12 DIRECT AND INVERSE VARIATION

12.1 INTRODUCTION

The value of variable is not constant and keeps on changing. There are many quantities whose value varies as per the circumstances. Some quantities have a relation with other quantities such that when one changes the other also changes. Such quantities are inter-related. This is called a variation. Variation is of two types: (i) Direct variation and (ii) Inverse variation.

12.2 DIRECT VARIATION

If two quantities are related in such a way that an increase in one quantity leads a corresponding increase in the other and also, a decrease in one quantity brings a corresponding decrease in the other, then this is a case of direct variation.

Two quantities x and y are said to be in direct variation, if $\Rightarrow x \propto y$

Here symbol '$\propto$' means varies as

or $x = ky$ or $\frac{x}{y} = k$ $\Rightarrow$ $\frac{x}{y} = \text{constant}$

Note : $\frac{x}{y}$ is always a positive number.

12.3 INVERSE VARIATION

Two quantities x and y said to vary in inverse variation, if x varies inversely as y $\Rightarrow x \propto \frac{1}{y}$ or $x = \frac{k}{y}$,

k is constant of proportionality, Also $xy = k$, there exists a relation of the type $xy = k$ between them, k is a constant called constant of proportionality, $k = xy$.

12.4 UNITARY METHOD

The method in which first we find the value of one unit and then the value of required number of units is known as **Unitary Method**.

12.4.1 Concept of Efficiency

When we say P is twice as efficient as Q. We mean to say that P does twice the work as Q in the same time. In other words we can also understand this as P will require half the time required by Q to complete the same work.

12.4.2 Concept of Mandays

The number of men multiplied by the number of days required to complete the work will give the number of MANDAYS. Total Mandays to complete a specific task will remain constant.

12.4.3 Concept of Wages

If more than one person are engaged on payment basis for doing a work, then wages distributed to each person are :

(a) in proportion of the 1 day work of each person.

(b) in proportion to the work done by each person.

(c) in inverse proportion to time needed by a person to complete the work.

12.5 TIME AND WORK

If A does a work in 'a' days then in 1 day A does $\frac{1}{a}$ of the work.

If B does a work in 'b' days then in 1 day B does $\frac{1}{b}$ of the work.

Then in 1 day, if A and B work together, their combined work is $\frac{1}{a}+\frac{1}{b}$ or $\frac{a+b}{ab}$.

The work will be completed when 1 unit of work is completed.

Now using Unitary Method

Time required to complete $\frac{a+b}{ab}$ work = 1 day

$\therefore$ Time required to complete 1 work = $\frac{1}{\frac{a+b}{ab}} = \frac{ab}{a+b}$

12.6 TIME AND DISTANCE

Motion / Movement occurs when a body of any shape and size changes its position with respect to any external stationary point.

The mathematical equation that describe the motion has three variables Speed, Time and Distance, which are connected by the following formula

Distance = Speed × Time

From the above equation, we can have the following conclusions :

(a) If speed is constant then distance and time are directly proportional to each other. i.e.

Distance $\propto$ Time.

(b) If time is constant, then distance and speed are directly proportional to each other. i.e.

Distance $\propto$ Speed

(c) When distance is constant then speed and time are inversely proportional to each other i.e.

$$\text{Speed} \propto \frac{1}{\text{Time}}$$

Normally speed is measured in km/hr or m/s.

Note : 1 km/hr = $\frac{1000\,\text{m}}{3600\,\text{s}} = \frac{5}{18}\,\text{m/s}$

or 1 m/s = $\frac{18}{5}\,\text{km/hr}$.

12.6.1 Average Speed

It is defined as the ratio of total distance covered to the total time taken by an object.
If an object travels $d_1, d_2, d_3, ..., d_n$ metres with different speeds $s_1, s_2, s_3......., s_n$ metres/sec in time $t_1, t_2, t_3,......, t_n$ seconds respectively, then average speed S_a is given by

$$S_a = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$$

12.6.2 Relative speed

Earlier we discussed movement of a body with respect to a stationary point. However we also need to determine the movement and its relationship with respect to a moving point / body.
The movement of a body / point with respect to another is called **relative movement**.

Relative speed of two independent bodies :

(a) If two bodies are moving in opposite direction at speeds s_1 and s_2 then their relative speed is $(s_1 + s_2)$

(b) If two bodies are moving in same direction at speeds s_1 and s_2 then
  (i) Relative speed is $(s_1 - s_2)$ if $s_1 > s_2$
  (ii) Relative speed is $(s_2 - s_1)$ if $s_2 > s_1$

The above concepts are applied in case of movements of trains. The following formula can be used to find time required by a train to cross different types of objects.

$$\text{Time to cross an object moving in the direction of train} = \frac{\text{Length of train} + \text{Length of object}}{\text{Speed of train} - \text{Speed of object}}$$

(i) If object has negligible length like a man, lamp post etc. We take length of object = 0

(ii) If object is moving in opposite direction of train, then put '–' sign in front of speed of object thus denominator will be (speed of train + speed of object)

(iii) We will also use 1 kmph = $\frac{5}{18}$ m/s to convert units of speed.

(iv) If two trains start at the same time from points A and B towards each other and after crosssing they take 'a' and 'b' seconds in reaching B and A respectively, then A's speed : B's speed = $\sqrt{b} : \sqrt{a}$.

12.7 BOATS AND STREAMS

When the water is not moving in a river or the speed of water in the river is zero, we say that it is **still water**.
When the water is moving with certain speed that we say it is stream water.
Let a swimmer or boat is moving in a river.

(i) If boat is moving in the direction of water stream, we call it as downstream movement.

(ii) If boat is moving in opposite direction of water stream, we call it as upstream movement.

(iii) When boat is moving down stream, the speed of water increases the speed of boat and

(iv) When boat is moving upstream, the speed of water opposes the speed of boat.

(v) If speed of boat in still water = S_B, Speed of current = S_C then
  Speed of boat in upstream = $(S_B - S_C)$
  Speed of boat in downstream = $(S_B + S_C)$

(vi) If the speed of boat in downstream = x and speed of boat upstream = y then Speed of boat in still water = $\frac{x+y}{2}$, Speed of water stream = $\frac{x-y}{2}$

SOLVED EXAMPLES

Example 1 :

Observe the following tables and find if x and y are directly proportional.

(i)

x	20	17	14	11	8	5	2
y	40	34	28	22	16	10	4

(ii)

x	6	10	14	18	22	26	30
y	4	8	12	16	20	24	28

(iii)

x	5	8	12	15	18	20
y	15	24	36	60	72	100

Solution :

(i) $\because$ $\frac{20}{40} = \frac{1}{2}$, $\frac{17}{34} = \frac{1}{2}$, $\frac{14}{28} = \frac{1}{2}$

$\frac{11}{22} = \frac{1}{2}$, $\frac{8}{16} = \frac{1}{2}$, $\frac{5}{10} = \frac{1}{2}$

$\frac{2}{4} = \frac{1}{2}$

i.e. Each ratio is the same.

$\therefore$ x and y are directly proportional

(ii) $\because$ $\frac{6}{4} = \frac{3}{2}$, $\frac{10}{8} = \frac{5}{4}$, $\frac{14}{12} = \frac{7}{6}$

$\frac{18}{16} = \frac{9}{8}$, $\frac{22}{20} = \frac{11}{10}$, $\frac{26}{24} = \frac{13}{12}$

$\frac{30}{28} = \frac{15}{14}$

i.e. All the ratios are not the same.

$\therefore$ x and y are not directly proportional.

(iii) $\because$ $\frac{5}{15} = \frac{1}{3}$, $\frac{8}{24} = \frac{1}{3}$, $\frac{12}{36} = \frac{1}{3}$

$\frac{15}{60} = \frac{1}{4}$, $\frac{18}{72} = \frac{1}{4}$, $\frac{20}{100} = \frac{1}{5}$

i.e. All the ratios are not the same.

$\therefore$ x and y are not directly proportional.

Example 2 :

If 15 books cost ₹ 35, what do ₹ 21 books cost ?

Solution :

Let cost of 21 books is ₹ x.

As we know, more number of books, more is the cost, so it is an example of direct variation

$\therefore$ 15 : 35 = 21 : x

$\Rightarrow$ $x = \dfrac{35 \times 21}{15}$ = ₹ 49

$\therefore$ Cost of 21 books will be ₹49.

Example 3 :

A fort had provisions for 150 men for 50 days. After 15 days 25 men left fort. How long will food last at the same rate for remaining men ?

Solution :

Let required number of days be 'x'. Here the number of men are reduced so the food will last longer. It is a case of indirect variation.

Now, after 15 days the remaining food will last for 150 men for (50 – 15) = 35 days.

Also 25 men left the fort so remaining men are (150 – 25) = 125

$\therefore$ $150 \times 35 = 125 \times x$

$\Rightarrow$ $x = \dfrac{150 \times 35}{125} = 42$ days

Example 4 :

Three quantities P,Q,R are such that PQ = KR, where K is constant. When P is kept constant, Q varies directly as R; When Q is constant, P varies directly as R and when R is constant P varies inversely as Q. Initially P was 3 and P : Q : R was 1 : 3 : 5. Find P when Q is 18 and R remains constant.

Solution :

Initial values of P, Q, R are 3, 9, 15 respectively

$\therefore$ $3 \times 9 = 15 \times K$

$\Rightarrow$ $K = \dfrac{27}{15} = \dfrac{9}{5}$

$\therefore$ equation is $PQ = \dfrac{9}{5}R$

Now for Q = 18 at constant R

$P \times 18 = \dfrac{9}{5} \times 15$

$P = \dfrac{9 \times 15}{5 \times 18} = \dfrac{5}{2} = 1.5$

Example 5 :

If 8 men can reap 80 hectares in 24 days, how many hectares can 36 men reap in 30 days.

Solution :

Let the required hectares be 'x'. We have the following situation

Men	Hectare	Days
8	80	24
36	x	30

Now keeping number of days constant, more men can reap more hectares

$\therefore$ 36 men can reap $\frac{80 \times 36}{8} = 360$ hectares in 24 days.

Now keeping number of men constant, In more number of days more hectares can be covered

$\therefore$ in 30 days 36 men can reap $\frac{360 \times 30}{24} = 450$ hectares

$\therefore$ x = 450 hectares.

Example 6 :

Three persons A, B and C can do a job alone in 10 days, 12 days and 15 days respectively, In how many days they can finish the job working together?

Solution :

One day work of A, B and C are $\frac{1}{10}, \frac{1}{12}$ and $\frac{1}{15}$ respectively.

One day work if they work together $= \left(\frac{1}{10} + \frac{1}{12} + \frac{1}{15}\right) = \frac{15}{60} = \frac{1}{4}$

$\therefore$ Time required to complete the job $= \frac{1}{1/4} = 4$ days

Example 7 :

Two persons A and B can do a piece of work alone in 10 days and 15 days respeactively, If A works for 2days and B works for 5 days, find the total amount of work done.

Solution :

1 days work of A and B are $\frac{1}{10}$ and $\frac{1}{15}$ respectively.

2 days work of A $= \frac{2}{10} = \frac{1}{5}$

5 day work of B $= \frac{5}{15} = \frac{1}{3}$

$\therefore$ they complete $\left(\frac{1}{5} + \frac{1}{3}\right) = \frac{8}{15}$ work if they work for 2 and 5 days respectively.

Example 8 :

A and B can do a piece of work in 12 days. B and C is 15 days, C and A in 20 days. How long would each take separately to do the same work ?

Solution :

We know that (A + B) + (B + C) + (C + A) = 2 (A + B + C)

$\therefore$ 2 (A + B + C)'s one day work = 1 day work of (A + B) + 1 day work of (B + C) + 1 day work of (C + A) = $\frac{1}{12}+\frac{1}{15}+\frac{1}{20}$

2 (A + B + C)' 1 day work = $\frac{5+4+3}{60}=\frac{12}{60}=\frac{1}{5}$

Now 1 day work of A = 1 day work of (A + B + C) – 1 day work of (B + C)

$$=\frac{1}{10}-\frac{1}{15}=\frac{3-2}{30}=\frac{1}{30}$$

$\therefore$ A can complete the work in 30 days.

Similarly 1 day work of B = $\frac{1}{10}-\frac{1}{20}=\frac{1}{20}$

$\therefore$ B can complete the work in 20 days.

1 day work of C = $\frac{1}{10}-\frac{1}{12}=\frac{6-5}{60}=\frac{1}{60}$

$\therefore$ C can complete the work in 60 days.

Example 9:

Jatin is thrice as good a workman as Suraj and is therfore able to finish a piece of work in 20 days less than Suraj. Find the time in which they can do it working together.

Solution :

If Jatin does a work in 1 day. Suraj will complete it in 3 days. So the differece is 3 – 1 = 2 = 1 × 2

Now for 20 days differnce i.e. (10 × 2). Jatin can do the job in 10 days and Suraj will take 10 × 3 = 30 days.

$\therefore$ 1 day work of Jatin = $\frac{1}{10}$; 1 day work of Suraj = $\frac{1}{30}$

1 day work of (Jatin + Suraj) = $\frac{1}{10}+\frac{1}{30}=\frac{4}{30}=\frac{2}{15}$

$\therefore$ Working together Jatin & Suraj can finish the job in $\frac{15}{2}$ or $7\frac{1}{2}$ days.

Example 10 :

If 5 men or 9 women can do a piece of work in 19 days. In how many days 3 men and 6 women will do the same work ?

Solution :

5 men = 9 women

$\therefore$ 1 men = $\frac{9}{5}$ women

$\therefore$ 3 men + 6 women = $3\times\frac{9}{5}+6=\frac{57}{5}$ women

$\because$ 9 women can do the work in 19 days

$\therefore$ $\frac{57}{5}$ women can do the work in $\frac{19\times9}{57}\times5 = 15$ days

Example 11 :

A car travels 300 km in 5.5 hrs and another 400 km in 8.5 hrs. Find the average speed of the car during the entire journey.

Solution :

Total distance trevelled = 300 + 400 = 700 km

Total time taken = 5.5 + 8.5 = 14 hrs.

$\therefore$ Average speed = $\frac{\text{Total distance travelled}}{\text{Total Time taken}} = \frac{700}{14}$ = 50 km/hr.

Example 12 :

An increase in the speed of car by 10 km per hour saves 1 hour in a journey of 200 km, find the initial speed of the car.

Solution :

Let the initial speed of car be x km per hour.

Time taken to cover distance 200 km, 't_1' = $\frac{200}{x}$

Time taken to cover 200 km with increased speed $t_2 = \frac{200}{x+10}$

Now, $t_1 - t_2 = 1$

$\therefore$ $\frac{200}{x} - \frac{200}{x+10} = 1 \Rightarrow x(x+10) = 2000 \Rightarrow x(x+10) = 40\times50$

From here x = 40 km/hr.

$\therefore$ Initial speed of the car is 40 km/hr

Example 13 :

A car travels a distance of 170 km in 2 hours partly at a speed of 100 km/hr and partly at 50 km/hr. Find the distace travelled at speed of 100 km/hr.

Solution :

Let distance travelled at 100 km/hr be 'x' km.

$\therefore$ Distance travelled at 50 km/hr is (170 – x) km.

Total time taken to cover 170 km is 2 hrs.

$$\therefore \quad \frac{x}{100} + \frac{170 - x}{50} = 2$$

$\Rightarrow$ x + 340 – 2x = 200

$\Rightarrow$ x = 140 km

$\therefore$ Distance travelled at 100 km/hr is 140 km.

Example 14 :

A truck travels a distance of 240 km in 6 hours, partly at a speed of 60 km/hr and partly at 30 km/hr. Find the time for which it travels at 60 km/hr.

Solution :

Let the truck travels for 't' hour at 60 km/hr.

$\therefore$ $60 \times t + 30 \times (6 - t) = 240$

$\Rightarrow$ $60t + 180 - 30t = 240$

$\Rightarrow$ $30t = 60$

$\Rightarrow$ t = 2hr.

$\therefore$ Truck travels 2 hours at 60 km/hr.

Example 15 :

A train 110 m long travels at 60 kmph. How long does it take to cross :

(a) a lamp post

(b) a dog running at 6 kmph in the direction of train.

(c) a dog running at 6 kmph in the opposite direction of train.

(d) a bridge 240 m long.

(e) another stationary train 170 m in length.

(f) another train of length 170 m running at 54 kmph in the same direction.

(g) another train of length 170 m running at 80 kmph in the opposite direction.

Solution :

(a) $60 \text{ kmph} = 60 \times \frac{5}{18} \text{ m/s}$

As lamp post is stationary

$$\therefore \quad \text{time required to cross} = \frac{\text{length of train}}{\text{speed of train}} = \frac{110}{60 \times \frac{5}{18}} = \frac{18 \times 110}{60 \times 5} = \frac{33}{5} = 6.6 \text{ seconds}$$

(b) Dog is running in the same direction.

$$\therefore \quad \text{Time required} = \frac{110}{(60-6) \times \frac{5}{18}} = \frac{110 \times 18}{54 \times 5} = 7.33 \text{ seconds}$$

(c) Dog is running in opposite direction.

$\therefore$ time required $= \dfrac{110}{[60-(-6)]\times\dfrac{5}{18}} = \dfrac{110\times18}{\cdot 66\times5} = 6$ seconds

(d) Bridge is stationary.

$\therefore$ time required to cross bridge $= \dfrac{(110+240)}{60\times\dfrac{5}{18}} = 21$ seconds.

(e) Train is stationary.

$\therefore$ time required to cross stationary train $= \dfrac{(110+170)}{60\times\dfrac{5}{18}} = 16.8$ seconds

(f) Train is running in the same direction

$\therefore$ time required $= \dfrac{110+170}{(60-54)\times\dfrac{5}{18}} = \dfrac{280\times18}{6\times5} = 168$ seconds

(g) Train is running in the opposite direction.

$\therefore$ time required $= \dfrac{110+170}{(60+80)\times\dfrac{5}{18}} = \dfrac{280\times18}{140\times5} = 7.2$ seconds

Example 16 :

Two trains were running in the same direction at 90 km/hr and 60 km/hr respectively, the faster train crossed a man in the slower train in 27 seconds. Find the length of the faster train.

Solution :

Let the length of faster train be L_F

Speed of faster train $V_f = 90$ km/hr $= 90\times\dfrac{5}{18} = 25$ m/s

Speed of slower train $V_s = 60$ km/hr $= 60\times\dfrac{5}{18} = \dfrac{50}{3}$ m/s

$\therefore$ Man is sitting in slower train.

Speed of man = speed of slower train (V_s)

so time taken by faster train to cross a man in slower train $= \dfrac{L_T}{V_f - V_s}$

$27 = \dfrac{L_T}{25-\dfrac{50}{3}} \Rightarrow 27 = \dfrac{3L_T}{25} \Rightarrow L_T = 225$ m

$\therefore$ Length of train = 225 m

Example 17:

A man rows upstream 24 km and downstream 36 km taking 6 hours each. Find the speed of current.

Solution :

Let man's rowing speed in still water = x km/hr

Let speed of current = y km/hr

Downstream speed = $x + y = \frac{36}{6} = 6$ ...(1)

Upstream speed = $x - y = \frac{24}{6} = 4$ ...(2)

(1) – (2)

$2y = 2 \Rightarrow y = 1$

$\therefore$ speed of current = 1 km/hr

Example 18:

A man can row 5 kmph in the still water. If the river is running at 3 kmph, it takes him 5 hours to row up to a place and come down. How far is the place ?

Solution :

Let the distance = d km

Time taken to row upstream 't_1' = $\frac{d}{5-3} = \frac{d}{2}$ ...(1)

Time taken to row downstream 't_2' = $\frac{d}{5+3} = \frac{d}{8}$ ...(2)

$t_1 + t_2 = 5$ (Given)

$\therefore \quad \frac{d}{2} + \frac{d}{8} = 5$

$\Rightarrow \quad \frac{4d + d}{8} = 5 \quad \Rightarrow \quad d = 8$ km

$\therefore$ Distance of the place = 8 km.

Example 19 :

A man rows 10 km upstream and back again to the starting point in 55 minutes. If the speed of stream is 2 km/hr, find the speed of rowing in still water.

Solution :

Let speed of rowing in still water = x km/hr

$\therefore \quad \frac{10}{x-2} + \frac{10}{x+2} = \frac{55}{60} \quad \Rightarrow \quad 10\left(\frac{x+2+x-2}{x^2-4}\right) = \frac{11}{12}$

$\Rightarrow \quad 240x = 11x^2 - 44 \quad \Rightarrow \quad 11x^2 - 240x - 44$

$\Rightarrow \quad 11x^2 - 242x + 2x - 44 = 0 \quad \Rightarrow \quad 11x(x - 22) + 2(x - 22) = 0$

$\Rightarrow \quad x = 22 \ \& \ x = -\frac{2}{11}$ (Not possible)

$\therefore$ Speed of rowing in still water = 22 km/hr

Example 20 :

A car travels 270 km in $4\frac{1}{2}$ hrs.

(i) How much time is required to cover 90 km.

(ii) Find the distance covered in 6 hours.

Solution :

(i) 270 km is covered in $4\frac{1}{2}$ or $\frac{9}{2}$ hr

$\therefore$ 1 km is covered in $\frac{9}{2\times 270}$ hr

$\therefore$ 90 km is covered in $\frac{9\times 90}{2\times 270}=\frac{3}{2}$ hr.

$\therefore$ Time required to cover 90 km is $\frac{3}{2}$ or $1\frac{1}{2}$ hours

(ii) Distance travelled in $\frac{9}{2}$ hr $=270$ km

$\therefore$ Distance travelled in 1 hr $=\frac{270}{9}\times 2$

$\therefore$ Distance travelled in 6 hr $=\frac{270\times 2\times 6}{9}=360$ km

$\therefore$ Car will travel 360 km in 6 hours.

CONCEPT APPLICATION LEVEL - I [NCERT Questions]

EXERCISE - 1

Q.1 Following are the car parking charges near a railway station upto

4 hours	**Rs. 60**
8 hours	**Rs. 100**
12 hours	**Rs. 140**
24 hours	**Rs. 180**

Check if the parking charges are in direct proportion to the parking time.

Sol. We have

$$\Rightarrow \quad \frac{60}{4} = \frac{15}{1}$$

$$\Rightarrow \quad \frac{100}{8} = \frac{25}{2}$$

$$\Rightarrow \quad \frac{140}{12} = \frac{35}{3}$$

$$\Rightarrow \quad \frac{180}{24} = \frac{15}{2}$$

Since all the values are not the same, therefore, the parking charges are not in direct proportion to the parking time.

Q.2 A mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table, find the parts of base that need to be added.

Parts of red pigment	1	4	7	12	20
Parts of base	8	.....	.....	.....	.....

Sol. Let the number of parts of red pigment be x and the number of parts of the base be y.

As the number of parts of red pigment increases, number of parts of the base also increases in the same ratio. It is a case of direct proportion.

We make use of the relation of the type

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

(i) Here

$x_1 = 1$

$y_1 = 8$ and $x_2 = 4$

Therefore,

$\frac{x_1}{y_1} = \frac{x_2}{y_2}$ gives

$\frac{1}{8} = \frac{4}{y_2} \quad \Rightarrow \quad y_2 = 8 \times 4 \quad \Rightarrow \quad y_2 = 32$

Hence, 32 parts of the base are needed to be added.

(ii) Here
$x_1 = 1$
$y_1 = 8$ and $x_3 = 7$

Therefore, $\frac{x_1}{y_1} = \frac{x_3}{y_3}$ gives

$\frac{1}{8} = \frac{7}{y_3} \quad \Rightarrow \quad y_3 = 8 \times 7 \quad \Rightarrow \quad y_3 = 56$

Hence, 56 parts of the base are needed to be added.

(iii) Here
$x_1 = 1$
$y_1 = 8$
and $x_4 = 12$

Therefore, $\frac{x_1}{y_1} = \frac{x_4}{y_4}$ gives

$\frac{1}{8} = \frac{12}{y_4} \quad \Rightarrow \quad y_4 = 12 \times 8 \quad \Rightarrow \quad y_4 = 96$

Hence, 96 parts of the base are needed to be added.

(iv) Here
$x_1 = 1$
$y_1 = 8$
and $x_5 = 20$

Therefore, $\frac{x_1}{y_1} = \frac{x_5}{y_5}$ gives

$\frac{1}{8} = \frac{20}{y_5} \quad \Rightarrow \quad y_5 = 8 \times 20 \quad \Rightarrow \quad y_5 = 160$

Hence, 160 parts of the base are needed to be added.

Q.3. In Question 2 above, if 1 part of a red pigment requires 75 mL of base, how much red pigment should we mix with 1800 mL of base ?

Sol. Let the number of parts of red pigment be x and the amount of base be y mL.

As the number of parts of red pigment increases, amount of base also increases in the same ratio. It is a case of direct proportion. We make use of the relation of the type.

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

Here $x_1 = 1$

$y_1 = 75$ and $y_2 = 1800$

Therefore, $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ gives

$$\frac{1}{75} = \frac{x_2}{1800}$$

$\Rightarrow \quad 75x_2 = 1800$

$\Rightarrow \quad x_2 = \frac{1800}{75}$

$\Rightarrow \quad x_2 = 24$

Hence, 24 parts of the red pigment should be mixed.

Q.4 A machine in a soft drink factory fills 840 bottles in six hours. How many bottles will it fill in five hours ?

Sol. Let the machine fill x bottles in five hours. We put the given information in the form of a table as shown below

Number of bottles filled	840	x
Number of hours	6	5

More the number of hours, more the number of bottles would be filled So, the number of bottles filled and the number of hours are directly proportional to each other.

So, $\frac{x_1}{x_2} = \frac{y_1}{y_2}$

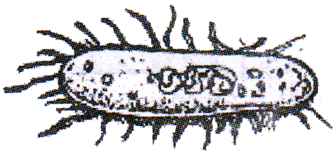
$\Rightarrow \quad \frac{840}{x_2} = \frac{6}{5} \quad \Rightarrow \quad 6x_2 = 840 \times 5 \quad \Rightarrow \quad x_2 = \frac{840 \times 5}{6}$

$\Rightarrow \quad x_2 = 700$

Hence, 700 bottles will be filled.

Q.5 A photograph of a bacteria enlarged 50,000 times attains a length of 5 cm as shown in the diagram. What is the actual length of the bacteria ? If the photograph is enlarged 20, 000 times only, what would be its enlarged length ?

Sol.



Actual length of the bacteria

$$= \frac{5}{50000}\text{ cm} \quad = \frac{1}{10000}\text{ cm} = 10^{-4}\text{ cm}$$

More the number of times a photograph of a bacteria is enlarged, more the length attained. So, the number of times a photograph of a bacteria is enlarged and the length attained are directly proportional to each other.

So, $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

$$\Rightarrow \quad \frac{50000}{5} = \frac{20000}{y_2} \qquad \Rightarrow \quad 50000\, y_2 = 5 \times 20000$$

$$\Rightarrow \quad y_2 = \frac{5 \times 20000}{50000} \qquad \Rightarrow \quad y_2 = 2$$

Hence, its enlarged length would be 2 cm.

Q.6 In a model of a ship, the mast is 9 cm high, while the mast of the actual ship is 12 m high. If the length of the ship is 28 m, how long is the model ship ?

Sol. Let the length of the model ship be x cm.

We form a table as shown below :

	Actual ship	Model
Length of the ship	28 m	x
Height of the mast	12 m	9 cm

More the length of the ship, more would be the length of its mast. Hence, this is a case of direct proportion. That is,

$$\Rightarrow \quad 12x = 28 \times 9 \qquad \Rightarrow \quad x = \frac{28 \times 9}{12}$$

$$\Rightarrow \quad x = 21$$

Hence, the length of the model ship is 21 cm.

Q.7 Suppose 2 kg of sugar contains 9×10^6 crystals. How many sugar crystals are there in (i) 5 kg of sugar? (ii) 1.2 kg of sugar ?

Sol. Suppose the amount of sugar is x kg and the number of crystals is y.

As the amount of sugar increases, the number of crystals also increases in the same ratio. It is a case of direct proportion. We make use of the relation of the type $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

(i) Here, $x_1 = 2$

$y_1 = 9 \times 10^6$

$x_2 = 5$

Therefore, $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ gives

$$\frac{2}{9\times10^6} = \frac{5}{y_2} \quad \Rightarrow \quad 2y_2 = 5 \times 9 \times 10^6 \quad \Rightarrow \quad y_2 = \frac{5\times9\times10^6}{2}$$

$$\Rightarrow \quad y_2 = 22.5 \times 10^6 \quad \Rightarrow \quad y_2 = 2.25 + 10^7$$

Hence, there are 225×10^5 crystals.

(ii) Here, $x_1 = 2$

$y_1 = 9 \times 10^6$

$x_3 = 1.2$

Therefore, $\frac{x_1}{y_1} = \frac{x_3}{y_3}$ gives

$$\frac{2}{9\times10^6} = \frac{1.2}{y_3}$$

$$\Rightarrow \quad 2y_3 = 1.2 \times 9 \times 10^6 \quad \Rightarrow \quad 2y_3 = 10.8 \times 10^6 \quad \Rightarrow \quad y_3 = \frac{10.8\times10^6}{2}$$

$$\Rightarrow \quad y_3 = 5.4 \times 10^6$$

Hence, there are 5.4×10^6 crystals.

Q.8 Rashmi has a road map with a scale of 1 cm representing 18 km. She drives on a road for 72 km. What would be her distance covered in the map ?

Sol. Let the distance covered in the map be x cm. Then,

$$1 : 18 = x : 72$$

$$\Rightarrow \quad \frac{1}{18} = \frac{x}{72} \quad \Rightarrow \quad x = \frac{72}{18} \quad \Rightarrow \quad x = 4$$

Hence, the distance covered in the map would be 4 cm.

Q.9 A 5 m 60 cm high vertical pole casts a shadow 3 m 20 cm long. Find at the same time (i) the length of the shadow cast by another pole 10 m 50 cm high (ii) the height of a pole which casts a shadow 5 m long.

Sol. Let the height of the vertical pole be x m and the length of the shadow be y m.

As the height of the vertical pole increases, the length of the shadow also increases in the same ratio. It is a case of direct proportion.

We make use of the relation of the type

$$\frac{x_1}{y_1} = \frac{x_2}{y_2}$$

(i) Here $x_1 = 5$ m 60 cm $= 5.60$ m; $y_1 = 3$ m 20 cm $= 3.20$ m

$x_2 = 10$ m 50 cm $= 10.50$ m

Therefore, $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ gives

$$\frac{5.6}{3.2} = \frac{10.5}{y_2} \quad \Rightarrow \quad 5.6\, y_2 = 3.2 \times 10.5 \quad \Rightarrow \quad y_2 = \frac{3.2 \times 10.5}{5.6}$$

$\Rightarrow$ $y_2 = 6$

Hence, the length of the shadow is 6 m

(ii) Here $x_1 = 5$ m 60 cm $= 560$ cm; $y_1 = 3$ m 20 cm $= 320$ cm

$y_3 = 5$ m $= 500$ cm

Therefore, $\frac{x_1}{y_1} = \frac{x_3}{y_3}$ gives

$$\frac{560}{320} = \frac{x_3}{500}$$

$$\Rightarrow \quad 320\, x_3 = 560 \times 500 \quad \Rightarrow \quad x_3 = \frac{560 \times 500}{320} \quad \Rightarrow \quad x_3 = 875$$

Hence, the height of the pole is 8 m 75 cm.

Q.10 A loaded truck travels 14 km in 25 minutes. If the speed remains the same, how far can it travel in 5 hours ?

Sol. Two quantities x and y which vary in direct proportion have the relation

$$x = ky \quad \text{or} \quad \frac{x}{y} = k$$

Here, $k = \frac{\text{number of km it can travel}}{\text{time in hours}} = \frac{14}{\left(\frac{25}{60}\right)} = \frac{14 \times 60}{25} = \frac{168}{5}$

Now, x is the distance travelled in 5 hours

Using the relation $x = ky$, we obtain

$$x = \frac{168}{5} \times 5 \quad \Rightarrow \quad x = 168$$

Hence, it can travel 168 km.

EXERCISE 2

Q.1 Which of the following are in inverse proportion ?

(i) The number of wokers on a job and the time to complete the job.

(ii) The time taken for a journey and the distance travelled in a uniform speed.

(iii) Area of cultivated land and the crop harvested.

(iv) The time taken for a fixed journey and the speed of the vehicle.

(v) the population of a country and the area of land per person.

Sol. (i) The number of workers on a jobs and the time to complete the job are in inverse proportion.

(ii) The time taken for a journey and the distance travelled in a uniform speed are not in inverse proportion.

(iii) Area of cultivated land and the crop harvested are not in inverse proportion.

(iv) The time taken for a fixed journey and the speed of the vehicle are in inverse proportion.

(v) The population of a country and the area of land per person are in inverse proportion.

Q.2 In a Television game show, the prize money of Rs. 1,00,000 is to be divided equally amongst the winners. Complete the following table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners ?

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in Rs)	1,00,000	50,000	...	...	...	...	...

Sol. $1 \times 1,00,000 = 4 \times x$

$\Rightarrow \quad x = \dfrac{1,00,000}{4} = 25,000 \qquad \Rightarrow \quad 1 \times 1,00,000 = 5 \times y$

$\Rightarrow \quad y = \dfrac{1,00,000}{5} = 20,000 \qquad \Rightarrow \quad 1 \times 1,00,000 = 8 \times z$

$\Rightarrow \quad z = \dfrac{1,00,000}{8} = 12,500 \qquad \Rightarrow \quad 1 \times 1,00,000 = 10 \times t$

$\Rightarrow \quad t = \dfrac{1,00,000}{10} = 10,000 \qquad \Rightarrow \quad 1 \times 1,00,000 = 20 \times w$

$\Rightarrow \quad w = \dfrac{1,00,000}{20} = 5,000$

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in Rs.)	1,00,000	50,000	25,000	20,000	12,500	10,000	5,000

Since $1 \times 1,00,000 = 2 \times 50,000 = 4 \times 25,000 = 5 \times 20,000 = 8 \times 12,500 = 10 \times 10,000 = 20 \times 5,000$, so we find that the prize money given to an individual winner is inversely proportional to the number of winners.

Q.3 Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spokes are equal. Help him by completing the following table.

Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90^o	60^o	...	...	...

(i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion ?

(ii) Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes.

(iii) How many spokes would be needed, if the angle between a pair of consecutive spokes is 40^o ?

Sol. $4 \times 90^o = 8 \times x$

$\Rightarrow \quad x = \dfrac{4 \times 90^o}{8} = 45^o \qquad \Rightarrow \quad 4 \times 90^o = 10 \times y$

$\Rightarrow \quad y = \dfrac{4 \times 90^o}{10} = 36^o \qquad \Rightarrow \quad 4 \times 90^o = 12 \times z$

$\Rightarrow \quad z = \dfrac{4 \times 90^o}{12} = 30^o$

Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90^o	60^o	45^o	36^o	30^o

(i) Yes ! The number of spokes and the angles formed between the pairs of consecutive spokes are in inverse proportion [$\therefore\ 4 \times 90^o = 6 \times 60^o = 8 \times 45^o = 10 \times 36^o = 12 \times 30^o$]

(ii) Let the angle between a pair of consecutive spokes on a wheel with 15 spokes be x^o.
Lesser the number of spokes, more will be the angle between a pair of consecutive spokes.
So, this is a case of inverse proportion.
Hence, $4 \times 90^o = 15 \times x$ [$x_1 y_1 = x_2 y_2$]

$\Rightarrow \quad x = \dfrac{4 \times 90^o}{15} \qquad \Rightarrow \quad x = 24^o$

Hence, the angle between a pair of consecutive spokes on a wheel with 15 spokes is 24^o.

(iii) Let x spokes be needed.
Lesser the number of spokes, more will be the angle between a pair of consecutive spokes. So, this is a case of inverse proportion.
Hence, $4 \times 90^o = x \times 40^o$ [$x_1 y_1 = x_2 y_2$]

$\Rightarrow \quad x = \dfrac{4 \times 90^o}{40^o} \qquad \Rightarrow \quad x = 9$

Hence, 9 spokes would be needed.

Q.4 If a box of sweets is divided among 24 children, they will get 5 sweets each. How many would each get, if the number of the children is reduced by 4 ?

Sol. Suppose that each would get x sweets.

Thus, we have the following table.

Number of children	24	$24 - 4 = 20$
Number of sweets	5	x

Lesser the number of children, more will be the number of sweets each would get. So, this is a case of inverse proportion.

Hence, $24 \times 5 = 20 \times x$

$\Rightarrow \quad x = \dfrac{24 \times 5}{20} \quad \Rightarrow \quad x = 6$

Hence, each would get 6 sweets.

Q.5 A farmer has enough food to feed 20 animals in his cattle for 6 days. How long would the food last if there were 10 more animals in his cattle ?

Sol. Suppose that the food would last for x days. We have the following table.

Number of animals	20	$20 + 10 = 30$
Number of days	6	x

We note that more the number of animals, lesser will be the number of days for which the food will last. Therefore, this is a case of inverse proportion.

So, $20 \times 6 = 30 \times x$

$\Rightarrow \quad x = \dfrac{20 \times 6}{30} \quad \Rightarrow \quad x = 4$

Hence, the food would last for 4 days.

Q.6 A contractor estimates that 3 persons could rewire Jasminder's house in 4 days. If, he uses 4 persons instead of three, how long should they take to complete the job ?

Sol. Suppose that they take x day to complete the job. We have the following table

Number of persons	3	4
Number of days	4	x

More the number of persons, lesser will be the number of days required to complete the job. So, this is a case of inverse proportion.

Hence, $3 \times 4 = 4 \times x$

$\Rightarrow \quad x = \dfrac{3 \times 4}{4} \quad \Rightarrow \quad x = 3$

Hence, they would take 3 days to complete the job.

Q.7 A batch of bottles were packed in 25 boxes with 12 bottles in each box. if the same batch is packed using 20 bottles in each box, how many boxes would be filled ?

Sol. Suppose that x boxes would be filled. We have the following table

Number of bottles	12	20
Number of boxes	25	x

Lesser the number of bottles, more will be number of boxes required to be filled. So, this is a case of inverse proportion.

Hence, $12 \times 25 = 20 \times x$

$$\Rightarrow \quad x = \frac{12 \times 25}{20} \qquad \Rightarrow \quad x = 15$$

Hence, 15 boxes would be filled.

Q.8 A factory requires 42 machines to produce a given number of articles in 63 Days. How many machines would be required to produce the same number of articles in 54 days ?

Sol. Suppose that x machines would be required. We have the following table.

Number of machines	42	x
Number of days	63	54

Lesser the number of machines, more will be the number of days to produce the same number of articles. So, this is a case of inverse proportion.

Hence, $42 \times 63 = x \times 54$

$$\Rightarrow \quad x = \frac{42 \times 63}{54} \qquad \Rightarrow \quad x = 49$$

Hence, 49 machines would be required.

Q.9 A car takes 2 hours to reach a destination by travelling at the speed of 60 km/h. How long will it take when the car travels at the speed of 80 km/h ?

Sol. Let it take x hours. We have the following table.

Speed (in km/h)	60	80
Number of hours	2	x

Lesser the speed, more the number of hours to reach the destination. So, this is a case of inverse proportion.

Hence, $60 \times 2 = 80 \times x$

$$\Rightarrow \quad x = \frac{60 \times 2}{80} \qquad \Rightarrow \quad x = \frac{3}{2} = 1\frac{1}{2}$$

Thus, $1\frac{1}{2}$ hours would be taken.

Q.10 Two persons could fit new windows in a house in 3 days.

(i) One of the persons fell ill before the work started. How long would the job take now ?

(ii) How many persons would be needed to fit the windows in one day ?

Sol. (i) Let the job would take x day.

We have the following table.

Number of persons	2	$2-1=1$
Number of days	3	x

Clearly, more the number of persons, lesser would be the number of days to do the job. So, the number of persons and number of days vary in inverse proportion.

So, $2 \times 3 = 1 \times x \quad \Rightarrow \quad x = 6$

Thus, the job would now take 6 days.

(ii) Let x persons be needed.

We have the following table.

Number of days	3	1
Number of persons	2	x

Clearly, more the number of persons, lesser would be the number of days to do the job. So, the number of perons and number of days vary in inverse proportion.

So, $3 \times 2 = 1 \times x \quad \Rightarrow \quad x = 6$

Thus, 6 persons would be needed.

Q.11. A school has 8 periods a day each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same ?

Sol. Let each period be x minutes long.

We have the following table

Number of periods	8	9
Length of each period (in minutes)	45	x

We note that more the number of periods, lesser would be the length of each period. Therefore, this is case of inverse proportion.

So, $8 \times 45 = 9 \times x$

$$\Rightarrow \quad x = \frac{8 \times 45}{9} \quad \Rightarrow \quad x = 40$$

Hence, each period would be 40 minutes long.

TRY THESE

Q.1 On a squared paper, draw five squares of different sides. Write the following information in a tabular form.

	Square – 1	Square – 2	Square – 3	Square – 4	Square – 5
Length of a side (L)					
Perimeter (P)					
$\frac{L}{P}$					
Area(A)					
$\frac{L}{A}$					

Find whether the length of a side is in direct proportion to :

(a) the perimeter of the square.

(b) the area of the square.

Sol.

	Square – 1	Square – 2	Square – 3	Square – 4	Square – 5
Length of a side (L)	1	2	3	4	5
Perimeter (P)	$4\times1=4$	$4\times2=8$	$4\times3=12$	$4\times4=16$	$4\times5=20$
$\frac{L}{P}$	$\frac{1}{4}=\frac{1}{4}$	$\frac{2}{8}=\frac{1}{4}$	$\frac{3}{12}=\frac{1}{4}$	$\frac{4}{16}=\frac{1}{4}$	$\frac{5}{20}=\frac{1}{4}$
Area(A)	$1^2=1$	$2^2=4$	$3^2=9$	$4^2=16$	$5^2=25$
$\frac{L}{A}$	$\frac{1}{1}=1$	$\frac{2}{4}=\frac{1}{2}$	$\frac{3}{9}=\frac{1}{3}$	$\frac{4}{16}=\frac{1}{4}$	$\frac{5}{25}=\frac{1}{5}$

(a) Since each $\frac{L}{P}$ is the same, so we find that the length of a side is in direct proportion to the perimeter of the square .

(b) Since all $\frac{L}{A}$ are not the same, therefore, we find that the length of a side is not in direct proportion to the area of the square.

Q.2 Observe the following tables and find which pair of variables (here x and y) are in inverse proportion.

(i)

x	50	40	30	20
y	5	6	7	8

(ii)

x	100	200	300	400
y	60	30	20	15

(iii)

x	90	60	45	30	20	5
y	10	15	20	25	30	35

Sol. (i) $x_1y_1 = 50 \times 5 = 250$

$x_2y_2 = 40 \times 6 = 240$

So, $x_1y_1 \neq x_2y_2$

Hence, x and y are not in invese proportion.

(ii) $x_1y_1 = 100 \times 60 = 6000$

$x_2y_2 = 200 \times 30 = 6000$

$x_3y_3 = 300 \times 20 = 6000$

$x_4y_4 = 400 \times 15 = 6000$

So, $x_1y_1 = x_2y_2 = x_3y_3 = x_4y_4$

Hence, x and y are in inverse proprtion

(iii) $x_1y_1 = 90 \times 10 = 900$

$x_2y_2 = 60 \times 15 = 900$

$x_3y_3 = 45 \times 20 = 900$

$x_4y_3 = 30 \times 25 = 750$

As $x_1y_1 = x_2y_2 = x_3y_3 \neq x_4y_4$

So, x and y are not in inverse proportion.

Note : When two quantities x and y are in direct proportion (or vary directly) they are also written as $x \propto y$. When two quantities x and y are in inverse proportion (or vary inversely) they are also written as $x \propto \frac{1}{y}$.

CONCEPT APPLICATION LEVEL - II

SECTION -A

➤ **FILL IN THE BLANKS**

Q.1 If A can do a piece of work in n days, then work done by A in 1 day is ___________ part of total work.

Q.2 A and B can together do a piece of work in 15 days B alone can do it in 20 days. A can do it alone in ___________ days.

Q.3 Speed 36 km/h = speed ___________ m/sec

Q.4 If $x > 0$, and $2x + 5 : x + 1 :: x + 4$ and $2x - 2$, then the value of x is ___________.

Q.5 The weight of 48 similar books is 30 kg. The weight of 8 similar books is ___________ kg.

Q.6 Shyam is twice as good workman as Ram. Ram can complete a work in 6 days. The time taken by shyam to complete the same work is ___________.

SECTION -B

➤ **MULTIPLE CHOICE QUESTIONS**

Q.1 If 30 men do a piece of work in 27 days, in what time can 18 men do same work ?
(A) 90 days (B) 45 days (C) 15 days (D) None of these

Q.2 If 18 binders bind 900 books in 10 days, how many binders will be required to bind 660 books in 12 days ?
(A) 14 (B) 13 (C) 22 (D) 11

Q.3 If a family of 7 persons can live on Rs. 8400 for 36 days, how long can a family of 9 persons live on Rs. 8100 ?
(A) 27 days (B) 37 days (C) 36 days (D) 24 days

Q.4 If I can walk a certain distance in 50 days when I rest 9 hour each day, how long will it take me to walk twice as fast if I walk twice as fast and rest twice as long each day
(A) 125 days (B) 120 days (C) 130 days (D) 124 days

Q.5 X and Y can do a piece of work in 72 days. Y and Z can do it in 120 days. X and Z can do it in 90 days. In how many days all the three together can do work ?
(A) 100 days (B) 150 days (C) 60 days (D) 80 days

Q.6 8 men and 2 children can do a work in 9 days. A child takes double the time to do a work than the man. In how many days 12 men can complete double the work ?
(A) $16\frac{1}{2}$ days (B) $10\frac{1}{2}$ days (C) 14 days (D) $13\frac{1}{2}$ days

Q.7 P is three times efficient then Q, and is therefore able to complete a work in 60 days earlier. The number of days that P and q together will take to complete the work is
(A) $22\frac{1}{2}$ (B) 30 (C) 25 (D) $27\frac{1}{2}$

Q.8 Two person A and B under take to do a piece of work for Rs. 4800. A could do it alone in 5 days and B could do it alone in 8 days. With the help of C and D they finished it in 3 days. If the alone work of C be twice that of 'D', the share of D is

(A) Rs. 60 (B) Rs. 20 (C) Rs. 40 (D) Rs. 80

Q.9 The work done by man, a woman and a boy are in the ratio 3 : 2 : 1. There are 24 men, 20 women and 16 boys in a factory yearly wages of 27 men, 40 women and 15 boys.

(A) ₹ 16366 (B) ₹ 16466 (C) ₹ 16066 (D) ₹ 16016

Q.10 Two pipes A and B can fill a tank in 24 minutes and 32 minutes respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 minutes ?

(A) 8 min (B) 9 min (C) 12 min (D) 10 min

Q.11 A man travels three-fifths of a distance ab at a speed of 3a and remaining at the speed of 2b. If he goes from B to A and back at speed of 5c in the same time then

(A) $\frac{1}{a}+\frac{1}{b}=\frac{2}{c}$ (B) $\frac{1}{a}+\frac{1}{b}=2c$ (C) a + b = c (D) None of these

Q.12 Twenty women can do a work in sixteen days 16 men can complete the same work in 15 days. What is the ratio between the capacity of a man and a woman ?

(A) 3 : 5 (B) 4 : 3 (C) 5 : 3 (D) 2 : 3

Q.13 In a camp there is provision for 1600 participants for 60 days. Actually 1200 participated how many days will the provision last for ?

(A) 70 days (B) 80 days (C) 83 days (D) 95 days

Q.14 4 men and 6 women can complete a work in 8 days while 3 men and 7 women can complete it in 10 days. In how many days will 10 women complete ?

(A) 28 days (B) 40 days (C) 42 days (D) 55 days

Q.15 The speed of a car increases by 2 kilometer after every one hour. If the distance travelled in the first one hour was 35 kilometers, then the total distance travelled in 12 hours was

(A) 460 km (B) 552 km (C) 483 km (D) 572 km

Q.16 The jogging track in a stadium is 726 m in circumference. Rakesh and Ismail start from the same point and walk in opposite direction at 4.5 kmph and 3.75 kmph respectively. They will meet for the first time in

(A) 4.7 min (B) 5.65 min (C) 4.97 min (D) 6.2 min

Q.17 Starting from his house, one day a student walk at a speed of $2\frac{1}{2}$ km/hr and reaches his school 6 minutes late. Next day he increases his speed by 1 km/hr and reaches the school 6 minutes early. How far is the school from his house ?

(A) 1.5 km (B) 1.75 km (C) 2.25 km (D) 2.5 km

Q.18 Two good trains each 500 m long are running in opposite direction on parallel tracks. Their speeds are 45 km/hr and 30 km/hr respectively. The time taken by the slower train to pass the driver of the faster train is

(A) 24 sec (B) 48 sec (C) 60 sec (D) 12 sec

Q.19 MS express left Nagpur for Mumbai at 14 : 30 hours, travelling at a speed of 60 km/hr and VB express left Nagpur for Mumbai on the same day at 16 : 30 hrs, travelling at a speed of 80 km/hr. How far away from Nagpur will the two trains meet.

(A) 150 km (B) 200 km (C) 400 km (D) 480 km

Q.20 If 'x' and 'y' are in a direct propostion then which of the following is correct ?

(A) $x - y = \text{constant}$ (B) $x + y = \text{constant}$ (C) $x \times y = \text{constant}$ (D) $\frac{x}{y} = \text{constant}$

Q.21 If 'x' and 'y' are in an inverse variation then which of the following is correct ?

(A) $x - y = \text{constant}$ (B) $x + y = \text{constant}$ (C) $xy = \text{constant}$ (D) $\frac{x}{y} = \text{constant}$

Q.22 If 'A' can finish a work in 'n' days and is twice as efficient as B then in how many days B can finish the hole work :

(A) $\frac{n}{2}$ (B) 2n (C) n (D) none of these

Q.23 If amount of work completed by 'A' in one day is $\frac{1}{n}$ then the whole work will be finished by 'A' is :

(A) n days (B) $1 - n$ days (C) $n - 1$ days (D) none of these

Q.24 "If speed is more that time to cover a fixed distance would be less". This is a case or :

(A) inverse variation (B) direct variation

(C) direct and indirect both variations (D) none of the above

Q.25 If x and y vary inversely. Then using the follwing table ?

x	5
y	30

The value of x for $y = 10$ is

(A) 10 (B) 40 (C) 15 (D) 20

Q.26 The ratio of girls to boys in a class is 2 : 3. The actual strength of the class is :

(A) 12 (B) 15 (C) 16 (D) 18

Q.27 If two quantities x and y are related to each other in such a way that $\frac{x}{y}$ remains a positive constant, then x and y are said to be in

(A) inverse variation (B) direct variation (C) variation (D) none of these

Q.28 The cost price of articles and number of articles are said to be in
(A) direct variation (B) inverse variation (C) variation (D) none of these

Q.29 Time taken to cover a distance by a car and speed of the car are said to be in
(A) direct variation (B) inverse variation (C) variation (D) none of these

Q.30 If 12 m of a uniform iron rod weights 42 kg, what will be the weight of 6 m of the same rod ?
(A) 20 kg (B) 21 kg (C) 84 kg (D) 42 kg

Q.31 A certain number of men can finish a piece of work in 100 days. If however, there were 10 men to be finished. How many men were originally there ?
(A) 90 (B) 100 (C) 110 (D) 120

Q.32 If 20 binders bind 1000 books in 10 days, then how many books will binded by 10 binders in 20 days?
(A) 2000 (B) 1000 (C) 1500 (D) 900

Q.33 A train 150 m long, is running at a speed of 90 km/hr. Then time taken by the train to cross a tree is
(A) 3 sec (B) 4 sec (C) 5 sec (D) 6 sec

Q.34 A train is running at a speed of 90 km/hr, crosses a pole in 10 seconds. The length of the train is
(A) 200 m (B) 250 m (C) 300 m (D) 350 m

Q.35 x and y are in inverse proportion. If y = 15 when x = 3, then value of y when x = 9, is
(A) 45 (B) 5 (C) 8 (D) 9

Q.36 x and y are in direct variation. If y = 10 when x = 5, then value of y when x = 10, is
(A) 2 (B) 5 (C) 10 (D) 20

Q.37 15 books weigh 6 kg. What will 6 books weigh ?
(A) 1.2 kg (B) 2.4 kg (C) 3.8 kg (D) 3 kg

Q.38 8 g of sandal wood cost ₹ 40. What will 10 g cost ?
(A) ₹ 30 (B) ₹ 36 (C) ₹ 48 (D) ₹ 50

Q.39 20 trucks can hold 150 metric tonnes. How much will 12 trucks hold ?
(A) 80 metric tonnes (B) 90 metric tonnes (C) 60 metric tonnes (D) 40 metric tonnes

Q.40 A boy runs 1 km in 10 minutes. How long will he take to run 600 m ?
(A) 2 minutes (B) 3 minutes (C) 4 minutes (D) 6 minutes

Q.41 A shot travels 90 m in 1 second. How long will it take to go 225 m ?
(A) 2 seconds (B) 2.5 seconds (C) 4 seconds (D) 3.5 seconds

Q.42 A train travels 60 km in 1 hour How long will it take to go 150 km ?
(A) 2 hours (B) 3 hours (C) 2.5 hours (D) 4 hours

Q.43 If 3 quintals of coal cost ₹ 6000, what is the cost of 120 kg ?
(A) ₹ 1200 (B) ₹ 2400 (C) ₹ 3600 (D) ₹ 4800

Q.44 The fare for a journey of 40 km is ₹ 25. How much can be travelled for ₹ 40 ?
(A) 32 km (B) 64 km (C) 50 km (D) 60 km

Q.45 A machine in a soft drink factory fills 600 bottles in 5 hours. How many bottles will it fill in 2 hours ?
(A) 120 (B) 180 (C) 150 (D) 240

Q.46 10 men can dig a trench in 15 days. How long will 3 men take ?
(A) 50 days (B) 60 days (C) 100 days (D) 75 days

Q.47 6 pipes are rquired to fill a tank in 1 hour. How long will it take if only 5 pipes of the same type are used?
(A) 75 minutes (B) 72 minutes (C) 80 minutes (D) 90 minutes.

Q.48 40 cows can graze a field in 16 days. How many cows will graze the same field in 10 days?
(A) 60 (B) 64 (C) 80 (D) 75

Q.49 The constant of variation, if $x \propto y$, from the following table is

x	6	12	15	21
y	2	4	5	7

(A) 1 (B) 2 (C) 3 (D) 4

Q.50 x and y vary inversely with each other. If x = 15 when y = 6, then the value of x when y = 15 is
(A) 2 (B) 4 (C) 5 (D) 4

Q.51 Two trains started at the same time from two towns 750 kms apart and travelled towards each other. The slower train travelled at an average speed of 60 km/hour and the faster one at 90 km/hour. After how many hours will they pass each other? **[IMO-2016]**
(A) 3 (B) 4 (C) 5 (D) 6

Q.52 A truck travelling at a speed of 40 km/hour left Delhi. An hour later, a car leaves Delhi and catches up with the truck after four hours. What was the average speed (in km/hr) of the car? **[IMO-2016]**
(A) 40 (B) 45 (C) 50 (D) 60

Q.53 The distance between two places R and S is 42 km. Kanika starts from R with a uniform speed of 4 km/h towards S and at the same time Yashika starts from S towards R also with some uniform speed. They meet each other after 6 hours. The speed of Yashika is **[IOM-2016]**
(A) 3 km/hour (B) 8 km/hour (C) 18 km/hour (D) 20 km/hour

Q.54 Two places P and Q are 162 km apart. A train leaves P for Q and simultaneously another train leaves Q for P. They meet at the end of 6 hours. If the former train travels 8 km/hour faster than the other, the speed of the train starting from Q is **[IOM-2016]**
(A) $9\frac{1}{2}$ km/hour (B) $8\frac{1}{2}$ km/hour (C) $10\frac{5}{6}$ km/hour (D) $12\frac{5}{6}$ km/hour

Q.55 A, B and C can do a work separately in 16, 32 and 48 days respectively. They started the work together but B left off 8 days and C left 6 days before the completion of work. In what time is the work finished? **[IOM-2016]**
(A) 14 days (B) 12 days (C) 9 days (D) 10 days

SECTION -C

➢ MATHCH THE FOLLOWING

Directions : Each equation contains statements given in two column which have to be matched. Statements (A, B, C, D....) in column I have to be matched with statement (p, q, r, s....) in column II

Q.1

	Column I		Column II
(A)	4 men or 4 women can do a work in 21 days. 8 men and 6 women will take ____ days to complete the work.	(p)	10
(B)	A train is running at the average speed 20 m/s, its speed in km/h is	(q)	6
(C)	A train crosses a pole in 20 seconds and a 300 metre long platform in 45 seconds. The speed of the train in km/h is	(r)	72
(D)	A man can row a boat 54 km/h with the stream and 34 km/h against the stream. The speed of stream in km/h, is	(s)	43.2

Q.2

	Column I		Column II
(A)	It takes 2 hours for a shirt to dry in sun. It will take _______ hrs to dry 25 such shirts.	(p)	1.5
(B)	A bus with stoppages, covers a distance at 50 km/h. The bus stops for _____ minutes per hour.	(q)	2
(C)	A is inversely proportional to B and A = 5 when B = 2. The value of A when B = $\frac{20}{3}$ is	(r)	30
(D)	12 persons takes 8 days to prepare 27 wooden doors. The number of days required by 72 persons to prepare 81 doors is _____.	(s)	4

ANSWER KEY

CONCEPT APPLICATION LEVEL - II

SECTION -A

Q.1	$\frac{1}{n}$	Q.2	60 days.	Q.3	10 m/sec.	Q.4	x = 2	Q.5	5kg
Q.6	3 days								

SECTION -B

Q.1	B	Q.2	D	Q.3	A	Q.4	A	Q.5	C	Q.6	D	Q.7	A
Q.8	C	Q.9	D	Q.10	A	Q.11	A	Q.12	B	Q.13	B	Q.14	B
Q.15	B	Q.16	C	Q.17	B	Q.18	B	Q.19	D	Q.20	D	Q.21	C
Q.22	B	Q.23	A	Q.24	A	Q.25	C	Q.26	B	Q.27	B	Q.28	A
Q.29	B	Q.30	B	Q.31	C	Q.32	B	Q.33	D	Q.34	B	Q.35	B
Q.36	D	Q.37	B	Q.38	D	Q.39	B	Q.40	D	Q.41	B	Q.42	C
Q.43	B	Q.44	B	Q.45	D	Q.46	A	Q.47	B	Q.48	B	Q.49	C
Q.50	D	Q.51	C	Q.52	C	Q.53	A	Q.54	A	Q.55	B		

SECTION -C

Q.1 (A) q, (B) r, (C) s, (D) p Q.2 (A) q, (B) r, (C) p (D) s