6 FACTORIZATION

THEORY

6.1 INTRODUCTION

In this chapter, we shall do the other way round, that is, we shall find two or more algebraic expressions whose product is equal to the given expression. The process of writing a given algebraic expression as the product of two or more expressions will be known as the factorization of the expression.

FACTORS If an algebraic expression is written as the product of numbers or algebraic expressions, then each of these numbers and expressions are called the factors of the given algebraic expression and the algebraic expression is called the product of these expressions.

Factorization : The process of writing a given algebraic expression as the product of two or more factors is called factorization.

Factors of a Monomial : Factors of a monomial consist of every literal, their product and number that will divide it exactly.

6.2 COMMON FACTORS AND GREATEST COMMON FACTOR OF MONOMIALS

Greatest common factor (GCF) or highest common factor (HCF) : The greatest common factor of given monomials is the common factor having greatest coefficient and highest power of the variables.

The following step-wise procedure will be helpful to find the GCFof two or more monomials.

Illustrations

Illustration 1

Find the greatest common factors of the monomials $14x^2y^3$, $21x^2y^2$, $35x^4y^5z$.

Solution

The numerical coefficients of the given monomials are 14, 21 and 35

The greatest common factor of 14, 21 and 35 is 7

The common literals appearing in the three monomials are x and y

The smallest power of 'x' in the three monomials = 2

The smallest power of 'y' in the three monomials = 2

The monomials of common literals with smallest power = x^2y^2

Hence, the greatest common factor = $7\,x^2y^2$.

6.3 FACTORISATION OF POLYNOMIALS

CASE I : When we have an Expression of the type ax + ay

By inspection, we find the greatest monomial factor which can divide each term of the expression.

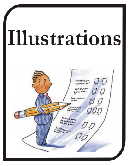

Illustration 2

Factorise $3x^2 - 9xy + 12\,xy^2$

Solution

$3x^2 = 3 \times x \times x$

$9xy = 3 \times 3 \times x \times y$

$12xy^2 = 3 \times 4 \times x \times y \times y$

H.C.F. is 3x

$\therefore \quad 3x^2 - 9xy + 12\,xy^2 = 3x\,(x - 3 \times y + 4 \times y \times y)$

$= 3x\,(x - 3y + 4y^2)$

Illustration 3

Factorise $x + 3 - 6xy - 18y$

Solution

$x + 3 - 6xy - 18\,y = (x + 3) - 6y\,(x + 3)$

$= (x + 3)\,(1 - 6y)$

Case II : Factorisation with the help of Algebraic Identities

Let us recall the following algebraic identities :

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$; $(x + y)\,(x - y) = x^2 - y^2$

$(x + a)(x + b) = x^2 + (a + b)\,x + ab$

Thus, we can say that

Factors of $x^2 + 2xy + y^2$ are $x + y$ and $x + y$

Factor of $x^2 - 2xy + y^2$ are $x - y$ and $x - y$

Factors of $x^2 + (a + b)\,x + ab$ are $x + a$ and $x + b$

On the basis of the above discussion let us deal with the following examples of factorisation.

A. Factorisation by using the Identities :

$x^2 \pm 2xy + y^2 = (x \pm y)^2$

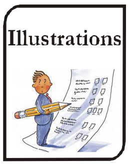

Illustration 4

Factorise : $25x^2 - 20x + 4$

Solution

$25x^2 \quad -20x \quad +4$

$\downarrow \qquad \uparrow \qquad \downarrow$

$= (5x)^2 - 2 \times 5x \times 2 + (2)^2$

$= (5x - 2)^2 = (5x - 2)\,(5x - 2)$

Note that in these two examples in second step two arrows are downward and one arrow is upward. This shows that in the second step first we write 1st and 3rd terms on the basis of given terms and then write the middle term to complete the formula and then compare it with given middle term.

B. Factorisation by Using the Identity $x^2 - y^2 = (x + y)(x - y)$

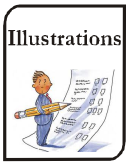

Illustration 5

Factorize $121x^2 - 81y^2$

Solution

$121x^2 - 81y^2 = (11x)^2 - (9y)^2$

(Using identity $x^2 - y^2 = (x - y)(x + y)$

$= (11x - 9y)(11x + 9y)$

C. Factorisation of Trinomial $x^2 + mx + n$

By splitting up the middle terms or factorisation by using the identity :
$x^2 + (a + b)x + ab = (x + a)(x + b)$.

We can find out two numbers a and b positive or negative, such that (a + b) is the same as the coefficient of x whereas the product ab is equal to the constant term in the given expression.

Let us consider examples to explain the above process.

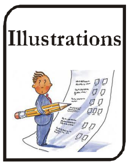

Illustration 6

Factorise $x^2 + 6x + 8$.

Solution

Here we have to find out two numbers a and b such that :

$a + b = 6$ (the coefficient of x)

$ab = 8$ (constant term)

Thus given polynomial can be written as $x^2 + 2x + 4x + 8$

$= (x^2 + 2x) + (4x + 8)$

[Here 4 terms obtained in 2nd step have been written as sum of two groups].

$= (x + 2)(x + 4)$

Again x + 2 which is common in both terms, has been taken out

or

$x^2 + 6x + 8$

Here $a = 4$ and $b = 2$

$\therefore \quad x^2 + (4 + 2)x + (x \times 2)$

$\Rightarrow \quad (x + 4)(x + 2)$

(Using Identity $x^2 + (a + b)x + ab = (x + a)(x + b)$

6.4 DIVISION OF POLYNOMIALS

6.4.1 Division of a Monomial by Another Monomial

To divide a monomial by another monomial, follow the following steps :

Step 1 : Find the quotient of the numeical coefficients.

Step 2 : Find the quotient of the variables.

Step 3 : Find the product of the results obtained in steps 1 and 2.

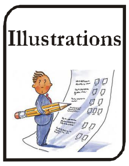

Illustration 7

Divide $108x^3 y^3z^7$ by $-120x^2y^2z^2$

Solution

$$\frac{108x^3y^3z^7}{-120x^2y^2z^2} = \frac{-9}{10}xyz^5$$

Thus, $108x^3y^3z^7 \div (-120x^2y^2z^2) = \frac{-9}{10}xyz^5$

Illustration 8

Divide $96x^3y^3z^2 - 36x^2y^2z^2 - 60xyz$ by $-12xyz$

Solution

$$(96x^3y^3z^2 - 36x^2y^2z^2 - 60xyz) \div (-12xyz)$$

$$= \frac{96x^3y^3z^2 - 36x^2y^2z^2 - 60xyz}{-12xyz}$$

$$= \frac{96x^3y^3z^2}{-12xyz} - \frac{36x^2y^2z^2}{-12xyz} - \frac{60xyz}{-12xyz} = -8x^2y^2z + 3xyz + 5$$

6.4.2 Division of a Polynomial by Another Polynomial

A. Factorisation Method

Consider $(3x^2 + 12x)$ divided by $x + 4$

We can write the factors fo $3x^2 + 12x$ as $3x(x + 4)$

Now $\frac{3x^2 + 12x}{x + 4} = \frac{3x(x+4)}{(x+4)} = 3x$

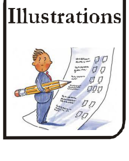

Illustration 9

Divide $9x^2 - 16y^2$ by $3x - 4y$

Solution

$$\frac{9x^2 - 16y^2}{3x - 4y} = \frac{(3x)^2 - (4y)^2}{3x - 4y} \quad \text{[Applying the identity } x^2 - y^2 = (x-y)(x+y)\text{]}$$

$$= \frac{(3x-4y)(3x+4y)}{3x-4y} = 3x + 4y$$

Illustration 10

Divide $x^2 - 9x + 14$ by $x - 2$

Solution

$$\frac{x^2 - 9x + 14}{x - 2} = \frac{x^2 - 7x - 2x + 14}{x - 2} = \frac{x(x-7) - 2(x-7)}{x - 2}$$

$$= \frac{(x-7)(x-2)}{x-2} = x - 7$$

B. Method of Long Division

1. Divide the first term (x^2) of the divident by the first term (x) of the divisor $x^2 \div x = x$
   Thus, x is the first term of the quotient
2. Multiply the divisor (x + 1) by the first term of the quotient obtained is step 1.
3. Write the like terms of the product $x(x + 1) = x^2 + x$ below the terms of the dividend such that like terms are placed below each other and subtract.
   $(x^2 + 3x + 2) - (x^2 + x) = 2x + 2$
4. Now, divide the first term of the remainder (2x) by the first term (x) of the divisor $2x \div x = 2$
   Thus, 2 is the next term of the quotient.
5. Multiply the divisor (x + 1) by the next term of the quotient (2) obtained in previous step.
6. Write the terms of the product $2(x + 1) = 2x + 2$ below terms of $2x + 2$ (remainder obtained in step 3) such that like terms are placed below each other and subtract
   $(2x + 2) - (2x + 2) = 0$
   Thus, the remainder is 0 and the quotient is x + 2
   To verify the result
   We know

Dividend = Divisor × Quotient + Remainder

R.H.S. = Divisor × Quotient = Remainder

$= (x + 1) \times (x + 2) + 0$

$= x(x + 2) + 1\,(x + 2) + 0$

$= x(x) + x(2) + x + 2$

$= x^2 + 2x + x + 2$

$= x^2 + 3x + 2$

L.H.S. = Dividend = $x^2 + 3x + 2$

As L.H.S. = R.H.S. Hence, verified, that the answer is correct.

$$\begin{array}{r|l} & x + 2 \\ \hline x + 1 & x^2 + 3x + 2 \\ & x^2 \pm x \\ \hline & 2x + 2 \\ & 2x \pm 2 \\ \hline & 0 \end{array}$$

SOLVED EXAMPLES

Example 1

Find the greatest common factor of $6x^3$ and $15\,x^2y$.

Solution

Highest common factor of 6 and 15 is 3 and the highest common factor of x^3 and x^2y is x^2. Hence, the highest common factor of $6x^3$ and $15x^2y$ is $3 \times x^2 = 3x^2$.

Example 2

Factorise ax + by + bx + ay

Solution

$ax + by + bx + ay = (ax + bx) + (by + ay)$
$= x(a + b) + y\,(b + a)$
$= (a + b)(x + y)$

Example 3

Factorise $4a^2 - 25$

Solution

$4a^2 - 25 = (2a)^2 - 5^2$
$= (2a + 5)(2a - 5)$ $[\because\ a^2 - b^2 = (a + b)\,(a - b)]$

Example 4

(i) $4x^2 + 12x + 9$ (ii) $16 - 24x + 9x^2$

Solution

(i) $4x^2 + 12x + 9$
$= (2x)^2 + 2 \times 2x \times 3 + 3^2$
$= (2x + 3)^2 = (2x + 3)(2x + 3)$ $[\because\ (a + b)^2 = a^2 + 2ab + b^2]$

(ii) $16 - 24x + 9x^2 = 4^2 - 2 \times 4 \times 3x + (3x)^2$
$= (4 - 3x)^2 = (4 - 3x)\,(4 - 3x)$ $[\because\ (a - b)^2 = a^2 - 2ab + b^2]$

Example 5

Factorise :

(i) $q^2 - 10q + 21$ (ii) $p^2 + 6p - 16$ (iii) $3x^2 - 9x - 12$

Solution

(i) We have $q^2 - 10q + 21$
Here $a + b = -10$ and $ab = 21$
For $a = -7$ and $b = -3$, we have
$a + b = -10$ and $ab = 21$.

So, $q^2 - 10q + 21 = q^2 - 7q - 3q + 21$
$= q(q - 7) - 3\,(q - 7)$
$= (q - 7)\,(q - 3)$

(ii) We have $p^2 + 6p - 16$
Here $a + b = 6$ and $ab = -16$
For $a = 8$ and $b = -2$, we have
$a + b = 6$ and $ab = -16$

So, $p^2 + 6p - 16 = p^2 + 8p - 2p - 16$
$= p(p + 8) - 2(P + 8)$
$= (p + 8)(p - 2)$

(iii) We have $3x^2 - 9x - 12 = 3(x^2 - 3x - 4)$
Here $a + b = -3$ and $ab = -4$
For $a = -4$ and $b = 1$, we have
$a + b = -3$ and $ab = -4$
So, $3(x^2 - 3x - 4) = 3(x^2 - 4x + x - 4)$
$= 3\{x(x - 4) + 1(x - 4)\}$
$= 3(x - 4)(x + 1)$

Example 6

Factorise completely $x^4 - y^4$

Solution

$x^4 - y^4 = (x^2)^2 - (y^2)^2$
$= (x^2 - y^2)(x^2 + y^2)$ [Using identity $a^2 - b^2 = (a - b)(a + b)$]
$= [(x)^2 - (y)^2](x^2 + y^2)$ [Using identity $a^2 - b^2 = (a - b)(a + b)$]

Example 7

Factorise :

$(p - q)^2 - (p + q)^2$

Solution

$(p - q)^2 - (p + q)^2 = (p - q + p + q)(p - q - p - q)$ [Using identity $a^2 - b^2 = (a + b)(a - b)$]
$= 2p(-2q) = -4pq$

Example 8

Factorise :

(i) $25a^2 - 4b^2 + 28bc - 49c^2$ (ii) $x^4 - (x - z)^4$ (iii) $a^4 - 2a^2b^2 + b^4$

Solution

(i) $25a^2 - 4b^2 + 28bc - 49c^2 = 25a^2 - (4b^2 - 28bc + 49c^2)$
$= 25a^2 - \{(2b)^2 - 2 \times (2b) \times (7c) + (7c)^2\}$
$= (5a)^2 - (2b - 7c)^2$
$= (5a + 2b - 7c)(5a - 2b + 7c)$

(ii) $x^4 - (x - z)^4 = [x^2]^2 - [(x - z)^2]$
$= \{x^2 - (x - z)^2\}\{x^2 + (x - z)^2\}$
$= \{(x - x + z)(x + x - z)\}\{x^2 + x^2 + z^2 - 2zx\}$
$= z(2x - z)(2x^2 + z^2 - 2xz)$

(iii) $a^4 - 2a^2b^2 + b^4 = (a^2)^2 - 2a^2b^2 + (b^2)^2$
$= (a^2 - b^2)^2 = \{(a - b)(a + b)\}^2$
$= (a - b)^2(a + b)^2$

Example 9

Factorise the expressions and divide them as directed :

(i) $(m^2 - 14m - 32) \div (m + 2)$ (ii) $39y^3 (50y^2 - 98) \div 26y^2 (5y + 7)$

Solution

(i) $(m^2 - 14m - 32) \div (m + 2)$

Dividend $= m^2 - 14\,m - 32$

$= m^2 - 16m + 2m - 32$

$= m\,(m - 16) + 2\,(m - 16)$

$= (m - 16)\,(m + 2)$

$\therefore$ $(m^2 - 14\,m - 32) \div (m + 2) = \dfrac{(m-16)(m+2)}{(m+2)} = m - 16$

(ii) Here, the dividend $= 39y^3\,(50y^2 - 98)$

$= 39y^3 \times 2\,(25y^2 - 49)$

$= 2 \times 3 \times 13 \times y^3\,[(5y)^2 - (7)^2]$

$= 2 \times 3 \times 13 \times y^3\,(5y + 7)(5y - 7)$

$\therefore$ $39y^3\,(50y^2 - 98) \div 26y^2(5y + 7) = \dfrac{2\times3\times13\times y^3(5y+7)(5y-7)}{2\times13\times y^2(5y+7)}$

$= 3y\,(5y - 7)$

Example 10

Divide $5x - 6 + x^2$ by $x - 1$ and verify that Dividend = divisor × quotient + remainder.

Solution

Step1. Write $5x - 6 + x^2$ in descending order as $x^2 + 5x - 6$.

$$\begin{array}{r|l} & x+6 \\ \hline x-1 & x^2+5x-6 \\ & x^2-x \\ & - \\ \hline & \quad 6x-6 \\ & \quad 6x-6 \\ & \qquad - \\ \hline & \qquad \times \end{array}$$

Step 2. Divide x^2 (first term of the dividend) by x (first term of the divisor). We get first term of the quotient as x.

Step3. Multiply $(x - 1)$ (while divisor) by x (first term of the quotient), we get $x^2 - x$ as product. Subtract $(x^2 - x)$ from $(x^2 + 5x - 6)$ (dividend), we get $(6x - 6)$.

Step 4. Divide 6x (first term of the remainder) by x, we get + 6 as quotient.

Step5. Multiply $(x - 1)$ (while divisor) by + 6 (second term of the quotient). Finally we see that in this case the remainder is zero.

Check : We know that

Divident = (Divisor × Quotient) + Remainder

$x^2 + 5x - 6 = (x - 1) \times (x + 6) + 0 = x \times (x + 6) - 1 \times (x + 6)$

$= x^2 + 6x - x - 6 + 0 = x^2 + 5x - 6$

$\therefore$ the division is correct.

Example 11

Divide :

(i) **$27x^3 - 64$ by $3x - 4$**

(ii) **$15x^4 + 6x^3 - 7x^2 + 11x - 21$ by $3x^2 + 4$**

Solution

$27x^3 - 64$ can be written as

$27x^3 + 0x^2 + 0x - 64$

Thus, quotient is $9x^2 + 12x + 16$

$$\begin{array}{r|l} & 9x^2 + 12x + 16 \\ 3x - 4 & 27x^3 + 0x^2 + 0x - 64 \\ & \underline{\_27x^3 \mp 36x^2} \\ & \quad 36x^2 + 0x - 64 \\ & \quad \underline{\_36x^2 \mp 48x} \\ & \qquad 48x - 64 \\ & \qquad \underline{\_48x \mp 64} \\ & \qquad \times \end{array}$$

Example 12

Factorise $10x^2 + 15xy^2 + 20z^2$

Solution

$10x^2 = 2 \times 5 \times x \times x$

$15xy^2 = 3 \times 5 \times x \times y \times y$

$20z^2 = 2 \times 2 \times 5 \times z \times z$

$\therefore$ $10x^2 + 15xy + 20z^2 = 2 \times 5 \times x \times y \times y + 2 \times 2 \times 5 \times z \times z$

The common factor is only 5.

$\therefore$ By taking out the common factors, we have

$10x^2 + 15\,xy^2 + 20z^2 = 5 \times (2 \times x \times x + 3 \times x \times y \times y + 4 \times z \times z)$

$15\,xy^2 = 3 \times 5 \times x \times y \times y$

$20\,z^2 = 2 \times 2 \times 5 \times z \times z$

$\therefore$ By taking out the common factors, we have

$10x^2 + 15xy^2 + 20z^2 = 5 \times (2 \times x \times x + 3 \times x \times y \times y + 2 \times 2 \times z \times z)$

$= 5\,(2x^2 + 3xy^2 + 4z^2)$

Example 13

Find the highest common factor in each of the following.

(a) **$45\,x^3y^2$ and $30\,x^4y$** **(b)** **$20\,a^2b^2$ and $25ab^2$**

Solution

(a) $45\,x^3y^2$ and $30\,x^4y$

$45x^3y^2 = 3 \times 3 \times 5 \times x \times x \times x \times y^2$

$30\,x^4y = 3 \times 2 \times 5 \times x \times x \times x \times x \times y$

Highest common factor $= 3 \times 5 \times x \times x \times x \times y = 15x^3y$

(b) $20\,a^2b^2$ and $25ab^2$

$20\,a^2b^2 = 2 \times 2 \times 5 \times a \times a \times b \times b$

$25ab^2 = 5 \times 5 \times a \times b \times b$

Highest common factor $= 5 \times a \times b \times b = 5ab^2$

Example 14

Factorise $9x^2 + 24xy + 16y^2$

Solution

We have, $9x^2 = (3x)^2$ and $16y^2 = (4y)^2$

$24xy = 2 \times 3x \times 4y$

$\therefore \quad 9x^2 + 24xy + 16y^2 = (3x)^2 + 2 \times 3x \times 4y + (4y)^2$

Comparing with $(a + b)^2 = a^2 + 2ab + b^2$

Here, $a = 3x$ and $b = 4y$

$\therefore \quad 9x^2 + 24xy + 16y^2 = (3x + 4y)^2$

Example 15

Factorise $16a^2 – 40xy + 25y^2 = (4a)^2 – 2 \times 4a \times 5y + (5y)^2$

Solution

We see that the first term and the last term are perfect squares and the second term has a negative sign.

$16a^2 = (4a)^2, 25y^2 = (5y)^2$

$40ay = 2 \times 4a \times 5y$

Clearly, this can be compared with

$(a – b)^2 = a^2 – 2ab + b^2$

Here, $a = 4a$ and $b = 5y$

$\therefore \quad 16a^2 – 40\ ay + 25y^2 = (4a)^2 – 2 \times 4a \times 5y + (5y)^2$ which is the factorised form of the given expression.

$\therefore \quad 16a^2 – 40ay + 25y^2 = (4a – 5y)^2$

Example 16

Factorise $25x^2 – 30xy + 9y^2 – 121$.

Solution

The first three terms taken together is given by

$25x^2 – 30xy + 9y^2 = (5x)^2 – 2 \times 5x \times 3y + (3y)^2 = (5x – 3y)^2$

[Using the identity $(a – b)^2 = a^2 – 2ab + b^2$ where $a = 5x$, $b = 3y$]

$\therefore \quad 25x^2 – 30\ xy + 9y^2 – 121 = (5x – 3y)^2 – (11)^2$

We now use the identity on difference of square i.e.,

$a^2 – b^2 = (a + b)(a – b)$ where $a = 5x – 3y$ and $b = 11$

$\therefore \quad (5x – 3y)^2 – (11)^2 = (5x – 3y + 11)\ (5x – 3y – 11)$

This is the required factorisation.

Example 17

Factorise $64p^4 – 25q^4$

Solution

$64p^4 = (8p^2)^2$ and $25\ q^4 = (5q^2)^2$

$64p^4 – 25q^4 = (8p^2)^2 – (5q^2)^2$

We use the identity on difference of squares, $a^2 – b^2 = (a + b)(a – b)$ [Here $a = 8p^2$, $b = 5q^2$]

$\therefore\ 64p^4 – 25q^4 = (8p^2 + 5q^2)\ (8p^2 – 5q^2)$

These are the factors of the given expression.

Example 18

Divide $4p^8 - 6p^6 + 5p^4$ by p^4

Solution

$4p^8 - 6p^6 + 5p^4 \div p^4$

$4p^8 - 6p^6 + 5p^4$ has a common factor p^4.

$\therefore\ 4p^8 - 6p^6 + 5p^4 = p^4\,(4p^4 - 6p^2 + 5)$

$$\therefore\ \frac{4p^8 - 6p^6 + 5p^4}{p^4} = p^4\,\frac{(4p^4 - 6p^2 + 5)}{p^4} = 4p^4 - 6p^2 + 5$$

Example 19

Divide $9x^2 - 16y^2$ by $3x - 4y$

Solution

$$\frac{9x^2 - 16y^2}{3x - 4y} = \frac{(3x)^2 - (4y)^2}{3x - 4y}$$ [Applying the identity $x^2 - y^2 = (x - y)(x + y)$]

$$= \frac{(3x - 4y)(3x + 4y)}{3x - 4y} = 3x + 4y$$

Example 20

Find the quotient and the remainder when $2x^4 - 3x^3 + x^2 + 1$ is divided by $x - 2$.

Solution

$$
\begin{array}{r|l}
 & 2x^3 + x^2 + 3x + 6 \\
\hline
x - 2 & 2x^4 - 3x^3 + x^2 + 1 \\
 & 2x^4 - 4x^3 \\
 & -\quad + \\
\hline
 & x^3 + x^2 \\
 & x^3 - 2x^2 \\
 & -\quad + \\
\hline
 & 3x^2 \\
 & 3x^2 - 6x \\
 & -\quad + \\
\hline
 & 6x + 1 \\
 & 6x - 12 \\
 & -\quad + \\
\hline
 & 13
\end{array}
$$

(Since dividend is having no term containing 'x', so space has been left blank for this term)

(Since after 'x^2' one space is blank in the dividend, therefore nothing has been brought down in this step)

Hence, quotient $= 2x^3 + x^2 + 3x + 6$ and remainder $= 13$

Example 21

Is $x^2 + 1$ a factor of $x^4 + 2x^3 - x^2 - 2x + 1$?

Solution

We divide $x^4 + 2x^3 - x^2 - 2x + 1$ by $x^2 + 1$

$$\begin{array}{r|l} & x^2 + 2x - 2 \\ \hline x^2+1 & x^4 + 2x^3 - x^2 - 2x + 1 \\ & -x^4 \qquad\quad + x^2 \\ & \quad - \qquad\quad - \\ \hline & \quad 2x^3 - 2x^2 - 2x + 1 \\ & \quad 2x^3 \qquad\quad + 2x \\ & \quad - \qquad\quad - \\ \hline & \qquad -2x^2 - 4x + 1 \\ & \qquad\quad -2x^2 \quad -2 \\ & \qquad\quad + \qquad + \\ \hline & \qquad\qquad -4x + 3 \end{array}$$

As the remainder $\neq 0$

Thus, $x^2 + 1$ is not a factor of $x^4 + 2x^3 - x^2 - 2x + 1$

Example 22

$(z + 5)^2 = z^2 + 25$, show the given statement is incorrect.

Solution

$(z + 5)^2 = z^2 + 25$

The given statement is incorrect.

$\because$ $(z + 5)^2 = z^2 + 2(z)(5) + (5)^2 = z^2 + 10z + 25$

$\therefore$ The correct statement is $(z + 5)^2 = z^2 + 10z + 25$

Example 23

$\frac{3x^2+1}{3x^2}$ =1 + 1 = 2, show the given statement is incorrect.

Solution

$$\frac{3x^2+1}{3x^2} = 1 + 1 = 2$$

$\Rightarrow$ The given statement is incorrect.

Since $\frac{3x^2+1}{3x^2} = \frac{3x^2}{3x^2} + \frac{1}{3x^2}$

$\therefore$ The correct statment is $\frac{3x^2+1}{3x^2} = 1 + \frac{1}{3x^2}$

Example 24

$\frac{7x+5}{5} = 7x$, **show the given statement is incorrect.**

Solution

$$\frac{7x+5}{5} = 7x$$

$$\because \quad \frac{7x+5}{5} = \frac{7x}{5} + \frac{5}{5} = \frac{7x}{5} + 1$$

$\therefore$ The correct statement is $\frac{7x+5}{5} = \frac{7x}{5} + 1$

Example 25

Factorise x – 9 + 9zy – xyz

Solution

By regrouping, we have

$x - 9 + 9zy - xyz = x - 9 - xyz + 9zy$

$= 1\,(x - 9) - yz(x - 9)$

$= (x - 9)(1 - yz)$

Example 26

Divide $63(p^4 + 5p^3 - 24p^2)$ by $9p(p + 8)$

Solution

We have $63(p^4 + 5p^3 - 24p^2) \div 9p(p + 8)$

$$= \frac{63(p^4 + 5p^3 - 24p^2)}{9p(p+8)} = \frac{63p^2(p^2 + 5p - 24)}{9p(p+8)}$$

$$= \frac{63p^2}{9p}\left[\frac{p^2 + 8p - 3p - 24}{p+8}\right] = 7p\left[\frac{p(p+8) - 3(p+8)}{p+8}\right]$$

$$= 7p\left[\frac{(p+8)(p-3)}{(p+8)}\right] = p\,(p - 3)$$

Example 27

Divide : $81x^3(50x - 98)$ by $27x^2\,(5x + 7)$

Solution

We have $50x^2 - 98 = 2(25x^2 - 49)$

$= 2[(5x)^2 - (7)^2]$

$= 2[(5x + 7)(5x - 7)]$ [Using $a^2 - b^2 = (a + b)(a - b)$]

Now, $\frac{81x^3[50x^2 - 98]}{27x^2[5x+7]} = \frac{81x^3}{27x^2}\left[\frac{2(5x-7)(5\times+7)}{(5x+7)}\right]$

$= 3x \times [2\,(5x - 7)] = 3x \times 2 \times (5x - 7)$

$= 6x\,(5x - 7)$

Example 28

Find the value of 'k' if the divison of $(kx^3 + 9x^2 + 4x - 10)$ by $(x + 3)$ leaves a remainder -22.

Solution

Let $P(x) = kx^3 + 9x^2 + 4x - 10$, $g(x) = (x + 3)$

Zero of g(x) is -3

$\therefore$ Remainder of division P(x) by g(x) is $P(-3)$

$\therefore$ $P(-3) = k(-3)^3 + 9(-3)^2 + 4(-3) - 10$

$p(-3) = -27k + 81 - 12 - 10 = -22$ (given)

$\therefore$ $-27k + 59 = -22$

$\Rightarrow$ $-27k = -81$

$\Rightarrow$ $k = 3$

Example 29

Factorise : $27x^3 + y^3 + z^3 - 9xyz$

Solution

$27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

$= (3x + y + z)[(3x)^2 + (y)^2 + (z)^2 - (3x)(y) - (y)(z) - (3x)z]$

$\Rightarrow$ $27x^3 + y^3 + z^3 = (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$

Example 30

Factorise : $\mathbf{(p - q)^3 + (q - r)^3 + (r - p)^3}$

Solution

Let $a = p - q$, $b = q - r$, $c = r - p$

We see that $a + b + c = (p - q) + (q - r) + (r - q) = 0$

$\therefore$ $a^3 + b^3 + c^3 = 3abc$

$\therefore$ $(p - q)^3 + (q - r)^3 + (r - p)^3 = 3(p - q)(q - r)(r - p)$

Example 31

Prove that : $\mathbf{\dfrac{0.96 \times 0.96 \times 0.96 + 0.04 \times 0.04 \times 0.04}{0.96 \times 0.96 - 0.96 \times 0.04 \times 0.04 \times 0.04} = 1}$

Solution

$$\frac{0.96 \times 0.96 \times 0.96 + 0.04 \times 0.04 \times 0.04}{0.96 \times 0.96 - 0.04 + 0.04 \times 0.04} = \frac{(0.96)^3 + (0.04)^3}{(0.96)^2 - (0.96)(0.04) + (0.04)^2}$$

$$= \frac{a^3 + b^3}{a^2 - ab + b^2} \text{ where } a = 0.96,\ b = 0.04$$

$$= \frac{(a + b)(a^2 - ab + b^2)}{(a^2 - ab + b^2)} = (a + b)$$

$0.96 + 0.04 = 1$

Example 32

Evaluate : (a) $(204)^2$ (b) $(148)^2$

Solution

(a) $(204)^2 = (200 + 4)^2 = 200^2 + 2 . 200 . 4 + 4^2$

$(204)^2 = 41616$

(b) $(148)^2 = (150 - 2)^2 = (150)^2 - 2.150.2 + 2^2$

$= 22500 - 600 + 4$

$(148)^2 = 21904$

Example 33

Evaluate the following products without direct multiplication.

(a) 103 × 107 (b) 104 × 96

Solution

(a) $103 \times 107 = (100 + 3)(100 + 7) = (100)^2 + (3 + 7) \times 100 + 3 \times 7$

[using $(x + a)(x + b) = x^2 + (a + b)x + ab$]

$= 1000 + 1000 + 21 = 11021$

(b) $104 \times 96 = (100 + 4)(100 - 4)$

$= 100^2 - 4^2$ [using $(x + a)(x - a) = x^2 - a^2$]

$= 9984$

Example 34

Without actual multiplication, find

(a) 109^2 (b) 95^2 (c) 127 × 127 – 73 × 73

Solution

(a) $109^2 = (100 + 9)^2 = 100^2 = 100^2 + 9^2 + 2 \times 100 \times 9$

$= 10000 + 81 + 1800 = 11881$

(b) $95^2 = (100 - 5)^2 = 100^2 + 5^2 - 2 \times 100 \times 5$

$= 10000 + 25 - 1000 = 9025$

(c) $127 \times 127 - 73 \times 73 = (127)^2 - (73)^2$

$= (127 + 73)(127 - 73) = 200 \times 54 = 10800$

Example 35

Factorize : $18a^2 + 12ab - 3a - 2b$

Solution

$18a^2 + 12ab - 3a - 2b = (6a)(3a) + (6a)(2b) - 3a - 2b$

$= 6a [3a + 2b] - 1 [3a + 2b]$

$= (6a - 1)(3a + 2b)$

Example 36

Factorize : $(t^2 - 1)^2 + (t^2 + 1)^2$

Solution

$(t^2 - 1) + (t^2 + 1)^2 = (t^2)^2 - 2t^2 + 1 + (t^2)^2 + 2t^2 + 1$

$= 2(t^4 + 1)$

Example 37

(i) $a^4 (a^2 + 20a + 84) \div a(a + 14)$ **(ii)** $a^2 b (625 a^4 - 81 b^4) \div (ab^2 (5a + 3b)$

Solution

(i) $a^4(a^2 + 20a + 84) \div a(a + 14) = \dfrac{a^4(a^2+20a+84)}{a(a+14)}$

$$= \frac{a^4(a^2+6a+14a+84)}{a(a+14)}$$

$$= \frac{a^4[a(a+6)+14(a+6)]}{a(a+14)}$$

$$= \frac{a^4(a+6)(a+14)}{a(a+14)}$$

$$= a^3 (a + 6)$$

(ii) $\dfrac{a^2b(625a^4-81b^4)}{ab^2(5a+3b)} = \dfrac{(ab)a((25a^2)^2-(9b^2)^2)}{(ab)(b)(5a+3b)}$

$$= \frac{a(25a^2-9b^2)(25a^2+9b^2)}{b(5a+3b)}$$

$$= \frac{a(5a+3b)(5a-3b)(25a^2+9b^2)}{b(5a+3b)}$$

$$= \frac{a(5a-3b)(25a^2+9b^2)}{b}$$

CONCEPT APPLICATION LEVEL - I [NCERT Questions]

EXERCISE - 1

Q.1 Find the common factors of the given terms :

(i)	**12x, 36**	**(ii)**	**2y, 22 xy**
(ii)	**14 pq, $28p^2q^2$**	**(iv)**	**2x, $3x^2$, 4**
(v)	**6abc, 24 ab^2, $12a^2b$**	**(vi)**	**$16x^3$, – $4x^2$, 32x**
(vii)	**10 pq, 20 qr, 30 rp**	**(viii)**	**$3x^2y^3$, $10x^3y^2$, $6x^2y^2z$**

Sol. (i) 12x, 36

$$12x = \underline{2}\times\underline{\underline{2}}\times\underset{\bullet}{3}\times x$$

$$36x = \underline{2}\times\underline{\underline{2}}\times\underset{\bullet}{3}\times 3$$

Common prime factors are 2 (Occurs twice) and 3.

∴ H.C.F. = 2 × 2 × 3 = 12

(ii) 2y, 22xy

$$2y = \underline{2}\times\underline{\underline{y}}$$

$$22xy = \underline{2}\times 11\times x\times\underline{\underline{y}}$$

Common factors are 2 and y.

∴ H.C.F. = 2 × y = 2y

(iii) 14 pq, 28 p^2q^2

$$14pq = \underline{2}\times\underline{\underline{7}}\times\underset{0}{p}\times\underset{w}{q}$$

$$28p^2q^2 = \underline{2}\times 2\times\underline{\underline{7}}\times\underset{0}{p}\times p\times\underset{w}{q}\times q$$

Common factors are 2, 7, p and q.

∴ H.C.F. = 2 × 7 × p × q = 14 pq

(iv) 2x, $3x^2$, 4

$2x = \underline{1} \times 2 \times x$

$3x^2 = \underline{1} \times 3 \times x \times x$

$4 = \underline{1} \times 2 \times 2$

Common factor is 1

∴ H.C.F. = 1

(v) 6abc, $24ab^2$, $12a^2b$

$$6abc = \underline{2}\times\underline{\underline{3}}\times\underset{0}{a}\times\underset{w}{b}\times c$$

$$24ab^2 = \underline{2}\times 2\times 2\times\underline{\underline{3}}\times\underset{0}{a}\times\underset{w}{b}\times c$$

$$12\ a^2b = \underline{2}\times 2\times\underline{\underline{3}}\times\underset{0}{a}\times a\times\underset{w}{b}$$

Common factors are 2, 3, a and b

∴ H.C.F. = 2 × 3 × a × b = 6ab

(vi) $16x^3, -4x^2, 32x$

$16x^3 = \underline{2} \times \underline{\underline{2}} \times 2 \times 2 \times \underset{0}{\underline{x}} \times x \times x$

$-4x^2 = -1 \times \underline{2} \times \underline{\underline{2}} \times \underset{0}{\underline{x}} \times x$

$32x = \underline{2} \times \underline{\underline{2}} \times 2 \times 2 \times 2 \times \underset{0}{\underline{x}}$

Common factors are 2 (occurs twice) and x (occurs once)

$\therefore$ H.C.F. = 2 × 2 × x = 4x

(vii) 10 pq, 20 qr, 30 rp

$10pq = \underline{2} \times \underline{\underline{5}} \times p \times q$

$20\ qr = \underline{2} \times 2 \times \underline{\underline{5}} \times q \times r$

$30rp = \underline{2} \times 3 \times \underline{\underline{5}} \times r \times p$

Common factors are 2 and 5.

$\therefore$ H.C.F. = 2 × 5 = 10

(viii) $3x^2y^3, 10x^3y^2, 6x^2y^2\,z$

$3x^2y^2 = 3 \times \underline{x} \times \underline{\underline{x}} \times \underset{0}{y} \times \underset{00}{y} \times y$

$10x^3y^2 = 2 \times 5 \times \underline{x} \times \underline{\underline{x}} \times x \times \underset{0}{y} \times \underset{00}{y}$

$6x^2y^2z = 2 \times 3 \times \underline{x} \times \underline{\underline{x}} \times \underset{0}{y} \times \underset{00}{y} \times z$

Common factors are x (occurs twice) and y(occurs twice)

$\therefore$ H.C.F. = $x \times x \times y \times y = x^2\,y^2$.

Q.2 Factorise the following expressions :

(i)	**$7x - 42$**	**(ii)**	**$6p - 12\,q$**
(iii)	**$7a^2 + 14\,a$**	**(iv)**	**$-16\,z + 20\,z^3$**
(v)	**$20\,\ell^2m + 30\,a\ell m$**	**(vi)**	**$5x^2y - 15xy^2$**
(vii)	**$10\,a^2 - 15b^2 + 20\,c^2$**	**(viii)**	**$-4a^2 + 4ab - 4ca$**
(ix)	**$x^2yz + xy^2z + xyz^2$**	**(x)**	**$ax^2y + bxy^2 + cxyz$**

Sol. (i) $7x - 42$

$7x = 7 \times x$

$42 = 2 \times 3 \times 7$

$\therefore$ $7x - 72 = 7 \times x - 2 \times 3 \times 7$

$= 7 \times (x - 2 \times 3)$

$= 7 \times (x - 6)$

(ii) $6p - 12p$

$6p = 2 \times 3 \times p$

$12q = 2 \times 2 \times 3 \times q$

$\therefore$ $6p - 12q = 2 \times 3 \times p - 2 \times 2 \times 3 \times q$

$= 2 \times 3 \times (p - 2 \times p)$

$= 6(p - 2q)$

(iii) $\mathbf{7a^2 + 14\,a}$

$7a^2 = 7 \times a \times a$

$14a = 2 \times 7 \times a$

$\therefore$ $7a^2 + 14a = 7 \times a \times a + 2 \times 7 \times a$

$= 7 \times a \times (a + 2) = 7a\,(a + 2)$

(iv) $\mathbf{-16z + 20\,z^3}$

$16z = 2 \times 2 \times 2 \times 2 \times z$

$20z^3 = 2 \times 2 \times 5 \times z \times z \times z$

$\therefore$ $-16z + 20z^3 = (-2 \times 2 \times 2 \times 2 \times z + 2 \times 2 \times 5 \times z \times z \times z)$

$= 4z\,(-4 + 5z^2)$

(v) $\mathbf{20\ell^2 m + 30\,a\ell m}$

$20\ell^2\,m = 2 \times 2 \times 5 \times \ell \times \ell \times m$

$30\,a\ell m = 2 \times 3 \times 5 \times a \times \ell \times m$

$20\ell^2 m + 30\,a\ell m = 2 \times 2 \times 5 \times \ell \times m \times m + 2 \times 3 \times 5 \times a \times \ell \times m$

$= 2 \times 5 \times \ell \times m \times (2 \times \ell + 3 \times a)$

$= 10\,\ell m\,(2\ell + 3a)$

(vi) $\mathbf{5x^2y - 15xy^2}$

$5x^2y = 5 \times x \times x \times y$

$15\,xy^2 = 3 \times 5 \times x \times y \times y$

$\therefore$ $5x^2y - 15xy^2 = 5 \times x \times x \times y - 3 \times 5 \times x \times y \times y$

$= 5 \times x \times y \times (x - 3 \times y)$

$= 5xy\,(x - 3y)$

(vii) $\mathbf{10a^2 - 15b^2 + 20\,c^2}$

$10a^2 = 2 \times 5 \times a \times a$

$15b^2 = 3 \times 5 \times b \times b$

$20c^2 = 2 \times 2 \times 5 \times c \times c$

$\therefore$ $10a^2 - 15b^2 + 20c^2 = 2 \times 5 \times a \times a - 3 \times 5 \times b \times b + 2 \times 2 \times 5 \times c \times c$

$= 5 \times (2 \times a \times a - 3 \times b \times b + 2 \times 2 \times c \times c)$

$= 5(2a^2 - 3b^2 + 4c^2)$

(viii) $\mathbf{-4a^2 + 4ab - 4ac}$

$4a^2 = 2 \times 2 \times a \times a$

$4ab = 2 \times 2 \times a \times b$

$4ca = 2 \times 2 \times c \times a$

$\therefore$ $-4a^2 + 4ab - 4ca + 2 \times 2 \times a \times b - 2 \times 2 \times c \times a$

$= 2 \times 2 \times a \times (-a + b - c)$

$= 4a(-a + b - c)$

(ix) $\mathbf{x^2yz = x \times x \times y \times z}$

$xy^2\,z = x \times y \times y \times z$

$xyz^2 = x \times y \times z \times z$

$\therefore$ $x^2yz + xy^2z + xyz^2 = x \times x \times y \times z + x \times y \times y \times z + x \times y \times z \times z$

$= x \times y \times z \times (x + y + z)$

$= xyz\,(x + y + z)$

(x) $ax^2y + bxy^2 + cxyz$

$ax^2y = a \times x \times x \times y$

$bxy^2 = b \times x \times y \times y$

$cxyz = c \times x \times y \times z$

$\therefore$ $ax^2y + bxy^2 + cxyz = a \times x \times x \times y + b \times x \times y \times y + c \times x \times y \times z$

$= x \times y \times (a \times x + b \times y + c \times z)$

$= xy(ax + by + cz)$

Q.3 Factorise

(i) $x^2 + xy + 8x + 8y$ **(ii)** $15xy - 6x + 5y - 2$

(iii) $ax + bx - ay - by$ **(iv)** $15pq + 15 + 9q + 25p$

(v) $z - 7 + 7xy - xyz.$

Sol. (i) $x^2 + xy + 8x + 8y = x(x + y) + 8(x + y)$

$= (x + y)(x + 8)$ Taking $(x + y)$ common

(ii) $15xy - 6x + 5y - 2 = 3x(5y - 2) + 1(5y - 2)$

$= (5y - 2)(3x + 1)$ Taking $(5y - 2)$ common

(iii) $ax + bx - ay - by = x(a + b) - y(a + b)$

$= (a + b)(x - y)$ Taking $(a + b)$ common

(iv) $15pq + 15 + 9q + 25p = 15pq + 9q + 25p + 15$

$= 3q(5p + 3) + 5(5p + 3)$

$= (5p + 3)(3q + 5)$ Taking $(5p + 3)$ common

(v) $z - 7 + 7xy - xyz = z - 7 - xyz + 7xy$

$= 1(z - 7) - xy(z - 7)$

$= (z - 7)(1 - xy)$ Taking $(z - 7)$ common

EXERCISE - 2

Q.1 Factorise the following expressions :

(i) $a^2 + 8a + 16$ **(ii)** $p^2 - 10p + 25$ **(iii)** $25m^2 + 30m + 9$

(iv) $49y^2 + 84yz + 36z^2$ **(v)** $4x^2 - 8x + 4$ **(vi)** $121b^2 - 88bc + 16c^2$

(vii) $(\ell + m)^2 - 4\ell m$ **(Hint : Expand $(\ell + m)^2$ first)**

(viii) $a^4 + 2a^2b^2 + b^4$

Sol. (i) $a^2 + 8a + 16 = (a)^2 + 2(a)(4) + (4)^2$

$= (a + 4)^2$ Applying Identity I

(ii) $p^2 - 10p + 25 = (p)^2 - 2(p)(5) + (5)^2$

$= (p - 5)^2$ Using Identity II

(iii) $25m^2 + 30m + 9 = (5m)^2 + 2(5m)(3) + (3)^2$

$= (5m + 3)^2$ Applying Identity I

(iv) $49y^2 + 84yz + 36z^2 = (7y)^2 + 2(7y)(6z) + (6z)^2$

$= (7y + 6z)^2$ Using Indentity I

(v) $4x^2 - 8x + 4 = 4(x^2 - 2x + 1)$

$= 4[(x)^2 - 2(x)(1) + (1)^2]$

$= 4(x - 1)^2$ Applying Identity II

(vi) $\mathbf{121b^2 - 88bc + 16c^2}$ $= (11b)^2 - 2(11b)(4c) + (4c)^2$
$= (11b - 4c)^2$ Using Identity II

(vii) $\mathbf{(\ell + m) - 4\ell m}$ $= (\ell^2 + 2\ell m + m^2) - 4\ell m$ Using Identity I
$= \ell^2 + (2\ell m - 4\ell m) + m^2$ Combining the line terms
$= \ell^2 - 2\ell m + m^2$
$= (\ell)^2 - 2(\ell)(m) + (m)^2$
$= (\ell - m)^2$ Applying Identity II

(viii) $\mathbf{a^4 + 2a^2b^2 + b^4}$ $= (a^2)^2 + 2(a^2)(b^2) + (b^2)^2$
$= (a^2 + b^2)^2$ Using Identity I

Q.2 Factorise :

(i) $\mathbf{4p^2 - 9q^2}$ **(ii)** $\mathbf{63a^2 - 112b^2}$ **(iii)** $\mathbf{49x^2 - 36}$
(iv) $\mathbf{16x^5 - 144x^3}$ **(v)** $\mathbf{(\ell + m) - (\ell - m)^2}$ **(vi)** $\mathbf{9x^2y^2 - 16}$
(vii) $\mathbf{(x^2 - 2xy + y^2) - z^2}$ **(viii)** $\mathbf{25a^2 - 4b^2 + 28bc - 49c^2}$

Sol. **(i)** $\mathbf{4p^2 - 9q^2}$ $= (2p)^2 - (3q)^2$
$= (2p - 3q)(2p + 3q)$ Using Identity III

(ii) $\mathbf{63a^2 - 112b^2}$ $= 7(9a^2 - 16b^2)$
$= \{(3a)^2 - (4b)^2\}$
$= 7(3a - 4b)(3a + 4b)$ Applying Identity III

(iii) $\mathbf{49x^2 - 36}$ $= (7x)^2 - (6)^2$
$= (7x - 6)(7x + 6)$ Using Identity III

(iv) $\mathbf{16x^5 - 144x^3}$ $= 16x^3(x^2 - 9)$
$= 16x^3\{(x)^2 - (3)^2\}$
$= 16x^3(x - 3)(x + 3)$ Using Identity III

(v) $\mathbf{(\ell + m)^2 - (\ell - m)^2}$ $= \{(\ell + m) - (\ell - m)\}\{(\ell + m) + (\ell - m)\}$
Applying Identity III
$= (2m)(2\ell)$
$= 4\ell m$

(vi) $\mathbf{9x^2y^2 - 16}$ $= (3xy)^2 - (4)^2$
$= (3xy - 4)(3xy + 4)$ Using Identity III

(vii) $\mathbf{(x^2 - 2xy + y^2) - z^2}$ $= (x - y)^2 - z^2$ Using Identity II

(viii) $\mathbf{25a^2 - 4b^2 + 28bc - 49c^2}$ $= 25a^2 - \{(2b)^2 - 2(2b)(7c) + (7c)^2\}$
$= (5a)^2 - (2b - 7c)^2$ Using Identity II
$= \{5a - (2b - 7c)\}\{5a + (2b - 7c)\}$
$= (5a - 2b + 7c)(5a + 2b - 7c)$

Q.3 Factorise the expressions :

(i) $ax^2 + bx$ **(ii)** $7p^2 + 21q^2$

(iii) $2x^3 + 2xy^2 + 2xz^2$ **(iv)** $am^2 + bm^2 + bn^2 + an^2$

(v) $(\ell m + \ell) + m + 1$ **(vi)** $y(y + z) + 9(y + z)$

(vii) $5y^2 - 20y - 8z + 2yz$ **(viii)** $10ab + 4a + 5b + 2$

(ix) $6xy - 4y + 6 - 9x$

Sol. **(i)** $ax^2 + bx = x(ax + b)$

(ii) $7p^2 + 21q^2 = 7(p^2 + 3q^2)$

(iii) $2x^3 + 2xy^2 + 2xz^2 = 2x(x^2 + y^2 + z^2)$

(iv) $am^2 + bm^2 + bn^2 + an^2 = am^2 + bm^2 + an^2 + bn^2$

$= m^2(a + b) + n^2(a + b)$

$= (a + b)(m^2 + n^2)$

(v) $(\ell m + \ell) + m + 1 = \ell(m + 1) + 1(m + 1)$

$= (m + 1)(\ell + 1)$

(vi) $y(y + z) + 9(y + z) = (y + z)(y + 9)$

(viii) $5y^2 - 20y - 8z + 2yz = 5y^2 - 20y + 2yz - 8z$

$= 5y(y - 4) + 2z(y - 4)$

$= (y - 4)(5y + 2z)$

(viii) $10ab + 4a + 5b + 2 = 2a(5a + 2) + 1(5b + 2)$

$= (5b + 2)(2a + 1)$

(ix) $6xy - 4y + 6 - 9x = 6xy - 4y - 9x + 6$

$= 2y(3x - 2) - 3(3x - 2)$

$= (3x - 2)(2y - 3)$

Q.4 Factorise :

(i) $a^4 - b^4$ **(ii)** $p^4 - 81$ **(iii)** $x^4 - (y + z)^4$

(iv) $x^4 - (x - z)^4$ **(v)** $a^4 - 2a^2b^2 + b^4$

Sol. **(i)** $a^4 - b^4 = (a^2)^2 - (b^{2m})^2$

$= (a^2 - b^2)(a^2 + b^2)$

$= (a - b)(a + b)(a^2 + b^2)$ Using Identity III

(ii) $p^4 - 81 = (p^2)^2 - (9)^2$

$= (p^2 - 9)(p^2 + 9)$ Using Identity III

$= \{(p)^2 - (3)^2 (p^2 + 9)\}$

$= (p - 3)(p + 3)(p^2 + 9)$ Using Identity III

(iii) $x^4 - (y + z)^4 = (x^2)^2 - \{(y + z)^2\}^2$

$= \{x^2 - (y + z)^2\}\{x^2 + (y + z)^2\}$ Using Identity III

$= \{x - (y + z)\}\{x + (y + z)\}$

$= \{x^2 + (y + z)^2\}$ Using Identity III

$= (x - y - z)(x + y + z)\{x^2 + (y + z)^2\}$

(iv) $\mathbf{x^4 - (x-z)^4} = (x^2)^2 - \{(x-z)^2\}^2$

$= \{x^2 - (x-z)^2\}\{x^2 + (x-z)^2\}$ Using Identity III

$= \{x - (x-z)\}\{x + (x-z)\}\{x^2 + (x-z)^2\}$ Applying Identity III

$= (x - x + z)(x + x - z)\{x^2 + (x-z)^2\}$

$= z(2x - z)\{x^2 + (x-z)^2\}$

$= z(2x - z)(x^2 + x^2 - 2xz + z^2)$ Using Identity II

$= z(2x - z)(2x^2 - 2xz + z^2)$

(v) $\mathbf{a^4 - 2a^2b^2 + b^4} = (a^2)^2 - 2(a^2)(b^2) + (b^2)^2$

$= (a^2 - b^2)^2$ Using Identity II

$= \{(a-b)^2 (a+b)^2\}$

Q.5 Factorise the following expressions.

(i) $\mathbf{p^2 + 6p + 8}$ **(ii)** $\mathbf{q^2 - 10q + 21}$ **(iii)** $\mathbf{p^2 + 6p - 16}$

Sol. (i) $\mathbf{p^2 + 6p + 8} = p^2 + 6p + 9 - 1$

$= \{(p)^2 + 2(p)(3) + (3)^2\} - (1)^2$

$= (p+3)^2 - (1)^2$ Using Identity I

$= (p + 3 - 1)(p + 3 + 1)$ Using Identity III

$= (p+2)(p+4)$

(ii) $\mathbf{q^2 - 10q + 21} = q^2 - 10q + 25 - 4$

$= \{(q)^2 - 2(q)(5) + (5)^2\} - 4$

$= (q-5)^2 - (2)^2$ Using Identity II

$= (q - 5 - 2)(q - 5 + 2)$ Using Identity III

$= (q-7)(q-3)$

(iii) $\mathbf{p^2 + 6p - 16} = p^2 + 6p + 9 - 25$

$= (p)^2 + 2(p)(3) + (3)^2 - (5)^2$

$= (p+3)^2 - (5)$ Using Identity I

$= (p + 3 - 5)(p + 3 + 5)$ Applying Identity III

$= (p-2)(p+8)$

EXERCISE - 3

Q.1 Carry out the following divisions :

(i) $\mathbf{28x^4 \div 56x}$ **(ii)** $\mathbf{-36y^3 \div 9y^2}$

(iii) $\mathbf{66pq^2r^3 \div 11qr^2}$ **(iv)** $\mathbf{34x^3y^3z^3 \div 51xy^2z^2}$

(v) $\mathbf{12a^8b^8 \div (-6a^6b^4)}$

Sol. (i) $\mathbf{28x^4 + 56x}$

$$28x^4 \div 56x = \frac{28x^4}{56x} = \frac{2\times2\times7\times x\times x\times x\times x}{2\times2\times2\times7\times x} = \frac{x\times x\times x}{2} = \frac{x^3}{2}$$

(ii) $\mathbf{-36y^3 \div 9y^2} = \frac{-36y^3}{9y^2} = \frac{-2\times2\times3\times3\times y\times y\times y}{3\times3\times y\times y}$

$= -2 \times 2 \times y = -4y$

(iii) $66pq^2r^3 \div 11\,qr^2 = \frac{66pq^2r^3}{11qr^2} = \frac{2\times3\times11\times p\times q\times q\times r\times r\times r}{11\times q\times r\times r}$

$= 2 \times 3 \times p \times q \times r = 6\,pqr$

(iv) $34x^3y^3z^3 \div 51xy^2z^3 = \frac{34x^3y^3z^3}{51xy^2z^3}$

$= \frac{2\times17\times x\times x\times x\times y\times y\times y\times z\times z\times z}{3\times17\times x\times y\times y\times z\times z\times z} = \frac{2\times x\times x\times y}{3} = \frac{2}{3}x^2y$

(v) $12a^8b^8 \div (-6a^6b^4) = \frac{12a^8b^8}{-6a^6b^4}$

$= \frac{2\times2\times3\times a\times a\times a\times a\times a\times a\times a\times a\times b\times b\times b\times b\times b\times b\times b\times b}{-2\times3\times a\times a\times a\times a\times a\times a\times b\times b\times b\times b}$

$= -2 \times a \times a \times b \times b \times b \times b$

$= -a^2b^4$

Q.2 Divide the given polynomial by the given monomial :

(i) $(5x^2 - 6x) \div 3x$ **(ii)** $(3x^8 - 4y^6 + 5y^4) \div y^4$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$ **(iv)** $(x^3 + 2x^2 + 3x) \div 2x$

(v) $(p^3q^6 - p^6q^3) \div p^3q^3$

Sol. (i) $(5x^2 - 6x) \div 3x = \frac{5x^2-6x}{3x} = \frac{5x^2}{3x} - \frac{6x}{3x} = \frac{5}{3}x - 2 = \frac{1}{3}(5x-6)$

(ii) $(3x^8 - 4y^6 + 5y^4) \div y^4 = \frac{3y^8-4y^6+5y^4}{y^4} = \frac{3y^8}{y^4} - \frac{4y^6}{y^4} + \frac{5y^4}{y^4} = 3y^4 - 4y^2 + 5$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2 = \frac{8(x^3y^2z^2+x^2y^3z^2+x^2y^2z^3)}{4x^2y^2z^2}$

$= \frac{8x^2y^2z^2(x+y+z)}{4x^2y^2z^2} = 2(x+y+z)$

(iv) $(x^3 + 2x^2 + 3x) \div 2x = \frac{x^3+2x^2+3x}{2x} = \frac{x\times(x^2+2x+3)}{2\times x} = \frac{1}{2}(x^2+2x+3)$

(v) $(p^3q^6 - p^6q^3) \div p^3q^3 = \frac{p^3q^6-p^6q^3}{p^3q^3} = \frac{p^3q^3(q^3-p^3)}{p^3q^3}$

$= q^3 - p^3.$

Q.3 Work out the following divisions :

(i) $(10x - 25) \div 5$

(ii) $(10x - 25) \div (2x - 5)$

(iii) $10y\,(6y + 21) \div 5(2y + 7)$

(iv) $9x^2y^2(3z - 24) \div 27\,xy(z - 8)$

(v) $96\,abc\,(3a - 12)\,(5b - 30) \div 144\,(a - 4)\,(b - 6).$

Sol. **(i)** $(10x - 25) \div 5 = \dfrac{5(2x-5)}{5} = 2x - 5$

(ii) $(10x - 25) \div (2x - 5) = \dfrac{10x-25}{2x-5} = \dfrac{5(2x-5)}{2x-5} = 5$

(iii) $10y\,(6y + 21) \div 5\,(2y + 7) = \dfrac{10y(6y+21)}{5(2y+7)} = \dfrac{10y\times 3(2y+7)}{5(2y+7)} = 6y$

(iv) $9x^2y^2(3z - 24) \div 27xy(z - 8) = \dfrac{9x^2y^2(3z-24)}{27xy(z-8)} = \dfrac{9x^2y^2\times 3(z-8)}{27xy(z-8)} = xy$

(v) $96abc\,(3 - 12)(5b - 30) \div 144\,(a - 4)(b - 6)$

$$= \frac{96abc(3a-12)(5b-30)}{144(a-4)(b-6)} = \frac{96abc(3a-4)\times 5(b-6)}{144(a-4)(b-6)} = 10\ abc.$$

Q.4 Divide as directed.

(i) $5(2x + 1)(3x + 5) \div (2x + 1)$

(ii) $26xy(x + 5)(y - 4) \div 13\,x(y - 4)$

(iii) $52pqr\,(p + q)(q + r)(r + p) \div 104\,pq(q + r)(r + p)$

(iv) $20\,(y + 4)\,(y^2 + 5y + 3) \div 5\,(y + 4)$

(v) $x(x + 1)(x + 2)(x + 3) \div x(x + 1)$

Sol. **(i)** $5(2x + 1)(3x + 5) \div (2x + 1) = \dfrac{5(2x+1)(3x+5)}{2x+1} = 5(3x + 5)$

(ii) $26xy(x + 5)(y - 4) \div 13\,x(y - 4) = \dfrac{26xy(x+5)(y-4)}{13x(y-4)} = 2y(x + 5)$

(iii) $52pqr\,(p + q)(q + r)(r + p) \div 104\,pq(q + r)(r + p)$

$$= \frac{52pqr(p+q)(q+r)(r+p)}{104pq(q+r)(r+p)} = \frac{1}{2}\ r(p + q)$$

(iv) $20\,(y + 4)\,(y^2 + 5y + 3) \div 5\,(y + 4) = \dfrac{20(y+4)(y^2+5y+3)}{5(y+4)} = 4(y^2 + 5y + 3)$

(v) $x(x + 1)(x + 2)(x + 3) \div x(x + 1) = \dfrac{x(x+1)(x+2)(x+3)}{x(x+1)} = (x+2)(x+3)$

Q.5 Factorise the expressions and divide them as directed.

(i) $(y^2 + 7y + 10) \div (y + 5)$ **(ii)** $(m^2 - 14m - 32) \div (m + 2)$

(iii) $(5p^2 - 25p + 20) \div (p - 1)$ **(iv)** $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

(v) $5pq(p^2 - q^2) \div 2p(p + q)$ **(vi)** $12xy\ (9x^2 - 16y^2) \div 4xy(3x + 4y)$

(vii) $39\ y^3(50\ y^2 - 98) \div 26y^2(5y + 7)$

Sol. **(i)** $(y^2 + 7y + 10) \div (y + 5) = \dfrac{y^2+7y+10}{y+5} = \dfrac{y^2+2y+5y+10}{y+5}$ Using Identity IV

$$= \frac{y(y+2)+5(y+2)}{y+5} = \frac{(y+2)(y+5)}{y+5} = y + 2$$

(ii) $(m^2 - 14m - 32) \div (m + 2) = \dfrac{m^2-14m-32}{m+2}$

$$= \frac{m^2-16m+2m-32}{m+2}$$ Using Identity IV

$$= \frac{m(m-16)+2m(m-16)}{m+2}$$

$$= \frac{(m-16)(m+2)}{m+2} = m - 16$$

(iii) $(5p^2 - 25p + 20) \div (p - 1) = \dfrac{5(p^2-5p+4)}{p-1}$

$$= \frac{5(p^2-p-4p+4)}{p-1}$$ Applying

$$= \frac{5\{p(p-1)-4(p-1)\}}{p-1}$$

$$= \frac{5(p-1)(p-4)}{p-1} = 5\ (p - 4)$$

(iv) $4yz(z^2 + 6z - 16) + 2y(z + 8)$

$$4yz(z^2+6z-16) \div 2y(z+8) = \frac{4yz(z^2+6z-16)}{2y(z+8)} = \frac{2z(z^2+6z-16)}{z+8}$$

$$= \frac{2z(z^2+8z-2z-16)}{z+8}$$ Using Identity IV

$$= \frac{2z[z(z+8)-2(z+8)]}{z+8}$$

$$= z\ (z - 2)$$

(v) $\mathbf{5pq(p^2 - q^2) \div 2p(p + q)} = \dfrac{5pq(p^2 - q^2)}{2p(p+q)}$

$= \dfrac{5pq(p+q)(p-q)}{2p(p+q)}$ Using Identity III

$= \dfrac{5}{2}\, q\,(p - q)$

(iv) $\mathbf{12xy\,(9x^2 - 16y^2) \div 4xy(3x + 4y)} = \dfrac{12xy(9x^2 - 16y^2)}{4xy(3x+4y)} = \dfrac{3(9x^2 - 16y^2)}{3x+4y}$

$= \dfrac{3\{(3x)^2 - (4y^2)\}}{3x+4y} = \dfrac{3(3x+4y)(3x-4y)}{3x+4y}$

$= 3(3x - 4y)$

(vii) $\mathbf{39y^3\,(50y^2 - 98) \div 26y^2(5y + 7)} = \dfrac{39y^3(50y^2 - 98)}{26y^2(5y+7)} = \dfrac{39y^3 \times 2 \times (25y^2 - 49)}{26y^2(5y+7)}$

$= \dfrac{39y^3 \times 2 \times \{(5y)^2 - (7)^2\}}{26y^2(5y+7)}$

$= \dfrac{39y^3 \times 2 \times (5y+7)(5y-7)}{26y^2(5y+7)}$ Using Identity III

$= 3y\,(5y - 7)$

EXERCISE - 4

Find and correct the errors in the following mathematical statements.

Q.1 $\mathbf{4(x - 5) = 4x - 5}$

Sol. $4(x - 5) = 4x - 20$

Q.2 $\mathbf{x(3x + 2) = 3x^2 + 2}$

Sol. $x(3x + 2) = 3x^2 + 2x$

Q.3 $\mathbf{2x + 3y = 5xy}$

Sol. $2x + 3y = 2x + 3y$

Q.4 $\mathbf{x + 2x + 3x = 5x}$

Sol. $x + 2x + 3x = 6x$

Q.5 $\mathbf{5y + 2y + y - 7y = 0}$

Sol. $5y + 2y + y - 7y = y$

Q.6 $\mathbf{3x + 2x = 5x^2}$

Sol. $3x + 2x = 5x$

Q.7 $(2x)^2 + 4(2x)_0 + 7 = 2x^2 + 8x + 7$

Sol. $(2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7$

Q.8 $(2x)^2 + 5x = 4x + 5x = 9x$

Sol. $(2x)^2 + 5x = 4x^2 + 5x$

Q.9 $(3x + 2)^2 = 3x^2 + 6x + 4$

Sol. $(3x + 2)^2 = 9x^2 + 12x + 4$

Q.10 **Substituting x = – 3 in**

(a) $x^2 + 5x + 4$ **gives** $(-3)^2 + 5(-3) + 4$

$= 9 + 2 + 4 = 15$

Sol. $x^2 + 5x + 4 = (-3)^2 - 5(-3) + 4$

$= 9 - 15 + 4$

$= -2$ and not 15

(b) $x^2 - 5x$ **gives** $(-3)^2 - 5(-3) + 4$

$= 9 - 15 + 4 = -2$

Sol. $x^2 - 5x + 4 = (-3)^2 - 5(-3) + 4$

$= 9 + 15 + 4$

$= 28$ and not -2

Q.11 $(y - 3)^2 = y^2 - 9$

Sol. $(y - 3)^2 = y^2 - 2(y)(3) + (3)^2$

$= y^2 - 6y + 9$ and not equal to $y^2 - 9$

Q.12 $(z + 5)^2 = z^2 + 25$

Sol. $(z + 5)^2 = z^2 + 2(z)(5) + (5)^2$

$= z^2 + 10z + 25$ and not equal to $z^2 + 25$

Q.13 $(2a + 3b)(a - b) = 2a^2 - 3b^2$

Sol. $2a + 3b(a - b) = 2a(a - b) + 3b(a - b)$

$= 2a^2 - 2ab + 3ba - 3b^2$

$= 2a^2 + ab - 3b^2$ and not equal to $2a^2 - 3b^2$

Q.14 $(a + 4)(a + 2) = a^2 + 8$

Sol. $(a + 4)(a + 2) = a(a + 2) + 4(a + 2)$

$= a^2 + 2a + 4a + 8$

$= a^2 + 6a + 8$ and not equal to $a^2 + 8$

Q.15 $(a - 4)(a - 2) = a^2 - 8$

Sol. $(a - 4)(a - 2) = a(a - 2) - 4(a - 2)$

$= a^2 - 2a - 4a + 8$

$= a^2 - 6a + 8$ and not equal to $a^2 - 8$

Q.16. $\frac{3x^2}{3x^2} = 0$

Sol. $\frac{3x^2}{3x^2} = 1$ and not equal to 0

Q.17. $\frac{3x^2+1}{3x^2}=1+1=2$

Sol. $\frac{3x^2+1}{3x^2}=\frac{3x^2}{3x^2}+\frac{1}{3x^2}\ =1+\frac{1}{3x^2}$ and no equal to 1 + 1 = 2

Q.18 $\frac{3x}{3x+2}=\frac{1}{2}$

Sol. $\frac{3x}{3x+2}=\frac{3x}{3x+2}$ and not equal to $\frac{1}{2}$

Q.19 $\frac{3}{4x+3}=\frac{1}{4x}$

Sol. $\frac{3}{4x+3}=\frac{3}{4x+3}$ and not equal to $\frac{1}{4x}$

Q.20 $\frac{4x+5}{4x}=5$

Sol. $\frac{4x+5}{4x}=\frac{4x}{4x}+\frac{5}{4x}$ and not equal to 5

Q.21 $\frac{7x+5}{5}=7x$

Sol. $\frac{7x+5}{5}=\frac{7x}{5}+\frac{5}{5}=\frac{7x}{5}+1$ and not equal to 7x

TRY THESE

Q.1 **Factorise :**

(i) **12x + 36** **(ii)** **22y – 33 z** **(iii)** **14 pq + 35 pqr**

Sol. **(i)** **12x + 6**

We have

$12x = 2 \times 2 \times 3 \times x$

$36 = 2 \times 2 \times 3 \times 3$

The two term have 2, 2 and 3 as common factors.

Therefore,

12x + 36

$= (2 \times 2 \times 3 \times x) + (2 \times 2 \times 3 \times 3)$

$= 2 \times 2 \times 3 \times (x + 3)$

Combining the terms

$= 12 \times (x + 3) = 12\ (x + 3)$ Required factor form

(ii) 22y – 33z

We have

$22y = 2 \times 11 \times y$

$33z = 3 \times 11 \times z$

The two terms have 11 as common factor.

Therefore,

$22y - 33z = (11 \times 2 \times y) - (11 \times 3 \times z)$

$= 11 \times [(2 \times y) - (3 \times z)]$

Combining the terms

$= 11 \times (2y - 3z)$

$= 11\,(2y - 3z)$

Required factor form

(iii) 14 pq + 35 pqr

we have

$14pq = 2 \times 7 \times p \times q$

$35pqr = 5 \times 7 \times p \times q \times r$

The terms have 7, p and q as common factors.

Therefore,

$14pq + 35pqr = 7 \times p \times q \times 2 + 7 \times p \times q \times 5 \times r$

$= 7 \times p \times q \times [2 + (5 \times r)] \Rightarrow 7pq\,(2 + 5r)$

Required factors form

Q.2 Divide :

(i) $24xy^2z^3$ by $6yz^2$ **(ii) $63a^2b^4c^6$ by $7a^2b^2c^3$**

Sol. (i) $24xy^2z^3$ by $6yz^2$ $= \dfrac{2\times2\times2\times3\times x\times y\times y\times z\times z\times z}{2\times3\times y\times z\times z}$

$$= \frac{2\times2\times x\times y\times z}{1} = 4xyz$$

(ii) $63a^2b^4c^6$ by $7a^2b^2c^3$

$$63a^2b^4c^6 \div 7a^2b^2c^3 = \frac{3\times3\times7\times a\times a\times b\times b\times b\times b\times c\times c\times c\times c\times c\times c}{7\times a\times a\times b\times b\times c\times c\times c}$$

$$= \frac{3\times3\times b\times b\times c\times c\times c}{1} = 9\,b^2c^3$$

CONCEPT APPLICATION LEVEL - II

SECTION - A

➢ **Fill in the blanks**

Q.1 Divide $\left(x^2+\frac{1}{x^2}+2\right)$ by $\left(x+\frac{1}{x}\right)$ ________________

Q.2 Express 10 xy (x + 3) as irreducible factor form ________________

Q.3 Divide $-15m^2n$ by -5 mn ________________

Q.4 Divide $a^2x^2 - 25$ by $(ax + 5)$ ________________

Q.5 Factorise : $x^4 - 1$. ________________

Q.6 The process of writing a given expression as the product of two or more factors is called ________________ Factorization.

Q.7 If 'a' is any rational number, then a × a × a × ________________ m times.

Q.8 $\frac{9x^2-16}{6x+8}$ is written in its lowest terms as = ________________ .

Q.9 $4x^2 + 6xy =$ ________________

Q.10 $x^2 + 11x + 24 =$ ________________

Q.11 $x^2 - 11x + 28 =$ ________________.

Q.12 $4x^2 - 169y^2 =$ ________________.

Q.13 $4x^2 + 28x + 49 =$ ________________.

SECTION - B

➢ **Multiple Choice Questions**

Q.1 Which of the following are the factor of $1 - x^2$?
(A) $(x + 1)(x - 1)$ (B) $(1 - x)(1 + x)$ (C) $(1 - x)(1 - x)$ (D) $(1 - x)(1 - x)$

Q.2 Which of the following is the common factor of :
5xy, 3pqr and 40 xyz ?
(A) 5 (B) 0 (C) xy (D) 1

Q.3 Which of the following is quotient obtained on dividing $-18xyz^2$ by $-3xz$?
(A) 6 yz (B) – 6 yz (C) $6xy^2$ (D) 6 xy

Q.4 Which of the following is quotient obtained on dividing $(x^2 - b)(x - a)$ by $-(x - a)$?
(A) $(x^2 - b)$ (B) $\frac{-(x^2-b)}{(x-a)}$ (C) $-(x^2 - b)$ (D) $-(x + a)$

Q.5 Which of the following is true ?
(A) $ab - a - b + 1 = (1 + a)(1 - b)$
(B) $ab - a - b + 1 = (a - 1)(b - 1)$
(C) $ab - a - b + 1 = (1 - a)(b - 1)$
(D) $ab - a - b + 1 = (a - 1)(1 - b)$

Q.6 Which of the following is equal to $x^3 - 225x$
(A) $x(1 - 15x)(1 + 15x)$
(B) $x(x - 15)(x + 15)$
(C) $x(1 - 15x)(1 - 15x)$
(D) $x(1 + 15x)(1 - 15x)$

Q.7 Which of the following is the quotient when $44x^2(x^2 - 5x - 24)$ is divided by $22x(x - 8)$:
(A) $x(x + 3)$ (B) $2x(x + 3)$ (C) $2(x - 3)$ (D) $x(x - 3)$

Q.8 By which of the following $a^4 - b^4$ be divided to get quotient $(a^2 + b^2)(a - b)$ and, remainder as 0.:
(A) $a^2 + b^2$ (B) $a - b$ (C) $a + b$ (D) $a^2 - b^2$

Q.9 Factorise : $\left(5x - \frac{1}{x}\right)^2 + 5\left(5x - \frac{1}{x}\right) + 6$

(A) $\left(5x - \frac{1}{x} + 3\right)\left(5x - \frac{1}{x} + 2\right)$ (B) $\left(5x - \frac{1}{x} - 3\right)\left(5x - \frac{1}{x} - 2\right)$

(C) $\left(5x - \frac{1}{x} + 3\right)\left(5x - \frac{1}{x} - 2\right)$ (D) $\left(5x - \frac{1}{x} - 3\right)\left(5x - \frac{1}{x} + 2\right)$

Q.10 Factors of $\left(x^3 + \frac{1}{x^3} - 2\right)$ are :

(A) $\left(x + \frac{1}{x} + 1\right)\left(x^2 + \frac{1}{x^2} + \frac{1}{x} + 1\right)$ (B) $\left(x + \frac{1}{x} - 1\right)\left(x^2 + \frac{1}{x^2} - \frac{1}{x} + x\right)$

(C) $\left(x + \frac{1}{x} - 1\right)\left(x^2 + \frac{1}{x^2} - \frac{1}{x} + x\right)$ (D) None of these

Q.11 $(x^{51} - 1)$ is always divisible by
(A) $(x + 1)$ (B) $(x - 1)$ (C) $(x + 2)$ (D) $(x - 2)$

Q.12 The factors of $y^2 + 2 + \frac{1}{y^2}$ are :

(A) $\left(y + \frac{1}{y}\right)^2$ (B) $\left(y - \frac{1}{y}\right)^2$ (C) $\left(y + \frac{1}{y} + 1\right)^2$ (D) $\left(y + \frac{1}{y} - 1\right)^2$

Q.13 When $16x^2 - 9y^2$ is resolved into factors, we get
(A) $(8x - 3y)^2$ (B) $(4x - 3y)(4x + 3y)$ (C) $(4x - 3y)^2$ (D) $(3x - 4y)(3x + 4y)$

Q.14 The factors of $y^2 + 7y + 10$ are
(A) $(y - 5)(y - 2)$ (B) $(y - 5)(y + 2)$ (C) $(y + 5)(y + 2)$ (D) $(y + 5)(y - 2)$

Q.15 Which of the following is a common factor is $15x^2$ and $18xy^2$?
(A) 5 (B) 3x (C) 5x (D) 6

Q.16 Which of the following is the common factor of $(2x - 3)$ and $(4x - 6)$?
(A) 2 (B) 3 (C) $2x - 3$ (D) $4x - 6$

Q.17 $55\ xy^2 \div 11\ xy =$ ________________
(A) 5y (B) 5x (C) $5xy^2$ (D) 5xy

Q.18 $\frac{5x + 10}{2} =$ ________________

(A) $5x + 5$ (B) $\frac{5x}{2} + 10$ (C) $\frac{5x}{2} + \frac{5}{2}$ (D) $\frac{5x}{2} + 5$

Q.19 Which of the following is/are the factors(s) of $25x^2 - 36y^2$?
(A) $5x + 6y$ (B) $5x - 6y$ (C) $25x^2 - 36y^2$ (D) All of these

Q.20 ac + ad + bc + bd = ____________________
(A) $(a + b)(b + d)$ (B) $(a + d)(b + c)$ (C) $(a + b)(c + d)$ (D) None of these

Q.21 Find the value of $9x^2 + 3x + 1$, when $x = -\frac{1}{3}$
(A) 1 (B) 2 (C) 3 (D) 4

Q.22 Factorize : $4t^4 + 4t^2 + 1$
(A) $(2t^2 + 1)^2$ (B) $(2t + 2)^2$ (C) $(2t^2 - 1)^2$ (D) $(4t + 4)^2$

Q.23 Factorize : $x^2y + xy^2 + 3x + 3y$
(A) $(xy + 3)(x + y)$ (B) $(xy + 3)(3x + y)$ (C) $(x + 2y)(2x + y)$ (D) $(xy + 3)(y + 3x)$

Q.24 Divide $x^2 - 9x + 14$ by $x - 2$
(A) $x - 7$ (B) $x - 8$ (C) $x - 5$ (D) $x - 2$

Q.25 Divide $4p^2q^4r^3 \div 12\,pqr$.
(A) $\frac{1}{3}pq^3r^2$ (B) pqr (C) $p^2q^3r^2$ (D) $3pq^3r^2$

Q.26 Divide $4(12x^4 - 25x^3 - 7x^2)$ by $8x(4x + 1)$
(A) $x(4x + 1)$ (B) $\frac{x}{2}(3x - 7)$ (C) $\frac{x(x+3)}{2}$ (D) None

Q.27 Divide $3x^3 + 7x^2 + 2x - 2$ by $x + 1$ and find the quotient.
(A) $(x + 3)(x + 4)$ (B) $3x^2 + 4x - 2$ (C) $x^2 + 5x - 6$ (D) None

SECTION - C

➢ **Match the Column :**

Q.1 Match the Column

	Column A		**Column B**
(A)	$x^4 + x^2y^2 + y^4$	(p)	$(x + 1)(x^2 + 1)$
(B)	$1 - x^2 + 2xy - y^2$	(q)	$(1 + x - y)(1 - x + y)$
(C)	$x^3 + x^2 + x + 1$	(r)	$(x^2 + xy + y^2)(x^2 - xy + y^2)$

Q.2 Match the Column

	Column-A		**Column B**
(A)	$\left(\frac{2}{3}a^2b\right)\left(\frac{-9}{4}ab^2\right)$	(p)	$\frac{-4}{9}p^5q^5$
(B)	$(-pq)(-2.3\,p^2q^2)$	(q)	$-0.45\,a^3b^3$
(C)	$(-1.5\,a^2b)(0.3\,ab^2)$	(r)	$\frac{-3}{2}a^3b^3$
(D)	$\left(\frac{-3}{7}p^3q^2\right)\left(\frac{-14}{9}pq^2\right)\left(\frac{-2}{3}pq\right)$	(s)	$-2.3\,p^3q^3$

ANSWER KEY

CONCEPT APPLICATION LEVEL - II

SECTION - A

Q.1 $x + \frac{1}{x}$ Q.2 $2 \times 5 \times x \times y\,(x + 3)$ Q.3 3m Q.4 $ax - 5$

Q.5 $(x^2 + 1)(x + 1)(x - 1)$. Q.6 prime Q.7 a^m Q.8 $\frac{3x - 4}{2}$

Q.9 $2x(2x + 3y)$ Q.10 $(x + 8)(x + 3)$ Q.11 $(x - 7)(x - 4)$

Q.12 $(2x + 13y)(2x - 13y)$ Q.13 $(2x + 7)(2x + 7)$

SECTION - B

Q.1	B	Q.2	D	Q.3	A	Q.4	C	Q.5	B	Q.6	B	Q.7	B
Q.8	C	Q.9	A	Q.10	D	Q.11	B	Q.12	A	Q.13	B	Q.14	C
Q.15	B	Q.16	C	Q.17	A	Q.18	D	Q.19	D	Q.20	C	Q.21	A
Q.22	A	Q.23	A	Q.24	A	Q.25	A	Q.26	BQ.27 B				

SECTION - C

Q.1 (A)-(r); (B)-(q); (C)-(p)

Q.2 (A)-(r); (B)-(s); (C)-(q); (D)-(p)