2 EXPONENTS & POWERS

THEORY

2.1 INTRODUCTION

Some of the numbers like 25700000000, 906472000000 are incredibly large and some numbers like 0.00001275, 0.000007 are unbelievably small.

It is diffcult to comprehend or visualise such numbers. They are cumbersome to read and write. Simpler ways have been devised to express very large numbers or very small numbers in a convenient form. This is known as the **'exponential form'**. When such numbers are expressed in the exponential form, they seem more meaningful.

Consider $7 \times 7 \times 7 = 7^3$. We read it as '7 raised to the power 3'. Here, 7 is the base and 3 is the exponent. Similarly, $10 \times 10 \times 10 \times 10 = 10^4$. We read it as '10 raised to the power 4'. Here, 10 is the base and 4 is the exponent.

2.2 POSITIVE INTEGRAL EXPONENT (POWER)

We know that $3 \times 3 \times 3 \times 3 \times 3$ can be written as 3^5. Here, 3 is the **base** and 5 is the **exponent** or **index**.3^5 is read as '3 raised to the power 5' or '3 to the power 5.'

$(-2) \times (-2) \times (-2) \times (-2) = (-2)^4$

$$\left(\frac{-5}{7}\right) \times \left(\frac{-5}{7}\right) \times \left(\frac{-5}{7}\right) = \left(\frac{-5}{7}\right)^3$$

Thus, if a is any rational number and n is a positive integer, then

$a \times a \times a \times \ldots\ldots\ldots\ldots \times a$ (n times) $= a^n$

a^n is called the n^{th} power of a and is also read as 'a raised to the power n'.

The rational number a is called the base and n is called the exponent (power or index).

This notation of writing the product of a rational number by itself several times is called the **exponential notation** or **power notation.**

Suppose, we have to find the value of $\left(\frac{3}{4}\right)^2$ and $\left(\frac{-2}{3}\right)^3$. We can write these as

$$\left(\frac{3}{4}\right)^2 = \frac{3}{4} \times \frac{3}{4} = \frac{3 \times 3}{4 \times 4} = \frac{9}{16}$$

And, $$\left(\frac{-2}{3}\right)^3 = \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) \times \left(\frac{-2}{3}\right) = \frac{(-2) \times (-2) \times (-2)}{3 \times 3 \times 3} = \frac{-8}{27}$$

From the above two examples, it is clear that if $\frac{p}{q}$ is any rational number, then

$$\left(\frac{p}{q}\right)^2 = \frac{p}{q} \times \frac{p}{q} = \frac{p \times p}{q \times q} = \frac{p^2}{q^2}$$

$$\left(\frac{p}{q}\right)^3 = \frac{p}{q} \times \frac{p}{q} \times \frac{p}{q} = \frac{p \times p \times p}{q \times q \times q} = \frac{p^3}{q^3}$$

Thus, for any rational number $\frac{p}{q}$ and for any positive integer n, we have $\boxed{\left(\frac{p}{q}\right)^n = \frac{p^n}{q^n}}$.

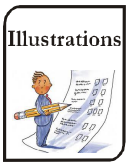

Illustration 1

Evaluate : (i) $\left(\frac{3}{7}\right)^3$ **(ii)** $\left(\frac{-4}{5}\right)^4$

Solution

(i) $\left(\frac{3}{7}\right)^3 = \frac{3^3}{7^3} = \frac{3 \times 3 \times 3}{7 \times 7 \times 7} = \frac{27}{343}$

(ii) $\left(\frac{-4}{5}\right)^4 = \frac{(-4)^4}{5^4} = \frac{-4 \times -4 \times -4 \times -4}{5 \times 5 \times 5 \times 5} = \frac{256}{325}$

Note : $(-1)^n = \begin{cases} -1, \text{if n is an odd positeve integer} \\ 1, \text{if n is an even positive int eger} \end{cases}$

2.3 NEGATIVE EXPONENTS

Observe and continue the following pattern :

$$5^{-1} = \frac{1}{5}$$

$$5^{-2} = \frac{1}{5} \div 5 = \frac{1}{5 \times 5} = \frac{1}{5^2}$$

$$5^{-3} = \frac{1}{5^2} \div 5 = \frac{1}{5 \times 5 \times 5} = \frac{1}{5^3}$$

$$5^{-4} = \frac{1}{5^3} \div 5 = \frac{1}{5 \times 5 \times 5 \times 5} = \frac{1}{5^4} \text{ and so on.}$$

Hence, 7^{-3} means $\frac{1}{7 \times 7 \times 7} = \frac{1}{7^3} = \frac{1}{343}$ and 10^{-4} means $\frac{1}{10 \times 10 \times 10 \times 10} = \frac{1}{10^4} = \frac{1}{10000}$

In general, for any non-zero integer x, $x^{-m} = \frac{1}{x^m}$, where m is a positive integer.

x^{-m} is the multiplicative inverse of x^m.

For example : Find the multiplicative inverse of **(i) 6^{-3} and (ii) 10^{-5}**

The multiplicative inverse of 6^{-3} is 6^3.

The multiplicative inverse of 10^{-5} is 10^5.

2.4 LAWS OF EXPONENTS

We know that,

$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$(a^m)^n = a^{mn}$$
$$a^m \times b^m = (ab)^m$$
$$a^0 = 1$$

where 'a' and 'b' are non-zero integers and 'm' and 'n' are whole numbers.

As a natural extension of these laws, we shall now learn and verify these laws for m and n as any integers.

1. $a^m \times a^n = a^{m+n}$

   Let us take m = 3 and n = –7

   Then, $a^m \times a^n = a^3 \times a^{-7} = a \times a \times a \times \frac{1}{a \times a \times a \times a \times a \times a \times a} = \frac{1}{a^4} = a^{-4}$

   $a^{m+n} = a^{3+(-7)} = a^{-4}$

   $\therefore \quad a^m \times a^n = a^{m+n}$

2. $a^m \div a^n = a^{m-n}$

   Let us take m = 8, n = –4

   $a^m \div a^n = a^8 \div a^{-4} = \frac{a^8}{a^{-4}} = a^8 \times a^4 = a^{8+4} = a^{12}$

   $a^{m-n} = a^{8-(-4)} = a^{8+4} = a^{12}$

   $\therefore \quad a^m \div a^n = a^{m-n}$

3. $(a^m)^n = a^{mn}$

   Put m = 3, n = –2

   $(a^m)^n = (a^3)^{-2} = \frac{1}{(a^3)^2} = \frac{1}{(a \times a \times a)^2} = \frac{1}{a \times a \times a \times a \times a \times a} = \frac{1}{a^6} = a^{-6}$

   $a^{mn} = a^{3 \times -2} = a^{-6}$

   $\therefore \quad (a^m)^n = a^{mn}$

4. $a^m \times b^m = (ab)^m$

   Put m = –3

   $a^{-3} \times b^{-3} = \frac{1}{a^3} \times \frac{1}{b^3} = \frac{1}{a \times a \times a} \times \frac{1}{b \times b \times b} = \frac{1}{ab} \times \frac{1}{ab} \times \frac{1}{ab} = \frac{1}{(ab)^3} = (ab)^{-3}$

   $\therefore \quad a^m \times b^m = (ab)^m$

5. $a^0 = 1$

We know that $3^{-5} \div 3^{-5} = 1$ (Any number except 0 divided by itself gives quotient 1)

i.e. $3^{-5-(-5)} = 1$ (by 2nd law)

$\Rightarrow 3^0 = 1$

Thus, any number except 0 raised to power 0 is 1.

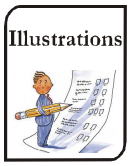

Illustration : 2

Evaluate :

(i) $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^{-3}$ **(ii)** $\left(\frac{25}{8}\right)^{-5} \div \left(\frac{25}{8}\right)^{-7}$ **(iii)** $\left[\left(\frac{2}{3}\right)^5\right]^{-2} \times \left(\frac{2}{3}\right)^7$

(iv) $2^{-3} \times 3^{-3}$ **(v)** $(6^0 - 5^0) \div (6^0 + 5^0)$

Solution

(i) $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^{-3} = \left(\frac{2}{3}\right)^{2+(-3)}$ (Applying $x^m \times x^n = x^{m+n}$)

$= \left(\frac{2}{3}\right)^{-1}$ (Applying $x^{-m} = \frac{1}{x^m}$)

$= \frac{3}{2}$

(ii) $\left(\frac{25}{8}\right)^{-5} \div \left(\frac{25}{8}\right)^{-7}$ (Applying $x^m \div x^n = x^{m-n}$)

$= \left(\frac{25}{8}\right)^{-5-(-7)} = \left(\frac{25}{8}\right)^{-5+7} = \left(\frac{25}{8}\right)^2 = \frac{625}{64}$

(iii) $\left[\left(\frac{2}{3}\right)^5\right]^{-2} \times \left(\frac{2}{3}\right)^7 = \left(\frac{2}{3}\right)^{5\times(-2)} \times \left(\frac{2}{3}\right)^7$ (Applying $x^m)^n = x^{m\times n}$)

$= \left(\frac{2}{3}\right)^{-10} \times \left(\frac{2}{3}\right)^7 = \left(\frac{2}{3}\right)^{-10+7} = \left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$

(iv) $2^{-3} \times 3^{-3} = (2 \times 3)^{-3} = 6^{-3} = \frac{1}{6^3} = \frac{1}{216}$

(v) $(6^0 - 5^0) \div (6^0 + 5^0)$

$= (1 - 1) \div (1 + 1)$ (Applying $x^o = 1$)

$= 0 \div 2 = 0$

Illustration : 3

Simplify $\left(\frac{1}{2}\right)^{-1}+\left(\frac{1}{3}\right)^{-2}+4^{-1}$

Solution :

$$\left(\frac{1}{2}\right)^{-1}+\left(\frac{1}{3}\right)^{-2}+4^{-1}=2+(3)^2+\frac{1}{4} \qquad \left(\text{Applying law } x^{-m}=\frac{1}{x^m}\right)$$

$$=2+9+\frac{1}{4}=11+\frac{1}{4}=\frac{44+1}{4}=\frac{45}{4}$$

Illustration : 4

By what number $\left(\frac{-2}{3}\right)^5$ be divided to obtain $\left(\frac{-2}{3}\right)^3$?

Solution :

$$\text{Quotient}=\left(\frac{-2}{3}\right)^3; \qquad \text{Dividend}=\left(\frac{-2}{3}\right)^5$$

Let the required number be x

$$\left(\frac{-2}{3}\right)^5\div x=\left(\frac{-2}{3}\right)^3 \quad\Rightarrow\quad \left(\frac{-2}{5}\right)^5\times\frac{1}{x}=\left(\frac{-2}{3}\right)^3$$

$$\left(\frac{-2}{3}\right)^5\div\left(\frac{-2}{3}\right)^3=x \quad\Rightarrow\quad x=\left(\frac{-2}{3}\right)^{5-3}=\left(\frac{-2}{3}\right)^2$$

Hence, the required number is $\left(\frac{-2}{3}\right)^2=\frac{4}{9}$

Illustration : 5

Solve for x : $2^{2x+3}=8^{5+x}$

Solution :

$2^{2x+3}=8^{5+x} \quad\Rightarrow\quad 2^{2x+3}=2^{3(5+x)}$

$2^{2x+3}=2^{15+3x}$

As the bases are same, on comparing powers

$2x+3=15+3x \quad\Rightarrow\quad 2x-3x=15-3$

$-x=12 \quad\Rightarrow\quad x=-12$

Illustration : 6

If $3^{2x}:3^{x+5}=1:27$ find x.

Solution :

$$\frac{3^{2x}}{3^{x+5}}=\frac{1}{27} \quad\Rightarrow\quad \frac{3^{2x}}{3^{x+15}}=\frac{1}{3^3}$$

$3^{2x}\times 3^3=3^{x+5} \quad\Rightarrow\quad 3^{2x+3}=3^{x+5}$

As base are same, on comparing pwers

$2x+3=x+5 \quad\Rightarrow\quad 2x-x=5-3 \quad\Rightarrow\quad x=2.$

2.5 USE OF EXPONENETS TO EXPRESS SMALL NUMBERS IN STANDARD FORM

In the previous class, we have read how to express large numbers in the standard form.
For Example, 2,73,00,000 = 2.73×10^7

85294.47 = 8.529447×10^4 etc.

Here, we shall learn how to express small numbers in the standard form.
We know that

$$0.1 = \frac{1}{10} = 10^{-1},\ 0.01 = \frac{1}{100} = 10^{-2},\ 0.001 = \frac{1}{1000} = 10^{-3} \text{ and so on}$$

Similarly, $0.05 = \frac{5}{100} = 5 \times 10^{-2}$

$$0.008 = \frac{8}{1000} = 8 \times 10^{-3}$$

$$0.0000021 = \frac{21}{10000000} = 21 \times 10^{-7} = 2.1 \times 10 \times 10^{-7} = 2.1 \times 10^{-6}$$

$$0.00000000837 = \frac{837}{100000000000} = 837 \times 10^{-11} = 8.37 \times 10^2 \times 10^{-11}$$
$$= 8.37 \times 10^2 \times 10^{-11} = 8.37 \times 10^{-9}$$

Illustration 7

Write the following numbers in the standard form :
(i) 0.000000564 (ii) 0.00000000000942 (iii) 602000000000000

Solution

(i) $0.000000564 = \frac{564}{1000000000} = 564 \times 10^{-9}$
$= 5.64 \times 10^2 \times 10^{-9} = 5.64 \times 10^{-7}$

(ii) $0.00000000000942 = \frac{942}{100000000000000} = 942 \times 10^{-14}$
$= 9.42 \times 10^2 \times 10^{-14} = 9.42 \times 10^{-12}$

(iii) $6020000000000000 = 6.02 \times 10^{15}$

Illustration 8

Express the following numbers in the usual form :
(i) 3.02×10^{-6} (ii) 3×10^{-8} (iii) 5.8×10^{12}

Solution

(i) $3.02 \times 10^{-6} = \frac{3.02}{10^6} = \frac{3.02}{1000000} = 0.00000302$

(ii) $3 \times 10^{-8} = \frac{3}{10^8} = \frac{3}{100000000} = 0.00000003$

(iii) $5.8 \times 10^{12} = 5.8 \times 1000000000000 = 5800000000000$

From the above examples, we notice that in the standard form :

(i) There is only one digit to the left of the decimal point.

(ii) When the given number is greater than 1, the power of 10 is either 0 or a positive integer, which is equal to the number of places by which the decimal point has been moved to the left.

(iii) When the given number is less than 1, the power of 10 is a negative integer, which is equal to the number of places by which the decimal point has been moved to the right.

A number in the from $k \times 10^m$, where $1 \le k < 10$ and m is an integer is said to be in the standard form.

2.6 EXPONENTIAL EQUATION

(i) If $a^m = a^n$, then m = n, if $a \neq 0$, $a \neq 1$ and $\neq -1$.

For Example :

(a) If $5^p = 5^3 \Rightarrow p = 3$ (b) If $4^p = 256$

$4^p = 4^4 \Rightarrow p = 4$

(ii) If $a^n = b^n$, then a = b (when n is odd).

For Example :

(a) If $5^7 = p^7$, then p = 5 (b) If $(5)^{2n-1} = (3 \times p)^{2n-1}$, then 5 = 3p or p = 5/3

(iii) If $a^n = b^n$, $n \neq 0$, then $a = \pm b$. (when n is even).

For Example :

$2^4 = x^4 \quad \Rightarrow \quad x = \pm 2$

2.7 UNIQUE PRIME FACTORIZATION THEOREM

If $p^m \times q^n \times r^s = p^a\, q^b\, r^c$, then m = a, n = b, s = c, where p, q and r are different primes.

For Example :

If $40500 = 2^a \times 5^b \times 3^c$, then find $a^a \times b^b \times c^c$.

$\therefore\ 40500 = 2^2 \times 5^3 \times 3^4 = 2^a \times 5^b \times 3^c$

$\therefore$ a = 2, b = 3 c = 4 (Using the above law).

$\therefore\ a^a \times b^b \times c^c = 2^2 \times 3^3 \times 4^4 = 27648.$

2	40500
2	20250
5	10125
5	2025
5	405
3	81
3	27
3	9
	3

SOLVED EXAMPLES

Example 1 :

Find the multiplicative inverse of the following :

(i) 2^{-4} (ii) 10^{-5} (iii) 7^{-2} (iv) 10^{-100}

Solution :

(i) 2^{-4}

The multiplicative inverse of $2^{-4} = \left(\frac{1}{2^{-4}}\right)$ is 2^4

(ii) 10^{-5}

The multiplicative inverse of $10^{-5} = \left(\frac{1}{10^{-5}}\right)$ is 10^5.

(iii) 7^{-2}

The multiplicative inverse of $7^{-2} = \left(\frac{1}{7^{-2}}\right)$ is 7^2

(iv) 10^{-100}

The multiplicative inverse of $10^{-100} = \left(\frac{1}{10^{-100}}\right)$ is 10^{100}

Example 2 :

Expand the following numbers using exponents:

(i) 1025.63 (i) 1256.249

Solution :

(i) $1025.63 = 1 \times 1000 + 0 \times 100 + 2 \times 10 + 5 \times 1 + 6 \times 10^{-1} + 3 \times 10^{-2}$

$= 1 \times 10^3 + 2 \times 10 + 5 \times 1 + 6 \times 10^{-1} + 3 \times 10^{-2}$

(ii) $1256.249 = 1 \times 1000 + 2 \times 100 + 5 \times 10 + 6 \times 1 + 2 \times 10^{-1} + 4 \times 10^{-2} + 9 \times 10^{-3}$

$= 1 \times 10^3 + 2 \times 10^2 + 5 \times 10 + 6 \times 1 + 2 \times 10^{-1} + 4 \times 10^{-2} + 9 \times 10^{-3}$

Example 3:

Simplify and write in exponential form :

(i) $(-2)^{-3} \times (-2)^{-4}$ (ii) $p^3 \times p^{-10}$ (iii) $3^2 \times 3^{-5} \times 3^6$

Solution :

(i) $(-2)^{-3} \times (-2)^{-4}$

$(-2)^{-3} \times (-2)^{-4} = (-2)^{(-3)+(-4)} = (-2)^{-7} = \{(-1) \times 2\}^{-7}$

$$= \frac{1}{\{(-1)\times 2\}^7} = \frac{1}{(-1)^7 \times (2)^7} = -\frac{1}{2^7}$$

(ii) $p^3 \times p^{-10}$

$p^3 \times p^{-10} = p^{3 + (-10)} = p^{-7} = \frac{1}{p^7}$

(iii) $3^2 \times 3^{-5} \times 3^6$

$3^2 \times 3^{-5} \times 3^6 = 3^{2 + (-5) + 6} = 3^3$

Remember : $a^n = 1$ only if $n = 0$. This will work for any a except $a = 1$ or $a = -1$.
For $a = 1$, $1^1 = 1^2 = 1^3 = 1^{-2} = ... = 1$ or $(1)^n = 1$ for infinitely many n.
For $a = -1$,
$(-1)^0 = (-1)^2 = (-1)^4 = (-1)^{-2} = ... = 1$ or
$(-1)^p = 1$ for any even integer p.

Example 4 :

Write the following numbers in standard form.

(i) 0.000000564 (ii) 0.0000021 (iii) 21600000 (iv) 15240000

Solution :

(i) $0.000000564 = \frac{564}{1000000000} = \frac{5.64}{10^7} = 5.64 \times 10^{-7}$

(ii) $0.0000021 = \frac{2.1}{1000000} = \frac{2.1}{10^6} = 2.1 \times 10^{-6}$

(iii) $21600000 = 2.16 \times 10^7$

(iv) $15240000 = 1.524 \times 10^7$

Example 5 :

Write all the facts given in the standard form.

Solution :

1. The distance from the Earth to the Sun is 1.496×10^{11} m
2. The speed of light is 3×10^8 m/sec
3. Thickness of Class VII Mathematics book is 2×10^1 mm.
4. The average diameter of a Red Blood cell is 7×10^{-6} mm
5. The thickness of human hair is in the range of 5×10^{-3} cm to 1×10^{-2} cm
6. The distance of moon from the Earth is 3.84467×10^8 m
7. The size of a plant cell is 1.275×10^{-5} m
8. Average radius of the Sun is 6.95×10^5 km
9. Mass of propellant in a space shuttle solid rocket booster is 5.036×10^5 kg
10. Thickness of a piece of paper is 1.6×10^{-3} cm
11. Diameter of a wire on a computer chip is 3×10^{-6} m
12. The height of Mount Everest is 8.848×10^3 m.

Example 6 :

Simplify and express in the exponential form : $\left(\frac{-7}{5}\right)^4 \times \left(\frac{-7}{5}\right)^6 \times \left(\frac{-7}{5}\right)$

(A) $\left(\frac{-7}{5}\right)^{11}$ (B) $\left(\frac{-7}{5}\right)^{10}$ (C) $\left(\frac{5}{7}\right)^9$ (D) $\left(\frac{-21}{15}\right)^6$

Solution : $\left(\frac{-7}{5}\right)^{4+6+1} = \left(\frac{-7}{5}\right)^{11}$

Example 7 :

Simplify : $(3^0 + 3^{-1}) \times 2^2$

(A) $\frac{16}{3}$ (B) 24 (C) 16 (D) $\frac{7}{3}$

Solution : $\left(1+\frac{1}{3}\right)\times 4 = \frac{4}{3}\times 4 = \frac{16}{3}$

Example 8 :

Simplify $\frac{3^{-5}\times 2^{-5}\times 5^3}{5^{-1}\times 6^{-5}}$

(A) 1 (B) 5^2 (C) 5^4 (D) $\left(\frac{10}{11}\right)^8$

Solution : $\frac{5^3\times 5^1\times 6^5}{2^2\times 3^5} = \frac{5^4\times 2^5\times 3^5}{2^5\times 3^5} = 5^4$

Example 9 :

Simplify : $5^{2x} \div 5^7 = 5^3$

(A) 1 (B) 5 (C) 2 (D) $\frac{21}{2}$

Solution :

$\frac{5^{2x}}{5^7} = 5^3 \Rightarrow 5^{2x} = 5^3 \times 5^7$

$\Rightarrow 5^{2x} = 5^{10}$

$\Rightarrow 2x = 10 \Rightarrow x = 5$

Example 10 :

Simplify : $\frac{2^{2x} \cdot 2}{2^{3x-2}} = 2^{-2}$

(A) 5 (B) –5 (C) 0 (D) $\frac{1}{2}$

Solution : $\frac{2^{2x+1}}{2^{3x-2}} = 2^{-2} \Rightarrow 2^{2x+1-3x+2} = 2^{-2}$

$\Rightarrow -x + 3 = -2 \Rightarrow x = 5$

Example 11 :

Simplify : $(3 \times 3^{5x}) \div 9^{2x} = \frac{1}{27}$

(A) –2 (B) 3 (C) 1 (D) –4

Solution :

$\frac{3^{1+5x}}{9^{2x}} = \frac{1}{27} \Rightarrow 3^{1+5x-4x} = 3^{-3} \Rightarrow 1 + x = -3 \Rightarrow x = -4$

Example 12 :

Simplify : $\left[\left(\frac{3}{4}\right)^5 \times \left(\frac{3}{4}\right)^3\right] \div \left(\frac{9}{16}\right)^4$

(A) 1 (B) 2 (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Solution :

$\left(\frac{3}{4}\right)^{5+3} - \left(\frac{3}{4}\right)^{2\times4} = \left(\frac{3}{4}\right)^{8-8} = 1$

CONCEPT APPLICATION LEVEL - I [NCERT Questions]

EXERCISE 1

Q.1 Evaluate :

(i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-5}$

Sol. (i) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

(ii) $(-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{16}$

(iii) $\left(\frac{1}{2}\right)^{-5} = \frac{1}{\left(\frac{1}{2}\right)^5} = \frac{1}{\frac{1^5}{2^5}} = \frac{1}{\left(\frac{1}{32}\right)} = 32$

Q.2 Simplify and express the result in power notation with positive exponent.

(i) $(-4)^5 \div (-4)^8$ (ii) $\left(\frac{1}{2^3}\right)^2$ (iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$ (v) $2^{-3} \times (-7)^{-3}$

Sol. (i) $(-4)^5 \div (-4)^8 = \frac{(-4)^5}{(-4)^8} = \frac{1}{(-4)^{8-5}} = \frac{1}{(-4)^3}$

(ii) $\left(\frac{1}{2^3}\right)^2 = \frac{1}{(2^3)^2} = \frac{1}{2^{3\times2}} = \frac{1}{2^6}$

(iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4 = \{(-1) \times 3\}^4 \times \left(\frac{5}{3}\right)^4 = (-1)^4 \times 3^4 \times \frac{5^4}{3^4} = (5)^4$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5} = \left(\frac{3^{-7}}{3^{-10}}\right) \times \frac{1}{3^5} = 3^{(-7)-(-10)} \times \frac{1}{3^5}$

$3^{-7+10} \times \frac{1}{3^5} = \frac{3^3}{3^5} = \frac{1}{3^{5-3}} = \frac{1}{(3)^2}$

(v) $2^{-3} \times (-7)^{-3} = \frac{1}{2^3} \times \frac{1}{(-7)^3} = \frac{1}{[2\times(-7)]^3} = \frac{1}{(-14)^3}$

Q.3 Find the value of :

(i) $(3^0 + 4^{-1}) \times 2^2$ (ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$ (v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

Sol. (i) $(3^0 + 4^{-1}) \times 2^2 = \left(1+\frac{1}{4}\right) \times 4 = \frac{5}{4} \times 4 = 5$

(ii) $(2^{-1} \times 4^{-1}) \div 2^{-2} = \{2^{-1} \times (2^2)^{-1}\} \div 2^{-2} = \{2^{-1} \times 2^{2 \times (-1)}\} \div 2^{-2}$

$= (2^{-1} \times 2^{-2}) \div 2^{-2} = 2^{-3} \div 2^{-2} = \frac{2^{-3}}{2^{-2}} = \frac{1}{2^{(-2)-(-3)}} = \frac{1}{2^{-2+3}} = \frac{1}{2^1} = \frac{1}{2}$

(iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} = \frac{1^{-2}}{2^{-2}} + \frac{1^{-2}}{3^{-2}} + \frac{1^{-2}}{4^{-2}} = \frac{2^2}{1^2} + \frac{3^2}{1^2} + \frac{4^2}{1^2}$

$= \frac{4}{1} + \frac{9}{1} + \frac{16}{1} = 4 + 9 + 16 = 29$

(iv) $[3^{-1} + 4^{-1} + 5^{-1}]^0 = \left[\frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right]^0 = \left(\frac{20+15+12}{60}\right)^0 = \left(\frac{47}{60}\right)^0 = 1$

(v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2 = \left(\frac{-2}{3}\right)^{(-2)\times 2} = \left(\frac{-2}{3}\right)^{-4} = \frac{(-2)^{-4}}{(3)^{-4}} = \frac{3^4}{(-2)^4}$

$= \frac{3 \times 3 \times 3 \times 3}{(-2) \times (-2) \times (-2) \times (-2)} = \frac{81}{16}$

Q.4 Evaluate :

(i) $\frac{8^{-1} \times 5^3}{2^{-4}}$ (ii) $(5^{-1} \times 2^{-1}) \times 6^{-1}$

Sol. (i) $\frac{8^{-1} \times 5^3}{2^{-4}} = \frac{\frac{1}{8^1} \times 125}{\frac{1}{2^4}} = \frac{\frac{1}{8} \times 125}{\frac{1}{16}} = \frac{1}{8} \times 125 \times \frac{16}{1} = 250$

(ii) $(5^{-1} \times 2^{-1}) \times 6^{-1} = \left(\frac{1}{5} \times \frac{1}{2}\right) \times \frac{1}{6} = \frac{1}{10} \times \frac{1}{6} = \frac{1}{60}$

Q.5 Find the value of m for which $5^m \div 5^{-3} = 5^5$.

Sol. $5^m \div 5^{-3} = 5^5 \Rightarrow \frac{5^m}{5^{-3}} = 5^5$

or $5^{m-(-3)} = 5^5 \Rightarrow 5^{m+3} = 5^5 \Rightarrow m + 3 = 5 \Rightarrow m = 5 - 3$

$\Rightarrow m = 2$

Q.6 Evaluate :

(i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$ (ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

Sol. (i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1} = \left(\frac{1^{-1}}{3^{-1}} - \frac{1^{-1}}{4^{-1}}\right)^1 = \left(\frac{3^1}{1^1} - \frac{4^1}{1^1}\right)^{-1} = \left(\frac{3}{1} - \frac{4}{1}\right)^{-1} = (3-4)^{-1}$

$= (-1)^{-1} = \frac{1}{(-1)^1} = \frac{1}{(-1)} = -1$

(ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-4}}{5^{-4}} = 5^{(-7)-(-4)} \times 8^{(-4)-(-7)} = 5^{-7+4} \times 8^{-4+7}$

$= 5^{-3} \times 8^3 = \frac{1}{5^3} \times 8^3 = \frac{8^3}{5^3} = \frac{512}{125}$

Q.7 Simplify

(i) $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} (t \neq 0)$ (ii) $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

Sol. (i) $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} (t \neq 0)$

$$\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} = \frac{25 \times \frac{1}{t^4}}{\frac{1}{5^3} \times 10 \times \frac{1}{t^8}} = \frac{\frac{25}{t^4}}{\frac{1}{125} \times 10 \times \frac{1}{t^8}} = \frac{\frac{25}{t^4}}{\frac{2}{25t^8}} = \frac{25}{t^4} \times \frac{25t^8}{2} = \frac{625\,t^{8-4}}{2} = \frac{625}{2} t^4$$

(ii) $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}} = \frac{3^{-5} \times (2 \times 5)^{-5} \times (5 \times 5 \times 5)}{5^{-7} \times (2 \times 3)^{-5}} = \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}} = \frac{5^{-5} \times 5^3}{5^{-7}}$

$= \frac{5^{(-5)+3}}{5^{-7}} = 5^{(-2)-(-7)} = 5^{-2+7} = 5^5$

EXERCISE 2

Q.1 Express the following numbers in standard form

(i) 0.0000000000085 (ii) 0.00000000000942

(iii) 602000000000000 (iv) 0.00000000837

(v) 31860000000

Sol. (i) $0.0000000000085 = \frac{85}{10^{13}} = \frac{8.5\times10}{10^{13}} = \frac{8.5\times10}{10^{13}} = \frac{8.5}{10^{13-1}} = \frac{8.5}{10^{12}} = 8.5\times10^{-12}$

(ii) $0.00000000000942 = \frac{942}{10^{14}} = \frac{9.42\times10^{2}}{10^{14}} = \frac{9.42}{10^{14-2}} = \frac{9.42}{10^{12}} = 9.42 \times 10^{-12}$

(iii) $602000000000000 = 6.02 \times 10^{15}$

(iv) $0.00000000837 = \frac{837}{10^{11}} = \frac{8.37\times10^{2}}{10^{11}} = \frac{8.37}{10^{11-2}} = \frac{8.37}{10^{9}} = 8.37 \times 10^{-9}$

(v) $31860000000 = 3.186 \times 10^{10}$

Q.2 Express the following numbers in usual form :

(i) 3.02×10^{-6} (ii) 4.5×10^{4} (iii) 3×10^{-8}

(iv) 1.0001×10^{9} (v) 5.8×10^{12} (vi) 3.61492×10^{6}

Sol. (i) $3.02 \times 10^{-6} = \frac{3.02}{10^{6}} = \frac{3.02}{1000000} = 0.00000302$

(ii) $4.5 \times 10^{4} = 4.5 \times 10000 = 45000$

(iii) $3 \times 10^{-8} = = \frac{3}{100000000} = 0.00000003$

(iv) $1.0001 \times 10^{9} = 1.0001 \times 1000000000 = 1000100000$

(v) $5.8 \times 10^{12} = 5.8 \times 1000000000000 = 5800000000000$

(vi) $3.61492 \times 10^{6} = 3.61492 \times 1000000 = 3614920$

Q.3 Express the number appearing in the following statements in standard form

(i) 1 micron is equal to $\frac{1}{1000000}$ m.

(ii) Charge of an electron is 0.000,000,000,000,000,000,16 coulomb.

(iii) Size of a bacteria is 0.0000005 m

(iv) Size of a plant cell is 0.00001275 m

(v) Thickness of a thick paper is 0.07 mm.

Sol. (i) $\frac{1}{1000000}\text{m} = \frac{1}{10^6}\text{m} = 1 \times 10^{-6}\text{ m}$

(ii) 0.000,000,000,000,000,000,16 Coulamb

$$= \frac{16}{10000000000000000000} = \frac{16}{10^{20}} = \frac{1.6\times10}{10^{20}} = \frac{1.6\times10^1}{10^{20}} = \frac{1.6}{10^{20-1}} = \frac{1.6}{10^{19}}$$

$$= 1.6 \times 10^{-19} \text{ coulomb}$$

(iii) $0.0000005\text{ m} = \frac{5}{10000000} = \frac{5}{10^7} = 5 \times 10^{-7}\text{ mm}$

(iv) $0.00001275 = \frac{1275}{100000000} = \frac{1275}{10^8} = \frac{1.275}{10^5} = 1.275 \times 10^{-5}\text{ mm}$

(v) $0.07\text{ mm} = \frac{7}{100} = \frac{7}{10^2} = 7 \times 10^{-2}\text{ mm}$

Q.4 In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack ?

Sol. Total thickness of books = 5×20 mm $= 100$ mm

Total thickness of paper sheets = 5×0.016 mm $= 0.080$ mm

$\therefore$ Total thickness of the stack = Total thickness of books + Total thickness of paper sheets

$= 100$ mm $+ 0.080$ mm $= (100 + 0.080)$ mm

$= 100.080$ mm $= 1.008 \times 10^2$ mm

CONCEPT APPLICATION LEVEL - II

SECTION - A

- **FILL IN THE BLANKS**

Q.1 Express 3^{-2} as a rational number _____.

Q.2 Express $\left[\left(\frac{3}{5}\right)^2\right]^{-4}$ with a positive exponent _____.

Q.3 Find the value of $\left(\frac{-11}{15}\right)^{-6} \times \left(\frac{-11}{15}\right)^{4} \times \left(\frac{-11}{15}\right)^{2}$. _____.

Q.4 Find the value of $(5^{-1})^{-1}$._____.

Q.5 Find the value of $(3^0 - 2^0) \times 5^2$._____.

Q.6 Find the value of $\left(\frac{1}{4}\right)^{-1} + \left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{3}\right)^{-1}$. _____.

Q.7 $\left(\frac{13}{14}\right)^5 \div (\_\_\_\_\_)^2 = \left(\frac{13}{14}\right)^3$.

Q.8 Find the value of $(4^{-1} + 8^{-1})$. _____.

Q.9 Express $\left(\frac{4}{5}\right)^{-3}$ with a positive exponent. _____.

Q.10 $[(-3)^{-2}]^{-1}$ = _____.

SECTION - B

- **MULTIPLE CHOICE QUESTIONS**

Q.1 The value of $\left(\frac{-1}{2}\right)^4$ is :

(A) $\frac{1}{8}$ (B) $\frac{-1}{8}$ (C) $\frac{1}{16}$ (D) $\frac{-1}{16}$

Q.2 $\left(\frac{1}{3}\right)^0$ is equal to

(A) $\frac{1}{3}$ (B) 1 (C) 3 (D) 0

Q.3 $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{3}\right)^3$ is equal to

(A) $\frac{1}{216}$ (B) $\frac{1}{125}$ (C) $\frac{1}{54}$ (D) None of these

Q.4 When simplified $7^0 \times 5^0$, we get

(A) 1 (B) 0 (C) 35 (D) None of these

Q.5 Which of the following is true?

(A) $\left[\left(\frac{a}{b}\right)^m\right]^n = \left(\frac{a}{b}\right)^{m+n}$ (B) $\left[\left(\frac{a}{b}\right)^m\right]^n = \left(\frac{a}{b}\right)^{mn}$

(C) $\left(\frac{a}{b}\right)^0 = 0$ (D) $\left(\frac{a}{b}\right)^n = \frac{1}{\left(\frac{a}{b}\right)^n}$

Q.6 If n is an odd positive integer, then $(-1)^n$ is equal to

(A) 1 (B) 0 (C) –1 (D) None of these

Q.7 Which of the following is not equal to $\frac{-8}{27}$?

(A) $\left(\frac{2}{3}\right)^{-3}$ (B) $-\left(\frac{2}{3}\right)^3$

(C) $\left(\frac{-2}{3}\right)^3$ (D) $\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)\times\left(\frac{-2}{3}\right)$

Q.8 $\left(\frac{2}{3}\right)^{-5}$ is equal to

(A) $\left(\frac{-2}{3}\right)^5$ (B) $\left(\frac{3}{2}\right)^5$ (C) $\frac{2\times(-5)}{3}$ (D) $\frac{2}{3\times5}$

Q.9 $\left(\frac{-3}{2}\right)^{-1}$ is equal to

(A) $\frac{2}{3}$ (B) $-\frac{2}{3}$ (C) $\frac{3}{2}$ (D) None of these

Q.10 $\left(-\frac{1}{5}\right)^3 \div \left(-\frac{1}{5}\right)^8$ is equal to

(A) $\left(-\frac{1}{5}\right)^5$ (B) $\left(-\frac{1}{5}\right)^{11}$ (C) $(-5)^5$ (D) $\left(\frac{1}{5}\right)^5$

Q.11 For a non-zero rational number a, $a^7 \div a^{12}$ is equal to
(A) a^5 (B) a^{-19} (C) a^{-5} (D) a^{19}

Q.12 The value of $\left(\frac{4}{3}\right)^{-6} \div \left(\frac{4}{3}\right)^{-6}$ is

(A) $\frac{-4}{3}$ (B) 1 (C) 0 (D) $\left(\frac{-4}{3}\right)^{-12}$

Q.13 For a non-zero rational number a, $(a^3)^{-2}$ is equal to
(A) a^6 (B) a^{-6} (C) a^{-9} (D) a^5

Q.14 The value of $(7^{-1} - 8^{-1})^{-1} - (3^{-1} - 4^{-1})^{-1}$ is
(A) 56 (B) 12 (C) 68 (D) 44

Q.15 The value of x to make the statement $\left(\frac{5}{9}\right)^{-2} \times \left(\frac{18}{25}\right)^{-2} = x^{-2}$ true, is

(A) $\frac{5}{2}$ (B) $\left(\frac{5}{2}\right)^{-2}$ (C) $\left(\frac{-5}{2}\right)^{4}$ (D) $\frac{2}{5}$

Q.16 If $\frac{x}{y} = \left(\frac{2}{5}\right)^{-3} \times \left(\frac{15}{8}\right)^{-3}$, then $\left(\frac{x}{y}\right)^{-1}$ is equal to

(A) $\frac{27}{64}$ (B) $\frac{81}{64}$ (C) $\frac{-81}{64}$ (D) $\frac{-16}{81}$

Q.17 If $x = \left(8^{\frac{2}{3}} \cdot 32^{-\frac{2}{5}}\right)$, then $x^{-5} =$

(A) $\frac{1}{32}$ (B) -1 (C) 1 (D) -5

Q.18 $81^{\frac{1}{4}} \times 9^{\frac{3}{2}} \times 27^{-\frac{4}{3}} =$

(A) 1 (B) 3 (C) 9 (D) $\frac{1}{3}$

Q.19 If $4(4x)^7 = 4^{6^2}$, then what is the value of x?
(A) 5 (B) 25 (C) 64 (D) 256

Q.20 $\sqrt[5]{0.03125} =$

(A) 0.25 (B) 0.5 (C) 0.126 (D) 0.15

Q.21 Find the value of $(0.000064)^{\frac{2}{3}} \div (0.0016)^{\frac{3}{4}}$

(A) 3^{-1} (B) 4^{-1} (C) 5^{-1} (D) 10^{-1}

Q.22 Find the value of $(6561)^{(0.125)} + (3125)^{0.2}$

(A) 4 (B) 6 (C) 8 (D) None of these

Q.23 Which of the following statements is true about a rational number $\frac{-3}{4}$?

(A) It lies on the right side of 0 on the number line.
(B) It lies on the left side of 0 on the number line.
(C) It is not possible to represent it on the number line.
(D) It cannot be determined on which side of 0, the number lies.

Q.24 If $\frac{-4}{x} = \frac{x}{16}$, then x is

(A) A rational number (B) An integer (C) A natural number (D) Not a rational number

Q.25 Form the number from this expanded form :

$$3 \times \frac{1}{10} + 4 \times \frac{1}{10^2} + 6 \times \frac{1}{10^4} + 3 \times \frac{1}{10^5}$$

(A) 3463 (B) 0.34063 (C) 34.63 (D) 34063

Q.26 Form the number from this expanded form :

$$9 \times 10^3 + 3 \times \frac{1}{10^2} + 4 \times \frac{1}{10^3}$$

(A) 934 (B) 9.034 (C) 0.934 (D) 9000.034

Q.27 Form the number from this expanded form :

$$7 \times 10^3 + 2 \times 10^2 + 1 \times 10 + 2 \times \frac{1}{10} + 3 \times \frac{1}{10^3}$$

(A) 7210.203 (B) 721.23 (C) 721.203 (D) 72.123

Q.28 Express the following numbers in usual form :
3.02×10^{-6}

(A) 0.000032 (B) 0.000232 (C) 0.00000302 (D)0.000302

Q.29 Express the following numbers in standard form :
732000000

(A) 7.32×10^8 (B) 732×10^6 (C) 73.2×10^7 (D) 0.732×10^9

Q.30 Express the following number in the standard form
0.00000053
(A) 0.53×10^{-6} (B) 5.3×10^{-7} (C) 53×10^{-8} (D) 530×10^{-9}

Q.31 Express $(125)^{-3}$ as a power with base 5.
(A) $(5)^{-3}$ (B) $(5)^{-6}$ (C) 5^0 (D) $(5)^{-9}$

Q.32 Express $(64)^{-2}$ as a power with base 4.
(A) $(4)^{-6}$ (B) $(4)^{-5}$ (C) 4^1 (D) 4^2

Q.33 Find the value of $(8^0 + 7^{-1}) \times 2^3$
(A) 120 (B) 64 (C) $\frac{72}{7}$ (D) $\frac{64}{7}$

Q.34 Find the value of $(3^{-1} \times 4^{-1}) \div 6^{-2}$
(A) 3 (B) $\frac{1}{3}$ (C) 72 (D) $\frac{9}{2}$

Q.35 Find the value of $\left(\frac{1}{3}\right)^{-3} + \left(\frac{1}{4}\right)^{-3} + \left(\frac{1}{2}\right)^{-3}$
(A) 99 (B) 729 (C) $\left(\frac{12}{13}\right)^3$ (D) $\frac{1}{99}$

Q.36 Find the value of $\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-3}$
(A) 75 (B) $\frac{1}{75}$ (C) $\frac{12}{13}$ (D) $\frac{1}{17}$

Q.37 Find the value of $\left\{\left(\frac{-6}{7}\right)^{-2}\right\}^{-2}$
(A) 1 (B) $\frac{-6}{7}$ (C) $\frac{1296}{2401}$ (D) $-\frac{1296}{2401}$

Q.38 $\left(\frac{-3}{5}\right)^{-1} \times \left(\frac{3}{5}\right)^2$
(A) $\frac{-5}{3}$ (B) $\frac{-3}{5}$ (C) $\frac{5}{3}$ (D) $\frac{3}{5}$

Q.39 By what number should $\left(\frac{-4}{7}\right)^2$ be multiplied to get $\left(\frac{49}{16}\right)^{-2}$?

(A) $\frac{7}{4}$ (B) $-\frac{7}{4}$ (C) $\frac{16}{49}$ (D) $-\frac{16}{49}$

Q.40 By what number should $(3)^{-2}$ be divided to get $(9)^{-3}$?

(A) 81 (B) $\frac{1}{81}$ (C) $\frac{1}{3}$ (D) $\frac{1}{9}$

Q.41 The product of two numbers is $\left(\frac{2}{5}\right)^{-3}$. If one of them is $\left(\frac{5}{4}\right)^{-2}$, find the other.

(A) $\frac{2^7}{5^5}$ (B) $\frac{2^2}{5^1}$ (C) $\frac{5^{-1}}{2^{-3}}$ (D) $\frac{5^5}{2^7}$

Q.42 If $(3^{2x+1}+9) \div 9 = 10$, find the value of x.

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) 1

Q.43 By what number should $(-5)^{-3}$ be multiplied to get $\left(\frac{1}{5}\right)^2$?

(A) 5 (B) –5 (C) $\frac{1}{5}$ (D) 5^2

Q.44 Find the value of $(2^{-1} \times 4^{-1}) \div 2^{-2}$

(A) 2 (B) $\frac{1}{2}$ (C) 4 (D) $\frac{1}{4}$

Q.45 Find the value of $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

(A) 29 (B) $\frac{1}{29}$ (C) $\frac{81}{144}$ (D) $\frac{169}{144}$

Q.46 Find the value of $\left\{5^{-1} + \left(\frac{1}{2}\right)^{-2} + 2^{-1}\right\}^0$

(A) 0 (B) 1 (C) $\frac{15}{2}$ (D) $\frac{47}{10}$

Q.47 By what number should $\left(\left(\frac{-2}{13}\right)^2\right)^{-3}$ be divided to obtain $\left(\frac{-2}{13}\right)^3$?

(A) $\left(\frac{-2}{13}\right)^9$ (B) $\left(\frac{13}{2}\right)^9$ (C) $\left(\frac{-13}{2}\right)^9$ (D) $\left(\frac{-13}{2}\right)^6$

Q.48 Simplify and express in the exponential form :

$\left(\frac{3}{11}\right)^2 \times \left(\frac{3}{11}\right)^5 \times \left(\frac{3}{11}\right)^9$

(A) $\left(\frac{3}{11}\right)^{16}$ (B) $\left(\frac{3}{11}\right)^{90}$ (C) $\left(\frac{27}{1331}\right)^9$ (D) $\left(\frac{6}{11}\right)^5$

Q.49 $\left(\frac{11}{7}\right)^9 \div \left(\frac{11}{7}\right)^7$

(A) $\left(\frac{7}{11}\right)^2$ (B) $\left(\frac{11}{7}\right)^2$ (C) $\left(\frac{11}{7}\right)^{16}$ (D) $\left(\frac{7}{11}\right)^{16}$

Q.50 $\left(\frac{2}{5}\right)^4 \times \left(\frac{15}{16}\right)^4$

(A) $\left(\frac{3}{8}\right)^4$ (B) $\left(\frac{3}{8}\right)^8$ (C) $\left(\frac{3}{8}\right)^{16}$ (D) $\left(\frac{30}{80}\right)^0$

Q.51 Subtract the sum of $\frac{-7}{8}$ and $\frac{-5}{6}$ from the sum of $\frac{2}{3}$ and $\frac{-8}{15}$.

(A) $\frac{221}{120}$ (B) $-\frac{221}{120}$ (C) $\frac{35}{48}$ (D) $\frac{16}{45}$

Q.52 Write the multiplicative inverse of $(-3)^3 \times \frac{1}{4^2}$

(A) 1 (B) $\frac{27}{16}$ (C) $\frac{-16}{27}$ (D) $-\frac{27}{16}$

Q.53 $\frac{1}{27} \div \frac{1}{3^4}$

(A) $\frac{1}{3}$ (B) 3 (C) 1 (D) 3^7

Q.54 Simplify : $[5^{-1} - 7^{-1}]^{-1}$

(A) $\frac{35}{2}$ (B) $\frac{-1}{2}$ (C) $\frac{2}{35}$ (D) -2

Q.55 Simplify : $\left[\left(\frac{2}{3}\right)^{-1} - \left(\frac{1}{2}\right)^{-1}\right]^{-1}$

(A) -2 (B) $\frac{-1}{2}$ (C) 1 (D) 6

Q.56 Find x, so that $(-6)^{x+1} \times (-6)^5 = (-6)^8$

(A) 0 (B) 1 (C) 2 (D) 3

Q.57 Simplify : $\left\{\left(\frac{1}{5}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

(A) $\frac{1}{4}$ (B) $\frac{17}{16}$ (C) $\frac{18}{17}$ (D) $\frac{25}{16}$

Q.58 Simplify : $\left(\frac{3}{8}\right)^{-7} \times \left(\frac{8}{3}\right)^{-2}$

(A) $\left(\frac{3}{8}\right)^5$ (B) $\left(\frac{3}{8}\right)^{-9}$ (C) $\left(\frac{8}{3}\right)^5$ (D) $\left(\frac{8}{3}\right)^{-9}$

Q.59 Simplify : $\frac{3^{-5} \times 10^{-4} \times 5^4}{5^2 \times 6^{-5}}$

(A) $\frac{2}{5^2}$ (B) $\frac{6}{25}$ (C) 0 (D) 50

Q.60 Solve for x.
$2^{2x} \times 2^{x+9} = (4^3)^6$

(A) 0 (B) 9 (C) $\frac{1}{3}$ (D) 4

Q.61 Solve for x : $27 \times 3^{5x-2} = \frac{1}{81}$.

(A) 0 (B) -1 (C) 2 (D) -3

Q.62 By what number $(4)^{-3}$ be multiplied so that the product becomes $\frac{1}{16}$?

(A) $\frac{1}{4^5}$ (B) 4^2 (C) $\frac{1}{16}$ (D) 4

Q.63 By what number should $\left(\frac{3}{4}\right)^{-3}$ be divided so that the quotient becomes 128.

(A) $\frac{1}{27}$ (B) $\frac{1}{54}$ (C) 27 (D) 54

Q.64 Simplify : $3^0 + 2^{-2}$

(A) $\frac{1}{25}$ (B) $\frac{5}{4}$ (C) $\frac{1}{4}$ (D) 5

Q.65 Find x so that $\left(\frac{7}{8}\right)^{-3} \times \left(\frac{7}{8}\right)^{5} = \left(\frac{7}{8}\right)^{x}$.

(A) 2 (B) 3 (C) 1 (D) –1

Q.66 Find x so that $\left(\frac{2}{3}\right)^{-2} \times \left(\frac{2}{3}\right)^{-9} = \left(\frac{2}{3}\right)^{2x+1}$.

(A) 6 (B) –6 (C) –8 (D) 4

Q.67 If $\frac{x}{y} = \left(\frac{3}{5}\right)^{-3} \times \left(\frac{5}{2}\right)^{-3}$, then find $\left(\frac{x}{y}\right)^{-1}$.

(A) $\frac{8}{27}$ (B) $\left(\frac{3}{2}\right)^{-6}$ (C) $\frac{27}{8}$ (D) $\frac{6}{25}$

Q.68 Find the value of x : $(4^{-1} + 8^{-1}) \times (3^{-1} - 9^{-1}) \div \frac{1}{12} = 5^x$.

(A) 0 (B) 1 (C) –6 (D) $\frac{1}{5}$

Q.69 Simplify : $\frac{\left[\left(\frac{2}{3}\right)^{-2} \div \left(\frac{2}{3}\right)^{-1}\right] \times \left(\frac{3}{2}\right)^{4}}{\left(\frac{3}{2}\right)^{2} \times \left(\frac{3}{2}\right)^{-3}}$.

(A) 1 (B) $\frac{1}{64}$ (C) $\frac{729}{64}$ (D) $\frac{64}{729}$

Q.70 Simplify : $(6^2 \times 3^{-2})\,(6^2 \div 3^{-2})$

(A) 16 (B) 296 (C) 324 (D) 1296

Q.71 Simplify : $(3^0 + 2^0) + (3^0 - 2^0)$

(A) 2 (B) 6 (C) 0 (D) 1

Q.72 Simplify : $\dfrac{27x^3yz^{-3}}{3x(y \times z)^3}$

(A) $9x^2$ (B) $\dfrac{9x^2}{y^2z^6}$ (C) $\dfrac{9x^3}{yz}$ (D) $\dfrac{9x^2y}{z^3}$

Q.73 Solve for x : $5^{-3x} : 5^{x+7} = 1 : 125$.

(A) –1 (B) 1 (C) 0 (D) 2

Q.74 Find the value of $\left(\dfrac{p}{q}\right)^3$ if $\left(\dfrac{7}{2}\right)^{-4} \times \left(\dfrac{4}{49}\right)^3 = \left(\dfrac{p}{q}\right)^{-10}$.

(A) $\dfrac{2}{7}$ (B) $\dfrac{7}{2}$ (C) $\dfrac{343}{8}$ (D) $\dfrac{8}{343}$

Q.75 Evaluate : $\left(\dfrac{5}{7}\right)^{2+3} \div \left(\dfrac{5}{7}\right)^5$

(A) 0 (B) 1 (C) –1 (D) $\left(\dfrac{5}{7}\right)^{10}$

Q.76 Simplify : $\left(\dfrac{-2}{3}\right)^4 \times \dfrac{27}{32}$

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{6}$ (D) 3

Q.77 Simplify : $\left[\left(\dfrac{1}{5}\right)^6 \div \left(\dfrac{1}{5}\right)^5\right] \div \dfrac{1}{5}$

(A) $\dfrac{1}{5}$ (B) $\left(\dfrac{1}{5}\right)^{10}$ (C) 5 (D) 1

Q.78 Simplify : $\left[\left(\dfrac{-2}{3}\right)^3 \times \left(\dfrac{-2}{3}\right)\right] \div \left(\dfrac{4}{9}\right)^2$

(A) 1 (B) $\dfrac{2}{3}$ (C) $\dfrac{3}{4}$ (D) $\dfrac{4}{9}$

Q.79 Simplify : $\frac{18^{-1}\times p^{-3}}{3^{-2}\times 10^{-2}\times p^{-5}}, (p\neq 0)$

(A) p^2 (B) 1 (C) $50\,p^2$ (D) $\frac{p^2}{9}$

Q.80 Simplify : $\frac{a^{-4}\times 25\times b^{-2}}{(ab)^{-3}\times 10^{-1}}, (a, b\neq 0)$

(A) $\frac{250\,b}{a}$ (B) $\frac{25}{10}ab$ (C) $250a$ (D) $\frac{25}{10}ab^2$

Q.81 Find m, if $\left(\frac{3}{7}\right)^5\times\left(\frac{3}{7}\right)^{-2}=\left(\frac{3}{7}\right)^m$

(A) 2 (B) 3 (C) 7 (D) 5

Q.82 Find m, if $\left(\frac{1}{8}\right)^3\div\left(\frac{1}{8}\right)^6=8^m$

(A) 3 (B) 8 (C) 1 (D) 2

Q.83 Find x, if $\left(\frac{2}{3}\right)^{-5}\times\left(\frac{2}{3}\right)^{12}=\left(\frac{2}{3}\right)^{3x-2}$

(A) –2 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 3

Q.84 Find the value of x^{-2}, if $x=\left(\frac{3}{7}\right)^{-5}\div\left(\frac{11}{14}\right)^0$.

(A) $\left(\frac{3}{7}\right)^{-10}$ (B) $\frac{3}{7}$ (C) $\left(\frac{7}{3}\right)^5$ (D) $\left(\frac{7}{3}\right)^{-10}$

Q.85 Simplify $\left(\frac{2}{3}\right)^3\times\left(\frac{2}{3}\right)^{-2}\times\left[\left(\frac{1}{2}\right)^2\right]^{-2}\times\frac{1}{24}$

(A) 1 (B) $\frac{2}{3}$ (C) $\frac{4}{9}$ (D) $\frac{8}{24}$

Q.86 Find the value of $\frac{x}{y}$, if $\left(\frac{3}{5}\right)^{-4} \times \left(\frac{15}{10}\right)^{-4} = \left(\frac{x}{y}\right)^{-4}$

(A) 1 (B) $\frac{9}{10}$ (C) $\frac{10}{9}$ (D) $\frac{1}{4}$

Q.87 Evaluate $(14^2 - 13^2)^{5/3}$.

(A) 1 (B) $2^{10/3}$ (C) 243 (D) $\frac{5}{3}$

Q.88 If $21168 = x^4 \times y^3 \times z^2$, find $(x + y + z)^{\left(\frac{y+z}{x+y}\right)}$, where x, y and z are positive integers.

(A) 144 (B) 169 (C) 1 (D) $9^{5/7}$

Q.89 Simplify: $\frac{5(81)^{n+1} - 3^{4n+5}}{3 \times 3^{4n} + (81)^n}$.

(A) $\frac{2}{81}$ (B) $\frac{81}{2}$ (C) $\frac{3}{81}$ (D) $\frac{81}{4}$

Q.90 If $\left(\frac{32}{243}\right)^n = \frac{8}{27}$, find $\left(\frac{n + 0.4}{1024}\right)^{-n}$.

(A) 96 (B) 1 (C) 64 (D) $\frac{3}{5}$

Q.91 If $16200 = 2^a \times 3^b \times 5^c$, then find a, b and c.

(A) a = 3, b = 4, c = 2 (B) a = 4, b = 2, c = 3
(C) a = 2, b = 3, c = 4 (D) a = 3, b = 2, c = 4

Q.92 Simplify: $\left(\frac{p^a}{p^b}\right)^c \times \left(\frac{p^b}{p^c}\right)^a \times \left(\frac{p^c}{p^a}\right)^b$.

(A) 0 (B) 1 (C) p^{abc} (D) $p^{\frac{1}{abc}}$

Q.93 If $\left[\sqrt[9]{\left(\frac{2}{3}\right)^5}\right]^{\sqrt{x-5}} = a^0$, find the value of x.

(A) 1 (B) 0 (C) 5 (D) $\frac{1}{5}$

Q.94 Simplify: $\dfrac{(p^{a+b})^5 \cdot (p^{b+c})^5 \cdot (p^{c+a})^5}{(p^a \cdot p^b \cdot p^c)^{10}}$

(A) $\dfrac{1}{p}$ (B) 1 (C) p^{5abc} (D) $p^{\frac{5}{abc}}$

Q.95 If $5^{3x+2} = 25 \times 5^{(4x-1)}$, find the value of x.

(A) 1 (B) 2 (C) –1 (D) 0

Q.96 Simplify: $\dfrac{(a+b+c)^{a+b} \cdot (a+b+c)^{b+c} \cdot (a+b+c)^{c+a}}{[(a+b+c)^a\ (a+b+c)^b\ (a+b+c)^c]^2}$

(A) 0 (B) 1 (C) (a + b + c) (D) $\dfrac{1}{a+b+c}$

Q.97 Simplify: $(a^{p+q})^{p-q} \cdot (a^{q+r})^{q-r} \cdot (a^{r+p})^{r-p}$

(A) a (B) a^{p+q+r} (C) 0 (D) 1

Q.98 Simplify: $\left[\left(\dfrac{a}{b}\right)^{\sqrt{p}+\sqrt{q}}\right]^{\sqrt{p}-\sqrt{q}} \left[\left(\dfrac{a}{b}\right)^{\sqrt{q}+\sqrt{r}}\right]^{\sqrt{q}-\sqrt{r}} \left[\left(\dfrac{a}{b}\right)^{\sqrt{r}+\sqrt{p}}\right]^{\sqrt{r}-\sqrt{p}}$

(A) $\left(\dfrac{a}{b}\right)$ (B) 1 (C) $\left(\dfrac{a}{b}\right)^{\sqrt{p}+\sqrt{q}+\sqrt{r}}$ (D) $\left(\dfrac{a}{b}\right)^{\sqrt{p}+\sqrt{q}-\sqrt{r}}$

Q.99 $\left(\dfrac{x^{-1}+y^{-1}}{x^{-1}}\right)^{-1} + \left(\dfrac{x^{-1}-y^{-1}}{x^{-1}}\right)^{-1}$

(A) $\dfrac{2y^2}{y^2-x^2}$ (B) $\dfrac{2x^2}{y^2+x^2}$ (C) $\dfrac{2(x+y)}{x}$ (D) $\dfrac{2(x+y)}{y}$

Q.100 If a, b, c are real numbers, then $\sqrt[5]{ab^{-1}} \times \sqrt[5]{bc^{-1}} \times \sqrt[5]{ca^{-1}}$.

(A) $(abc)^{-1/5}$ (B) 1 (C) $(abc)^{-5}$ (D) $\dfrac{1}{abc}$

ANSWER KEY

CONCEPT APPLICATION LEVEL - II

SECTION - A

Q.1 $\left(\frac{1}{9}\right)$ Q.2 $\left(\frac{5}{3}\right)^8$ Q.3 1 Q.4 5 Q.5 0 Q.6 9 Q.7 $\frac{13}{14}$

Q.8 $\frac{3}{8}$ Q.9 $\left(\frac{5}{4}\right)^3$ Q.10 9

SECTION - B

Q.1	C	Q.2	B	Q.3	A	Q.4	A	Q.5	B	Q.6	C	Q.7	A
Q.8	B	Q.9	B	Q.10	C	Q.11	C	Q.12	B	Q.13	B	Q.14	D
Q.15	D	Q.16	A	Q.17	C	Q.18	A	Q.19	D	Q.20	B	Q.21	C
Q.22	C	Q.23	B	Q.24	D	Q.25	B	Q.26	D	Q.27	A	Q.28	C
Q.29	A	Q.30	B	Q.31	D	Q.32	A	Q.33	D	Q.34	A	Q.35	A
Q.36	A	Q.37	C	Q.38	B	Q.39	C	Q.40	A	Q.41	D	Q.42	A
Q.43	B	Q.44	B	Q.45	A	Q.46	B	Q.47	C	Q.48	A	Q.49	B
Q.50	A	Q.51	A	Q.52	C	Q.53	B	Q.54	A	Q.55	A	Q.56	C
Q.57	B	Q.58	C	Q.59	A	Q.60	B	Q.61	B	Q.62	D	Q.63	B
Q.64	B	Q.65	A	Q.66	B	Q.67	C	Q.68	A	Q.69	C	Q.70	D
Q.71	A	Q.72	B	Q.73	A	Q.74	C	Q.75	B	Q.76	C	Q.77	D
Q.78	A	Q.79	C	Q.80	A	Q.81	B	Q.82	A	Q.83	D	Q.84	D
Q.85	C	Q.86	B	Q.87	C	Q.88	A	Q.89	B	Q.90	C	Q.91	A
Q.92	B	Q.93	C	Q.94	B	Q.95	A	Q.96	B	Q.97	D	Q.98	B
Q.99	A	Q.100	B										