4x - 1) - 6\left(4x - \frac{1-3x}{2}\right) = \frac{x-7}{2}$

Multiplying both sides by 6, we get [6 is the L.C.M. of 3, 2]

$4(4x - 1) - 6\left(4x - \frac{1-3x}{2}\right) = 3(x - 7)$

or $16x - 4 - 24x + 3(1 - 3x) = 3x - 21$

or $16x - 4 - 24x + 3 - 9x = 3x - 21$

or $-17x - 1 = 3x - 21$

or $-17x - 3x = -21 + 1$ ... (Transposing)

or $-20x = -20$

or $\frac{-20x}{-20} = \frac{-20}{-20}$ ... [Dividing by (–20)]

$\therefore$ $x = 1$

Example 3 :

The length of rectangle exceeds its breadth by 4cm. If the length and breadth are each increased by 3cm, the area of the new rectangle will be 81 cm^2 more than that of the given rectangle. Find the length and breadth of the given rectangle.

Solution :

Let the breadth of the given rectangle be x cm.

Then, Length = (x + 4)cm

$\therefore$ Area = Length × Breadth = $(x + 4) \times x = x^2 + 4x$

When length and breadth are each increased by 3cm. Then,

New length = (x + 4 + 3)cm = (x + 7) cm and New breadth =(x + 3)cm

$\therefore$ Area of new rectangle = Length × Breadth

$= (x + 7)(x + 3) = x(x + 3) + 7(x + 3)$

$= x^2 + 3x + 7x + 21 = x^2 + 10x + 21$

It is given that the area of new rectangle is 81cm^2 more than the given rectangle.

$\therefore \quad x^2 + 10x + 21 = x^2 + 4x + 81 \quad \Rightarrow \quad x^2 + 10x - x^2 - 4x = 81 - 21$

$\Rightarrow \quad 6x = 60 \quad \Rightarrow \quad x = \frac{60}{6} = 10$

Thus, length of the given rectangle

= (x + 4)cm = (10 + 4)cm = 14cm

and breadth of the given rectangle = 10cm

Check : Area of the given rectangle $= (x^2 + 4x)cm^2 = (10^2 + 4 \times 10)cm^2 = 140cm^2$

Area of new rectangle $= (x^2 + 10x + 21)cm^2 = (10^2 + 10 \times 10 + 21)\ cm^2 = 221\ cm^2$

Clearly, area of the new rectangle is 81cm^2 more than that of the given rectangle, which is same as given in the problem. Hence, our answer is correct.

Example 4 :

The ages (in years) of Ramesh and Rahim are in the ratio 5 : 7. If Ramesh were 9 years older and Rahim 9 years younger, the age of Ramesh would have been twice the age of Rahim. Find their ages.

Solution :

Let the age (in years), of Ramesh and Rahim be 5x and 7x respectively.

Had Ramesh been 9 years older, then his age would have been (5x + 9) years.

Had Rahim been 9 years younger, then his age would have been (7x – 9) years.

According to the condition given in the problem, we have

$5x + 9 = 2(7x - 9) \quad \Rightarrow \quad 5x + 9 = 14x - 18$

$\Rightarrow \quad 5x - 14x = -18 - 9 \quad \Rightarrow \quad -9x = -27 \quad \Rightarrow \quad x = \frac{-27}{-9} = 3$

Thus, age of Ramesh = 5x years = (5 × 3) years = 15 years

and Rahim's age = 7x years = (7 × 3) years = 21 years

Ramesh's age = 15 years.

Rahims' age = 21 years

$\therefore$ Ratio of their ages = 15 : 21 = 5: 7,

which is the same as given in the problem. If Ramesh were 9 years older, then Ramesh's age(15 + 9) years = 24years. If Rahim were 9 years younger, then Rahim's age = 21 – 9 = 12 years.

Clearly, Ramesh's age i.e., 24 years is twice the Rahim's age, i.e., 12years. This is the same as given in the problem. Thus, our answer is correct.

Example 5 :

The distance between two stations A and B is230 km. Two motor cyclists start simutaneously from A to B in the opposite directions and the distance between them after three hours is 20 km. If the speed of one motor cyclist is less than that of the other by 10km/hr, find the speed of each motor cyclist.

Solution :

Let the speed of the faster motor cyclist be x km/hr and let he starts from A.

Then, the speed of the other motor cyclist = (x – 10) km/hr.

The other motor cyclist starts from B.

Let M and N be the positons of the two motor cyclists after three hours. Then,

AM = distance travelled by the faster motor cyclist in 3 hours with the speed of x km/hr =3x km

[Using : Distance = Speed × Time]

BN = distance travelled by the other motor cyclist in 3 hours with the speed of (x – 10)km.

= 3(x – 10)km.

It is given that the distance MN between the two motor cyclists after 3 hours is 20km.

Now, AB = 230 km.

$\Rightarrow$ AM + MN + NB = 230 $\Rightarrow$ $3x + 20 + 3(x - 10) = 230$

$\Rightarrow$ $3x + 20 + 3x - 30 = 230$ $\Rightarrow$ $6x - 10 = 230$

$\Rightarrow$ $6x = 240$ $\Rightarrow$ $x = \frac{240}{6} = 40$

Thus, speed of the faster motor cyclist = 40 km/hr

Speed of the second motor cyclist = (x – 10) km/hr = (40 – 10) km/hr = 30 km/hr

Check : The distance AM travelled by the faster motor cyclist in 3 hours = (40 × 3) km = 120 km.

The distance BN travelled by the second motor cyclist in 3 hours = (30 × 3) = 90 km.

$\therefore$ MN = Distance between the two motor cyclist after 3 hours.

AB = AM + MN + NB $\Rightarrow$ MN

= (AB – AM – BN)

= (230 – 120 – 90) km = 20 km,

which is the same as given in the problem.

Hence, our solution is correct.

Example 6 :

A steamer goes downstream and covers the distance between two parts in 4 hours while it covers the same distance upstream in 5 hours. If the speed of the stream is 2km/hr, find the speed of the steamer in still water.

Solution :

Let the speed of the steamer in still water be x km/hr.

We have, speed of the stream = 2km/hr

Speed downstream =(x + 2) km/hr

Speed upstream = (x – 2) km/hr

$\therefore$ Distance covered in 4 hours while going downstream = 4(x + 2)km

and distance covered in 5 hours while going upstream = 5(x – 2)km

According to the given condition,

$4(x + 2) = 5(x - 2) \quad \Rightarrow \quad 4x + 8 = 5x - 10$

$\Rightarrow \quad 4x - 5x = -10 - 8 \quad \Rightarrow \quad -x = -18$

$\Rightarrow \quad x = 18$

Hence, the speed of the steamer in the still water is 18km/hr.

Check : Speed of the steamer in still water = 18 km/hr

Speed downstream = (18 + 2) km/hr = 20 km/hr

Speed upstream = (18 – 2) km/hr = 16 km/hr

and Distance covered in 4 hours while going downstream = (20 × 4)km = 80 km

and Distance covered in 5 hours while going upstream = (16 × 5)km = 80 km

Clearly, both the distances are equal.

Hence, our answer is correct.

Example 7 :

Three prizes are to be distributed in a quiz contest. The vlaue of second prize is five-sixth the value of the first prize and the value of the third prize is four-fifths of the second prize. If the total value of three prizes is Rs. 150, find the value of each prize.

Solution :

Let the value of the first prize be Rs. x,

Then,

Value of the second prize = Rs. $\frac{5}{6}x$.

and Value of third prize = Four fifths the value of second prize = Rs. $\frac{4}{5} \times \left(\frac{5}{6}x\right)$ = Rs. $\frac{4}{6}x = \frac{2}{3}x$

$\therefore$ Total value of three prizes = Rs. $\left(x + \frac{5}{6}x + \frac{2}{3}x\right)$

Since, Total value of three prizes is given as Rs. 150.

$\therefore \quad x + \frac{5}{6}x + \frac{2}{3}x = 150 \quad \Rightarrow \quad 6x + 5x + 2 \times 2x = 6 \times 150$

$\Rightarrow \quad 15x = 900 \quad \Rightarrow \quad x = \frac{900}{15} = 60$

$\therefore$ Value of first prize = Rs. 60

Value of second prize = Rs. $\left(\frac{5}{6} \times 60\right)$ = Rs.50

and Value of third prize = Rs. $\left(\frac{2}{3} \times 60\right)$ = Rs. 40

Example 8 :

Divide Rs. 500 between Rita and Seema such that one-third share of Rita and half share of Sema are equal.

Solution :

Let the share of Rita = Rs. x;

$\therefore$ Share of Seema = Rs. (500 – x)

Now $\frac{1}{3}$ of Rita's share = $\frac{1}{3}$ of Rs x or Rs. $\frac{x}{3}$

$\frac{1}{2}$ of Seema's share = $\frac{1}{2}$ × (500 – x)

$\Rightarrow$ Rs. $\frac{500-x}{2}$

$\therefore$ By the given condition $\frac{x}{3} = \frac{500-x}{2}$

Multiplying both sides by 6, we get

$$6 \times \frac{x}{3} = 6 \times \frac{500-x}{2}$$

$\Rightarrow$ 2x = 3(500 – x) $\Rightarrow$ 2x = 1500 –3x

$\Rightarrow$ 2x + 3x = 1500 ... (Transposing 3x to the LHS)

$\Rightarrow$ 5x = 1500 $\Rightarrow$ $\frac{5x}{5} = \frac{1500}{5}$ ...(Dividing by 5)

$\Rightarrow$ x = 300

$\therefore$ Rita's share = Rs. 300

Seema's share = Rs. (500 – 300) = Rs. 200

Example 9 :

70 coins of 10-paise and 50-paise are mixed in a purse. If the total vlaue of the money in the purse is Rs.19, find the number of each type of coins.

Solution :

Suppose the number of 10-paise coins = x

$\therefore$ The number of 50-paise coins = 70 – x

Value of 10-paise coins = Rs. $\frac{x}{10}$

Value of 50-paise coins = Rs. $\frac{70-x}{2}$

Total value of coins = $\frac{x}{10} + \frac{70-x}{2}$

$\therefore$ $\frac{x}{10} + \frac{70-x}{2} = 19$

Multiplying both sides by 10, we get

$x + 5(70 - x) = 19 \times 10$ ... (LCM of 10, 2 = 10)

or $x + 350 - 5x = 190$

or $-4x = 190 - 350$

or $-4x = -160$

or $\dfrac{-4x}{-4} = \dfrac{-160}{-4}$

$\therefore$ $x = 40$

$\therefore$ Number of 10-paise coins = 40

Number of 50-paise coins = 70 – 40 = 30

Example 10 :

The sum of 3 consecutive integers is 30. Find the consecutive integers.

Solution :

Let us take 3 consecutive integers as x, x + 1 and x + 2.

Their sum is 30.

$\therefore x + (x + 1) + (x + 2) = 30 \quad \Rightarrow \quad x + x + 1 + x + 2 = 30$

$\Rightarrow \quad 3x + 3 = 30 \quad \Rightarrow \quad 3x = 30 - 3$ (transposing 3 to RHS)

$\Rightarrow \quad 3x = 27 \quad \Rightarrow \quad x = \dfrac{27}{3}$ (dividing both sides by 3)

$\therefore$ $x = 9$

Thus, the numbers are x, x + 1, x + 2, i.e., 9, 10, 11.

Hence, the numbers are 9, 10 and 11.

CONCEPT APPLICATION LEVEL - I [NCERT Questions]

EXERCISE - 1

Q.1 Solve the following equations :

(1) $x - 2 = 7$ (2) $y + 3 = 10$ (3) $6 = z + 2$ (4) $\frac{3}{7} + x = \frac{17}{7}$

(5) $6x = 12$ (6) $\frac{t}{5} = 10$ (7) $\frac{2x}{3} = 18$ (8) $1.6 = \frac{y}{1.5}$

(9) $7x - 9 = 16$ (10) $14y - 8 = 13$ (11) $17 + 6p = 9$ (12) $\frac{x}{3} + 1 = \frac{7}{15}$

Sol. (1) $x - 2 = 7$

We have $x - 2 = 7$

$\Rightarrow x = 7 + 2$ (Transposing -2 to R.H.S.)

$\Rightarrow x = 9$

This is the required solution.

Check : L.H.S. $= x - 2 = 9 - 2 = 7 =$ R.H.S. (as required)

(2) $y + 3 = 10$

We have $y + 3 = 10$

$\Rightarrow y = 10 - 3$ (Transposing 3 to R.H.S.)

$\Rightarrow y = 7$

This is the required solution.

Check : L.H.S. $= y + 3 = 7 + 3 = 10 =$ R.H.S. (as required)

(3) $6 = z + 2$

We have $6 = z + 2$

$\Rightarrow 6 - 2 = z$ (Transposing 2 to L.H.S.)

$\Rightarrow 4 = z$

This is the required solution.

Check : R.H.S. $= z + 2 = 4 + 2 = 6 =$ L.H.S. (as required)

(4) $\frac{3}{7} + x = \frac{17}{7}$

We have $\frac{3}{7} + x = \frac{17}{7}$

$\Rightarrow x = \frac{17}{7} - \frac{3}{7}$ (Transposing $\frac{3}{7}$ to R.H.S.)

$\Rightarrow x = \frac{17-3}{7}$ $\Rightarrow$ $x = \frac{14}{7} = 2$

This is the required solution.

Check : L.H.S. $= \frac{3}{7} + x = \frac{3}{7} + 2 = \frac{17}{7} =$ R.H.S. (As required)

(5) **$6x = 12$**

We have $6x = 12$

$\Rightarrow$ $x = \frac{12}{6} = 2$

This is the required solution.

Check : L.H.S. $= 6x = 6 \times 2 = 12 =$ R.H.S. (as required)

(6) $\mathbf{\frac{t}{5} = 10}$

We have $\frac{t}{5} = 10$

$\Rightarrow$ $t = 10 \times 5 = 50$ (Multiplying both sides by 5)

This is the required solution.

Check : L.H.S. $= \frac{t}{5} = \frac{50}{5} = 10 =$ R.H.S. (as required)

(7) $\mathbf{\frac{2x}{3} = 18}$

We have $\frac{2x}{3} = 18$

$\Rightarrow$ $2x = 18 \times 3$ (Multiplying both sides by 3)

$\Rightarrow$ $2x = 54$

$\Rightarrow$ $x = \frac{54}{2} = 27$

This is the required solution.

Check : L.H.S. $= \frac{2x}{3} = \frac{2 \times 27}{3} = 18 =$ R.H.S. (as required)

(8) $\mathbf{1.6 = \frac{y}{1.5}}$

We have $1.6 = \frac{y}{1.5}$

$\Rightarrow$ $1.6 \times 1.5 = y$ (Multiplying both sides by 1.5)

$\Rightarrow$ $2.4 = y$

$\Rightarrow$ $y = 2.4$

This is the required solution.

Check : R.H.S. $= \frac{y}{1.5} = \frac{2.4}{1.5} = 1.6 =$ R.H.S. (as required)

(9) 7x – 9 = 16

We have $7x - 9 = 16$

$\Rightarrow$ $7x = 16 + 9$ (Transposing – 9 to R.H.S.)

$\Rightarrow$ $7x = 25$

$\Rightarrow$ $x = \frac{25}{7}$ (Dividing both sides by 7)

This is the required solution.

Check : L.H.S. $= 7x - 9 = 7 \times \frac{25}{7} - 9$

$= 25 - 9 = 16 =$ R.H.S. (as required)

(10) 14y – 8 = 13

We have $14y - 8 = 13$

$\Rightarrow$ $17y = 13 + 8$ (Transposing – 8 to R.H.S.)

$\Rightarrow$ $14y = 21$

$\Rightarrow$ $y = \frac{21}{14}$ (Dividing both sides by 14)

$\Rightarrow$ $y = \frac{21 \div 7}{14 \div 7} = \frac{3}{2}$

This is the required solution.

Check : L.H.S. $= 14y - 8 = 14\left(\frac{3}{2}\right) - 8$

$= 21 - 8 = 13 =$ R.H.S. (as required)

(11) 17 + 6p = 9

We have $17 + 6p = 9$

$\Rightarrow$ $6p = 9 - 17$ (Transposing 17 to R.H.S.)

$\Rightarrow$ $6p = 8$

$\Rightarrow$ $p = \frac{-8}{6}$ (Dividing both sides by 6)

$\Rightarrow$ $p = \frac{-8 \div 2}{6 \div 2}$

$\Rightarrow$ $p = \frac{-4}{3}$

This is the required solution.

Check : L.H.S. $= 17 + 6p$

$= 17 + 6\left(\frac{-4}{3}\right)$

$= 17 - 8 = 9 =$ R.H.S. (as required)

(12) $\frac{x}{3}+1=\frac{7}{15}$

We have $\frac{x}{3}+1=\frac{7}{15}$

$\Rightarrow \quad \frac{x}{3}=\frac{7}{15}-1$ (Transposing 1 to R.H.S.)

$\Rightarrow \quad \frac{x}{3}=\frac{7-15}{15} \quad \Rightarrow \quad \frac{x}{3}=\frac{-8}{15}$

$\Rightarrow \quad x=\left(-\frac{8}{15}\right)\times 3$ (Multiplying both sides by 3)

$\Rightarrow \quad x=\frac{-8}{5}$

This is the required solution.

Check : L.H.S. $=\frac{x}{3}+1=\frac{-8}{15}+1$

$=-\frac{8+15}{15}=\frac{7}{15}=$ R.H.S. (as required)

EXERCISE - 2

Q.1 If you subtract $\frac{1}{2}$ from a number and multiply the result by $\frac{1}{2}$, you get $\frac{1}{8}$. What is the number?

Sol. Let the number be x.
According to the question,

$\frac{1}{2}\left(x-\frac{1}{2}\right)=\frac{1}{8}$

$\Rightarrow \quad \frac{1}{2}\left(\frac{2x-1}{2}\right)=\frac{1}{8}$

$\Rightarrow \quad \frac{2x-1}{4}=\frac{1}{8} \quad \Rightarrow \quad \frac{2x-1}{4}\times 8=\frac{1}{8}\times 8$ (Multiplying both sides by 8)

$\Rightarrow \quad (2x-1)2=1$

$\Rightarrow \quad 4x-2=1 \quad \Rightarrow \quad 4x=1+2$ (Transposing – 2 to R.H.S.)

$\Rightarrow \quad 4x=3 \quad \Rightarrow \quad x=\frac{3}{4}$ (Dividing both sides by 4)

Hence, the desired number is $\frac{3}{4}$.

Check : $\frac{1}{2}\left(\frac{3}{4}-\frac{1}{2}\right)=\frac{1}{2}\left(\frac{3-2}{4}\right)=\frac{1}{2}\left(\frac{1}{4}\right)=\frac{1}{8}$ (as desired)

Q.2 The perimeter of a rectangular swimming pool is 154 m. Its length is 2m more than twice its breadth. What are the length and the breadth of the pool?

Sol. Let the breadth of the pool be x m.

$\because$ Its length is 2 m more than twice its breadth.

$\therefore$ Length of the tank = (2x + 2) = m

$\therefore$ Perimeter of the tank = 2 × (Length + Breadth)

= 2 × (2x + 2 + x) = 2 × (3x + 2) = (6x + 4) m

$\because$ The perimeter of a rectangular swimming pool is 154 m.

$\therefore$ $6x + 4 = 154$

$\Rightarrow$ $6x = 154 - 4$ (Transposing 4 to R.H.S.)

$\Rightarrow$ $6x = 150$

$\Rightarrow$ $x = \frac{150}{6} = 25$ (Dividing both sides by 6)

$\Rightarrow$ $2x + 2 = 2 \times 25 + 2 = 50 + 2 = 52$

Hence, the length and breadth of the pool are 52 m and 25 m respectively.

Check : $52 = 2 \times 25 + 2$ (as desired)

$2 \times (52 + 25) = 2 \times 77 = 154$

Q.3 The base of an isosceles triangle is $\frac{4}{3}$ cm .The perimeter of the triangle is $4\frac{2}{15}$ cm. What is the length of either of the remaining equal sides?

Sol. Let the length of either of the remaining equal sides be x.

Base = $\frac{4}{3}$ cm

$\therefore$ Perimeter of the triangle = $\left(x + x + \frac{4}{3}\right)$cm = $\left(2x + \frac{4}{3}\right)$cm

According to the question,

$$2x + \frac{4}{3} = 4\frac{2}{15}$$

$\Rightarrow$ $2x + \frac{4}{3} = \frac{62}{15}$ $\Rightarrow$ $2x = \frac{62}{15} - \frac{4}{3}$ (Transposing $\frac{4}{3}$ to R.H.S.)

$\Rightarrow$ $2x = \frac{62 - 20}{15}$

$\Rightarrow$ $2x = \frac{42}{15}$ $\Rightarrow$ $x = \frac{42}{15 \times 2}$ (Dividing both sides by 2)

$\Rightarrow$ $x = \frac{21}{15}$ $\Rightarrow$ $x = \frac{21 \div 3}{15 \div 3}$ (Dividing the numerator and denominator by 3)

$\Rightarrow$ $x = \frac{7}{5} = 1\frac{2}{5}$

Hence, the length of either of the remaining equal sides is $1\frac{2}{5}$ cm.

Check : Perimeter $= \left(1\frac{2}{5}+1\frac{2}{5}+\frac{4}{3}\right)$ cm $= \left(\frac{7}{5}+\frac{7}{5}+\frac{4}{3}\right)$ cm $= \frac{21+21+20}{15}$ cm

$= \frac{62}{15}$ cm $= 4\frac{2}{15}$ cm (as desired)

Q.4 Sum of two numbers is 95. If one exceeds the other by 15, find the numbers.

Sol. Let the smaller number be x.

Then, the larger number = x + 15

$\because$ Sum of two numbers is 95

$\therefore$ $x + (x + 15) = 95$ $\Rightarrow$ $2x + 15 = 95$

$\Rightarrow$ $2x = 95 - 15$ (Transposing 15 to RHS)

$\Rightarrow$ $2x = 80$ $\Rightarrow$ $x = \frac{80}{2} = 40$ (Dividing both sides by 2)

$\Rightarrow$ $x + 15 = 40 + 15 = 55$

Hence, the desired numbers are 40 and 55.

Check : $55 = 40 + 15$ (as desired)

$40 + 55 = 95$

Q.5 Two numbers are in the ratio 5 : 3. If they differ by 18, what are the number?

Sol. Let the two numbers be 5x and 3x.

$\because$ They differ by 18

$\therefore$ $5x - 3x = 18$ $\Rightarrow$ $2x = 18$

$\Rightarrow$ $x = \frac{18}{2} = 9$ (Dividing both sides by 2)

$\Rightarrow$ $5x = 5 \times 9 = 45$ and $3x = 3 \times 9 = 27$

Hence, the desired numbers are 45 and 27.

Check : $45 : 27 = \frac{45}{27} = \frac{45 \div 9}{27 \div 9} = \frac{45}{27} = \frac{45 \div 9}{27 \div 9} = \frac{5}{3} = 5 : 3$ (as desired)

$45 - 27 = 18$

Q.6 Three consecutive integers add upto 51. What are these integers?

Sol. Let the three consecutive integers be x, + 1 and x + 2.

$\because$ They add upto 51.

$\therefore$ $x + (x + 1) + (x + 2) = 51$

$\Rightarrow$ $3x + 3 = 51$ $\Rightarrow$ $3x = 51 - 3$ (Transposing 3 to RHS)

$\Rightarrow$ $3x = 48$ $\Rightarrow$ $x = \frac{48}{3} = 16$ (Dividing both sides by 3)

$\Rightarrow$ $x + 1 = 16 + 1 = 17$ and $x + 2 = 16 + 2 = 18$

Hence, the desired integers are 16, 17 and 18.

Check : $17 = 16 + 1$

$18 = 17 + 1$ (as desired)

$16 + 17 + 18 = 51$

Q.7 The sum of three consecutive multiples of 8 is 888. Find multiples.

Sol. Let the three consecutive multiples of 8 be 8x, 8(x + 1) and 8(x + 2).

$\because$ Their sum is 888.

$\therefore$ $8x + 8(x + 1) + 8(x + 2) = 888$

$\Rightarrow$ $8\{x + (x + 1) + (x + 2)\} = 888$

$\Rightarrow$ $8(3x + 3) = 888$

$\Rightarrow$ $3x + 3 = \frac{888}{8}$ (Dividing both sides by 8)

$\Rightarrow$ $3x + 3 = 111$

$\Rightarrow$ $3(x + 1) = 111$ $\Rightarrow$ $x + 1 = \frac{111}{3}$ (Dividing both sides by 3)

$\Rightarrow$ $x + 1 = 37$ $\Rightarrow$ $x = 37 - 1$ (Transposing 1 to RHS)

$\Rightarrow$ $x = 36$ $\Rightarrow$ $8x = 8 \times 36 = 288$

and $8(x + 2) = 8(36 + 2) = 8 \times 38 = 304$

Hence, the desired multiples are 288, 296 and 304.

Check : $288 = 8 \times 36$

$296 = 8 \times 37 = 8 \times (36 + 1) = 8 \times 36 + 8 = 288 + 8$

$304 = 8 \times 38 = 8 \times (37 + 1) = 8 \times 37 + 8 = 296 + 8$

$288 + 296 + 304 = 888$ (as desired)

Q.8 Three consecutive integers are such that when they are taken in increasing order and multiplied by 2, 3 and 4 respectively, they add upto 74. Find these numbers.

Sol. Let the three consecutive integers be x, x + 1 and x + 2.

$\because$ When taken in increasing order and multiplied by 2, 3 and 4 respectively, then add upto 74.

$\therefore$ $2x + 3(x + 1) + 4(x + 2) = 74$

$\Rightarrow$ $2x + 3x + 3 + 4x + 8 = 74$

$\Rightarrow$ $9x + 11 = 74$ $\Rightarrow$ $9x = 74 - 11$ (Transposing 11 to RHS)

$\Rightarrow$ $9x = 63$ $\Rightarrow$ $x = \frac{63}{9} = 7$ (Dividing both sides by 9)

$\Rightarrow$ $x + 1 = 7 + 1 = 8$ and $x + 2 = 7 + 2 = 9$

Hence, the desired numbers are 7, 8 and 9.

Check : $8 = 7 + 1$

$9 = 7 + 2$ (as desired)

$2 \times 7 + 3 \times 8 + 4 \times 9 = 14 + 24 + 36 = 74$

Q.9 The ages of Rahul and Haroon are in the ratio 5 : 7. Four years later the sum of their ages will be 56 years. What are their present ages?

Sol. Let the present ages of Rahul and Haroon be 5x years and 7x years respectively.

Four years later

Age of Rahul = $(5x + 4)$ years

Age of Haroon = $(7x + 4)$ years

$\because$ Four years later the sum of their ages will be 56 years.

$\therefore$ $(5x + 4) + (7x + 4) = 56$

$\Rightarrow$ $12x + 8 = 56$ $\Rightarrow$ $12x = 56 - 8$ (Transposing 8 to RHS)

$\Rightarrow$ $12x = 48$ $\Rightarrow$ $x = \frac{48}{12} = 4$ (Dividing both sides by 12)

$\Rightarrow$ $5x = 5 \times 4 = 20$ and $7x = 7 \times 4 = 28$

Hence, their present ages are 20 years and 28 years.

Check : $20 : 28 = \frac{20 \div 4}{28 \div 4} = \frac{5}{7} = 5 : 7$ (as desired)

$20 + 4 = 24$

$28 + 4 = 32$

$24 + 32 = 56$

Q.10 The number of boys and girls in a class are in the ratio 7 : 5. The number of boys is 8 more than the number of girls. What is the total class strength?

Sol. Let the number of boys and girls in a class be 7x and 5x respectively.

$\because$ The number of boys in 8 more than the number of girls.

$\therefore$ $7x - 5x = 8$ (Transposing 5x to RHS)

$\Rightarrow$ $2x = 8$ $\Rightarrow$ $x = \frac{8}{2} = 4$ (Dividing both sides by 2)

$\Rightarrow$ $7x = 7 \times 4 = 28$ and $5x = 5 \times 4 = 20$

$\therefore$ Total class strength = Number of boys + Number of girls

$= 28 + 20 = 48$

Hence, the total class strength in 48.

Check : $28 : 20 = \frac{28}{20} = \frac{28 \div 4}{20 \div 4} = \frac{7}{5} = 7 : 5$ (as desired)

$28 = 20 + 8$

Total class strength = $28 + 20 = 48$

Q.11 Baichung's father is 26 years younger than Baichung's grandfather and 29 years older than Baichung. The sum of the ages of all the three is 135 years. What is the age of each one of them?

Sol. Let the age of Baichung be x years.

Then, the age of Baichung's father = (x + 29) years

and the age of Baichung's grandfather = (x + 29 + 26) years

= (x + 55) years

$\because$ The sum of the ages of all three is 135 years.

$\therefore$ $x + (x + 29) + (x + 55) = 135$

$\Rightarrow$ $3x + 84 = 135$

$\Rightarrow$ $3x = 135 - 84$ (Transposing 84 to RHS)

$\Rightarrow$ $3x = 51$

$\Rightarrow$ $x = \frac{51}{3} = 17$ (Dividing both sides by 3)

$\Rightarrow$ $x + 29 = 17 + 29 = 46$

and $x + 55 = 17 + 55 = 72$

Hence, the ages of Baichung, Baichung's father and Baichung's grandfather are 17 years, 46 years and 72 years respectively.

Check : $46 = 17 + 29$

$72 = 46 + 26$ (as desired)

$17 + 46 + 72 = 135$

Q.12 Fifteen years from now Ravi's age will be four times his present age. What is Ravi's present age?

Sol. Let the present age of Ravi be x years.

Then, age of Ravi fifteen years from now = (x + 15) years

$\because$ Fifteen years from now Ravi's age will be four times his present age.

$\therefore$ $x + 15 = 4x$

$\Rightarrow$ $15 = 4x - x$ (Transposing x to RHS)

$\Rightarrow$ $15 = 3x$

$\Rightarrow$ $3x = 15$

$\Rightarrow$ $x = \frac{15}{3} = 5$ (Dividing both sides by 3)

Hence, the present age of Ravi is 5 years.

Check : $5 + 15 = 20$ (as desired)

$20 = 4 \times 5$

Q.13 A rational number is such that when you multiply it by $\frac{5}{2}$ and add $\frac{2}{3}$ to the product, you get $-\frac{7}{12}$. What is the number?

Sol. Let the number be x.

According to the question,

$\frac{5}{2}x + \frac{2}{3} = -\frac{7}{12}$ $\Rightarrow$ $\frac{5}{2}x = -\frac{2}{3} - \frac{7}{12}$ (Transposing $\frac{2}{3}$ to RHS)

$\Rightarrow$ $\frac{5}{2}x = \frac{-8-7}{12}$

$\Rightarrow$ $\frac{5}{2}x = -\frac{15}{12}$ $\Rightarrow$ $x = -\frac{15}{12} \times \frac{2}{5}$ (Multiplying both sides by $\frac{2}{3}$)

$\Rightarrow$ $x = -\frac{1}{2}$

Hence, the desired rational number is $-\frac{1}{2}$.

Check: $\left(-\frac{1}{2}\right) \times \frac{5}{2} + \frac{2}{3} = -\frac{5}{4} + \frac{2}{3} = \frac{-15+8}{12} = -\frac{7}{12}$ (as desired)

Q.14 Lakshmi is a cashier in a bank she has currency notes of denominations ₹100, ₹50 and ₹10, respectively. The ratio of the number of these notes is 2 : 3 : 5. The total cash with Lakshmi is ₹4,00,000. How many notes of each denomination does she have?

Sol. Let the number of currency notes of denominations ₹100, ₹50 and ₹10 be 2x, 3x and 5x respectively.

The amount she has

(i) from 100 rupee notes = 2x × 100 = 200x

(ii) from 50 rupee notes = 3x × 50 = 150x

(iii) from 10 rupee notes = 5x × 10 = 50x

Hence, the total money she has = 200x + 150x + 50x = 400x

But the total cash with her is ₹4,00,000.

Therefore,

$400x = 4,00,000$

$\Rightarrow$ $x = \frac{4,00,000}{400}$ (Dividing both sides by 400)

$\Rightarrow$ $2x = 2 \times 1000 = 2000$

$\Rightarrow$ $3x = 3 \times 1000 = 3000$

and $5x = 5 \times 1000 = 5000$

Hence, she has 2000, 3000 and 5000 notes of denominations ₹100, ₹50 and ₹10 respectively.

Check : 2000 : 3000 : 5000 = 2 : 3 : 5

2000 × 100 = 2,00,000

3000 × 50 = 1,50,000 (as desired)

5000 × 10 = 50,000

2,00,000 + 1,50,000 + 50,000 = 4,00,000

Q.15 I have a total of ₹300 in coins of denomination ₹1, ₹2 and ₹5. The number of ₹2 coins is 3 times the number of ₹5 coins. The total number of coins is 160. How many coins of each denomination are with me?

Sol. Let the number of ₹5 coins be x.

Then, the number of ₹2 coins = 3x

$\because$ The total number of coins is 160

$\therefore$ The number of coins of ₹1= 160 – (x + 3x) = 160 – 4x

The amount I have

from ₹5 coins = 5 × x = 5x

from ₹2 coins = 2 × 3x = 6x

from ₹1 coin = (160 – 4x) × 1 = 160 – 4x

$\because$ I have a total of ₹300 in coins of denomination ₹1, ₹2 and ₹5.

$\therefore$ 160 – 4x + 5x + 6x = 300

$\Rightarrow$ 160 + 7x = 300

$\Rightarrow$ 7x = 300 – 160 (Transposing 160 to RHS)

$\Rightarrow$ 7x = 140

$\Rightarrow$ $x = \frac{140}{7} = 20$ (Dividing both sides by 7)

$\Rightarrow$ 3x = 20 × 3 = 60

and 16x – 4x = 160 – 4 × 20 = 160 – 80

Hence I have 80, 60 and 20 coins of denomination ₹1, 2 and ₹5 respectively.

Check : 60 = 20 × 3

80 + 60 + 20 = 160 (as desired)

80 × 1 + 60 × 2 + 20 × 5 = 80 + 120 + 100 = 300

Q.16 The organisers of an essay competition decided that a winner in the competition gets a prize of ₹100 and a participant who does not win gets a prize of ₹25. The total prize money distributed is ₹3,000. Find the number of winners, if the total number of participants is 63.

Sol. Let the number of winners be x.

$\because$ The total number of participants is 63.

$\therefore$ The number of non-winners = 63 – x

Prize money got by winners

= ₹x × 100 = ₹100x

Prize money got by non-winners

= ₹(63 – x) × 25

= ₹(1575 – 25x)

$\because$ The total prize money distributed is ₹3000.

$\therefore$ 100x + (1575 – 25x) = 3000

$\Rightarrow$ 75x + 1575 = 3000 $\Rightarrow$ 75x = 3000 – 1575 (Transposing 1575 to RHS)

$\Rightarrow$ 75x = 1425 $\Rightarrow$ $x = \frac{1425}{75} = 19$ (Dividing both sides by 75)

Hence, the number of winners is 19.

Check : 19 + (63 + 19) = 63

19 × 100 + (63 – 19) × 25

= 1900 + 44 × 25 (as desired)

= 1900 + 1100 = 3000

EXERCISE - 3

Solve the following equations and check your results :

1. $3x = 2x + 18$ **2.** $5t - 3 = 3t - 5$ **3.** $5x + 9 = 5 + 3x$

4. $4z + 3 = 6 + 2z$ **5.** $2x - 1 = 14 - x$ **6.** $8x + 4 = 3(x - 1) + 7$

7. $x = \frac{4}{5}(x + 10)$ **8.** $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$ **9.** $2y + \frac{5}{3} = \frac{26}{3} - y$

10. $3m = 5m - \frac{8}{5}$

Sol. 1. $3x = 2x + 18$

We have $3x = 2x + 18$ (Transposing 2x to LHS)

$3x - 2x = 18$

$\Rightarrow x = 18$

This is the requried solution.

Check : L.H.S. $= 3x = 3 \times 18 = 54$

R.H.S. $= 2x + 18 = 2 \times 18 + 18 = 36 + 18 = 54$

Therefore, L.H.S. = R.H.S. (as desired)

2. $5t - 3 = 3t - 5$

We have $5t - 3 = 3t - 5$ (Transposing 3t to LHS and –3 to RHS)

$5t - 3t = -5 + 3$

$\Rightarrow 2t = -2$

$\Rightarrow t = -\frac{2}{2} = -1$ (Dividing both sides by 2)

This is the requried solution.

Check : L.H.S. $= 5(-1) - 3$

$= -5 - 3 = -8$

R.H.S. $= 3t - 5 = 3(-1) - 5$

$= -3 - 5 = -8$

Therefore, L.H.S. = R.H.S. (As desired)

3. $5x + 9 = 5 + 3x$

We have $5x + 9 = 5 + 3x$ (Transposing 3x to LHS and 9 to RHS)

$5x - 3x = 5 - 9$

$\Rightarrow 2x = -4$

$\Rightarrow x = -\frac{4}{2} = -2$ (Dividing both sides by 2)

This is the required solution.

Check : L.H.S. $= 5(-2) + 9 = -10 + 9 = -1$

R.H.S. $= 5 + 3(-2) = 5 - 6 = -1$

Therefore, L.H.S. = R.H.S. (as desired)

4. $\mathbf{4z + 3 = 6 + 2z}$

We have $4z + 3 = 6 + 2z$ (Transposing 2z to LHS and 3 to RHS)

$4z - 2z = 6 - 3$

$\Rightarrow$ $2z = 3$

$\Rightarrow$ $z = \frac{3}{2}$ (Dividing both sides by 2)

This is the required solution.

Check : L.H.S. $= 4\left(\frac{3}{2}\right) + 3 = 6 + 3 = 9$

R.H.S. $= 6 + 2z = 6 + 2\left(\frac{3}{2}\right) = 6 + 3 = 9$

Therefore, L.H.S. = R.H.S. (as desired)

5. $\mathbf{2x - 1 = 14 - x}$

We have $2x - 1 = 14 - x$

$\Rightarrow$ $2x + x = 14 + 1$ (Transposing $-x$ to LHS and -1 to RHS)

$\Rightarrow$ $3x = 15$

$\Rightarrow$ $x = \frac{15}{3} = 5$ (Dividing both sides by 3)

This is the required solution.

Check : L.H.S.$= 2x - 1 = 2(5) - 1 = 10 - 1 = 9$

R.H.S. $= 14 - x = 14 - 5 = 9$

Therefore, L.H.S. = R.H.S. (as desired)

6. $\mathbf{8x + 4 = 3(x - 1) + 7}$

We have $8x + 4 = 3(x - 1) + 7$

$\Rightarrow$ $8x + 4 = 3x - 3 + 7$

$\Rightarrow$ $8x + 4 = 3x + 4$

$\Rightarrow$ $8x - 3x = 4 - 4$ (Transposing 3x to LHS and 4 to RHS)

$\Rightarrow$ $5x = 0$

$\Rightarrow$ $x = \frac{0}{5} = 0$ (Dividing both sides by 5)

This is the required solution.

Check : L.H.S. $= 8x + 4 = 8(0) + 4 = 4$

R.H.S. $= 3(x - 1) + 7 = 3(0 + 1) + 7 = 3(-1) + 7 = -3 + 4 = 4$

Therefore, L.H.S. = R.H.S. (as desired)

7. $\mathbf{x = \frac{4}{5}(x + 10)}$

We have $x = \frac{4}{5}(x+10)$

$\Rightarrow$ $5x = 4(x + 10)$ (Multiplying both sides by 5)

$\Rightarrow$ $5x - 4x = 40$ (Transposing 4x to L.H.S.)

$\Rightarrow$ $x = 40$

This is the required solution.

Check : L.H.S. = 40

R.H.S. = $\frac{4}{5}(x+10) = \frac{4}{5}(40+10) = \frac{4}{5}(50) = 4(10) = 40$

Therefore, L.H.S. = R.H.S. (as desired)

8. $\mathbf{\frac{2x}{3} + 1 = \frac{7x}{15} + 3}$

We have $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$

$\Rightarrow$ $\frac{2x}{3} - \frac{7x}{15} = 3 - 1$ (Transposing $\frac{7x}{15}$ to L.H.S. and 1 to R.H.S.)

$\Rightarrow$ $\frac{2x}{3} - \frac{7x}{15} = 2$

$\Rightarrow$ $15\left(\frac{2x}{3} - \frac{7x}{15}\right) = 2 \times 15$ (Multiplying both sides by 15)

$\Rightarrow$ $10x - 7x = 30$

$\Rightarrow$ $3x = 30$

$\Rightarrow$ $x = \frac{30}{3} = 10$ (Dividing both sides by 3)

This is the required solution

Check : L.H.S. = $\frac{2x}{3} + 1 = \frac{2}{3}(10) + 1 = \frac{20}{3} + 1 = \frac{20+3}{3} = \frac{23}{3}$

R.H.S. = $\frac{7x}{15} + 3 = \frac{7}{15}(10) + 3 = \frac{70}{15} + 3 = \frac{70 \div 5}{15 \div 5} + 3 = \frac{14}{3} + 3 = \frac{14+9}{3} = \frac{23}{3}$

Therefore L.H.S. = R.H.S. (as desired)

9. $\mathbf{2y + \frac{5}{3} = \frac{26}{3} - y}$

We have $2y + \frac{5}{3} = \frac{26}{3} - y$ (Transposing –y to LHS and $\frac{5}{3}$ to RHS)

$2y + y = \frac{26}{3} - \frac{5}{3}$

$\Rightarrow \quad 3y = \frac{26-5}{3} \quad \Rightarrow \quad 3y = 7$

$\Rightarrow \quad y = \frac{7}{3}$ (Dividing both sides by 3)

This is the required solution

Check : L.H.S. = $2\left(\frac{7}{3}\right) + \frac{5}{3} = \frac{14}{3} + \frac{5}{3}$

R.H.S. = $\frac{26}{3} - \frac{7}{3} = \frac{26-7}{3} = \frac{19}{3}$

Therefore, LHS = RHS (as desired)

10. $\mathbf{3m = 5m - \frac{8}{5}}$

We have $3m = 5m - \frac{8}{5}$

$\Rightarrow \quad 3m - 5m = -\frac{8}{5}$ (Transposing 5m to LHS)

$\Rightarrow \quad -2m = -\frac{8}{5}$

$\Rightarrow \quad m = \frac{-8}{5 \times (-2)} = \frac{4}{5}$ (Dividing both sides by –2)

This is the required solution.

Check : LHS = $3 \times \frac{4}{5} = \frac{12}{5}$

RHS = $5\left(\frac{4}{5}\right) - \frac{8}{5} = 4 - \frac{8}{5} = \frac{20-8}{5} = \frac{12}{5}$

Therefore, LHS = RHS (as desired)

EXERCISE - 4

Q.1 Amina thinks of a number and subtracts $\frac{5}{2}$ from it. She multiplies the result by 8. The result now obtained is 3 times the same number she thought of. What is the number?

Sol. Let the number be x.

Then, according to the question,

$$\left(x - \frac{5}{2}\right)8 = 3x$$

$\Rightarrow \quad 8x - \frac{5}{2} \times 8 = 3x$

$\Rightarrow \quad 8x - 20 = 3x$

$\Rightarrow \quad 8x - 3x = 20$ (Transposing 3x to LHS and –20 to RHS)

$\Rightarrow \quad 5x = 20$

$\Rightarrow \quad x = \frac{20}{5} = 4$ (Dividing both sides by 5)

Hence, the required number is 4.

Check : $\left(4 - \frac{5}{2}\right)8 = \frac{8-5}{2} \times 8 = \frac{3}{2} \times 8 = 3 \times 4 = 12 = 3x = 3 \times 4 = 12$

Hence, the result is verified.

Q.2 A positive number is 5 times another number. If 21 is added to both the numbers, then one of the new numbers becomes twice the other new number. What are the numbers?

Sol. Let the numbers be x and 5x.

If 21 is added to both the numbers, then first new number.

$= x + 21$

and second new number $= 5x + 21$

Case I ***When first new number is twice the second new number.***

Then, $x + 21 = 2(5x + 21)$

$\Rightarrow \quad x + 21 = 10x + 42$

$\Rightarrow \quad x - 10x = 42 - 21$

$\Rightarrow \quad x - 10x = 42 - 21$ (Transposing 10x to LHS and 21 to RHS)

$\Rightarrow \quad -9x = 21 \quad \Rightarrow \quad x = -\frac{21}{9}$ (Dividing both sides by –3)

$\Rightarrow \quad x = \frac{-21 \div 3}{9 \div 3} \quad \Rightarrow \quad x = -\frac{7}{3}$

$\Rightarrow \quad 5x = 5 \times \left(-\frac{7}{3}\right) = -\frac{35}{3}$

Hence, the numbers are $-\frac{7}{3}$ and $-\frac{35}{3}$.

This case is in admissible as the required are positive.

Case II ***When second new number is twice the first new number.***

Then, $5x + 21 = 2(x + 21)$

$\Rightarrow$ $5x + 21 = 2x + 42$

$\Rightarrow$ $5x - 2x = 42 - 21$ (Transposing 2x to LHS and 21 to RHS)

$\Rightarrow$ $3x = 21$ $\Rightarrow$ $x = \frac{21}{3} = 7$ (Dividing both sides by 3)

$\Rightarrow$ $5x = 5 \times 7 = 35$

Hence, the numbers are 7 and 35.

Check : $35 = 7 \times 5$

$7 \times 21 = 28$

$35 + 21 = 56$

$56 = 28 \times 2$

Hence, the result is verified.

Q.3 Sum of the digits of a two digit number is 9. When we interchange the digits, it is found that the resulting new number is greater than the original number by 27. What is the two-digit number?

Sol. Let the units digit of the two-digit number be x.

Then, the tens digit of the two-digit number $= 9 - x$

($\because$ Sum of the digit of the two-digit number is 9)

$\therefore$ Original number $= 10(9 - x) + x$

$= 90 - 10x + x$

$= 90 - 9x$

When we interchange the digits, then

Units digit $= 9 - x$

and tens digit $= x$

$\therefore$ Resulting number $= 10x + (9 - x) = 9x + 9$

According to the question,

$(9x + 9) = (90 - 9x) + 27$

$\Rightarrow$ $9x + 9x = 90 + 27 - 9$ (Transposing –9x to LHS and 9 to RHS)

$\Rightarrow$ $18x = 108$

$\Rightarrow$ $x = \frac{108}{18} = 16$ (Dividing both sides by 18)

$\Rightarrow$ $9 - x = 9 - 6 = 3$

Hence, the required two-digit number is 36.

Check : $3 + 6 = 9$

$63 = 36 + 27$

Hence, the result is verified.

Q.4 One of the two digits of a two digit number is three times the other digit. If you interchange the digits of this two-digit number and add the resulting number to the original number, you get 88. What is the original number?

Sol. **Case I** Units digits = x

Then, tens digit = 3x

$\therefore$ Original number $= (3x) \times 10 + x = 30x + x + 31x$

On interchanging the digits

Units digit = 3x and tens digit = x

$\therefore$ Resulting number = x × 10 + 3x = 10x + 3x = 13x

According to the question,

$13x + 31x = 88$

$\Rightarrow \quad 44x = 88$

$\Rightarrow \quad x = \frac{88}{44} = 2$ (Dividing both sides by 44)

$\Rightarrow \quad 3x = 3 \times 2 = 6$

Hence, the original number is 62.

Check : $6 = 3 \times 2$

$62 + 26 = 88$

Hence, the result is verified.

Case II Tens digits = x

Then, units digit = 3x

$\therefore$ Original number = x × 10 + 3x = 10x + 3x = 13x

Units digit = x

and tens digit = 3x

$\therefore$ Resulting number = (3x) × 10 × x = 30x + x = 31x

According to the question,

$31x + 13x = 88$

$\Rightarrow \quad 44x = 88$

$\Rightarrow \quad x = \frac{88}{44} = 2$ (Dividing both sides by 44)

$\Rightarrow \quad 3x = 6$

Hence, the original number is 26.

Check : $6 = 3 \times 2$

$26 + 62 = 88$

Hence, the result is verified.

Q.5 Shobo's mother's present age is six times Shobo's present age. Shobo's age five years from now will be one third of his mother's present age. What are their present ages?

Sol. Let the present age of Shobo be x year. Then, Shobo's mother's present age = 6x years.

Five years from now

Shobo's age = (x + 5) years

According to the question

$x + 5 = \frac{1}{3}(6x) \quad \Rightarrow \quad x + 5 = 2x$

$\Rightarrow \quad 5 = 2x - x$ (Transposing x to RHS)

$\Rightarrow \quad 5 = x \quad \Rightarrow \quad x = 5$

$\Rightarrow \quad 6x = 6 \times 5 = 30$

Hence, their, present ages are 5 years and 30 year

Check : $30 = 5 \times 6$

$5 + 5 = 10$

$10 = \frac{1}{3}(30)$

Hence, the result is verified.

Q.6 There is a narrow rectangular plot, reserved for a school, in Mahuli village. The length and breadth of the plot are in the ratio 11 : 4. At the ratio of ₹100 per metre it will cost the village panchayat ₹75000 to fence the plot. What are the dimensions of the plot?

Sol. Let the length and breadth of the plot be 11x m and 4x m respectively.

Then, perimeter of the plot

$= 2 \times (\text{Length} + \text{Breadth})$

$= 2 \times (11x + 4x)\text{ m} = 2 \times (15x)\text{ m} = 30x\text{ m}$

$\therefore$ Cost of fencing the plot = ₹(30x) × 100 = ₹3000x

According to the question,

$3000x = 75000$

$\Rightarrow \quad x = \frac{75000}{3000} = 25$ (Dividing both sides by 3000)

$\therefore$ Length of the plot $= 11x = 11 \times 25 = 275$ m

and breadth of the plot $= 4x = 4 \times 25 = 100$ m

Hence, the dimensions of the plot are 275 m and 100 m.

Check : $275 : 100 = \frac{275}{100} = \frac{275 \div 25}{100 \div 25} = \frac{11}{4} = 11 : 4$

$2(275 + 100) \times 100 = 2(375) \times 100 = 750 \times 100 = 75000$

Hence, the result is verified.

Q.7 Hason buys two kinds of cloth materials for school uniforms, shirt material that costs him ₹50 per metre and trouser material that costs him ₹90 per metre. For every 2 meters of the trouser material he buys 3 metres of the shirt material. He sells the materials at 12% and 10% profit respectively. His total sale is ₹36,600. How much trouser material did he buy?

Sol. Suppose that he bought x metres of trouser material.

$\because$ For every 2 metres of trouser material, he buys = 3 metres of shirt material.

$\therefore$ For every x metres of trouser material he buys = $\frac{3x}{2}$ metres of shirt material.

$\therefore$ Cost of trouser material = x × 90 = ₹90x

Cost of shirt material = $\frac{3x}{2} \times 50$ = ₹75x

Profit of 10% on trouser material = ₹90 × $\frac{10}{100}$ = ₹9x

Profit of 12% on shirt material = ₹75 × $\frac{12}{100}$ = ₹9x

$\therefore$ S.P. of trouser material = ₹90x + ₹9x = ₹99x

S.P. of shirt material = ₹75x + ₹9x = ₹84x

$\therefore$ Total sale price = S.P. of trouser material + S.P. of shirt material

= ₹99x + ₹84x = ₹183x

$\because$ His total sale is ₹36,600

$\therefore$ $183x = 36,600$

$\Rightarrow \quad x = \frac{36600}{183} = 200$ (Dividing both sides by 183)

Hence, he bought 200 m of trouser material.

Check : $\frac{3x}{2} = \frac{3}{2} \times 200 = 300 \quad \left(200 \times 90 + 200 \times 90 \times \frac{10}{100}\right) + \left(300 \times 50 + 300 \times 50 \times \frac{12}{100}\right)$

$= (18000 + 1800) + (15000 + 1800) = 19800 + 16800 = 36600$

Hence, the result is verified.

Q.8 Half of a herd of deer are grazing in the field and three fourths of the remaining are playing nearby. The rest 9 are drinking water from the pound. Find the number of deer in the herd.

Sol. Let the number of deer in the herd be x.

Then, number of deer grazing in the field = $\frac{x}{2}$

$\therefore$ Number of remaining deer = $x - \frac{x}{2} = \frac{x}{2}$

$\therefore$ Number of deer playing nearby = $\frac{3\left(\frac{x}{2}\right)}{4} = \frac{3x}{8}$

Number of deer drinking water from the pond.

$$= x - \left(\frac{x}{2} + \frac{3x}{8}\right) = x - \left(\frac{4x + 3x}{8}\right) = x - \frac{7x}{8} = \frac{8x - 7x}{8} = \frac{x}{8}$$

According to the question, $\frac{x}{8} = 9$

$\Rightarrow$ $x = 9 \times 8 = 72$ (Multiplying both sides by 8)

Hence, the number of deer in the herd is 72.

Check : $\frac{72}{2} = 36.$

$\frac{3}{4}(36) = 27$

$72 - (36 + 27) = 72 - 63 = 9$

Hence, the result is verified.

Q.9 A grandfather is ten times older than his granddaughter. He is also 54 years older than her. Find their present ages.

Sol. Let the present age of granddaughter be x years.

Then, the present age of grandfather is 10x years.

According to the question,

$10x = x + 54$

$\Rightarrow$ $10x - x = 54$ (Transposing x to LHS)

$\Rightarrow$ $9x = 54$

$\Rightarrow$ $x = \frac{54}{9} = 6$ (Dividing both sides by 9)

$\Rightarrow$ $10x = 10 \times 6 = 60$

Hence, their present ages are 60 years and 6 years.

Check : $60 = 6 \times 10$

$60 = 6 + 54$

Hence, the result is verified.

Q.10 Aman's age is three times his son's ages. Ten years ago he was five times his son's age. Find their present ages.

Sol. Let the present age of son be x years.

Then, the present age of Aman = 3x years.

Ten years ago

Age of Aman = $(3x - 10)$ years

Age of son = $(x - 10)$ years

According to the question,

$3x - 10 = 5(x - 10)$

$\Rightarrow \quad 3x - 10 = 5x - 50$

$\Rightarrow \quad -10 + 50 = 5x - 3x$ (Transposing 3x to RHS and –50 to LHS)

$\Rightarrow \quad 40 = 2x$

$\Rightarrow \quad 2x = 40$

$\Rightarrow \quad x = \frac{40}{2} = 20$ (Dividing both sides by 2)

$\Rightarrow \quad 3x = 3 \times 20 = 60$

Hence, the present ages of Aman and his son are 60 years and 20 years respectively.

Check : $60 = 3 \times 20$

$60 - 10 = 50$

$20 - 10 = 10$

$50 = 10 \times 5$

Hence, the result is verified.

EXERCISE - 5

Q.1 Solve the following linear equations

(1) $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$

(2) $\frac{n}{2} - \frac{3n}{4} + \frac{5n}{6} = 21$

(3) $x + 7 - \frac{8x}{3} = \frac{17}{6} - \frac{5x}{2}$

(4) $\frac{x-5}{3} = \frac{x-3}{5}$

(5) $\frac{3t-2}{4} - \frac{2t+3}{3} = \frac{2}{3} - t$

(6) $m - \frac{m-1}{2} = 1 - \frac{m-2}{3}$

(7) $3(t - 3) = 5(2t + 1)$

(8) $15(y - 4) - 2(y - 9) + 5(y + 6) = 0$

(9) $3(5z - 7) - 2(9z - 11) = 4(8z - 13) - 17$

(10) $0.25(4f - 3) = 0.05(10f - 9)$

Sol. (1) $\mathbf{\frac{x}{2}-\frac{1}{5}=\frac{x}{3}+\frac{1}{4}}$

We have $\frac{x}{2}-\frac{1}{5}=\frac{x}{3}+\frac{1}{4}$

It is a linear equation since it involves linear expression only.

$\Rightarrow \quad \frac{x}{2}-\frac{x}{3}=\frac{1}{4}+\frac{1}{5}$ (Transposing $\frac{x}{3}$ to LHS and $-\frac{1}{5}$ to RHS)

$\Rightarrow \quad \frac{3x-2x}{6}=\frac{5+4}{20}$

$\Rightarrow \quad \frac{x}{6}=\frac{9}{20} \quad \Rightarrow \quad x=\frac{9}{20}\times 6$ (Multiplying both sides by 6)

$\Rightarrow \quad x=\frac{27}{10}$

This is the required solution.

Check : LHS = $\frac{1}{2}\times\frac{27}{10}-\frac{1}{5} = \frac{27}{20}-\frac{1}{5} = \frac{27-4}{20} = \frac{23}{20}$

RHS = $\frac{1}{3}\times\frac{27}{10}+\frac{1}{4} = \frac{9}{10}+\frac{1}{4}$ $\frac{18+5}{20} = \frac{23}{20}$

Therefore, LHS = RHS (as desired)

(2) $\mathbf{\frac{n}{2}-\frac{3n}{4}+\frac{5n}{6}=21}$

We have $\frac{n}{2}-\frac{3n}{4}+\frac{5n}{6}=21$

It is a linear equation since it involves linear expressions only.

$\Rightarrow \quad \frac{6n-9n+10n}{12}=21$ [LCM (2, 4, 6) = 12]

$\Rightarrow \quad \frac{7n}{12}=21$

$\Rightarrow \quad n=21\times\frac{12}{7}=36$ (Multiplying both sides by $\frac{12}{7}$)

This is the required solution.

Check : LHS $=\frac{1}{2}\times 36-\frac{3}{4}\times 36+\frac{5}{6}\times 36$

$= 18 - 27 + 30 = 21 = \text{RHS}$ (as desired)

(3) $\mathbf{x+7-\frac{8x}{3}=\frac{17}{6}-\frac{5x}{2}}$

We have $x+7-\frac{8x}{3}=\frac{17}{6}-\frac{5x}{2}$

It is a linear equation since it involves linear expressions only.

$\Rightarrow \quad x-\frac{8x}{3}+\frac{5x}{2}=\frac{17}{6}-7$ (Transposing $\frac{-5x}{2}$ to LHS and 7 to RHS)

$\Rightarrow \quad \frac{6x-16x+15}{6}=\frac{17-42}{6}$

$\Rightarrow \quad \frac{5x}{6}=\frac{-25}{6}$

$\Rightarrow \quad x=\frac{-25}{6}\times\frac{6}{5}$ (Multiplying both sides by $\frac{6}{5}$)

$\Rightarrow \quad x=-5$

This is the requried solution.

Check : LHS $=-5+7-\frac{8}{3}(-5)=-5+7+\frac{40}{3}=2+\frac{40}{3}=\frac{6+40}{3}=\frac{46}{3}$

RHS $=\frac{17}{6}-\frac{5}{2}(-5)=\frac{17}{6}+\frac{25}{2}=\frac{92}{6}=\frac{92\div 2}{6\div 2}=\frac{46}{3}$

Therefore, LHS = RHS (as desired)

(4) $\mathbf{\frac{x-5}{3}=\frac{x-3}{5}}$

We have $\frac{x-5}{3}=\frac{x-3}{5}$

It is a linear equation since it involves linear expression only.

$\Rightarrow \quad \frac{x}{3}-\frac{5}{3}=\frac{x}{5}-\frac{3}{5}$

$\Rightarrow \quad \frac{x}{3}-\frac{x}{5}=\frac{5}{3}-\frac{3}{5}$ (Transposing $\frac{x}{5}$ to LHS and $\frac{-5}{3}$ to RHS)

$\Rightarrow \quad \frac{5x-3x}{15}=\frac{25x-9}{15}$

$\Rightarrow \quad \frac{2x}{15}=\frac{16}{15}$

$\Rightarrow \quad x=\frac{16}{15}\times\frac{15}{2}=8$ (Multiplying both sides by $\frac{15}{2}$)

This is the required solution.

Check : LHS = $\frac{8-5}{3} = \frac{3}{2} = 1$

RHS = $\frac{8-3}{5} = \frac{5}{5} = 1$

Therefore LHS = RHS (as desired)

(5) $\mathbf{\frac{3t-2}{4} - \frac{2t+3}{3} = \frac{2}{3} - t}$

We have $\frac{3t-2}{4} - \frac{2t+3}{3} = \frac{2}{3} - t$

It is a linear equation since it involves linear expression only.

$\Rightarrow \quad \frac{3}{4}t - \frac{2}{4} - \frac{2}{3}t - \frac{3}{3} = \frac{2}{3} - t$

$\Rightarrow \quad \frac{3}{4}t - \frac{1}{2} - \frac{2}{3}t - 1 = \frac{2}{3} - t$

$\Rightarrow \quad \frac{3}{4}t - \frac{2}{3}t + t = \frac{2}{3} + \frac{1}{2} + 1$ (Transposing –t to LHS $-\frac{1}{2}$ and –1 to RHS)

$\Rightarrow \quad \frac{9t - 8t + 12t}{12} = \frac{4+3+6}{6}$

$\Rightarrow \quad \frac{13t}{12} = \frac{13}{6}$

$\Rightarrow \quad t = \frac{13}{6} \times \frac{12}{13} = 2$ (Multiplying both sides by $\frac{12}{13}$)

This is the required solution.

Check : LHS $= \frac{3}{4} \times 2 - \frac{2}{4} - \frac{2}{3} \times 2 - \frac{3}{3} = \frac{3}{2} - \frac{1}{2} - \frac{4}{3} - 1 = \frac{9-3-8-9}{6}$

$= -\frac{8}{6} = -\frac{8 \div 2}{6 \div 2} = -\frac{4}{3}$

RHS $= \frac{2}{3} - 2 = \frac{2-6}{3} = \frac{-4}{3}$

Therefore LHS = RHS (as desired)

(6) $\mathbf{m - \frac{m-1}{2} = 1 - \frac{m-2}{3}}$

We have $m - \frac{m-1}{2} = 1 - \frac{m-2}{3}$

It is a linear equation since it involves linear expression only.

$\Rightarrow \quad m - \frac{m}{2} + \frac{1}{2} = 1 - \frac{m}{3} + \frac{2}{3}$

$\Rightarrow \quad m - \frac{m}{2} + \frac{m}{3} = 1 + \frac{2}{3} - \frac{1}{2}$ (Transposing $-\frac{m}{3}$ to LHS and $\frac{1}{2}$ to RHS)

$\Rightarrow \quad \frac{6m - 3m + 2m}{6} = \frac{6 + 4 - 3}{6}$

$\Rightarrow \quad \frac{5m}{6} = \frac{7}{6}$

$\Rightarrow \quad m = \frac{7}{6} \times \frac{6}{5} = \frac{7}{5}$ (Multiplying both sides by $\frac{6}{5}$)

This is the required solution.

Check : LHS $= \frac{7}{5} - \frac{1}{2} \times \frac{7}{5} + \frac{1}{2} = \frac{7}{5} - \frac{7}{10} + \frac{1}{2} = \frac{14 - 7 + 5}{10} = \frac{12}{10} = \frac{6}{5}$

RHS $= 1 - \frac{1}{3} \times \frac{7}{5} + \frac{2}{3} = 1 - \frac{7}{15} + \frac{2}{3} = \frac{15 - 7 + 10}{15} = \frac{18}{15} = \frac{6}{5}$

Therefore LHS = RHS (as desired)

(7) 3(t – 3) = 5(2t + 1)

We have $3(t - 3) = 5(2t + 1)$

$\Rightarrow \quad 3t - 9 = 10t + 5$

$\Rightarrow \quad 3t - 10t = 5 + 9$ (Transposing 10t to LHS and –9 to RHS)

$\Rightarrow \quad -7t = 14$

$\Rightarrow \quad t = -\frac{14}{7} = -2$ (Multiplying both sides by $\frac{6}{5}$)

This is the required solution.

Check : LHS $= 3(t - 3) = 3(-2 - 3) = 3(-5) = -15$

RHS $= 5(2t + 1) = 5(2 \times (-2) + 1) = 5(-4 + 1) = 5(-3) = -15 =$ LHS

Therefore LHS = RHS (as desired)

(8) 15(y – 4) – 2 (y – 9) + 5 (y + 6) = 0

We have $15(y - 4) - 2(y - 9) + 5(y + 6) = 0$

$\Rightarrow \quad 15y - 60 - 2y + 18 + 5y + 30 = 0$

$\Rightarrow \quad 18y - 12 = 0$

$\Rightarrow \quad 18y = 12$ (Transposing –12 to RHS)

$\Rightarrow \quad y = \frac{12}{18} = \frac{2}{3}$ (Dividing both sides by 18)

This is the required solution.

Check : LHS $= 15(y-4) - 2(y-9) + 5(y+6)$

$$= 15\left(\frac{2}{3}-4\right) - 2\left(\frac{2}{3}-9\right) + 5\left(\frac{2}{3}+6\right)$$

$$= 15\left(\frac{2-12}{3}\right) - 2\left(\frac{2-27}{3}\right) + 5\left(\frac{2+18}{3}\right) = 15\left(-\frac{10}{3}\right) - 2\left(-\frac{25}{3}\right) + 5\left(\frac{20}{3}\right)$$

$$= -15 + \frac{50}{3} + \frac{100}{3} = \frac{-150+50+100}{3} = \frac{0}{3} = 0 = \text{RHS}$$

Therefore LHS = RHS (as desired)

(9) 3(5z – 7) – 2 (9z – 11) = 4 (8z – 13) – 17

We have $3(5z-7) - 2(9z-11) = 4(8z-13) - 17$

$\Rightarrow \quad 5z - 21 - 18z + 22 = 32z - 52 - 17$

$\Rightarrow \quad -3z + 1 = 32z - 52 - 17$

$\Rightarrow \quad -3z - 32z = -69 - 1$ (Transposing 32z to LHS and 1 to RHS)

$\Rightarrow \quad -35z = -70$

$\Rightarrow \quad z = \dfrac{-70}{-35} = 2$ (Dividing both sides by –35)

This is the required solution.

Check : LHS $= 3(5x-7) - 2(9z-11) = 3(5\times 2 - 7) - 2(9\times 2 - 11)$

$= 3(10-7) - 2(18-11) = 3(3) - 2(7)$

$= 9 - 14 = -5$

RHS $= 4(8z-13) - 17 = 4(8\times 2 - 13) - 17$

$= 4(16-13) - 17 = 4(3) - 17$

$= 12 - 17 = -5$

Therefore LHS = RHS (as desired)

(10) 0.25 (4f – 3) = 0.05 (10f – 9)

We have $0.25(4f-3) = 0.05(10f-9)$

$\Rightarrow \quad f - 0.75 = 0.5f - 0.45$

$\Rightarrow \quad f - 0.5f = -0.45 + 0.75$ (Transposing 0.5f to LHS and –0.75 to RHS)

$\Rightarrow \quad 0.5f = 0.30$

$\Rightarrow \quad f = \dfrac{0.30}{0.5} = 0.6$ (Dividing both sides by 0.5)

This is the required solution.

Check : LHS $= 0.25(4f-3) = 0.25(4\times 0.6 - 3)$

$= 0.25(2.4-3) = 0.25(-0.6)$

$= -0.15$

RHS $= 0.05(10f-9) = 0.05(10\times 0.6 - 9)$

$= 0.05(6-9) = 0.05(-3)$

$= -0.15$

Therefore LHS = RHS (as desired)

EXERCISE - 6

Q.1 Solve the following equations

(1) $\frac{8x-3}{3x}=2$ **(2)** $\frac{9x}{7-6x}=15$ **(3)** $\frac{z}{z+15}=\frac{4}{9}$

(4) $\frac{3y+4}{2-6y}=\frac{-2}{5}$ **(5)** $\frac{7y+4}{y+2}=\frac{-4}{3}$

Sol. (1) $\frac{8x-3}{3x}=2$

We have $\frac{8x-3}{3x}=2$ (Multiplying both sides by 3x)

$\Rightarrow \quad \frac{8x-3}{3x}\times 3x = 2\times 3x$

$\Rightarrow \quad 8x - 3 = 6x$

$\Rightarrow \quad 8x - 6x = 3$ (Transposing 6x to LHS and –3 to RHS)

$\Rightarrow \quad 2x = 3 \quad \Rightarrow \quad x = \frac{3}{2}$ (Dividing both sides by 2)

This is the required solution.

Check : LHS $= \frac{8\times\frac{3}{2}-3}{3\times\frac{3}{2}} = \frac{12-3}{\frac{9}{2}} = \frac{9}{\frac{9}{2}} = 9\times\frac{2}{9} = 2 =$ RHS (as desired)

(2) $\frac{9x}{7-6x}=15$

We have $\frac{9x}{7-6x}=15$ [Multiplying both sides by (7 – 6x)]

$\Rightarrow \quad \frac{9x}{7-6x}\times(7-6x) = 15\times(7-6x)$

$\Rightarrow \quad 9x = 105 - 90x$

$\Rightarrow \quad 9x + 90x = 105$ (Transposing –90x to LHS)

$\Rightarrow \quad 99x = 105$

$\Rightarrow \quad x = \frac{105}{99} = \frac{35}{33}$ (Dividing both sides by 99)

This is the required solution.

Check : LHS $= \frac{9\times\frac{35}{33}}{7-6\times\frac{35}{33}} = \frac{\frac{105}{11}}{7-\frac{70}{11}} = \frac{\frac{105}{11}}{\frac{77-70}{11}} = \frac{\frac{105}{11}}{\frac{7}{11}}$

$= \frac{105}{11}\times\frac{11}{7} = 15 =$ RHS (as desired)

(3) $\mathbf{\frac{z}{z+15}=\frac{4}{9}}$

We have $\frac{z}{z+15}=\frac{4}{9}$

$\Rightarrow \quad \frac{z}{z+15}\times(z+15)=\frac{4}{9}\times(z+15)$ [Multiplying both sides by $z+15$]

$\Rightarrow \quad z=\frac{4z+60}{9}$

$\Rightarrow \quad 9z=\frac{4z+60}{9}\times 9$ [Multiplying both sides by 9]

$\Rightarrow \quad 9z = 4z + 60$

$\Rightarrow \quad 9z - 4z = 60$ (Transposing 4z to LHS)

$\Rightarrow \quad 5z = 60$

$\Rightarrow \quad z=\frac{60}{5}=12$ (Dividing both sides by 5)

This is the required solution.

Check : LHS $=\frac{12}{12+15}=\frac{12}{27}=\frac{12\div 3}{27\div 3}=\frac{4}{9}=$ RHS (as desired)

(4) $\mathbf{\frac{3y+4}{2-6y}=\frac{-2}{5}}$

We have $\frac{3y+4}{2-6y}=\frac{-2}{5}$ [Multiplying both sides by $2-6y$]

$\Rightarrow \quad \left(\frac{3y+4}{2-6y}\right)\times(2-6y)=\frac{-2}{5}\times(2-6y)$

$\Rightarrow \quad 3y+4=\frac{-2}{5}(2-6y)$

$\Rightarrow \quad 5(3y + 4) = -2(2 - 6y)$ [Multiplying both sides by 5]

$\Rightarrow \quad 15y + 20 = -4 + 12y$

$\Rightarrow \quad 15y - 12y - 4 - 20$ (Transposing 12y to LHS and 20 to RHS)

$\Rightarrow \quad 3y = -24$

$\Rightarrow \quad y=\frac{-24}{3} = -8$ (Dividing both sides by 3)

This is the required solution.

Check : LHS $=\frac{3(-8)+4}{2-6(-8)}=\frac{-24+4}{2+48}=\frac{-20}{50}=\frac{-20\div 10}{50\div 10}$

$=\frac{-2}{5}=$ RHS (as desired)

(5) $\mathbf{\dfrac{7y+4}{y+2} = \dfrac{-4}{3}}$

We have $\dfrac{7y+4}{y+2} = \dfrac{-4}{3}$ [Multiplying both sides by $y+2$]

$\Rightarrow \quad \dfrac{7y+4}{y+2} \times (y+2) = \dfrac{-4}{3} \times (y+2)$

$\Rightarrow \quad 7y+4 = \dfrac{-4y+8}{3}$

$\Rightarrow \quad (7y + 4) \times 3 = -4\,(4y + 8)$ [Multiplying both sides by 3]

$\Rightarrow \quad 21y + 12 = -4y - 8$

$\Rightarrow \quad 21y + 4y = -8 - 12$ (Transposing $-4y$ to LHS and 12 to RHS)

$\Rightarrow \quad 25y = -20$

$\Rightarrow \quad y = \dfrac{-20}{25}$ (Dividing both sides by 25)

$\Rightarrow \quad y = \dfrac{-20 \div 5}{25 \div 5} = \dfrac{-4}{5}$

This is the required solution.

Check : LHS $= \dfrac{7\left(-\frac{4}{5}\right)+4}{\frac{-4}{5}+2} = \dfrac{\frac{-28+20}{5}}{\frac{-4+10}{5}} = \dfrac{\frac{-8}{5}}{\frac{6}{5}}$

$= \dfrac{-8}{5} \times \dfrac{5}{6} = \dfrac{-8}{6} = \dfrac{-8 \div 2}{6 \div 2} = \dfrac{-4}{3} =$ RHS (as desired)

Q.6 The ages of Hari and Harry are in the ratio 5 : 7. Four years from now the ratio of their ages will be 3 : 4. Find their present ages.

Sol. Let the present ages of Hari and Harry be 5x years and 7x years respectively.

Four years from now

Age of Hari = (5x + 4) years

Age of Harry = (7x + 4) years

According to the question,

$\dfrac{5x+4}{7x+4} = \dfrac{3}{4}$

Multiplying both sides by $7x+4$, we get

$\left(\dfrac{5x+4}{7x+4}\right) \times (7x+4) = \dfrac{3}{4} \times (7x+4)$

$\Rightarrow \quad 5x+4 = \dfrac{3(7x+4)}{4}$

Multiplying both sides by 4, we get

$4(5x + 4) = 3(7x + 4)$

$\Rightarrow$ $20x + 16 = 21x + 12$

$\Rightarrow$ $16 - 12 = 21x - 20x$ (Transposing 20x to RHS and 12 to LHS)

$\Rightarrow$ $4 = x$

$\Rightarrow$ $x = 4$

$\Rightarrow$ $5x = 5 \times 4 = 20$

and $7x = 7 \times 4 = 28$

Hence, their present ages are 20 years and 28 years respectively.

Check : $20 : 28 = \frac{20}{28} = 5 : 7$

Four years from now.

$20 + 4 = 24$

$28 + 4 = 32$

$24 : 32 = \frac{24}{32} = \frac{24 \div 8}{32 \div 8} = \frac{3}{4} = 3 : 4$

Hence, the result is verified.

Q.7 The denominator of a rational number is greater than its numerator by 8. If the numerator is increased by 17 and the denominator is decreased by 1, the number obtained is $\frac{3}{2}$. Find the rational number.

Sol. Let the rational number be $\frac{x}{x+8}$.

According to the question,

$\frac{x+17}{x+8-1} = \frac{3}{2} \quad \Rightarrow \quad \frac{x+17}{x+7} = \frac{3}{2}$

Cross multiplying, we get

$2(x + 17) = 3(x + 7)$

$\Rightarrow$ $2x + 34 = 3x + 21$

$\Rightarrow$ $34 - 21 = 3x - 2x$ (Transposing 21 to LHS and 2x to RHS)

$\Rightarrow$ $13 = x$

$\Rightarrow$ $x = 13$

$\Rightarrow$ $x + 8 = 13 + 8 = 21$

Hence, the rational number is $\frac{13}{21}$.

Check : $21 = 13 + 8$

$\frac{13+17}{21-1} = \frac{30}{20} = \frac{30 \div 10}{20 \div 10} = \frac{3}{2}$

Hence, the result is verified.

CONCEPT APPLICATION LEVEL - II

SECTION - A

- **FILL IN THE BLANKS**

Q.1 Every linear equation in two variable has __________ solutions.

Q.2 If $\frac{1}{2} - x = \frac{1}{2}$, then x = __________

Q.3 Solution of the equation $\frac{p+1}{5} = -\frac{3(p-1)}{10} + 2$ is __________

Q.4 Solution of $0.18(5x-4) = 0.5x + 0.8$ is __________.

SECTION - B

- **MULTIPLE CHOICE QUESTIONS**

Q.1 If $\frac{x+a}{x-a} - \frac{x-b}{x+b} = \frac{2(a+b)}{x}$, then x =

(A) $\frac{a}{a-b}$ (B) $\frac{b}{a-b}$ (C) $\frac{ab}{a-b}$ (D) $\frac{ab}{b-a}$

Q.2 Solve $\frac{2x-3}{2} - \frac{x+1}{3} = \frac{3x-8}{4}$.

(A) 1 (B) 2 (C) $\frac{4}{5}$ (D) $\frac{5}{8}$

Q.3 Solve : $\frac{7y+4}{y+2} = \frac{-4}{3}$.

(A) $\frac{5}{8}$ (B) $\frac{-4}{5}$ (C) 1 (D) $\frac{-5}{8}$

Q.4 The value of x, for which $\frac{1-x}{2-x} = 0$ is

(A) 1/2 (B) –1 (C) 1 (D) –1/2

Q.5 If $5(x-3)-4(x-2) = 0$ then the value of x is

(A) 7 (B) –7 (C) 8 (D) –8

Q.6 The sum of two number is 45 and their ratio is 7 : 8. The numbers are

(A) 28 : 32 (B) 35 : 40 (C) 21 : 24 (D) none of these

Q.7 Five times the number increased by 4 is equal 39. The number is

(A) 4 (B) 5 (C) 7 (D) 6

Q.8 If $\frac{5x}{6}+\frac{3x}{4}=\frac{19}{12}$, then the value of x is
(A) –1 (B) –2 (C) 1 (D) 2

Q.9 The solution of the equation (p + 2) (p – 3) + (p – 3) (p – 4) = p(2p – 5) is
(A) 2 (B) 7 (C) 5 (D) None

Q.10 The equation $\frac{12x+1}{4}=\frac{15x-1}{5}+\frac{2x-5}{3x-1}$ is true for
(A) $x = 1$ (B) $x = 2$ (C) $x = 5$ (D) $x = 7$

Q.11 Pick up the correct value x for which $\frac{x}{0.5}-\frac{1}{0.05}+\frac{x}{0.005}-\frac{1}{0.0005}=0$
(A) $x = 0$ (B) $x = 1$ (C) $x = 10$ (D) None

Q.12 A boat covers a certain distance downstream in 3 hours, and it covers the same distance upstream in 5 hours. If the speed of the boat in still water is 8 km/hr, then the speed of the stream is
(A) 1 km/hr (B) 1.5 km/hr (C) 2 km/hr (D) 3 km/hr

Q.13 The angle A of a triangle ABC is equal to the sum of the two other angles. Also the ratio of the angle B to angle C is 4 : 5. The three angles are
(A) 90°, 40°, 50° (B) 90°, 55°, 35° (C) 90°, 60°, 30° (D) None of these

Q.14 The age of Reena and Tina are in the ratio 3 : 4. Five years ago their age were in the ratio 2 : 3. Find their present ages.
(A) 15 , 20 (B) 20 , 80 (C) 30 , 40 (D) 12 , 16

Q.15 The perimeter of a rectangle is72m. Its length is 10m more than the breadth. Find the length and beadth of the rectangle.
(A) 13 , 23 (B) 14 , 24 (C) 15, 25 (D) 12 , 22

Q.16 A number plus two-third of itself, plus half to itself, plus one-seventh of itself equals 97. Find the number.
(A) 24 (B) 46 (C) 42 (D) 62

Q.17 A labourer was engaged for a month (30 days) on the condition that he will receive Rs. 60 each day he works and will be fined Rs. 10 each day he is absent. At the end of the month he recieved Rs. 1380. How many days did he work ?
(A) 25 (B) 28 (C) 24 (D) 23

Q.18 Rohan spent, $\frac{1}{2}$ of his pocket money on lunch, $\frac{1}{3}$ on conveyance and gave the remaining money to his sister Sweety. If Sweety got Rs. 5, how much pocket money did Rohan get ? How much did he spend on lunch ?
(A) 15 (B) 13 (C) 14 (D) 12

Q.19 Sum of the digits of a 2-digit number is 9. When the digits are reversed (interchanged), it is found that the resulting number is greater than the original number by 27. Find the number.
(A) 63 (B) 45 (C) 54 (D) 36

Q.20 Two third of a number increased by 19 gives the result as 29. Find the number.
(A) 25 (B) 15 (C) 12 (D) 14

Q.21 Divide 18 into two parts such that the larger part divided by the smaller part gives the quotient 2. What is the larger part ?
(A) 6 (B) 12 (C) 9 (D) 18

Q.22 Rahul has 260 coins of Re. 1, Rs. 2 and Rs. 5 alltogether. The total value of the money is Rs. 309. The number of Rs. 2 coins is three times the number of Rs. 5 coins. Find the number of 1 Rs coins.
(A) 232 (B) 200 (C) 210 (D) 243

Q.23 The numerator of a fraction is six more than the denominator. If the numerator is increased by 5 and the denominator is decreased by 1, the fraction becomes $\frac{3}{2}$. Find the denominator.
(A) 25 (B) $\frac{27}{29}$ (C) $\frac{1}{25}$ (D) $\frac{29}{30}$

Q.24 Solve : $\frac{x^2-(x+2)(x+3)}{x+3}=\frac{2}{3}$.
(A) $\frac{24}{17}$ (B) $\frac{-24}{17}$ (C) $\frac{28}{17}$ (D) $\frac{-28}{17}$

Q.25 The diagonal of a rectangle is 5 cm and one of it sides is 4 cm. Its area is
(A) 20 cm^2 (B) 12 cm^2 (C) 10 cm^2 (D) None

Q.26 The sum of two digits of a two digit number is 12. If the digits are reversed, then the number so formed exceeds the original number by 18. Find the original number.
(A) 57 (B) 85 (C) 75 (D None

Q.27 The sum of the digits of a two-digits is 14. When the digits of this number are reversed, the new number formed is greater than the original number by 36. Find the original number.
(A) 59 (B) 95 (C) 68 (D) 86

Q.28 The length of a rectangle is 16m less than 2 times it width. If its perimeter is 112m, find its length and width.
(A) 24 , 36 (B) 24 , 32 (C) 32 , 36 (D) 32 , 24

Q.29 Half of a heard of deer are grazing in the field and three-fourths of the remaining are playing nearby. The rest 9 are drinking water from the pond. Find the number of deer in the herd.
(A) 70 (B) 69 (C) 72 (D) 75

Q.30 Lakshmi is a cashier in a bank. She has currency notes of denominations of Rs. 100, Rs. 50 and Rs. 10 respectively. The ratio of the number of these notes is 2 : 3 : 5. The total cash with Laxmi is Rs. 4,00,000. How many notes of each denomination does she have ?
(A) 5000 (B) 3000 (C) 2000 (D) 10000

Q.31 The organisers of an essay competition decided that a winner in the competition gets a prize of Rs. 100 and a participant who does not win gets a prize of Rs. 25. The total prize money distributed is Rs. 3000. Find the number of winners, if the total number of participants is 63.
(A) 54 (B) 55 (C) 56 (D) 19

Q.32 The sum of the digits of a 2-digit number is 6. On reversing its digits, the new number, is 18 less than the original number. Find the number.
(A) 24 (B) 42 (C) 51 (D) 15

Q.33 The numerator of a fraction is 6 less than the denominator. If 1 is added to both numerator and denominator, the fraction becomes $\frac{1}{2}$. Find the fraction.
(A) $\frac{5}{11}$ (B) $\frac{12}{13}$ (C) $\frac{14}{15}$ (D) $\frac{6}{13}$

Q.34 At present the sum of Mala's age and her daughter's age is 44 years. After 2 years, daughter's age will be three times that of her daughter's age. Find their present ages.
(A) 33 , 11 (B) 35 , 9 (C) 32 , 12 (D) 34 , 10

Q.35 Rooplal left one-third of his property to his son, one-fourth to his daughter and the remaining to his wife. If the wife's share was worth Rs. 32000, how much money did Roopalal have ?
(A) 76800 (B) 77880 (C) 78000 (D) 76000

Q.36 Solve for x if $kx + a = mx + b$
(A) $\frac{a-b}{k-m}$ (B) $\frac{b-a}{k-m}$ (C) $\frac{b-a}{m-k}$ (D) $\frac{a-b}{k}$

Q.37 Solve for x : $(\sqrt{5}+5)\,x + 4 = 2\sqrt{\mathbf{5}} + 8$.
(A) $\frac{2\sqrt{5}+4}{\sqrt{5}+5}$ (B) $\frac{5+4\sqrt{5}}{\sqrt{5}+5}$ (C) $\frac{2\sqrt{5}+4}{5+5\sqrt{5}}$ (D) $\frac{2\sqrt{5}+4}{4\sqrt{5}+20}$

Q.38 Solve for y : $\frac{1}{2}(3y+1) - \frac{1}{3}(5y+2) = y - 1$.
(A) $\frac{5}{8}$ (B) $\frac{5}{7}$ (C) $\frac{7}{9}$ (D) $\frac{8}{11}$

Q.39 Solve for x : $\frac{6x-7}{2x+1} = \frac{3x+1}{x+5}$.
(A) 5 (B) 3 (C) 2 (D) 1

Q.40 A number consists of two digits. The digit at ten's place is two times the digit at the unit's place. The number formed by reversing the digits, is 27 less than the original number. Find the original number.
(A) 63 (B) 36 (C) 42 (D) 84

Q.41 Divide 300 into two parts so that half of one part may be less than the other by 48. Find the larger part.
(A) 132 (B) 168 (C) 160 (D) 170

Q.42 The sum of two-digit number and the number obtained by reversing the order of its digits is 165. If the digits differ by 3, find the number.
(A) 96 (B) 69 (C) both (A) and (B) (D) None of these

Q.43 An altitude of a triangle is five-third the length of its corresponding base. If the altitude was increased by 4 cm and the base is decreased by 2cm, the area of the triangle would remain the same. Find the altitude of the triangle.
(A) 30 (B) 35 (C) 20 (D) 25

SECTION - C

- **COMPREHENSION**

I Suppose a number I divide it by 15 and then divide the quotient by 16. Then multiply the final quotient by 30 from the product so obtained. I subtract the number which I supposed, the result is –7.

Q.1 Form an equation

(A) $\frac{x}{15\times16}\times30-x=-7$ (B) $\frac{16}{15}x\times(30-x)=-7$

(C) $\frac{15}{16}x\times(30-x)=-7$ (D) None

Q.2 Find the value of the number
(A) $x = 4$ (B) $x = 8$ (C) $x = 1$ (D) $x = 0$

Q.3 Find the value of p. If $px + x = 8$ and the number obtained from (ii) will satisfy the equation
(A) p = 0 (B) p = 8 (C) p = 1 (D) None

SECTION - D

- **MATCH THE COLUMN**

Q.1

	Column I		**Column II**
(A)	Solution of $2x - 3 = 7$ is	(p)	-8
(B)	Solution of $\frac{15}{4}-7x=9$ is	(q)	0.6
(C)	Solution of $1.6=\frac{y}{1.5}$ is	(r)	$-\frac{3}{4}$
(D)	Solution of $0.25(4t-3) = 0.05\,(10t-9)$ is	(s)	5
(E)	Solution of $\frac{3x+4}{2-6x}=-\frac{2}{5}$ is	(t)	2.4

ANSWER KEY

CONCEPT APPLICATION LEVEL - II

SECTION - A

Q.1 infinitely many Q.2 0 Q.3 $\frac{21}{5}$ Q.4 3.8

SECTION - B

Q.1	D	Q.2	B	Q.3	B	Q.4	C	Q.5	A	Q.6	C	Q.7	C
Q.8	C	Q.9	A	Q.10	D	Q.11	C	Q.12	C	Q.13	A	Q.14	A
Q.15	A	Q.16	C	Q.17	C	Q.18	A	Q.19	D	Q.20	B	Q.21	B
Q.22	A	Q.23	A	Q.24	B	Q.25	B	Q.26	A	Q.27	A	Q.28	B
Q.29	C	Q.30	A	Q.31	D	Q.32	B	Q.33	A	Q.34	D	Q.35	A
Q.36	B	Q.37	A	Q.38	B	Q.39	C	Q.40	A	Q.41	B	Q.42	C
Q.43	C												

SECTION - C

Q.1 A Q.2 B Q.3 A

SECTION - D

Q.1 (A)-(s); (B)-(r); (C)-(t); (D)-(q); (E)-(p)